*Progress In Electromagnetics Research Letters, Vol. 79, 103–108, 2018*

# Using Homogeneous Equivalent Parameters in Finite Element Models of Curved Metamaterial Structures

**Richard Mattish[1], Fabio Alves[2], and Dragoslav Grbovic[2, *]**

**Abstract**—We report on the experimental verification of the employment of equivalent parameters in a 2D finite element model to describe absorptivity of curve-shaped, large-scale metamaterial structures. Equivalent homogeneous optical parameters were retrieved from experimental measurements of flat metamaterial sheets with square resonators of 8 and 9 mm and used in a 2D FE model to obtain the absorptivity of curved structures with similar metamaterial unit cells. The curved structures were experimentally characterized and showed good agreement with the model. The tremendous simplification made possible by simulating complex structures as homogeneous materials makes the method very attractive for designing large-scale electromagnetic shields and absorbers.

## 1. INTRODUCTION

Microwave (MW) radiation is abundant in our environment. Vast applications range from communications to radar and security to weaponry (High Power Microwave weapons). In a microwave dense environment, modern electronics and devices can be susceptible to interference or damage. It is therefore desirable to develop technologies to protect electronics from microwave radiation. Metamaterials are a promising technology that appear capable of doing just that.

A metamaterial is any artificial composite material made up of multiple macroscopic elements in a repeating pattern, which gains its bulk electromagnetic properties from its structural configuration rather than solely from its constituent materials. It is the ability to engineer a material with desired properties not already found in traditional materials that makes metamaterials research an area of increasing interest. Applications of metamaterials include information and communications technology, and security, medical, and radar imaging, to name a few [1–3]. As it pertains to the shielding from microwave interference, it is the ability to make planar metamaterials that exhibit near 100% absorption at a specific frequency or frequencies that makes them a viable shielding option [4–6]. Many different configurations of electromagnetic metamaterials have been fabricated and tested, and some examples are split-ring resonators [7–9], circular fishnet designs [10], resistive film arrays [11], and conductive square arrays [12–14]. For our research, the latter design was used and consisted of a matrix of copper squares on top of a dielectric layer, backed by a continuous copper ground plane as shown schematically in Fig. 1(a).

Depending on the size and geometry of the metamaterial object, modeling can become computationally challenging. In addition, analytical representation of metamaterial configured as perfect absorbers can be difficult. Tretyakov [15] describes several methods to model some planar metamaterial configurations using approximate surface impedance. Most of the planar arrays of particles studied in the Tretyakov (2003) are meshes, wires or loaded patches, where polarizability and coupling parameters can be obtained without much effort. For the configuration described in this paper, where the

---

*Received 3 July 2018, Accepted 15 October 2018, Scheduled 23 October 2018*

* Corresponding author: Dragoslav Grbovic (dgrbovic@nps.edu).

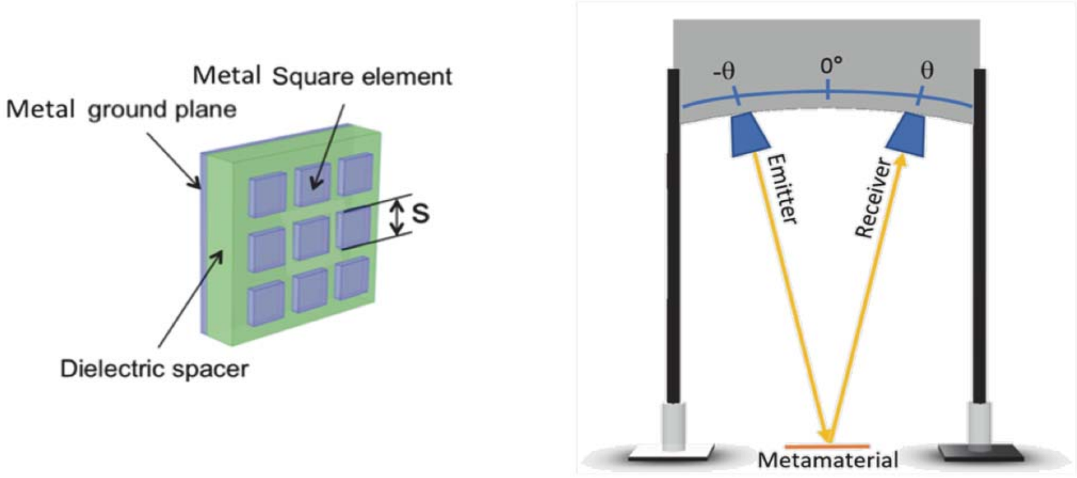
[1] Department of Physics and Chemistry, Bob Jones University, Greenville, SC 29614, USA. [2] Department of Physics, Naval Postgraduate School, Monterey, CA 93943, USA.

metal elements are not connected to each other nether to the ground plane, the necessary parameters for the analytic models are not easily obtained. Therefore, a method of parametrizing material properties of the metamaterial was developed [16]. The metamaterial is treated as a homogeneous material with frequency-dependent, effective bulk values for relative permittivity ($\varepsilon_r$), relative permeability ($\mu_r$), and conductivity ($\sigma$). This method allows a metamaterial to be modeled more easily. Until now, this method has only been tested on planar surfaces and had not yet been experimentally verified for any non-planar geometries [16].

In this paper, we report the successful application of the method developed by Hewitt et al. [16] to non-planar surfaces. We experimentally verify that the method extends to curved metamaterials as well. The parameterization method was used to extract equivalent parameters from reflectivity measurements and subsequently model them in curved geometries. COMSOL Multiphysics was used to predict spectral responses of the curved metamaterials using the equivalent bulk properties. The predicted absorptivities were then compared with experimental measurements of the metamaterials in a non-planar geometry.

## 2. METHOD

The microwave metamaterial developed by our research group is composed of a periodic array of copper square elements spaced by a thin layer of flexible polyimide from a homogeneous layer of copper (ground plane) as shown in Fig. 1(a). Both the copper ground plane and copper squares have an average thickness of 25 µm, and the polyimide has an average thickness of 100 µm. This configuration significantly reduces transmissivity and makes the structure mostly reflective to the incident microwaves except for a resonant frequency. The metal ground plane, which is thicker than the skin depth, allows no transmission. This gives the structure selective absorptivity and makes it attractive for use in various applications, such as sensing (e.g., in self-driving cars), increased efficiency in solar panel technology, and cloaking [16].



(a) (b)

**Figure 1.** (a) Schematic diagram of the metamaterials analyzed in this work. The two square resonator (metal square element) sizes are represented by $S$ and are 8 and 9 mm. (b) Schematic diagram of the NRL Arch used to measure the reflectivity of the metamaterials.

The metamaterial has a total thickness of just 150 µm, making it very flexible (similar to a heavy weight of paper). The flexibility of the metamaterial allows it to be bent in non-planar geometries very easily. Two different samples of metamaterials were tested: one with 8 mm squares and one with 9 mm squares. The squares were spaced 0.5 mm apart (meaning 9 mm pitch for the 8 mm squares and 10 mm pitch for the 9 mm squares).

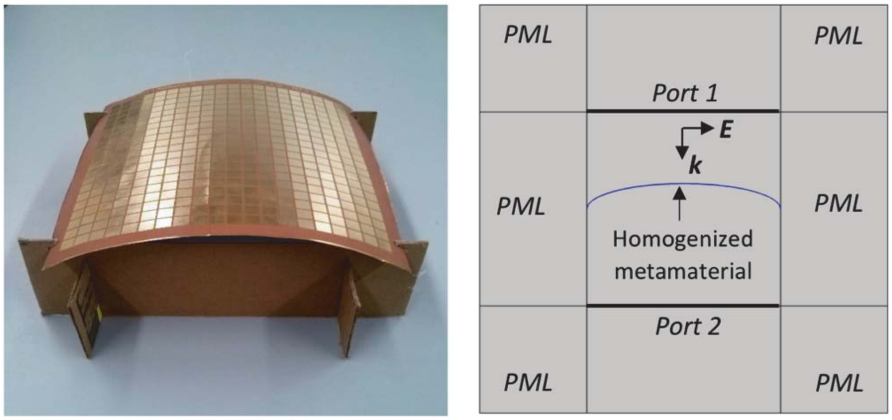
To collect experimental data for the reflectivity of our metamaterials, an NRL Arch was employed as is shown in Fig. 1(b). Two identical Cobham H-1498 horn antennas were attached to the NRL Arch at equal angular displacements with reference to the center of the arch. The transmitter antenna (left) was connected to an HP 8350B sweep oscillator which was used to generate a microwave signal with frequencies between 5 GHz and 15 GHz. The receiver antenna (right) was connected to an Agilent E4407B spectrum analyzer which was used to collect the reflected signal from the sample. Because all of the metamaterial samples characterized had a ground plane thicker than the skin depth throughout the region of interest, transmissivity was assumed negligible. Therefore, the absorptivity, $A$, can be calculated as:

$$A = 1 - \frac{R_{\text{metamaterial}}}{R_{\text{mirror}}}, \tag{1}$$

where $R_{\text{metamaterial}}$ is the signal reflected by the metamaterial, and $R_{\text{mirror}}$ (background signal) is the signal reflected by an almost perfect mirror in the spectral region of interest. The mirror has to have the same dimensions and position as the metamaterial sample in the experimental setup (Fig. 1(b)). To simplify the experimental procedure, the background was collected by turning over the metamaterial sample and the ground plane was used as a mirror.

Using the reflectivity obtained experimentally for a particular metamaterial lying flat, complex frequency-dependent material parameters ($\varepsilon_r$, $\mu_r$ and $\sigma$) were retrieved for the metamaterial at each frequency. Using the method presented in [16], the metamaterial was treated as a homogeneous material with frequency-dependent bulk-equivalent material properties and was modeled in 2D in COMSOL. The 2D simulation significantly reduces the computational cost and allows for larger objects to be simulated. This is only possible, however, when the surface being simulated is symmetric with respect to the in-plane. The intrinsic 3D characteristic of the metamaterial structures is embedded in the homogeneous equivalent properties, allowing this simplification. Due to the complex nature of metamaterials, 3D finite element models are limited to small portions of the sample or basically the periodic unit cell [13, 14, 16–18]. In applications where the sample is much larger than the wavelengths and oddly shaped, a homogenized equivalent would make the simulation much simpler.

Absorptivity of both flexible metamaterials, with 8 and 9 mm square resonators, were simulated for a curved geometry (Fig. 2(a)) using a COMSOL 2D model, detailed in Fig. 2(b), with a radius of curvature, $r_c$ of 157 mm. There, the blue line represents the homogenized metamaterial sample which was set as a generic material whose properties are the homogeneous equivalents. All other domains are considered air. A 2-port configuration was used where a 1 W plane wave, swept from 5 to 15 GHz, leaves port 1 and reaches the sample. All external domains are set as perfectly matched layers, which



(a) (b)

**Figure 2.** (a) Picture of the flexible metamaterial held in a curved shape (radius of curvature of 157 mm) and characterized in the experimental setup described in Fig. 1(b). (b) Schematic diagram of the 2D FE model used to simulate the curved metamaterials.

absorb all scattered radiation. A direct computation of the scattering parameter $S_{21}$ shows that there is negligible transmission, while the absorptivity was obtained by integrating the resistive losses over the sample domain [14].

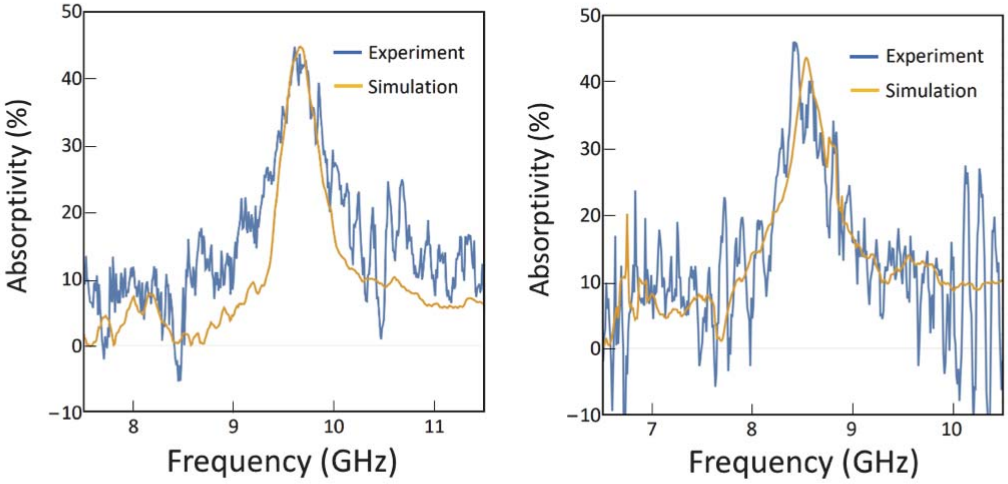
Prior to this paper, the method presented in [16] had not been experimentally tested for anything outside of a planar geometry. In order to validate this method and the results produced by the 2D finite element model of the homogenized metamaterial in a non-planar geometry, both flexible structures were held in a fixed curved position as shown in Fig. 2(a) and experimentally characterized over the same frequency range of the simulation. One horn antenna, which served as the microwave emitter, was fixed at $-5^\circ$ on the NRL Arch. The receiver antenna was moved from $5^\circ$ to $30^\circ$, scanning the region to capture most of the reflected signal. Measurements were taken in $5^\circ$ increments. Using the experimental data along with Equation (1), the absorption for both flexible metamaterial samples was experimentally obtained, allowing the simulation to be directly compared to experimental results.

## 3. RESULTS AND DISCUSSION

The results for the absorption of the flexible 8 mm metamaterial obtained from both experimental measurements as well as the simulated absorption from the COMSOL 2D finite element model of the homogenized metamaterial are shown in Fig. 3(a). The experimental absorption is shown in blue, and the simulated absorption is shown in yellow. As can be seen, the simulated absorption agrees well with experimental measurements, indicating that the 2D homogenized model is a good representation of the metamaterial structure. Both the experimental and simulated absorption plots show an absorption peak at 9.6 GHz.

The results for the absorption of the flexible 9 mm metamaterial are shown and compared in Fig. 3(b). While the agreement between the simulated absorption and the experimental measurements are not as close as it was for the 8 mm metamaterial, it is still clear that they are in good agreement. Both curves give an absorption peak of approximately 8.5 GHz.

The experimental absorptivity plot clearly shows the interference fringes from transmitter-receiver crosstalk on top of the resonant characteristic. The slight difference in absorption peaks between simulation and experiment for the 9 mm metamaterial (Fig. 3(b)) is most likely due to the interference fringes.



(a) (b)

**Figure 3.** (a) Comparison of simulated and experimental absorptivity for the curved 8 mm metamaterial. (b) Comparison of simulated and experimental absorptivity for the curved 9 mm metamaterial. Simulations were performed with the homogeneous equivalent parameters of the corresponding metamaterial structures.

## 4. SUMMARY

In summary, we have experimentally demonstrated that homogeneous-equivalent parameters, relative permittivity, relative permeability, and conductivity, retrieved from planar metamaterials can be used to accurately model curved metamaterials with the same periodic unit cell. Furthermore, since the intrinsic effects of the 3D nature of the metamaterial unit cells are embedded in their homogeneous equivalent parameters, a 2D model can be used, radically simplifying the simulations. Metamaterials with square resonators of sizes 8 and 9 mm were used, and their absorptivities resonating around 9.6 GHz and 8.5 GHz, respectively, were obtained using both methods in good agreement. These results indicate a great potential to use 2D finite element modeling to accurately simulate the performance of metamaterials in large-scale complex geometries and experimentally validates the application of 2D FE models to geometries other than the one that reflection data was originally obtained from.

## ACKNOWLEDGMENT

This work is supported by the Office of Naval Research (ONR) under Grant (or Contract) No. N0001418WX00095 and was conducted entirely at the Naval Postgraduate School's Applied Physics department's facilities. The authors would like to thank Chester Hewitt and Edward Wulff for their help and assistance.

## REFERENCES

1. Dean, P., A. Valavanis, J. Keeley, K. Bertling, Y. L. Lim, R. Alhathlool, A. D. Burnett, L. H. Li, S. P. Khanna, D. Indjin, T. Taimre, A. D. Rakic, E. H. Linfield, and A. G. Davies, "Terahertz imaging using quantum cascade lasers — A review of systems and applications," *J. Phys. D*, Vol. 47, 374008, 2014.
2. Qiao, S., Y. Zhang, S. Liang, L. Sun, H. Sun, G. Xu, Y. Zhao, and Z. Yang, "Multi-band terahertz active device with complementary metamaterial," *J. Appl. Phys.*, Vol. 118, 123106, 2015.
3. Vendik, I. B. and O. G. Vendik, "Metamaterials and their applications in microwaves: A review," *Tech. Phys.*, Vol. 58, 1–24, 2013.
4. Boggi, S., R. Alonso, and W. G. Fano, "Shielding effectiveness of a metamaterial measured at microwave range of frequency, known as wire screen metamaterial (WSM)," *Progress In Electromagnetics Research M*, Vol. 63, 33–46, 2018.
5. Tsutaoka, T., K. Hatakeyama, and T. Kasagi, "Possibilities for the EM absorber and shielding by use of metamaterials," *2009 International Symposium on Electromagnetic Compatibility*, Kyoto, Japan, Jul. 20–24, 2009.
6. Solovey, A., "Theoretical limitations on shielding and reflective properties of microwave metamaterial absorbers," *2017 International workshop on Antenna Technology: Small Antennas, Innovative Structure, and Applications (iWat)*, 148–151, Athens, 2017.
7. Aydin, K., I. Bulu, K. Guven, M. Kafesaki, C. M. Soukoulis, and E. Ozbay, "Investigation of magnetic resonances for different split-ring resonator parameters and designs," *New J. Phys.*, Vol. 7, 168–183, 2005.
8. Guo, H., N. Liu, L. Fu, T. P. Meyrath, T. Zentgraf, H. Schweizer, and H. Giessen, "Resonance hybridization in double split-ring resonator metamaterials," *Optics Express*, Vol. 15, 12095–12101, 2007.
9. Gwinner, M. C., E. Koroknay, L. Fu, P. Patoka, W. Kandulski, M. Giersig, and H. Giessen, "Periodic large-area metallic split-ring resonator metamaterial fabrication based on shadow nanosphere lithography," *Small*, Vol. 5, 400–406, 2009.
10. Ozturk, Y. and A. E. Yilmaz, "Multiband and perfect absorber with circular fishnet metamaterial and its variations," *ACES*, Vol. 31, 1445–1451, 2016.
11. Shen, Y., Z. Pei, Y. Pang, J. Wang, A. Zhang, and S. Qu, "An extremely wideband and lightweight metamaterial absorber," *J. Appl. Phys.*, Vol. 117, 224503, 2015.

12. Grbovic, D., F. Alves, B. Kearney, B. Waxer, R. Perez, and G. Omictin, "Metal-organic hybrid resonant terahertz absorbers with SU-8 photoresist dielectric layer," *J. Micro/Nanolith. MEMS MOEMS*, Vol. 12, No. 4, 041204, 2013.
13. Alves, F., B. Kearney, D. Grbovic, N. V. Lavrik, and G. Karunasiri, "Strong terahertz absorption using $SiO_2$/Al based metamaterial structures," *Appl. Phys. Letters*, Vol. 100, 111104, 2012.
14. Kearney, B., F. Alves, D. Grbovic, and G. Karunasiri, "Al/$SiO_x$/Al single and multiband metamaterial absorbers for terahertz sensor applications," *Opt. Engineering*, Vol. 52, No. 1, 013801, 2013.
15. Tretyakov, S., *Analytical Modeling in Applied Electromagnetics*, Artech House, 2003.
16. Hewitt, C., F. Alves, J. Luscombe, and D. Grbovic, "Application of equivalent medium parameters in finite element models of microwave metamaterials," *J. Appl. Phys.*, Vol. 123, 115101, 2018.
17. Liu, R., C. Ji, Z. Zhao, and T. Zhou, "Metamaterials: Reshape and rethink," *Engineering*, Vol. 1, 179–184, 2015.
18. Alves, F., B. Kearney, D. Grbovic, and G. Karunasiri, "Narrowband terahertz emitters using metamaterial films," *Optics Express*, Vol. 20, 171863, 2012.