

# Two-Stage Channel Estimation Assisted by Correlation Exploitation for Amplify and Forward Relay Networks with Multiple Transmit and Receive Antennas

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**Abstract**—This paper proposes a new two-stage channel estimation (TSCE) method to estimate the cascaded channels in amplify-and-forward (AF) one-way relay network (OWRN) with multiple transmit and receive antennas. Different from existing estimation methods, the proposed TSCE estimates the cascaded channel matrix by utilizing the special structure of the received signal in the destination and by exploiting the correlations among the cascaded channel matrix entries. The TSCE not only obtains the channel state information (CSI) when receiving training sequences, but also improves the accuracy of CSI when receiving the data sequences. Simulation results demonstrate that the proposed TSCE can improve estimation accuracy compared with traditional channel estimation schemes.

## 1. INTRODUCTION

Recently, relays and multi-input multi-output (MIMO) have been identified as two key technologies to improve communications over a wireless channel [1–3]. With multiple transmit and multiple receive antennas, MIMO systems can obtain either a diversity gain or a capacity gain compared with the single-input single-output (SISO) systems [4]. Meanwhile, relays can substantially improve the wireless coverage and link reliability for users subject to limited power and spectral resources [5, 6]. Therefore, there have been many research efforts on MIMO relay systems.

There are usually two types of relays: Amplify-and-forward (AF) and decode-and-forward (DF). The DF relays need to decode and re-encode the received signal before their retransmission, while AF relays only amplify and forward the received signal, hence reducing the complexity of the relay nodes. To perform self-interference cancellation and coherent decoding [7], the cascaded channel state information (CSI) of AF relay systems is highly desired. In [8], a simple least square (LS) channel estimation method is proposed based on block-based training. The superimposed channel training algorithm is proposed to estimate the time-varying channel matrices for MIMO relay systems in [9]. For AF one-way relay network (OWRN) with single antenna at both the source and the destination, some cascaded channel estimation schemes have been developed in [10–12], which directly estimate the cascaded channels and address the problem of optimal training design. For AF OWRN with multiple antennas, [13, 14] use the singular value decomposition (SVD) method to estimate the cascaded channels.

This paper focuses on the cascaded channel estimation of the two-hop AF OWRN with multiple transmit and receive antennas, under flat fading channel scenario. Note that the frequency selective channel scenario TSCE can be deduced from the investigation in this paper. The proposed method can be divided into two stages. In the First stage, we employ the traditional channel estimation methods to estimate the cascaded channels coarsely with the help of training sequence. In this paper, we use the LS channel estimation method as an example of the traditional methods, and other methods, such as linear

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*Received 3 August 2018, Accepted 8 October 2018, Scheduled 18 October 2018*

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minimum mean-square-error (LMMSE), can also be applied to the proposed TSCE. In the second stage, by exploiting the special structure of the received signals in the destination, the TSCE tries to improve the accuracy of channel estimates when receiving the data sequences. Considering the special structure (correlation relationship) of the received signals, we exploit a simple averaging filter to estimate the channels between relay and destination up to scaling, which can be utilized to improve the accuracy of the estimated CSI. Note that the exploited simple averaging filter in the second stage of TSCE may not be optimal in the statistical meaning, but it is enough to indicate the validity of the proposed system, and further attempt can be made to design a statistically optimum estimator. To the best of our knowledge, this paper is the first to try to exploit the correlations in the cascaded channel matrix when receiving data sequence. Compared with the LS estimation that ignores the special structure of the received signal and correlations among cascaded channel matrix entries, the proposed TSCE is superior in the performance of normalized mean square error (NMSE), which is shown by simulations.

The remaining part of this paper is organized as follows. The system model of AF OWRN with multiple transmit and receive antennas is introduced in Section 2, followed by the proposed channel estimation scheme described in Section 3. Simulations in Section 4 show the effectiveness of the proposed scheme, and our summary is provided in Section 5.

## 2. SYSTEM MODEL

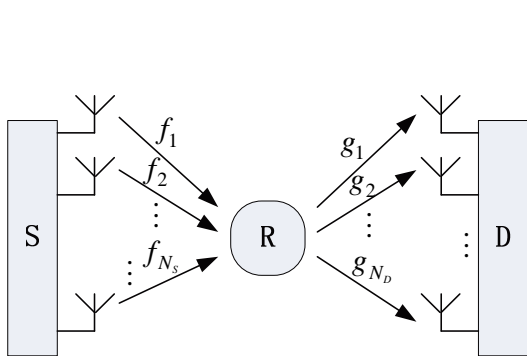
This paper considers an AF OWRN system in which one source node equipped with  $N_S$  antennas transmits to one destination node with  $N_D$  antennas, and a relay node equipped with single antenna is used for the communication, as shown in Fig. 1. Assume that there is no direct link between the source and the destination in this relay system, which is appropriate when the direct link from source to destination is far and weak enough to be neglected. The channels from the  $k$ th source antenna to the relay and from relay to the  $l$ th destination antenna are denoted by  $f_k$  and  $g_l$ , respectively. The channels are assumed to be i.i.d. circularly symmetric complex Gaussian (CSCG) with zero-mean and unit-variance, i.e.,

$$f_k, g_l \sim \mathcal{CN}(0, 1).$$

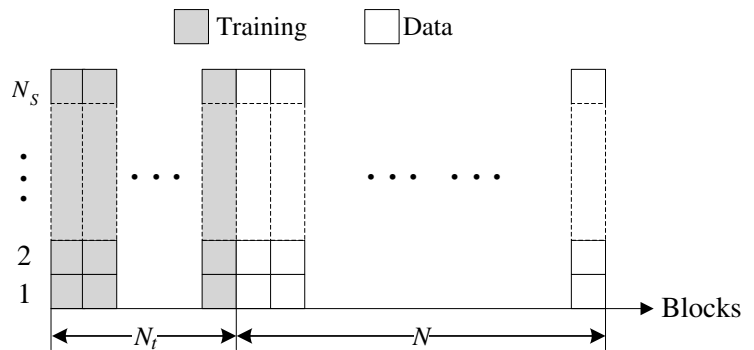
The transmitted signal frame is shown in Fig. 2. In each frame,  $N_t$  training blocks and  $N$  data blocks are transmitted, and thus the frame size is

$$N_f = N_t + N.$$

Note that one signal block includes  $N_S$  symbols for the  $N_S$  transmit antennas. Also, assume that the channels remain constant during one frame. In this paper, we not only use the training sequence to estimate the channels as the traditional methods, but also utilize the data sequence to improve the estimation results.



**Figure 1.** AF OWRN with multiple transmit and multiple receive antennas.



**Figure 2.** Transmitted signal frame structure.

Define

$$\begin{aligned}\mathbf{s}(n) &= [ s_1(n) \quad s_2(n) \quad \dots \quad s_{N_S}(n) ]^T, \\ \mathbf{y}(n) &= [ y_1(n) \quad y_2(n) \quad \dots \quad y_{N_D}(n) ]^T, \\ \mathbf{f} &= [ f_1 \quad f_2 \quad \dots \quad f_{N_S} ]^T, \\ \mathbf{g} &= [ g_1 \quad g_2 \quad \dots \quad g_{N_D} ]^T,\end{aligned}\tag{1}$$

which are the  $n$ th transmitted signal block at the source,  $n$ th received signal block at the destination, source-relay channel vector and the relay-destination channel vector, respectively, where the superscript ' $T$ ' denotes the transpose. In this scenario, the received signal block can be described as

$$\mathbf{y}(n) = \mathbf{g}\mathbf{f}^T\mathbf{s}(n) + \mathbf{g}n_R(n) + \mathbf{n}_D(n),\tag{2}$$

where  $n_R(n)$  and

$$\mathbf{n}_D(n) = [ n_D^1(n) \quad n_D^2(n) \quad \dots \quad n_D^{N_D}(n) ]^T$$

are the noise at the relay and destination, respectively. All noises are assumed to be CSCG with zero-mean and variance  $N_0$ . Define the cascaded channel matrix as

$$\mathbf{H} = \mathbf{g}\mathbf{f}^T.\tag{3}$$

In the following, we try to estimate the cascaded channel matrix  $\mathbf{H}$  at the destination in a new two-stage way.

### 3. THE PROPOSED TSCE METHOD

This section introduces the proposed channel estimation method. In the first stage, the LS method is employed to estimate the cascaded channels during the training period. In the second stage, by taking into consideration the special structure of the received signals in the destination during the communicating period, we estimate the channels between relay and destination up to scaling, which is then used to exploit the correlations in the channel matrix using a simple averaging filter.

#### 3.1. The First Stage of TSCE

In the first stage of TSCE, the LS method is employed to estimate the channel coarsely. During the training period, the system model can be given as

$$\mathbf{Y} = \mathbf{H}\mathbf{S} + \mathbf{g}\mathbf{n}_R^T + \mathbf{N}_D,\tag{4}$$

where

$$\mathbf{S} = [ \mathbf{s}(1) \quad \mathbf{s}(2) \quad \dots \quad \mathbf{s}(N_t) ]$$

is the  $N_S \times N_t$  training matrix,

$$\mathbf{Y} = [ \mathbf{y}(1) \quad \mathbf{y}(2) \quad \dots \quad \mathbf{y}(N_t) ]$$

is the  $N_D \times N_t$  received matrix,

$$\mathbf{n}_R = [ n_R(1) \quad n_R(2) \quad \dots \quad n_R(N_t) ]^T$$

is the noise vector at relay, and

$$\mathbf{N}_D = [ \mathbf{n}_D(1) \quad \mathbf{n}_D(2) \quad \dots \quad \mathbf{n}_D(N_t) ]$$

is the noise matrix at destination. Then, the LS estimation of  $\mathbf{H}$  can be given as

$$\hat{\mathbf{H}}_{\text{LS}} = \mathbf{Y}\mathbf{S}^\dagger = \mathbf{H} + \mathbf{g}\mathbf{n}_R^T\mathbf{S}^\dagger + \mathbf{N}_D\mathbf{S}^\dagger = \mathbf{g} \left( \mathbf{f}^T + \mathbf{n}_R^T\mathbf{S}^\dagger \right) + \mathbf{N}_D\mathbf{S}^\dagger = \mathbf{g}(\mathbf{h})^T + \mathbf{N}_D\mathbf{S}^\dagger,\tag{5}$$

where  $\mathbf{S}^\dagger$  is the pseudo-inverse of  $\mathbf{S}$ , and

$$\mathbf{h} = \left( \mathbf{f}^T + \mathbf{n}_R^T\mathbf{S}^\dagger \right)^T.$$

### 3.2. The Second Stage of TSCE

In the first stage of TSCE, only the training sequences is used to estimate the channel. In the following, we also utilize the received data sequences to improve the channel estimation accuracy of AF OWRN.

On one hand, as seen in Eq. (5), the accuracy of LS estimation  $\hat{\mathbf{H}}$  is affected by two noises: one at the relay and the other at the destination. On the other hand, based on Eq. (5), if we define

$$\mathbf{\Pi} = \mathbf{g}(\mathbf{h})^T = \begin{bmatrix} g_1 h_1 & g_1 h_2 & \dots & g_1 h_{N_S} \\ g_2 h_1 & g_2 h_2 & \dots & g_2 h_{N_S} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N_D} h_1 & g_{N_D} h_2 & \dots & g_{N_D} h_{N_S} \end{bmatrix}, \quad (6)$$

then the rank of  $\mathbf{\Pi}$  is obviously 1, and thus, the rows of  $\mathbf{\Pi}$  are correlated. Therefore, this correlation can be exploited to eliminate the noise at the destination.

In order to exploit this correlation, the channel vector  $\mathbf{g}$  should at least be decided up to scaling. Due to the special structure of the received signal at the destination, the requirement can be approached. Equation (2) can be rewritten as

$$\begin{aligned} y_1(n) &= g_1 [\mathbf{f}^T \mathbf{s}(n) + n_R(n)] + n_D^1(n), \\ y_2(n) &= g_2 [\mathbf{f}^T \mathbf{s}(n) + n_R(n)] + n_D^2(n) \\ &\vdots \\ y_{N_D}(n) &= g_{N_D} [\mathbf{f}^T \mathbf{s}(n) + n_R(n)] + n_D^{N_D}(n) \end{aligned} \quad (7)$$

From Eq. (7), if we ignore the effects of noise  $n_D^l(n)$ ,  $l = 1, 2, \dots, N_D$  at the destination and use  $1/y_1(n)$  to multiply each sub-equation in Eq. (7), then the following estimation can be obtained

$$\begin{aligned} g_1/g_1 &= y_1(n)/y_1(n), \\ g_2/g_1 &\approx y_2(n)/y_1(n), \\ &\vdots \\ g_{N_D}/g_1 &\approx y_{N_D}(n)/y_1(n). \end{aligned} \quad (8)$$

Define

$$\mathbf{g}' = [g'_1 \quad g'_2 \quad \dots \quad g'_{N_D}]^T,$$

where  $g'_l = g_l/g_1$ ,  $l = 1, 2, \dots, N_D$ . Then,

$$\hat{\mathbf{g}}'(n) = [1 \quad y_2(n)/y_1(n) \quad \dots \quad y_{N_D}(n)/y_1(n)]^T$$

is a coarse estimation of  $\mathbf{g}'$  based on the  $n$ th received block  $\mathbf{y}(n)$  using Equation (8). Note that the results in Equation (8) can be obtained by using both the training and data blocks. Then, the accurate estimation of  $\mathbf{g}'$  can be obtained by averaging the blocks in one whole frame as

$$\hat{\mathbf{g}}' = \frac{1}{N_f} \sum \hat{\mathbf{g}}'(n), \quad (9)$$

where  $\hat{\mathbf{g}}' = [\hat{g}'_1 \quad \hat{g}'_2 \quad \dots \quad \hat{g}'_{N_D}]^T$ . Usually, the frame size  $N_f$  is large, so the estimation  $\hat{\mathbf{g}}'$  is very accurate.

Then, Equation (6) can be rewritten as

$$\mathbf{\Pi} = \mathbf{g}'(\mathbf{h}')^T = [g'_1 \mathbf{h}' \quad g'_2 \mathbf{h}' \quad \dots \quad g'_{N_D} \mathbf{h}']^T, \quad (10)$$

where  $\mathbf{h}' = g_1 \mathbf{h}$ . At the  $i$ th antenna of destination node,  $\mathbf{h}'$  can be estimated using the minimum mean squared error (MMSE) method as

$$\hat{\mathbf{h}}'(i) = \frac{(g'_i)^*}{g'_i (g'_i)^* + \sigma_D^2} \hat{\mathbf{H}}(i), \quad (11)$$

where the superscript “\*” denotes the complex conjugate;  $\hat{\mathbf{H}}(i)$  is the  $i$ th row of  $\hat{\mathbf{H}}$ ;  $\hat{\mathbf{h}}'(i)$  is the estimation of  $\mathbf{h}'$  based on  $\hat{\mathbf{H}}(i)$ ;  $\sigma_D^2 = N_0$  is the noise variance at destination. Then, an accurate estimation of  $\mathbf{h}'$  can be obtained by averaging  $\hat{\mathbf{h}}'(i)$ ,  $i = 1, 2, \dots, N_D$ , as

$$\hat{\mathbf{h}}' = \frac{1}{N_D} \sum \hat{\mathbf{h}}'(i). \quad (12)$$

Finally, the proposed channel estimation result can be given as

$$\hat{\mathbf{H}}_{\text{TSCE}} = \hat{\mathbf{\Pi}} = \hat{\mathbf{g}}'(\hat{\mathbf{h}}')^T. \quad (13)$$

In this way, the correlation between the rows of  $\mathbf{\Pi}$  is exploited to eliminate the noise at the destination. It should be noted that only the noise at the destination can be filtered by using TSCE, and the noise at the relay cannot be eliminated. However, as shown in the next section, the proposed method still achieves significant improvement.

#### 4. SIMULATION RESULTS

In this section, we compare the performance of TSCE and LS techniques in terms of the NMSE, defined as

$$\text{NMSE} = \frac{E \left[ \left\| \mathbf{H}[n] - \hat{\mathbf{H}}_{\text{TSCE}}[n] \right\|^2 \right]}{E \left[ \left\| \mathbf{H}[n] \right\|^2 \right]}, \quad (14)$$

where

$$\begin{aligned} \mathbf{H}[n] &= [\mathbf{H}[n, 0], \mathbf{H}[n, 1], \dots, \mathbf{H}[n, K-1]]^T \\ \hat{\mathbf{H}}[n] &= [\hat{\mathbf{H}}[n, 0], \hat{\mathbf{H}}[n, 1], \dots, \hat{\mathbf{H}}[n, K-1]]^T. \end{aligned} \quad (15)$$

are the real and estimated channels (TSCE or LS), respectively.

As this paper focuses on the estimation rules, optimizations of the training pilot and power allocation are not considered. The signal to noise ratio is defined as [10]

$$\text{SNR} = \frac{P_s}{N_0},$$

where  $P_s$  is the average transmitting power of the source node. In this paper, the average signal power is normalized to  $P_s = 1$ , and the SNR changes with noise power  $N_0$ , which is assumed to be known. Assume that the antenna number  $N_S = N_D$ , the training length  $N_t$  is the same as antenna number, and  $N_f = 200$  in this paper. Before presenting the simulation results, we first conclude the parameters of the simulations in Table 1.

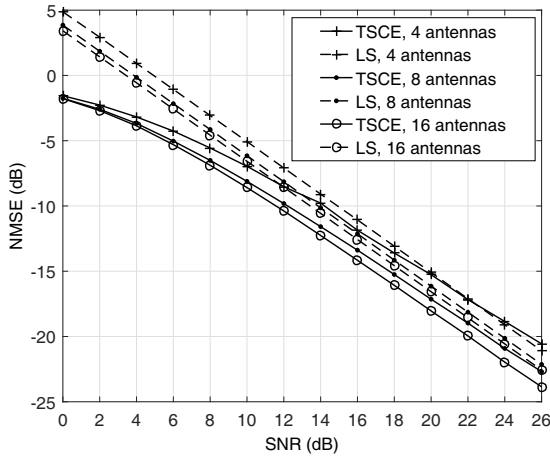
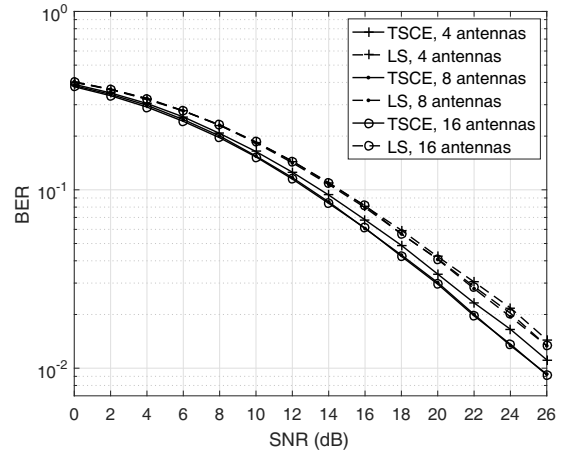
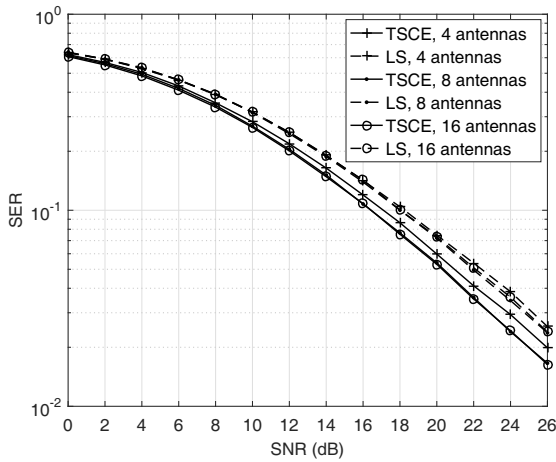
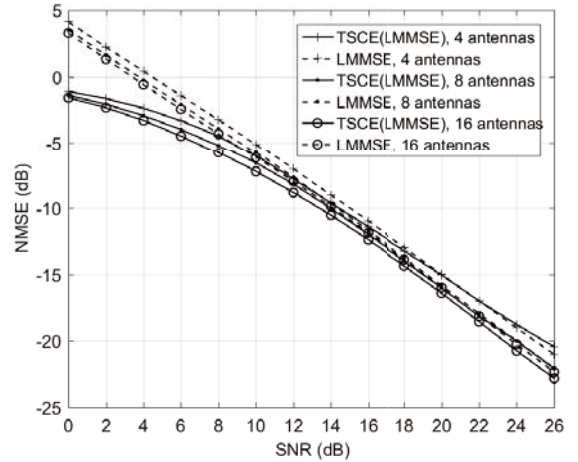
Figure 3 gives the NMSE performance comparison between the proposed channel estimator and LS estimator for different antennas. The simulation adopts 1000 independent Monte-Carlo runs for averaging. In the simulation, the proposed channel estimator is better than the LS one, especially when SNR is lower. Also, as the number of antennas increases, the NMSE performance gain of the proposed channel estimator increases. This is because when the antenna number increases,  $\mathbf{\Pi}$  has more correlated rows, which can be utilized to improve the estimation accuracy.

Figure 4 compares the bit-error-rate (BER) performance of the proposed TSCE and LS method for an AF OWRN system with different antenna numbers, and Fig. 5 compares the corresponding symbol-error-rate (SER) performance. The simulations show that the BER and SER of the proposed TSCE are also better than those of LS method, which is also a verification for TSCE performance.

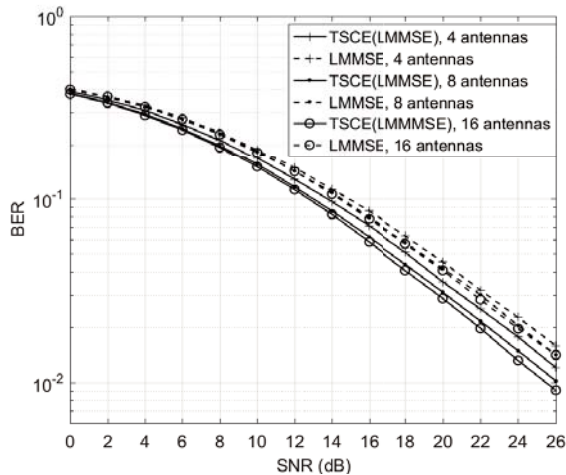
It should be noted that in the first stage of TSCE, the traditional channel estimation method is employed to estimate the cascaded channels coarsely with the help of training sequence, and the LS method is used as an example of the traditional methods in this paper. Furthermore, other methods can also be applied to the proposed TSCE. For the wireless channels in cellular systems, training sequence based MMSE estimators are often used for optimal channel estimation. However,

**Table 1.** Simulation parameters.

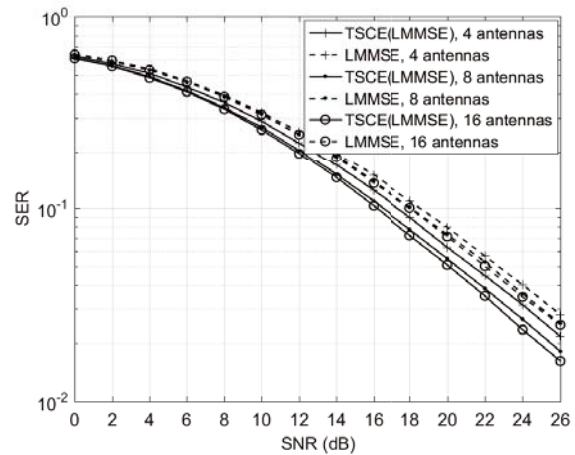
$N_S$	Transmit Antenna Number	4, 8, 16
$N_R$	Relay Antenna Number	1
$N_D$	Receive Antenna Number	$N_D = N_S$
$N_f$	Frame Length	200
$N_t$	Training Blocks	$N_t = N_S$
$N$	Data Blocks	$N = N_f - N_t$
$f_k, g_l$	Channels	$f_k, g_l \sim \mathcal{CN}(0, 1)$
$P_s$	Average Signal Power	1
$N_0$	Noise Power	
$SNR$	Signal to Noise Ratio	$P_s/N_0$

**Figure 3.** Channel estimation NMSEs versus SNR for TSCE and LS methods with different antennas.**Figure 4.** BERs versus SNR for TSCE and LS methods with different antennas.**Figure 5.** SERs versus SNR for TSCE and LS methods with different antenna**Figure 6.** Channel estimation NMSEs versus SNR for TSCE (LMMSE) and LMMSE methods with different antennas.

the AF relay channel is a cascade of two links: source-to-relay and relay-to-destination links, which results in the non-Gaussian property of the overall source-relay-destination channel and the noise at the destination [15]. Therefore, the MMSE estimator design for the AF relay channels is very complex and difficult for practical application. For this reason, sub-optimal LMMSE estimation is often considered in AF relay system. Here, we also compare the performance of the TSCE (with LMMSE as the first stage method) and the LMMSE estimators [16] in terms of the NMSE. Fig. 6 gives the NMSE performance comparison between the proposed channel estimator and the LMMSE estimator for different antennas. Fig. 7 compares the BER performance of the proposed TSCE (LMMSE) and the LMMSE method for an AF OWRN system with different antenna numbers, and Fig. 8 compares the corresponding SER performance. It can be observed that the proposed TSCE method with the LMMSE estimation as the first state can also improve the estimation accuracy.



**Figure 7.** BERs versus SNR for TSCE (LMMSE) and LMMSE methods with different antennas.



**Figure 8.** SERs versus SNR for TSCE (LMMSE) and LMMSE methods with different antennas.

## 5. CONCLUSIONS

This paper introduces a new two-stage scheme to estimate the cascaded channels of AF OWRN with multiple transmit and receive antennas. The first stage uses traditional LS (or LMMSE etc.) estimation during training period. In the second stage, the receiving data sequence is utilized to exploit the special structure of the cascaded channel matrix, which can improve the accuracy of channel estimation. The benefits of the proposed channel estimator compared with traditional LS and LMMSE estimators are confirmed by simulation results.

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