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Direct Matrix Synthesis for In-Line Diplexers with Transmission Zeros Generated by Frequency Variant Couplings

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Abstract—This paper presents a direct matrix synthesis for in-line diplexers constructed by general Chebyshev channel filters. The finite transmission zeros of the channel filters are generated and independently controlled by a set of frequency-variant couplings (FVC) sections. The network only involves resonators cascaded one by one without any auxiliary elements (such as cross-coupled or extracted-pole structures), and this paper provides the best synthesis solution in configuration simplicity for narrowband contiguous diplexers. For the channel filters, considering both the couplings and capacitances matrices of a traditional low-pass prototype, a generalized transformation on the admittance matrix is introduced as the basis of the synthesis, which allows more than one cross-coupling to be annihilated in a single step, while generating an FVC section simultaneously. Two examples of diplexer are synthesized to show the validation of the method presented in this paper.

1. INTRODUCTION

Microwave diplexers and multiplexers are extensively employed in wireless and satellite communication systems to combine RF signals of different frequency bands into one channel with specified frequency selectivity and isolation requirements. A very common and simple way to combine the multiple channel signals is to directly connect all the channel filters to a star-junction with one common port. Such a connecting scheme makes the multiplexing network simple while maintaining a good microwave performance. The most critical issue in synthesizing such a multiplexer is how to take the interaction among all the channels into account, especially when the frequency bands are spaced close to each other. An analytical approach to synthesis of such a multiplexer is highly desirable in the industry.

The research efforts to analytically synthesize a multiplexer with a star-junction have never rested over the past three decades. In the early years, the classical circuit synthesis approach was adopted [1, 2]. To compensate for the interaction among the channel filters, the parameters of separately designed channel filters are subject to an appropriate adjustment, but the number of channels and coupling topologies of channel filters are limited and the synthesis result deteriorates as the frequency bands get closer to each other.

In recent years, a more effective and flexible way to synthesize diplexers is proposed in [3]. The relationship between the overall diplexer parameters and those of separate channel filters is derived by circuit analysis first. Suitable polynomials describing the characteristics of the diplexer are evaluated by insisting on reflection zeroes assigned a priori, and the transfer and reflection functions of each channel filter are derived accordingly. At last the channel filters are synthesized separately by using a well-known coupling matrix synthesis approach [4–7]. The nonresonant node (NRN) type of junction and the resonant node type of junction are analyzed separately and treated differently in the synthesis approach. In [8], the method for a diplexer with a resonant junction is extended to the synthesis of starjunction

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multiplexers. In such a type of approach, the roots of high-order polynomials need to be identified. The root-finding process will be a decisive factor that affects the accuracy of the method, and thus the applicable aggregate number of system poles of a multiplexer system becomes limited. Another existing approach to the diplexer and multiplexer synthesis is to directly apply the optimization method to obtain the coupling matrix synthesis of diplexer and multiplexers, such as in [9–11]. Recently, a favorable direct synthesis method is reported in [12], but is available for a very particular in-line condition where only one TZ can be realized. Lacking a general direct synthesis approach, whether a selected in-line network can realize required frequency response is still not predictable.

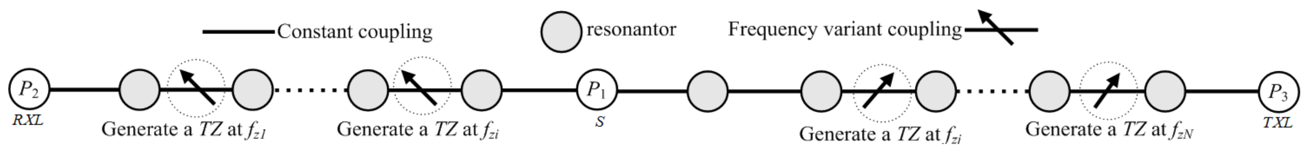

Figure 1. Ideal prototype for a class of in-line topology diplexers with multiple FVC sections, which can generate TZs at f_{z1}, f_{z2}, ..., $f_{z,NZ}$.

In this paper, a general direct synthesis approach is presented for synthesizing in-line diplexers with multiple TZs (as shown in Fig. 1). Concerning the capacitance matrix and coupling matrix together, a new transformation for the admittance matrix is proposed, which presents new possibilities that can transform a basic triplet section into a FVC section and annihilate more than one cross coupling in a single step. With a specific procedure of transformations, an in-line topology containing a set of frequency-variant couplings is ultimately decided from cascaded triplet topology diplexer.

This paper is organized as follows. Basic theory on the admittance matrix transformation process is detailed in Section 2. Two synthesis examples of diplexer are shown in Section 3. One diplexer is the type-I junction, and the other is type-II junction. The validation is shown from the two examples. Section 4 provides the conclusion.

2. BASIC THEORY

The diplexer is a three-port network. The coupling matrix could be used for the analysis of coupled resonator diplexers. The relation between S-parameters and "$N+3$" coupling matrix can be expressed as [13, 14]

$$S_{11} = 1 + 2j\left[A^{-1}\right]_{P_1P_1} \qquad S_{21} = -2j\left[A^{-1}\right]_{P_2P_1} \qquad S_{31} = -2j\left[A^{-1}\right]_{P_3P_1} \tag{1}$$

where the matrix A is given by

$$A = M + \omega \cdot C - jG \tag{2}$$

here, M is the coupling matrix, ω the normalized frequency, C the capacitance matrix, $C(k,k) = 1$ except $C(p_1p_1) = C(p_2p_2) = C(p_3p_3) = 0$, G the terminal load matrix, and $G(k,k) = 0$ except $G(p_1p_1) = G(p_2p_2) = G(p_3p_3) = 1$. The proposed approach starts with the $N+3$ admittance matrix A for an Nth order low-pass cross-coupled prototype. The start topology of the diplexer is the CT section, it can be obtained by [3].

To obtain an in-line diplexer configuration constructed by FVC sections, it is known that transformations on coupling matrix are always required, such as the manipulations discussed in [4] and [6]. While in [12], the transformation on capacitance matrix are also taken into account. Concluding both situations, in this paper, a generalized transformation (represented by matrix T) on admittance matrix is introduced as

$$\begin{aligned} A_{n+1} &= T_{n+1}^T A_n T_{n+1} \\ &= T_{n+1}^T (M_n + \omega C_n - jG) T_{n+1} \\ &= T_{n+1}^T M_n T_{n+1} + \omega T_{n+1}^T C_n T_{n+1} - jG \\ &= M_{n+1} + \omega C_{n+1} - jG \end{aligned} \tag{3}$$

where $n = 0, 1, 2, \ldots$ represents the corresponding parameters of the nth transform operation. The proposed transformation T is discussed in three conditions.

1) Similarity Transformation from CT Section to BOX Section

Similarity transformation where T refers to the rotation matrix R,

$$R = \begin{bmatrix} 1 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \cos\theta_r & \ldots & -\sin\theta_r & \ldots & 0 \\ \ldots & \ldots & 0 & \ldots & 0 & \ldots & \ldots \\ 0 & \ldots & \sin\theta_r & \ldots & \cos\theta_r & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & \ldots & 0 & \ldots & 1 \end{bmatrix}_{N+3,N+3} \tag{4}$$

Stipulating the rotation pivot as $[i, j]$ $(i, j = 1, 2, \ldots, N)$ and the rotation angle as θ_r, R is observed to be unity matrix except for the entries $R_{ii} = R_{jj} = \cos\theta_r$, $R_{ij} = -\sin\theta_r$ and $R_{ji} = -\sin\theta_r$.

In this case, the values of the coupling matrix are changed in concordance with the well-known rotation process given in [13], such as Fig. 1(a) and Fig. 1(b). The formulation of the rotation angle that transform the CT section to BOX is given as [15]

$$\theta_r = \frac{1}{2}\tan^{-1}\left[\frac{2M_{ij}}{M_{jj} - M_{ii}}\right] + \frac{k\pi}{2} \tag{5}$$

where, k is an arbitrary integer. i is the starts resonator of the CT section, $j = i + 1$.

2) Rescaling the BOX Section

Applying the rescaling transformation, i.e., $T = U$, to redefine the capacitance element in the basic triplet section. Taking into account the rescaling factor of resonator i as α_i, as Fig. 1(b) shows, the entry U_{ii} in U can be evaluated by formulation

$$U_{ii} = \sqrt{\alpha_i} = \sqrt{-\frac{M_n(i-1, i+1)\, M_n(i+1, i+2)}{M_n(i-1, i)\, M_n(i, i+2)}} \tag{6}$$

The operation thus produce new admittance matrix A_{n+1} by the following:

$$\begin{aligned} A_{n+1} &= U_{n+1}^T A_n U_{n+1} \\ &= U_{n+1}^T (M_n + \omega C_n - jG) U_{n+1} \\ &= U_{n+1}^T M_n U_{n+1} + \omega U_{n+1}^T C_n U_{n+1} - jG \\ &= M_{n+1} + \omega C_{n+1} - jG \end{aligned} \tag{7}$$

thus,

$$M_{n+1} = U_{n+1}^T M_n U_{n+1} \tag{8a}$$

$$C_{n+1} = U_{n+1}^T C_n U_{n+1} \tag{8b}$$

Note, the topology of the diplexer in this step is not changed.

3) BOX Section to FVC Section

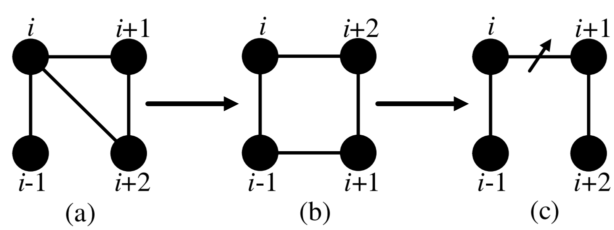

Figure 2. The fundamental transformation process. (a) Basic triplet, (b) basic box section, (c) basic FVC section.

Implementing a further similarity transformation on the rescaled network, which ultimately realizes an in-line frequency-variant coupling as Fig. 1(c). The pivot is also $[i, j]$, and the rotation angle θ_b is given by

$$\theta_b = \tan^{-1}\left(-\frac{M_n(i-1, i+1)}{\sqrt{\alpha_i} M_n(i-1, i)}\right) = \tan^{-1}\left(\frac{\sqrt{\alpha_i} M_n(i+1, i+2)}{M_n(i, i+2)}\right) \tag{9}$$

in this step, the rotation affects the capacitance matrix as formulation (8).

Where the relevant capacitance value in C_{n+1} become $(k = p_1, p_2, p_3, 1, 2, \ldots, N)$

$$\begin{aligned}
C_{n+1}(i,k) &= \cos\theta_b C_n(i,k) - \sin\theta_b C_n(j,k) \\
C_{n+1}(j,k) &= \sin\theta_b C_n(i,k) + \cos\theta_b C_n(j,k) \\
C_{n+1}(k,i) &= \cos\theta_b C_n(k,i) - \sin\theta_b C_n(k,j) \\
C_{n+1}(k,j) &= \sin\theta_b C_n(k,i) + \cos\theta_b C_n(k,j) \\
C_{n+1}(i,i) &= \cos^2\theta_b C_n(i,i) + \sin^2\theta_b C_n(j,j) - 2\sin\theta_b\cos\theta_b C_n(i,j) \\
C_{n+1}(j,j) &= \sin^2\theta_b C_n(i,i) + \cos^2\theta_b C_n(j,j) + 2\sin\theta_b\cos\theta_b C_n(i,j) \\
C_{n+1}(i,j) &= C_n(i,j)(\cos^2\theta_b - \sin^2\theta_b) + \sin\theta_b\cos\theta_b(C_n(i,i) - C_n(j,j))
\end{aligned} \tag{10}$$

In this step, a frequency slope $C_{n+1}(i,j)$ is created. Comparing the resulting in-line topology with the triplet structure in Fig. 2(a), it is apparent that the number of coupling paths for realizing a TZ is reduced. This shows exactly the benefits of in-line topologies.

The algorithm translates cascaded triplets to the FVC sections is shown as the following:

$M = M_{CT}$

for $k = 1 : n-2$

if $M_{k,k+2} \sim= 0$

translate the CT to BOX section, annihilates $M_{i,i+1}$, generates the elements $M_{i,i+2}$, $(i, i+1)$ as pivot

scaling the node $M_{i,i}$, $U(i,i)$ is shown as formulation (7)

$(i, i+1)$ as pivot, annihilates $M_{i,i+2}$, generates the FVC elements $C\ (i, i+1)$

end

end

3. EXPERIMENT RESULTS

For verification of the diplexer synthesis method presented in this paper, two examples are synthesized in this section.

3.1. Example One (Type-I Junction)

The first synthesized example with type-I junction [3] is carried out in the normalized frequency domain, and the specifications is shown as following:

RX channel filter: The finite transmission zeros in the normalized frequency domain are -2.5606, -1.7908, 1.1282, 0.3005. The return loss is 23 dB. the order of the RX channel filter is 10.

TX channel filter: The finite transmission zeros in the normalized frequency domain are -0.6824, -0.2383, 1.9658. The return loss is 23 dB. the order of the TX channel filter is 9.

First, synthesize the diplexer as the method presented in [3, 6] and obtain the cascaded triplet sections topology diplexer as shown in Fig. 3. The coupling coefficients of the cascaded triplets diplexer are also shown in Fig. 3. Then, using the method presented in this paper translates the cascaded triplets into the in-line topology as shown in Fig. 4. The coupling coefficients of the in-line topology diplexer are also shown in Fig. 4. The details of the transformations are shown in Table 1. The responses of the topologies in Fig. 3 and Fig. 4 are shown in Fig. 5. The polynomial response can be calculated by Eq. (2) in [3]. It is shown that the matrix response agrees with the polynomials response well.

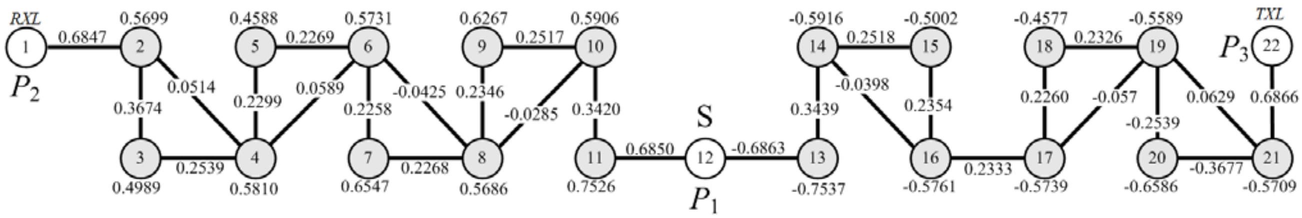

Figure 3. The cascaded triplet topology of the diplexer for example 1.

Table 1. First example: details of the transformation from CT to FVC.

RX channel filter				TX channel filter			
step	pivot	Angle (rad)	notes	step	pivot	Angle (rad)	notes
1	[2, 3]	2.4044	Triplet 234 to BOX 1234	13	[14, 15]	3.8372	Triplet 14,15,16 to BOX 13,14,15,16
2			Rescale U (2,2)	14			Rescale U(14, 14)
3	[2, 3]	0.8382	BOX 1234 to FVC 23	15	[14, 15]	−0.7793	BOX 13,14,15,16 to FVC 14,15
4	[4, 5]	2.4861	Triplet 456 to BOX 3456	16	[17, 18]	3.8012	Triplet 17,18,19 to BOX 16,17,18,19
5			Rescale U(4,4)	17			Rescale U(17, 17)
6	[4, 5]	0.7841	BOX 3456 to FVC 45	18	[17, 18]	−0.7796	BOX 16,17,18,19 to FVC 17,18
7	[6, 7]	3.8376	Triplet 678 to BOX 5678	19	[19, 20]	3.8301	Triplet 19,20,21 to BOX 18,19,20,21
8			Rescale U(6,6)	20			Rescale U(19,19)
9	[6, 7]	−0.7887	BOX 5678 to FVC 67	21	[19, 20]	−0.7730	BOX 18,19,20,21 to FVC 19,20
10	[8, 9]	3.8654	Triplet 89,10 to BOX 789,10				
11			Rescale U(8,8)				
12	[8, 9]	−0.7801	BOX 789,10 to FVC 89				

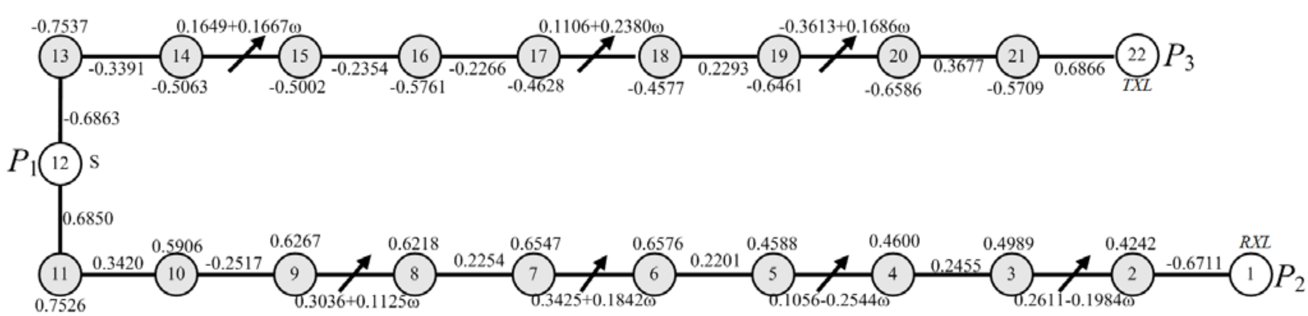

Figure 4. The in-line topology of the diplexer for example 1.

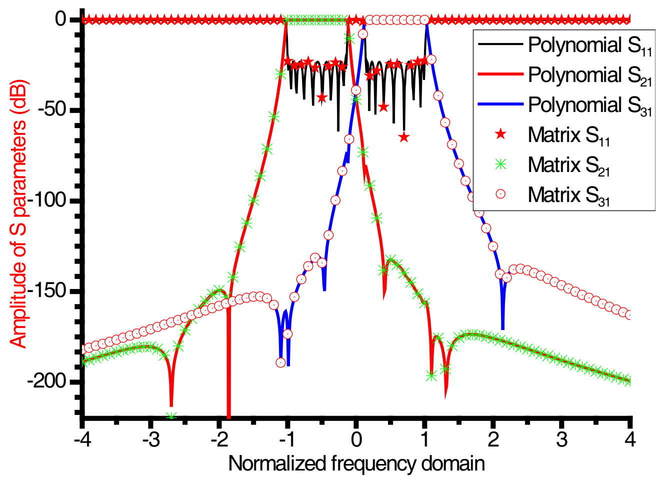

Figure 5. The response of the diplexer for example 1.

3.2. Example Two (Type-II Junction)

The second synthesized example with the type-II junction [3] is carried out in the normalized frequency domain, and the specifications is shown as following:

RX channel filter: The finite transmission zeros in the normalized frequency domain are 0.3005, 1.1282, −2.5606. The return loss is 20 dB. the order of the RX channel filter is 7.

TX channel filter: The finite transmission zeros in the normalized frequency domain are −0.6824, −0.2383, 1.9658. The return loss is 20 dB. the order of the TX channel filter is 7.

First, synthesize the diplexer as the method presented in [3, 6] and obtain the cascaded triplet sections topology diplexer as shown in Fig. 6. The coupling coefficients of the cascaded triplets diplexer are also shown in Fig. 6. Then, using the method presented in this paper translates the cascaded triplets into the in-line topology as shown in Fig. 7. The coupling coefficients of the in-line topology diplexer are also shown in Fig. 7. The details of the transformations are shown in Table 2. The responses of the topologies in Fig. 6 and Fig. 7 are shown in Fig. 8. It is shown that the matrix response agree with the polynomials response well.

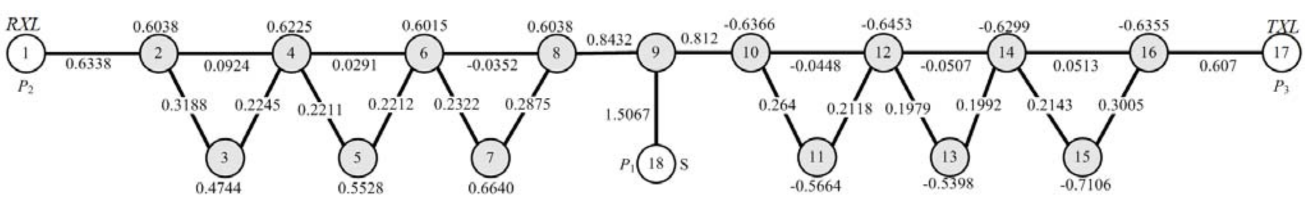

Figure 6. The cascaded triplet topology of the diplexer for example 2.

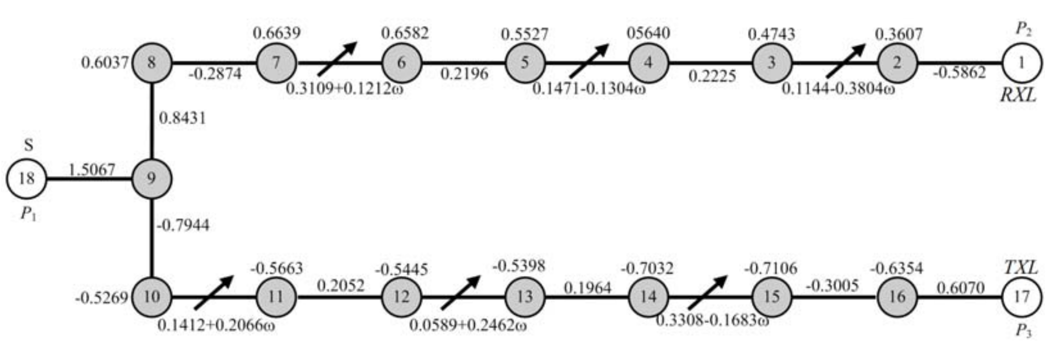

Figure 7. The in-line topology of the diplexer for example 2.

Table 2. Second example: details of the transformation from CT to FVC.

RX channel filter				TX channel filter			
step	pivot	Angle (rad)	notes	step	pivot	Angle (rad)	notes
1	$[2, 3]$	2.4563	Triplet 234 to BOX 1234	10	$[10, 11]$	3.8608	Triplet 10,11,12 to BOX 1234
2			Rescale U(2,2)	11			Rescale U(10,10)
3	$[2, 3]$	0.8882	BOX 1234 to FVC 23	12	$[10, 11]$	-0.8241	BOX 9,10,11,12 to FVC 10,11
4	$[4, 5]$	2.4344	Triplet 456 to BOX 3456	13	$[12, 13]$	3.7967	Triplet 12,13,14 to BOX 11,12,13,14
5			Rescale U(4,4)	14			Rescale U(12,12)
6	$[4, 5]$	0.7725	BOX 3456 to FVC 45	15	$[12, 13]$	-0.7793	BOX 11,12,13,14 to FVC 12,13
7	$[6, 7]$	3.8601	Triplet 678 to BOX 5678	16	$[14, 15]$	2.4494	Triplet 14,15,16 to BOX 13,14,15,16
8			Rescale U(6,6)	17			Rescale U(14,14)
9	$[6, 7]$	-0.7792	BOX 5678 to FVC 67	18	$[14, 15]$	0.7766	BOX 13,14,15,16 to FVC 14,15

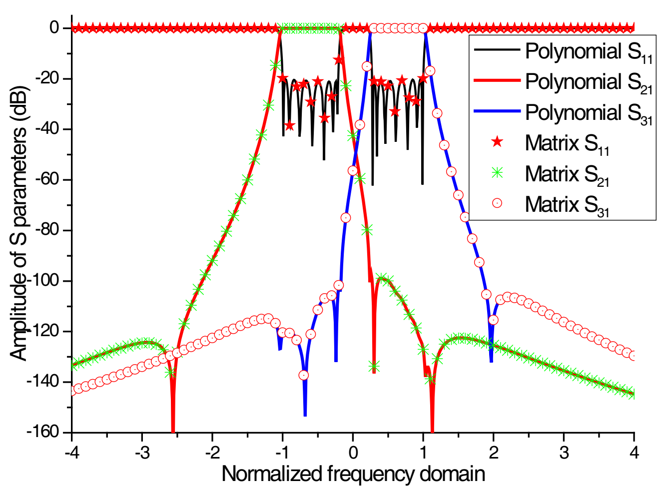

Figure 8. The response of the diplexer for example 2.

4. CONCLUSION

A direct matrix synthesis for in-line diplexers constructed by general Chebyshev channel filters is presented in this paper. The finite transmission zeros of the channel filters are generated and independently controlled by a set of basic frequency variant couplings (FVC). Two examples are synthesized by this method. Excellent agreement between the response computed from characteristic polynomials and the response computed from couplings matrix is obtained from the proposed method. The star-junction multiplexer can also be synthesized by this method.

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