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Suitable Impedance Boundary Condition Applied to the Enhancement of the Electric Field Radiated by a High Frequency Surface Wave Radar

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Abstract—Efficiency of high frequency surface wave radars may be improved by inserting a metamaterial in the vicinity of transmitting antennas that will reinforce the propagation of surface waves. This paper deals with the first and second order derivations of the surface impedance boundary conditions (IBC) applied to model such a metamaterial, which is equivalent to a bounded ground with a low negative permittivity. The goal of this paper is to extend an approach previously based on the classical Leontovich IBC which is usually restrained to high permittivity grounds. As shown here, a simplification in the expression of the surface impedance is possible in the case of a planar and homogeneous surface. That allows to have a first order impedance boundary condition substituted for the required second order impedance boundary condition.

1. INTRODUCTION

High Frequency Surface Wave Radars (HFSWR) are currently deployed worldwide [1, 2] and particularly in the European Union [3] as a part of the Integrated Maritime Surveillance Systems (IMSS). These systems aim to bring global and operational means for maritime security and surveillance [4]. Those radars are considered as an optimum and low cost solution for monitoring the Exclusive Economic Zone. HFSWR are based on the property of high frequency waves to be propagated beyond the line-of-sight. Indeed, at those frequencies (i.e., from 3 MHz to 30 MHz), electromagnetic waves have the ability to be guided over the surface of the sea with reasonable losses. Fig. 1(a) shows a biconical antenna used for Onera's demonstrator of the HFSWR located at Levant Island in France. We can see in Fig. 1(b) typical tracking results in the Mediterranean sea.

Today, a substantial effort is required to increase the efficiency of HFSWR while reducing their cost. During the last decade, improvements of HFSWR systems have dealt with compactness [5], clutter removal [6] and architecture or detection scheme [7]. Actually, most of the electromagnetic power of transmitting antennas is radiated towards the sky and a minor part of it is radiated at low elevation angle. Therefore, an increase of the surface wave power radiated by the transmitting antennas is an essential contribution to improve the efficiency of HFSWR. But this improvement may go through the use a low negative permittivity medium which is not properly characterized by the standard approach. Hence, there is still an issue regarding this solution since the simulation results could be a calculation artifact [8].

In order to clarify the surface wave improvement using such a metamaterial, we propose an accurate analytic formulation to compute the electromagnetic field radiated by the transmitting antennas whatever the permittivity of the antenna ground is. We have mainly based our approach on the

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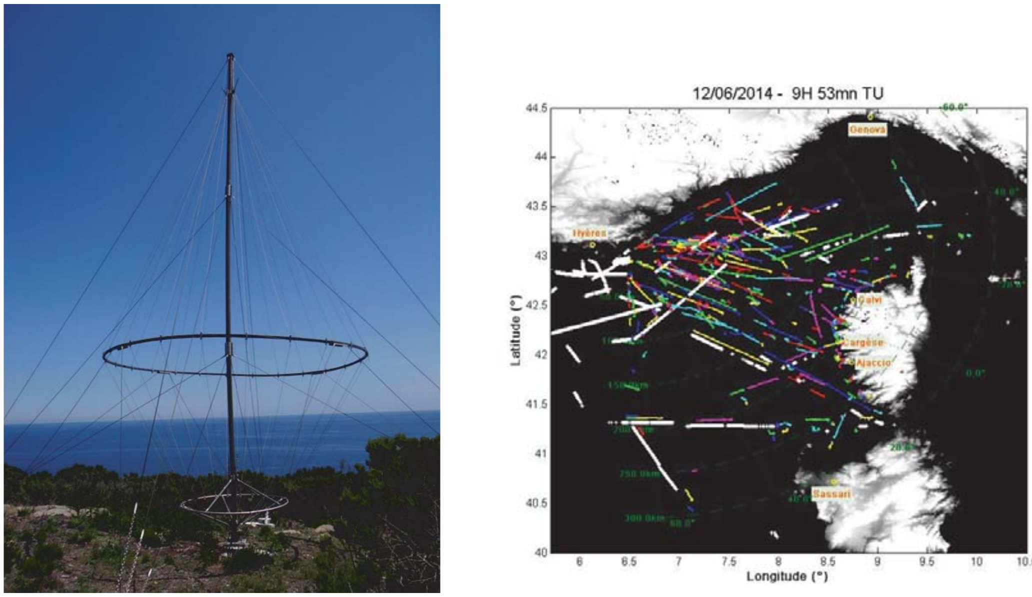


(a) (b)

Figure 1. (a) A biconical antenna and (b) results of tracking in the Mediterranean sea.

works published by Karp and Karal [9]. They have derived a decomposition of the electromagnetic field radiated by a magnetic line source located above a plane structure. We have also taken into account the results obtained by Kane and Karp [10] who have introduced a second order impedance boundary condition. The latter approach is not limited to large permittivity values like the Leontovich one [11].

In Section 2, we show that a metamaterial can effectively propagate a strong surface wave. In Section 3, we describe the geometry of the studied configuration and explain the principle of surface impedance boundary condition. In Section 4, we derive the field radiated by an infinite line source of vertical electric Hertzian dipoles, using a first order Impedance Boundary Condition (IBC), when the source is over a soil with low negative permittivity. In Section 5, we recall the second order IBC which allows to consider ground with a permittivity close to unity. Finally, we will draw a conclusion.

2. ENHANCED SURFACE WAVE

The first way to enhance a part of the electromagnetic field radiated at low elevation angle may be to optimize the shape of the transmitting antenna, but it appears that it is not efficient enough to significantly improve the excitation of the surface wave [12, 13]. Recently, Petrillo et al. [14] have shown that it is not possible to excite a strong surface wave when the transmitting antennas are located over a ground characterized by a positive permittivity. They nevertheless have suggested that inserting a metamaterial with negative permittivity in the vicinity of the transmitting antennas could overcome this difficulty. Fig. 2(a) highlights the improvement obtained when the antenna is located over a metamaterial ground. Obviously, in a realistic configuration, the metamaterial cannot entirely replace the medium of propagation (sandy land and sea water). So, the idea is to insert, between the transmitter and the natural propagation medium, a finite length part of metamaterial to propagate a strong surface wave (Fig. 2(b)) in order to focus the energy toward the sea. However, the analytic formulation developed in [14] is limited to a ground with a large permittivity due to the Leontovich condition [11]. Hence a doubt arises on the validity of the results. In practical case, the negative permittivity will be supplied by a metamaterial which is a dispersive medium. The permittivity could then be large or weak depending on the configuration and the operating frequencies. Moreover, there remains a possibility that the obtained results are due to a calculation artifact. As a consequence, we need to develop another formulation to consider the case of the soil with a permittivity close to unity, as it may be the case with a metamaterial.

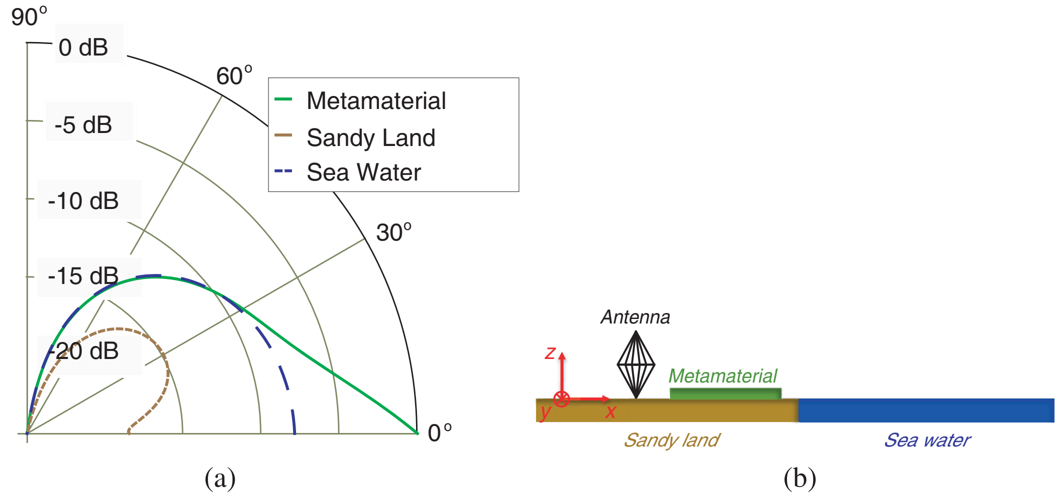


Figure 2. (a) shows the distribution of the vertical component of the electric field radiated by a Hertzian dipole located over different grounds, at 10 MHz and at 50 m from the antenna and (b) depicts the side view of the new HFSWR configuration.

3. IMPEDANCE BOUNDARY CONDITIONS

During the last century, many authors [11, 15] tried to derive the field radiated by a Hertzian dipole over a good conductor and developed an IBC as a substitute of the conductor. In [11] the main assumptions are that the soil is plane and exhibits a permittivity with a large magnitude. This last point leads to neglect the second order variation of the field along the interface. In the same time, Rytov [15] extended this formulation of IBC for a curved surface but still assuming a ground with a large permittivity. In 1959, Karp and Karal [9] proposed a General Impedance Boundary Condition (GIBC) to model a dielectric medium which can propagate more than one surface wave as, for example, a dielectric slab with a large thickness. But this derivation has been performed considering a magnetic line source which has not the characteristics of the antenna used in the HFSWR applications. The condition has also been used in [16] to study the problem of a planar metal-backed uniform dielectric layer. Kane and Karp [10] applied the second order GIBC to study the reflection of plane waves by a soil with a permittivity close to unity. In consequence, starting from the results obtained by Kane, Karal and Karp [9, 10], we will describe, in this section, the required steps to establish the first and second IBC in case of an infinite line source of vertical Hertzian dipoles. The electromagnetic behaviour of such a source is in better relation to that of the antennas used in HFSWR.

3.1. Geometry

The geometry is depicted in Fig. 3 in a Cartesian coordinate system with a time dependence $e^{-\mathrm{i}\omega t}$. A plane interface separates the two media at $z = 0$. Medium 1 in the upper side ($z > 0$) is air characterized by ε_0, the vacuum permittivity, and μ_0, the vacuum permeability. Medium 2 in the lower side ($z < 0$) is a dielectric medium whose relative complex permittivity is $\underline{\varepsilon_r}$, expressed as:

$$\underline{\varepsilon_r} = \varepsilon_{rr} + \mathrm{i}\frac{\sigma}{\omega\varepsilon_0} \tag{1}$$

where ω is the angular frequency; ε_{rr} and σ are respectively the real part of the relative permittivity and the conductivity of the medium.

We consider an infinite line source of vertical electric Hertzian dipoles located along the y axis ($x = z = 0$). It carries a current density in A/m^2:

$$J_z = I_e \delta(x)\, \delta(z) \tag{2}$$

where I_e is a current equal to 1 A thereafter.

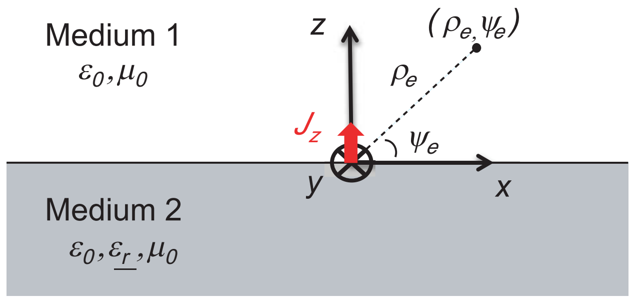


Figure 3. Geometry under consideration.

The geometry is invariant in y, and hence the source radiates a Transverse Magnetic (TM) mode of propagation. As we are interested in the surface wave mode, we will focus on the propagation along the x axis. Thus the following assumptions are considered:

$$\frac{\partial E_x}{\partial y} = 0, \quad \frac{\partial E_z}{\partial y} = 0, \quad \frac{\partial H_y}{\partial y} = 0 \tag{3}$$

$$E_y = 0, \quad H_x = 0, \quad H_z = 0 \tag{4}$$

The next step consists in solving the Helmholtz equation in H_y which can be expressed in medium 1 as:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_0^2\right) H_y(x, z) = \frac{\partial J_z}{\partial x} \tag{5}$$

where k_0 is the vacuum wave number and the right part of the equation stands for the source. From Maxwell-Ampère's and Maxwell-Faraday's equations, we can express the components of the electric field as a function of the y-component of the magnetic field:

$$E_x = +\frac{1}{\mathrm{i}\omega\varepsilon_0}\frac{\partial H_y}{\partial z} \tag{6}$$

$$E_z = -\frac{1}{\mathrm{i}\omega\varepsilon_0}\left(\frac{\partial H_y}{\partial x} - J_z\right) \tag{7}$$

It is not necessary to express the Helmholtz equation in medium 2 as the fields in medium 1 can be derived by applying an IBC at $z = 0$.

3.2. First and Second Order IBC Derivation

The surface impedance concept enables to replace medium 2 by an IBC which reduces the complexity of the computation. This condition is an approximated relation between the tangential components of the electric and magnetic fields at the interface ($z = 0$).

The first author who introduced the IBC is Leontovich [11]. Assuming that $|\underline{\varepsilon_r}| \gg 1$, this first order condition can be expressed as:

$$\hat{n} \times \vec{E} = Z\hat{n} \times \left(\hat{n} \times \vec{H}\right) \tag{8}$$

where $\hat{n}$ is the outgoing normal to the interface and $Z = \sqrt{\frac{\mu_0}{\varepsilon_0\underline{\varepsilon_r}}}$ is the impedance of medium 2. In our case, Equation (8) is reduced to:

$$E_x = -ZH_y \tag{9}$$

Combining Equations (6) and (9), the first order IBC can be expressed as:

$$\left(\frac{\partial}{\partial z} + \lambda_s\right) H_y(x, z) = 0, \quad z = 0 \tag{10}$$

where λ_s is an imaginary wave number along the z axis that can be written as:

$$\lambda_s = \mathrm{i}\omega\varepsilon_0 Z = \mathrm{i}k_0\sqrt{\frac{1}{\underline{\varepsilon_r}}} \tag{11}$$

To obviate the constrain on the magnitude of the permittivity, Kane and Karp [10] introduced a second order IBC in 1964. This condition is an expansion of the field components in Taylor series [17] that can be written as:

$$\left(\Lambda_{s2}\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial z} + \lambda_s\right) H_y\left(x, z\right) = 0, \quad z = 0 \tag{12}$$

where

$$\Lambda_{s2} = \frac{\mathrm{i}}{2k_0(\underline{\varepsilon_r})^{3/2}} \tag{13}$$

Considering Equations (7), (10) and (12), we can include both boundary conditions in one General IBC, named GIBC, which depends only on the normal component of the electric field and can be written as:

$$\sum_{m=0}^{N} \frac{a_m}{(\mathrm{i}k_0)^m}\frac{\partial^m}{\partial z^m} E_z(x, z) = 0, \quad z = 0 \tag{14}$$

where N is the order of the IBC. We have gathered the different types of conditions in Table 1.

Table 1. Order of surface impedance condition.

N	Name of condition	Nature of soil
0	Perfect Electric Conductor Condition	PEC
1	Leontovich Boundary Condition	$\lvert\underline{\varepsilon_r}\rvert \gg 1$
2	High Order Condition	$\lvert\underline{\varepsilon_r}\rvert > 1$

4. FORMULATION FOR A FIRST ORDER IBC

As mentioned before, the IBC has been extensively studied for magnetic line source which obviously does not match our needs for surface wave propagation. Here after we have derived the IBC using an infinite line source of vertical electric Hertzian dipoles. We can derive the result in closed-form by using an approach similar to that applied to a horizontal magnetic line source [9]. The decomposition of the field is performed by applying an IBC at the interface. Firstly, we will start with the Leontovich Condition (10) and compare our results to the results generated by the full-wave commercial software FEKO. Secondly, we will improve the first order IBC by a slight modification to consider grounds with a permittivity close to unity.

The magnetic field can be decomposed in three terms as follows:

$$H_y\left(x, z\right) = H_y^{(d)}\left(x, z\right) + H_y^{(r)}\left(x, z\right) + H_y^{(sur)}\left(x, z\right) \tag{15}$$

where $H_y^{(d)}(x, z)$ and $H_y^{(r)}(x, z)$ are respectively the direct and reflected fields whose summation defines the sky wave mode, called $H_y^{(sky)}(x, z)$. $H_y^{(sur)}(x, z)$ is the surface wave mode.

4.1. Formulation

To express the magnetic field radiated by an infinite line source of electric vertical Hertzian dipoles, we solve the Helmholtz Equation (5) and use the Leontovich Condition (10) to replace medium 2.

We combine both equations to find the contribution of the sky wave mode and we define an auxiliary function $v(x, z)$ as:

$$\left(\frac{\partial}{\partial z}+\lambda_s\right)H_y\left(x,z\right)=v\left(x,z\right) \tag{16}$$

One particular solution is:

$$H_{yp}\left(x,z\right)=e^{-\lambda_s z}\int\limits_{-\infty}^{z}e^{\lambda_s\eta}v\left(x,\eta\right)d\eta \tag{17}$$

The auxiliary function $v(x,z)$ is a solution of the Helmholtz equation and can be written as:

$$v\left(x,z\right)=\frac{\mathrm{i}k_0}{4}\frac{\partial}{\partial z}\left[H_1^{(1)}\left(k_0\rho_e\right)\cos\psi_e\right] \tag{18}$$

where $H_1^{(1)}$ is the Hankel function of order one of the first kind; $\rho_e=\sqrt{x^2+z^2}$ is the distance between the origin and the observation point; $\cos\psi_e=\frac{x}{\sqrt{x^2+z^2}}$.

We combine Equations (17) and (18), and integrate by parts to obtain the sky wave mode:

$$H_y^{(sky)}\left(x,z\right)=+\frac{\mathrm{i}k_0}{4}H_1^{(1)}\left(k_0\rho_e\right)\cos\psi_e-\frac{\mathrm{i}k_0\lambda_s}{4}e^{-\lambda_s z}\int\limits_{-\infty}^{z}e^{\lambda_s\eta}H_1^{(1)}\left(k_0\rho_\eta\right)\cos\psi_\eta d\eta \tag{19}$$

where $\rho_\eta=\sqrt{x^2+\eta^2}$ and $\cos\psi_\eta=\frac{x}{\sqrt{x^2+\eta^2}}$.

The surface wave mode is the solution of the homogeneous Helmholtz Equation (5) and is obtained from a separation of variables. The solution can be written as:

$$H_y^{(sur)}\left(x,z\right)=Ae^{\mathrm{i}k_z z}e^{\mathrm{i}\sqrt{{k_0}^2-{k_z}^2}|x|} \tag{20}$$

where k_z is the wave number along the z axis. Now, we proceed by determining the expressions of k_z and A. Firstly, we substitute the first order IBC in Equation (10) into Equation (20), and the value of k_z can be expressed as:

$$k_z=\mathrm{i}\lambda_s \tag{21}$$

Secondly, we apply the continuity condition of the tangential component E_x of the electric field across the line $x=0$ and the value of A can be expressed as:

$$A=\frac{\lambda_s}{2} \tag{22}$$

The total magnetic field is the sum of Equations (19) and (20). It can be expressed as:

$$H_y\left(x,z\right)=+\frac{\mathrm{i}k_0}{4}H_1^{(1)}\left(k_0\rho_e\right)\cos\psi_e-\frac{\mathrm{i}k_0\lambda_s}{4}e^{-\lambda_s z}\int\limits_{-\infty}^{z}e^{\lambda_s\eta}H_1^{(1)}\left(k_0\rho_\eta\right)\cos\psi_\eta\mathrm{d}\eta+\frac{\lambda_s}{2}e^{-\lambda_s z}e^{\mathrm{i}\sqrt{{k_0}^2+{\lambda_s}^2}|x|} \tag{23}$$

Looking at Equation (23) the first term is the direct field. The second term is a continuous spectrum and corresponds to the reflected field. These two terms constitute the sky wave contribution to the total magnetic field. The third term can be identified as a surface wave propagating along the interface.

4.2. Numerical Results

In this section and in the forthcoming results, we are interested in the decay of the magnetic field along the interface ($z=0$). We will compare the surface wave contribution with the total magnetic field. Two kinds of soil will be considered: one with a large permittivity and one with a low permittivity.

In the first step, to validate the formulation based on the IBC derivation, we have compared our formulation using a first order condition with that considered in [14] and with a line of vertical Hertzian dipoles based on Norton's formulation [18]. Whatever the kind of soil, the results are in perfect agreement for the surface wave contribution as well as for the total magnetic field. Nevertheless, Norton's formulation is not valid when the soil has a permittivity close to the unity.

In order to validate our approach with that kind of soil, we have compared our results with those obtained through a full-wave simulation performed with the commercial software FEKO. To perform the computation with FEKO, we have simulated a quasi infinite line source, defined with 600 vertical electric Hertzian dipoles, each dipole having a length equal to $\lambda/300$, the gap between each dipole being of $\lambda/5$.

Figure 4(a) shows the decay of the H_y component when medium 2 is sea water. This medium has a large value of permittivity ($\varepsilon_{rr} = 81$, $\sigma = 5\,\mathrm{S/m}$). In such a case, first order and FEKO solutions are identical. We can also notice that the contribution of the surface wave is entirely masked as the total magnetic field is equal to the sky wave contribution whatever the distance is. This complies with the usual propagation properties of wave over a soil with a positive permittivity.

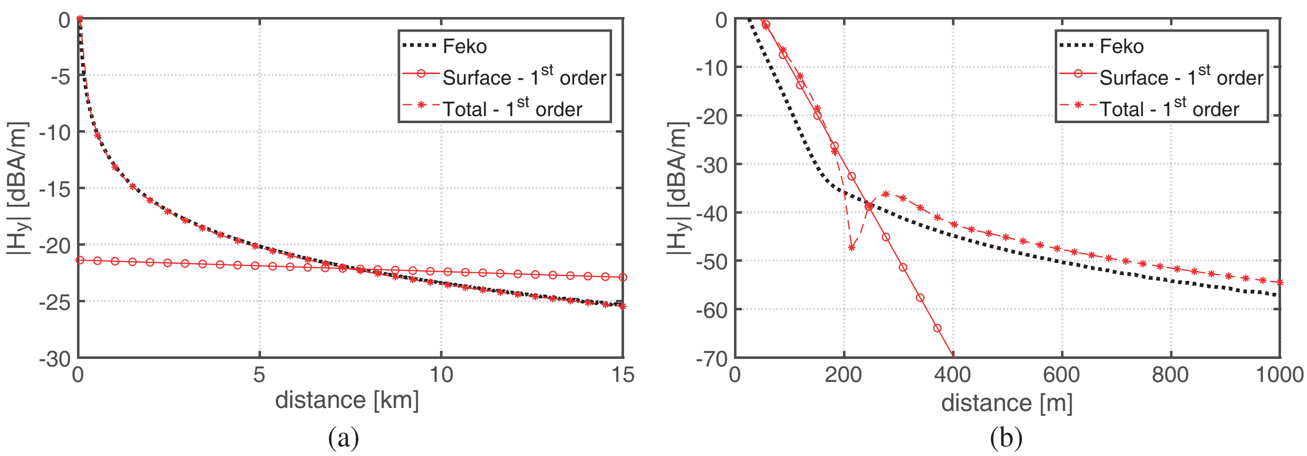


Figure 4. Decay of the magnetic field radiated by an infinite line source of Hertzian dipoles over soils (a) with a large positive permittivity and (b) with a low negative permittivity at 10 MHz. (a) $\varepsilon_{rr} = 81$; $\sigma = 5\,\mathrm{S/m}$. (b) $\varepsilon_{rr} = -2$; $\sigma = 0.001\,\mathrm{S/m}$.

On the contrary, when the line source is located over a ground with a negative permittivity, as shown in Fig. 4(b), there are two modes of propagation depending on the distance to the source. Close to it, the total magnetic field is equal to the surface wave, and at a larger distance the total magnetic field is equal to the sky wave. Similar results have been observed on a metallic plane at optical frequencies for which the interface behaves like a metamaterial [19]. In this case, the first order and FEKO solutions are not identical. Indeed, the first order solution is not accurate when the soil has a permittivity close to the unity.

4.3. Extension of the First IBC Validity Range

As shown previously, the Leontovich condition is not applicable to a ground characterized by a low permittivity. But assuming that the soil is planar and homogeneous, the first order IBC can be improved by a slight modification.

Considering only the surface wave, the solution in medium 1 for the magnetic field is already known and expressed in Equation (20):

$$H_{y1}(x,z) = A_1 e^{\mathrm{i}k_{z1}z} e^{\mathrm{i}k_x x} \tag{24}$$

In medium 1, the tangential component E_x of the electric field is easily deduced from Equation (6) and can be expressed as:

$$E_{x1}(x,z) = A_1 \frac{k_{z1}}{\omega\varepsilon_0} e^{\mathrm{i}k_{z1}z} e^{\mathrm{i}k_x x} \tag{25}$$

In medium 2, the magnetic and electric fields are given by:

$$H_{y2}(x,z) = A_2 e^{-\mathrm{i}k_{z2}z} e^{\mathrm{i}k_x x} \tag{26}$$

$$E_{x2}(x,z) = -A_2 \frac{k_{z2}}{\omega\varepsilon_0\underline{\varepsilon_r}} e^{-\mathrm{i}k_{z2}z} e^{\mathrm{i}k_x x} \tag{27}$$

At the interface ($z = 0$), the continuity of H_y imposes that $A_1 = A_2$ and then the continuity of E_x implies that:

$$\frac{k_{z1}}{k_{z2}} = \frac{1}{\underline{\varepsilon_r}} \tag{28}$$

From the dispersion relation in medium 1 ($k_0^2 = k_x^2 + k_{z1}^2$) and in medium 2 ($\underline{\varepsilon_r} k_0^2 = k_x^2 + k_{z2}^2$), we find the expression of k_x:

$$k_x = k_0 \sqrt{\frac{\underline{\varepsilon_r}}{1 + \underline{\varepsilon_r}}} \tag{29}$$

and then the expression of k_{z2}:

$$k_{z2} = k_0 \sqrt{\frac{\underline{\varepsilon_r}^2}{1 + \underline{\varepsilon_r}}} \tag{30}$$

The general expression of the surface impedance in TM mode is given by Equation (9) at the interface ($z = 0$). Then, we can deduce the expression of the modified surface impedance Z_{mod} as:

$$Z_{mod} = \frac{k_{z2}}{\omega\varepsilon_0\underline{\varepsilon_r}} \tag{31}$$

We combine Equations (30) and (31), and the modified surface impedance can also be written as:

$$Z_{mod} = \frac{k_0}{\omega\varepsilon_0} \sqrt{\frac{1}{1 + \underline{\varepsilon_r}}} \tag{32}$$

and the modified imaginary wave number along the z axis that can be written as:

$$\lambda_{smod} = \mathrm{i}k_0 \sqrt{\frac{1}{1 + \underline{\varepsilon_r}}} \tag{33}$$

Now we replace λ_s by λ_{smod} in Equation (23) and obtain the new expression of the magnetic field radiated by an infinite line source of Hertzian dipoles using a modified first order solution.

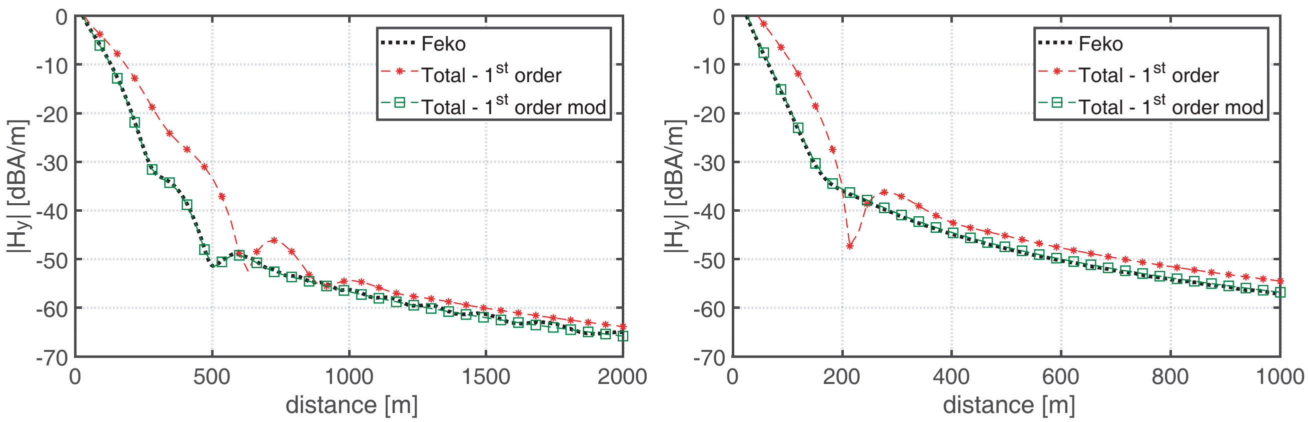
4.4. Numerical Results

In this section, we are interested in the comparison between the first order solution, the first order modified solution and the FEKO's one in Fig. 5. We compare the values of the total magnetic field obtained when using this approach and the second order IBC, in the cases of soils with low negative permittivities ($\varepsilon_{rr} = -2$ or -4 and $\sigma = 0.001\,\mathrm{S/m}$). In both cases, we have a very good agreement between the full-wave results and the first order modified ones. So, it appears that this modified surface impedance can be applied as a substitute of the second order IBC in the case of a planar and homogeneous interface.

Nevertheless, if the soil is not planar, for example to simulate a realistic rough surface of the sea, the first order modified solution cannot be applied. It is the reason why, in the next section, we have extended our formulation using a second order IBC. It will also provide us with the opportunity to compare the second order results to the first order modified ones.

5. EXTENDED FORMULATION USING A SECOND ORDER IBC DERIVATION

In this part, we will extend the decomposition validity by applying a second order condition (12) which is appropriate to handle a lower medium with a permittivity close to unity and also to consider a non planar and inhomogeneous soil for future works.



(a) (b)

Figure 5. (a) and (b) Decay of the magnetic field radiated by an infinite line source of Hertzian dipoles over soils with low negative permittivities at 10 MHz. (a) $\varepsilon_{rr} = -4$; $\sigma = 0.001$ S/m. (b) $\varepsilon_{rr} = -2$; $\sigma = 0.001$ S/m.

5.1. Formulation

Now, we use Equation (12) as the IBC. For calculation convenience, we switch to the Fourier domain to compute the magnetic field. The second order IBC in the Fourier domain can be written as:

$$\left(\frac{\partial}{\partial z} + \gamma_s\right)\hat{H}_y(s,z) = 0, \quad z = 0 \tag{34}$$

where $\gamma_s = \lambda_s - \Lambda_{s2}s^2$; s is the spectral variable associated to the x variable, and the symbol $\hat{}$ denotes variable expressed in the Fourier domain. It can be noticed that the second order IBC has, in the Fourier domain, an expression similar to that of the first order in the spatial domain (10).

Then the Helmholtz equation becomes:

$$\left(\frac{\partial^2}{\partial z^2} + K^2\right)\hat{H}_y(s,z) = -\mathrm{i}s\delta(z) \tag{35}$$

where $K = \sqrt{k_0{}^2 - s^2}$. Applying the same approach as the one used for the first order, we obtain the sky wave mode in the Fourier domain that can be written as:

$$\hat{H}_y^{(sky)}(s,z) = \frac{s}{2K}\left(e^{\mathrm{i}Kz} - \gamma_s e^{-\gamma_s z}\int_{-\infty}^{z} e^{\gamma_s\eta}e^{\mathrm{i}K\eta}d\eta\right) \tag{36}$$

In the spatial domain, the sky wave mode is then given by:

$$H_y^{(sky)}(x,z) = +\frac{\mathrm{i}k_0}{4}H_1^{(1)}(k_0\rho_e)\cos\psi_e - \frac{1}{4\pi}\int_{-\infty}^{+\infty}\frac{s\gamma_s e^{-\gamma_s z}}{\sqrt{k_0^2 - s^2}}\int_{-\infty}^{z} e^{\gamma_s\eta}e^{\mathrm{i}\sqrt{k_0^2-s^2}\eta}d\eta e^{\mathrm{i}sx}\mathrm{d}s \tag{37}$$

The direct field does not depend on the choice of the IBC, then the first term is found equal to that of Equation (23).

As in the first order resolution, the surface wave mode is the solution of the homogeneous Helmholtz Equation (5) whose solution is given by Equation (20). We use the same approach to find A and k_z when applying the second order IBC of Equation (12) in Equation (20). Finally, they can be written as:

$$k_z = \mathrm{i}\beta_s \tag{38}$$

$$A = \frac{\beta_s}{2} \tag{39}$$

where

$$\beta_s = -\frac{1 \pm \mathrm{i}\sqrt{-1 - 4\lambda_s \Lambda_{s2} + 4(k_0 \Lambda_{s2})^2}}{2\Lambda_{s2}} \tag{40}$$

where the $\pm$ sign refers to the sign of the real part of the ground permittivity. The total magnetic field can then be written as:

$$H_y(x,z) = +\frac{\mathrm{i}k_0}{4} H_1^{(1)}(k_0 \rho_e)\cos\psi_e - \frac{1}{4\pi}\int_{-\infty}^{+\infty} \frac{s\gamma_s e^{-\gamma_s z}}{\sqrt{k_0^2 - s^2}} \int_{-\infty}^{z} e^{\gamma_s \eta} e^{\mathrm{i}\sqrt{k_0^2 - s^2}\eta} d\eta e^{\mathrm{i}sx} \mathrm{d}s + \frac{\beta_\mathrm{s}}{2} \mathrm{e}^{-\beta_\mathrm{s} \mathrm{z}} \mathrm{e}^{\mathrm{i}\sqrt{\mathrm{k_0}^2 + \beta_\mathrm{s}^2}|\mathrm{x}|} \tag{41}$$

5.2. Numerical Results

In this section, we are interested in the decay of the magnetic field along the interface ($z = 0$). We will compare the surface wave contribution with the total magnetic field given by the second order solution and FEKO's one. Two kinds of soil have been considered.

The results shown in Fig. 6 are obtained over grounds with a negative permittivity ($\varepsilon_{rr} = -2$ or -4 and $\sigma = 0.001\,\mathrm{S/m}$) and all solutions are in very good agreement.

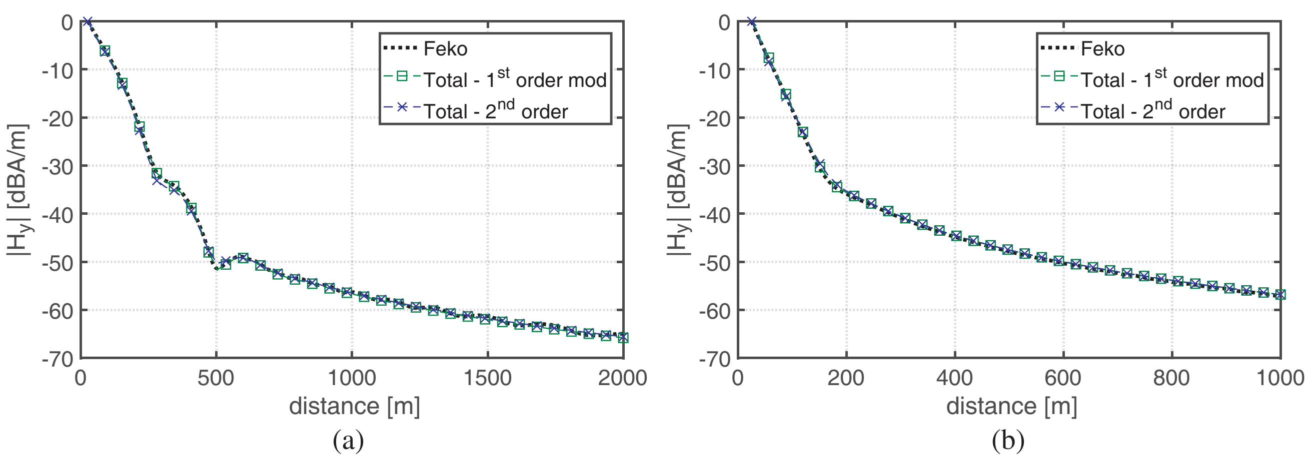


Figure 6. (a) and (b) Decay of the magnetic field radiated by an infinite line source of vertical electric Hertzian dipoles over soils: with low negative permittivities at 10 MHz. (a) $\varepsilon_{rr} = -4$; $\sigma = 0.001\,\mathrm{S/m}$. (b) $\varepsilon_{rr} = -2$; $\sigma = 0.001\,\mathrm{S/m}$.

Figure 7 shows, at two distances from the source, the influence of the real part of the permittivity on the relative discrepancies between the magnetic fields obtained with first and second order conditions. As expected with [10] closer is the permittivity to unity, larger is the relative error.

This section may be ended with two main conclusions. The first and the main one is that only a negative permittivity allows the propagation of a strong surface wave. The second one claims that, having a ground with a low permittivity, the Leontovich condition is questionable since it significantly differs from a higher order model.

In the near future, the second order solution, or the first order modified solution in the case of a planar interface, may be applied to model the abrupt change of the impedance along the line separating the metamaterial and the sea surface as shown in Fig. 2(b). Such a model can rely on Clemmow's formulation [20] or Wait's formulation [21] as already attempted in [8] using the classical Leontovich impedance expression. The aim, as previously explained, is to enhance the surface wave radiation without increasing too much the scattering of sky waves.

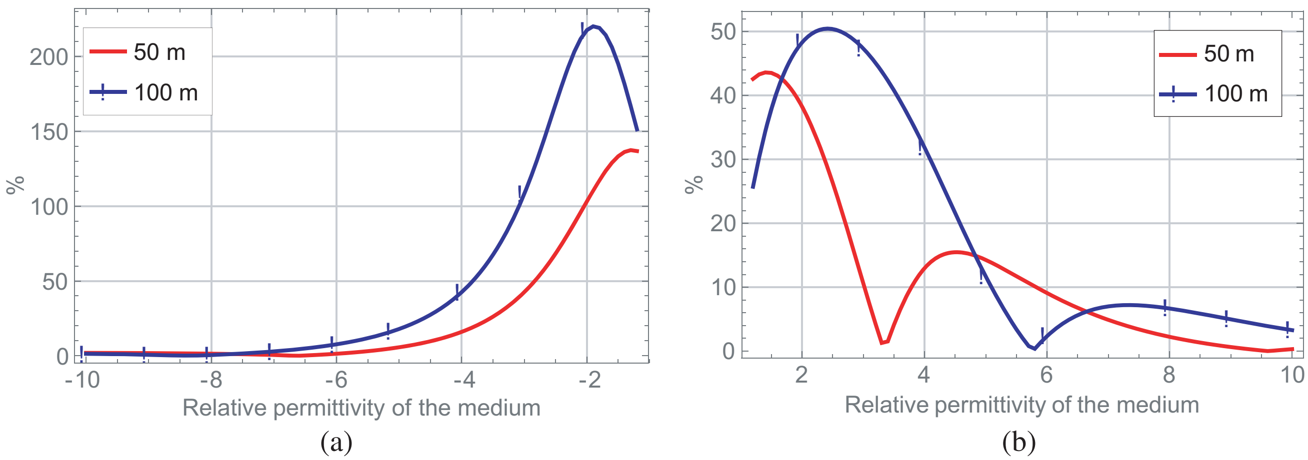


Figure 7. Relative error between the magnetic fields obtained with first and second order conditions at 50 m and 100 m (a) with a negative real part of permittivity and (b) with a positive real part of permittivity.

6. CONCLUSION

To improve the performance of a HFSWR, it is possible to insert a metamaterial medium between the transmitting antennas and the sea in order to reinforce the propagation of strong surface waves [14]. Nevertheless, the design of such a material highlights the limits of accuracy of the standard IBC when a low permittivity medium is considered. In this paper, we have proposed an accurate and quite simple analytic formulation of the impedance boundary condition (IBC) derived from the first order IBC, suitable to compute the electromagnetic field radiated by the transmitting antennas over a ground with a low positive permittivity but also with a negative permittivity, which characterizes such a metamaterial medium. As needed, this approach leads to an electromagnetic field decomposed in three terms, the direct and reflected fields and the surface wave field. Moreover, the results obtained with a second order IBC, which can deal with non-planar interfaces, provides more accurate results than those obtained when applying the usual first order IBC. Last, the second order solution, or the first order modified solution on a planar interface, will be applied in a next step to the abrupt impedance transition between the metamaterial and the sea.

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