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Design and Optimization of Quasi-Constant Mutual Inductance for Asymmetric Two-Coil Wireless Power Transfer System with Lateral Misalignments

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Abstract—Magnetic resonant wireless power transfer (WPT) is an emerging technology that may create new applications for wireless power charging. However, the output voltage fluctuations resulting from lateral misalignments are main obstructing factors for promoting this technology. In this paper, an asymmetric two-coil WPT system is presented. The mathematical model of the proposed topology with lateral misalignments is built based on equivalent circuit method. The expression of the output voltage is then derived by solving the system equivalent equations. In addition, a method of optimization parameters is proposed. The mutual inductance between the receiving coil and transmission coil is nearly constant by the proposed method with lateral misalignments. Therefore, the output voltage can be kept nearly constant. The asymmetric two-coil WPT system via magnetic resonance coupling is designed. Simulated and experimental results validating the proposed method are given.

1. INTRODUCTION

Wireless power transfer (WPT) methods for electric vehicles (EVs) are receiving increasing attention in international research community due to their convenience and high efficiency [1–4]. However, in practice, inevitable lateral and vertical misalignments between the transmission coil and the receiving coil lead to variations of the mutual inductance between the transmitting coil and receiving coil. The output voltage may be changed as the mutual inductance between the transmitting coil and coils is changed [5].

In order to deal with the problem that the variation of mutual inductance results in the output voltage fluctuations, DC/DC converter is used to maintain the output voltage constant with variation of mutual inductance. The output voltage can be kept constant by changing the duty cycle of DC-DC converter [6]. However, DC-DC converters are added to wireless charging systems, and the system efficiency is decreased inevitably because DC-DC converters have power losses. There are other methods using new coil structures to deal with the adverse effect of misalignments on output voltage: 1) A narrow-width I-type power supply rail and pickups were proposed [7]. When the pickup moved away from the center of the pole along with x-direction (x-direction is shown in Fig. 1), the experimental results showed that the output power was reduced, and the output power reached nearly 1 kW, whereas the output power exceeded 20 kW as the pickup was laterally moved in y-direction (y-direction is shown in Fig. 1) within 20 cm and became half at 24 cm from the center of the power supply rail. And the system efficiency was 74%; 2) A large lateral displacement of 30 cm was experimentally obtained by using an ultraslim S-type power supply rail with an air gap of 20 cm [8]. The load power reached -3 db point having half of its maximum load power with the lateral tolerance of 30 cm; 3) A new homogeneous

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WPT technique was proposed and implemented to maintain the output voltage constant for lateral and vertical misalignments systems [9, 10]. The homogeneous magnetic field should be paid with reduction of the quality factor of the transmitter because the transmitter is composed of many small coils (58 mm × 58 mm × 22.5 mm) [10]. The system efficiency is reduced when the quality factor of the transmitter is decreased [9]; 4) Reference [11] proposed a new coil structure, including a big power supply coil set (110 cm × 80 cm) and a small pickup coil set. When the pickup coil laterally deviated from the center of the power supply coil by 39 cm, the output voltage was reduced by about 29% from the maximum output voltage 150 V. The misalignments were improved by this proposed structure. However, the output voltage fluctuations are also large. Therefore, it is necessary to further reduce output voltage pulsation. In this paper, an asymmetric two-coil WPT system is presented. The proposed topology is composed of a big transmitting resonance coil and a small receiving resonance coil. In addition, a method of optimization parameters is proposed. The mutual inductance between the receiving and transmitting coils is nearly constant with lateral misalignments by using the proposed optimization method. Therefore, the output voltage and efficiency can be kept nearly constant by the proposed method.

The remainder of the paper is outlined as follows. Section 2 describes the asymmetric two-coil WPT system and builds the model of the proposed system. Section 3 analyses the effects of the parameters of each coil on the mutual inductance between the receiving and transmitting coils. Section 4 proposes an optimization method. Section 5 presents simulation results, the experimental setup, and measurement results with lateral misalignments. Section 6 draws the conclusion.

2. ASYMMETRIC TWO-COIL MODEL

2.1. Calculated Mutual Inductance

Figure 1 shows two loops with lateral misalignments between loop_1 and loop_2. D is the transmission distance, a the radius of loop_1, b the radius of loop_2, and Δ the lateral misalignments between loop_1 and loop_2.

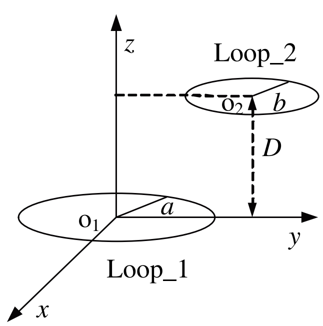


Figure 1. Lateral misalignments between loop_1 and loop_2.

The mutual inductance of two loops whose axes are parallel can be expressed by the single integral [12–14]

$$M_1(a,b,\Delta,D) = \pi\mu_0\sqrt{ab}\int_0^{\infty} J_1\left(x\sqrt{a/b}\right) J_1\left(x\sqrt{a/b}\right) \times J_0\left(x\frac{\Delta}{\sqrt{ab}}\right)\exp\left(-x\frac{D}{\sqrt{ab}}\right)dx \tag{1}$$

where J_0 and J_1 are the Bessel functions of zeroth-order and first-order, respectively.

For a coil composed of multi-turn, the mutual inductance between the transmitting and receiving coils can be calculated using

$$M_{ab} = \sum_{i=1}^{N_A}\sum_{j=1}^{N_B} M_1\left(a_i, b_j, D, \Delta\right) \tag{2}$$

where $N_{\rm A}$, $a_{\rm i}$ are the turns of the transmitting coil and the radius of the transmitting coil, respectively. $N_{\rm B}$, $b_{\rm j}$ are the turns of the receiving coil and the radius of the receiving coil, respectively.

As shown in Eqs. (1) and (2), it can be seen that mutual inductance is dependent on a, b, D, and Δ. When a, b, and D are fixed, the mutual inductance is only related to Δ. The bigger the value of Δ is, the smaller the mutual inductance becomes.

2.2. Asymmetric Two-Coil WPT System

The asymmetric two-coil WPT system with lateral misalignments is composed of two resonance coils: transmitting and receiving resonance coils, labeled as Tx and Rx, as shown in Fig. 2. The diameter of Tx is larger than that of Rx. D is the distance between Tx and Rx. C_1 and C_2 are the external compensating capacitances.

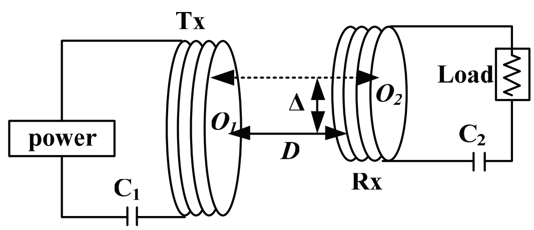


Figure 2. The simplified schematic of the WPT system with lateral misalignments.

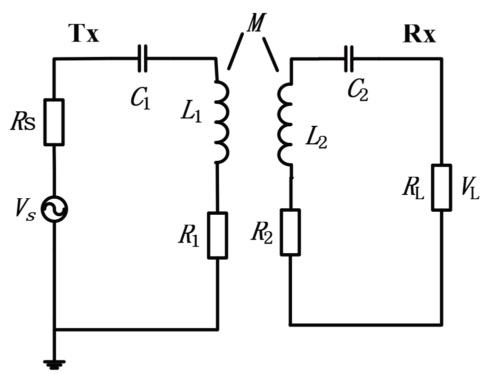


Figure 3. Equivalent circuit model for the two-coil WPT system.

The asymmetric two-coil WPT system can be represented in terms of lumped circuit elements (L, C, and R), as shown in Fig. 3. $V_{\rm s}$ is the voltage of the power source, R_1 the parasitic resistor of Tx, R_2 the parasitic resistor of Rx, $R_{\rm s}$ the internal resistor of the power source, $R_{\rm L}$ the load resistor, L_1 the inductance of Tx, L_2 the inductance of Rx, and M the mutual inductance between Tx and Rx. By applying Kirchhoff's voltage law (KVL), the WPT system is presented as follows:

$$\begin{cases} (R_S + Z_1)I_{\rm Tx} + j\omega M I_{\rm Rx} = V_s \\ j\omega M I_{\rm Tx} + (R_L + Z_2)I_{\rm Rx} = 0 \end{cases} \tag{3}$$

$$\begin{cases} Z_1 = R_1 + j\omega L_1 + 1/(j\omega C_1) \\ Z_2 = R_2 + j\omega L_2 + 1/(j\omega C_2) \end{cases} \tag{4}$$

where $I_{\rm Tx}$ is the current of Tx, $I_{\rm Rx}$ the current of Rx, and ω the operating angular frequency.

The currents of Tx and Rx can be obtained by solving Eqs. (3) and (4)

$$\begin{cases} I_{\rm Tx} = \dfrac{V_s(R_L + Z_2)}{(R_s + Z_1)(R_L + Z_2) + (\omega M)^2} \\ I_{\rm Rx} = -\dfrac{j\omega M V_s}{(R_s + Z_1)(R_L + Z_2) + (\omega M)^2} \end{cases} \tag{5}$$

The output voltage of the two-coil WPT system is as follows:

$$V_L = \frac{-j\omega M V_s R_L}{(R_s + Z_1)(R_L + Z_2) + (\omega M)^2} \tag{6}$$

When the WPT system operates at resonant state, we have $Z_1 = R_1$ and $Z_2 = R_2$. The output voltage of the two-coil WPT system can be simplified as follows:

$$V_L = \frac{-j\omega M V_s R_L}{(R_s + R_1)(R_L + R_2) + (\omega M)^2} \tag{7}$$

The expression of efficiency in Eq. (8) is as follows:

$$\eta = \frac{I_{\mathrm{Rx}}^2 R_L}{V_s I_{Tx}} = \frac{(\omega M)^2 R_L}{(R_s + R_1)(R_L + R_2)^2 + (\omega M)^2 (R_L + R_2)} \tag{8}$$

As shown in Eqs. (7) and (8), the output voltage and efficiency are changed as the mutual inductances M are changed (lateral misalignments between Tx and Rx results in the variation of the mutual inductance) when the voltage of the power source V_{s}, the internal resistor of the power source R_{s}, the load resistor R_{L}, the parasitic resistors of the coil R_1 and R_2 are nearly constant for given coils.

3. ANALYSIS OF MUTUAL INDUCTANCE

In this section, the effects of the parameters of the coil (e.g., the number of turns of each coil, the out diameter of each coil) on the mutual inductance between Tx and Rx are displayed. According to Eqs. (1) and (2), the simulation results are shown in Fig. 4 with the help of MATLAB.

Fig. 4(a) shows the mutual inductance versus the radius of Rx with different Δ when the number of turns of Tx is equal to 20 and the out diameter of Tx equal to 0.60 m. It can be seen that the mutual inductance increases with the increase in the radius of Rx, and the mutual inductance difference between $\Delta = 10$ and $\Delta = 0$ increases with the increase in the radius of Rx. Therefore, the radius of Rx should be decreased with lateral misalignments in order to obtain constant mutual inductance.

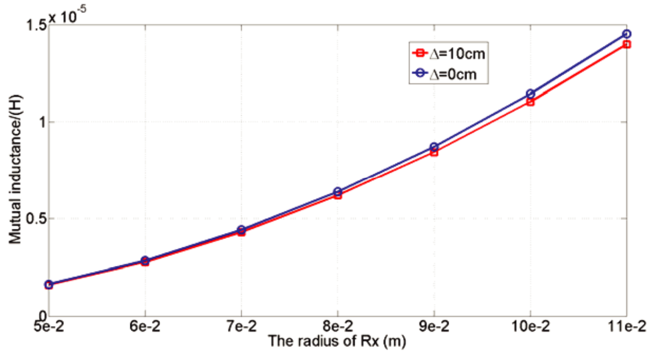
Fig. 4(b) shows the mutual inductance versus the radius of Tx with different Δ when the number of turns of Rx is equal to 10 and the out diameter of Rx equal to 0.20 m. It can be seen that the mutual inductance decreases with the increase in the radius of Tx, and the difference between the mutual inductance with $\Delta = 10$ and the mutual inductance with $\Delta = 0$ decreases with the increase in the radius of Tx. Therefore, the radius of Tx should be increased with lateral misalignments in order to obtain constant mutual inductance.

Fig. 4(c) shows the mutual inductance versus the number of turns of Tx with different Δ when the number of turns of Rx is equal to 10 and the out diameter of Rx equal to 0.20 m. It can be seen that the mutual inductance increases with the increase in the number of turns of Tx, and the mutual inductance difference between $\Delta = 10$ and $\Delta = 0$ increases with the increase in the number of turns of Tx. Therefore, the number of turns of Tx should be decreased with lateral misalignments in order to obtain constant mutual inductance.

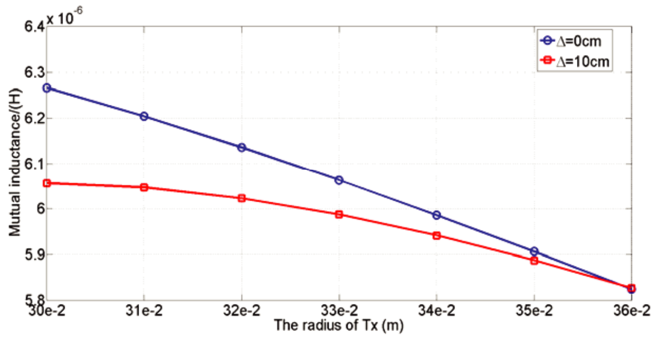
Fig. 4(d) shows the mutual inductance versus the number of turns of Rx with different Δ when the number of turns of Tx is equal to 20 and the out diameter of Tx equal to 0.60 m. It can be seen that the mutual inductance increases with the increase in the number of turns of Rx, and the mutual inductance difference between $\Delta = 10$ and $\Delta = 0$ decreases with the increase in the number of turns of Rx. Therefore, the number of turns of Rx should be increased with lateral misalignments in order to obtain constant mutual inductance. In the above analysis, the mutual inductance may be changed with the variations of lateral misalignments and the parameters of each coil. In next section, a method of optimizing the parameters of each coil will be proposed to maintain the mutual inductance constant in next section.

4. OPTIMIZATION METHOD

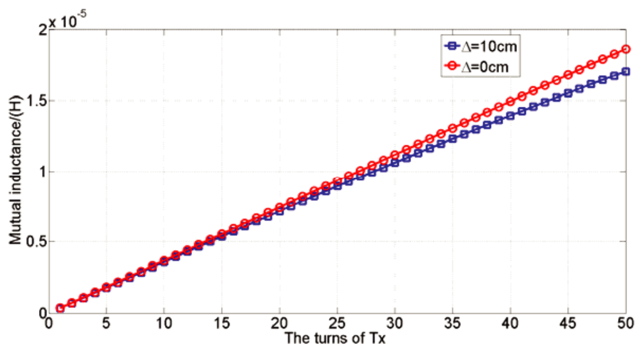
It can be seen from Eq. (7) that the output voltage V_{L} is only dependent on the mutual inductance between transmitting and receiving coils when the parameters (R, L, C) of each coil and the load are



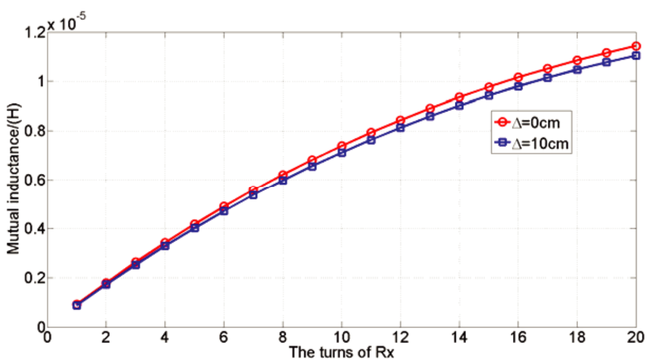
(a) Mutual inductance versus the radius of Rx with Δ=10 and Δ=0



(b) Mutual inductance versus the radius of Tx with Δ=10 and Δ=0



(c) Mutual inductance versus the turns of Tx with Δ=10 and Δ=0



(d) Mutual inductance versus the turns of Rx with Δ=10 and Δ=0

Figure 4. Simulation results.

given. The mutual inductance may be changed with the variations of lateral misalignments and the parameters of each coil (number of turns and outer diameter). In this section, a method of parameters' optimization is proposed. The mutual inductance can be kept nearly constant with lateral misalignments by using the proposed method.

The process of the proposed optimization method is as follows:

1) Parameters setting: the resonant angular frequency is set to 85 kHz. The mutual inductance is set to be larger than 6.0 μH. The transmission distance is set to 15 cm. The diameter of copper is set to 1.75 mm. The outer diameter of Tx is changed from 0.25 m to 0.30 m in a step of 2.0 mm. The outer diameter of Rx is changed from 0.2 m to 0.25 m in a step of 2.0 mm. The number of turns of Tx is changed from 17 to 40 in a step of 1. The number of turns of Rx is changed from 6 to 10 in a step of 1.

2) Calculated mutual inductance: According to Eqs. (1) and (2), the mutual inductance between Tx and Rx can be obtained with different lateral misalignments. M_{10} is the mutual inductance between Tx and Rx when lateral misalignment equals 10 cm; M_0 is the mutual inductance between Tx and Rx when lateral misalignment equals 0 cm.

3) Compared to ε ($\varepsilon = (M_0 - M_{10})/M_0$) at a given outer diameter of Tx, outer diameter of Rx, the number of turns of Tx and the number of turns of Rx, the optimum ε is obtained (The smaller ε is, the smaller the variations of mutual inductance becomes).

4) According to the optimum ε, the optimum parameters of Tx and Rx can be obtained. The detailed optimization process is shown in Fig. 5.

Figure 6 shows the optimum parameters of each coil (N_a, N_b, a, b). It can be seen that the optimum N_a is equal to 17, optimum N_b equal to 10, optimum a equal to 0.3 m, and optimum b equal to 0.1 m. Fig. 7 shows optimum mutual inductance M_0, M_{10} and ε. It is clearly seen that all ε are smaller than 5%. The optimum ε is 3.1%, and all mutual inductances are larger than 6.0 μH. In the above analysis, it can be seen that the radius of Tx and the number of turns of Rx should be increased, and the radius of Rx and the number of turns of Tx should be decreased in order to obtain constant mutual inductance.

5. EXPERIMENTAL RESULTS

To validate the proposed structure and optimization method, the prototype model of the system has been built, as shown in Fig. 8. It is composed of a DC voltage source, transmitting resonant coil, receiving resonant coil, H-bridge inverter, full-bridge rectifier, and load. The value of the DC voltage source is 48 V. The DC voltage source is shown in Fig. 8(a). H-bridge inverter is used at the transmitter side to provide AC excitation, as shown in Fig. 8(b). It contains four MOSFETs (IRF3207), and a full-bridge rectifier is used at the receiver side to convert AC to DC. In future research, the power level of the prototype will be increased, and those full-bridge rectifiers will continue to be used in high power system. The full-bridge rectifier contains four diodes (HFA15TB60) with a voltage rating of 600 V, which can also be used in high power system. The transmitting resonant coil is shown in Fig. 8(c). The receiver resonant coils are shown in Fig. 8(d). The overall setup is shown in Fig. 8(e). The outer diameter of the transmitting resonant coil is 0.6 m with a pitch of 0 cm for approximately 17 turns according to the optimization method. The outer diameter of each receiving resonant coil is 0.2 m with a pitch of 0 cm for approximately 10 turns according to the optimization method. All coils are made from 300-strand AWG 38Litz-wire. An impedance analyzer is used to extract the parameters in Eqs. (3) and (4). The original resonant frequency is set to 85.0 kHz. The load resistor is set to 10 Ω. The transmission distance is 15 cm. The parameters of the resonant coils are listed in Table 1. The moving direction of the receiver is noted as y-axis, and the coordinates are marked in Fig. 1.

Figure 9 shows the measured mutual inductance between Tx and Rx placed at different positions. It can be seen that the mutual inductance M is changed from 6.89 μH to 6.63 μH as the misalignment is varied from 0 cm to 10 cm. The difference between M_0 and M_{10} is 0.26 μH. ε is equal to 3.77% according to $\varepsilon = (M_0 - M_{10})/M_0$. The variations of mutual inductance are very smooth as the misalignment is varied from 0 cm to 10 cm.

Figure 10 shows the measured output voltage versus misalignments. It can be seen that the output voltage is changed from 14.4 V to 14.6 V as the misalignment is varied from 0 cm to 10 cm. The output voltage is nearly constant as the misalignment is varied from 0 cm to 10 cm. Fig. 11 shows the measured and calculated efficiencies versus misalignments. It can be seen that the efficiency is changed from

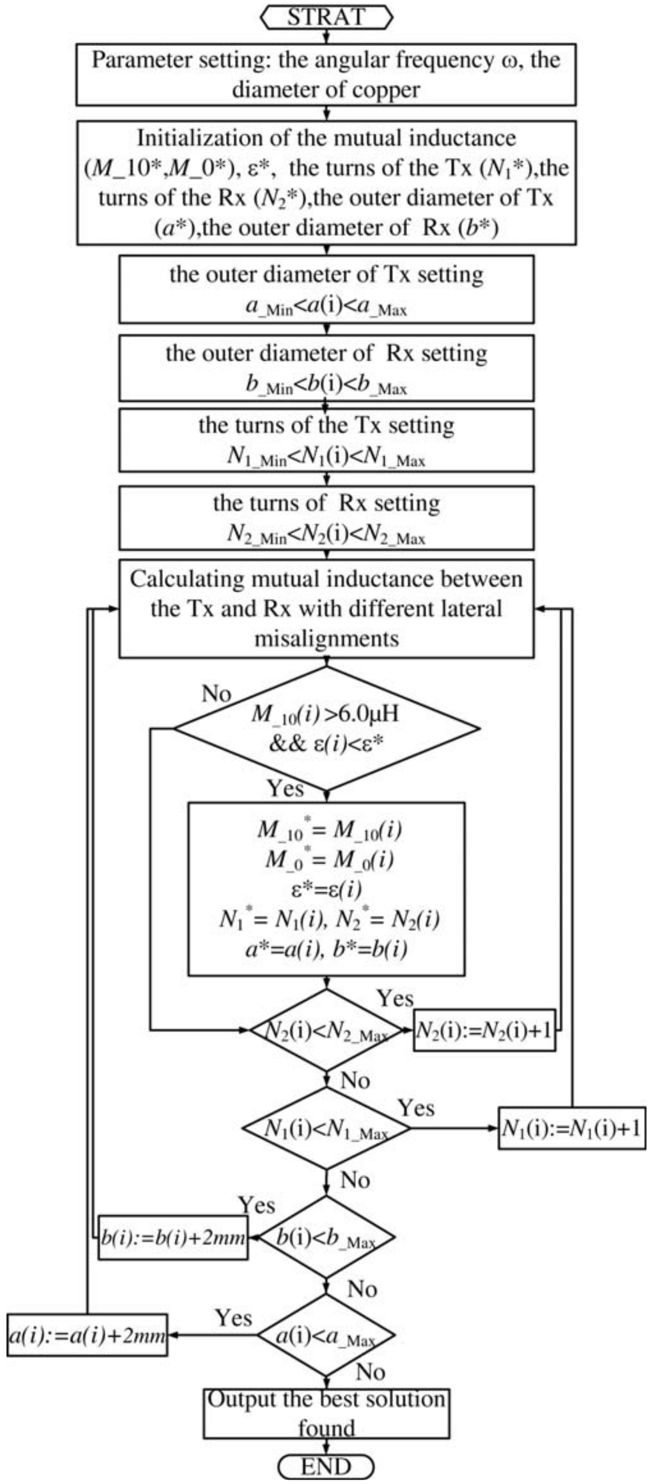


Figure 5. The flowchart of the proposed optimization method.

65.6% to 63.5% as the misalignment is varied from 0 cm to 10 cm. The efficiency is also nearly constant as the misalignment is varied from 0 cm to 10 cm. The validity of the proposed optimization method is verified by simulated and experimental results.

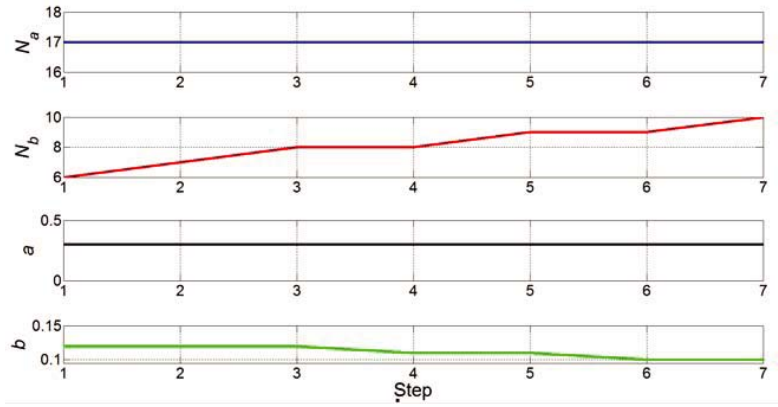


Figure 6. The optimum parameters of each coil. N_{a} is the the number of turns of Tx, N_{b} is the number of turns of Rx, a is the out radius of Tx, b is the out radius of Rx.

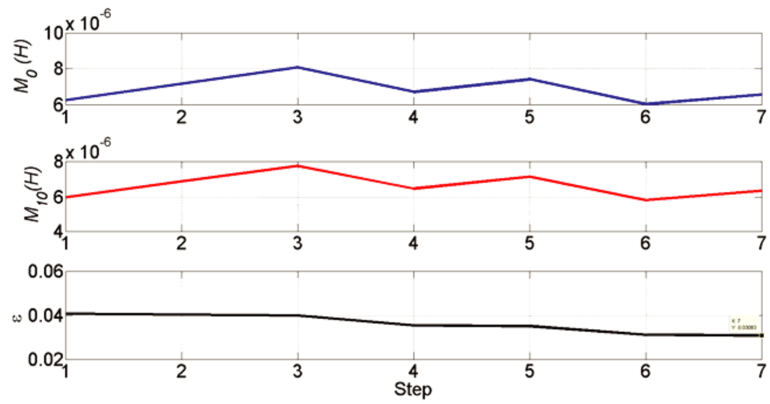
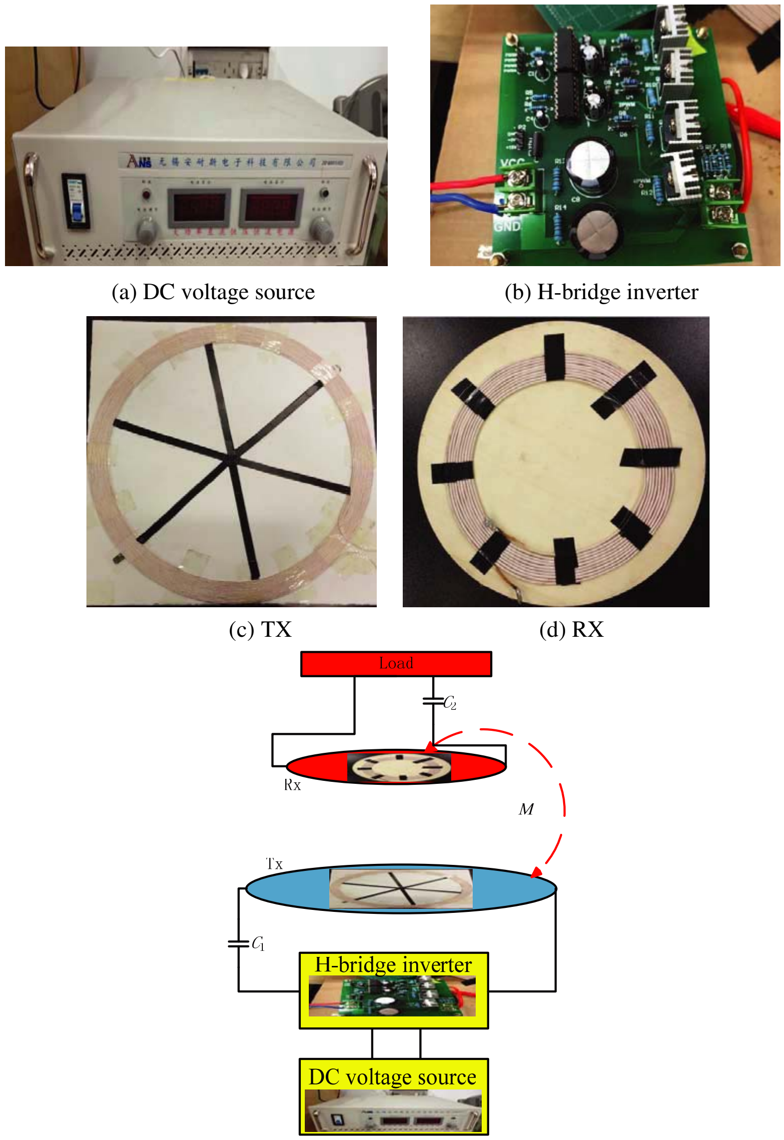


Figure 7. The optimum mutual inductance M_0 , M_{10} and ε. M_{10} is the mutual inductance between Tx and Rx when lateral misalignment equals to 10 cm; M_0 is the mutual inductance between Tx and Rx when lateral misalignment equals to 0 cm, ε is equal to $(M_0 - M_{10})/M_0$.

Table 1. Parameters of the resonant coils.

Symbol	Quantity	Value
L_1	the inductance of Tx	333.7 µH
L_2	the inductance of Rx	30.5 µH
C_1	the compensation capacitance of Tx	10.5 nF
C_2	the compensation capacitance of Rx	115.0 nF
R_1	the parasitic resistor of Tx	0.45 Ω
R_2	the parasitic resistor of Rx	0.06 Ω
f_0	the original resonant frequency	85.0 kHz



(a) DC voltage source (b) H-bridge inverter

(c) TX (d) RX

(e) the overall setup

Figure 8. Experimental setup.

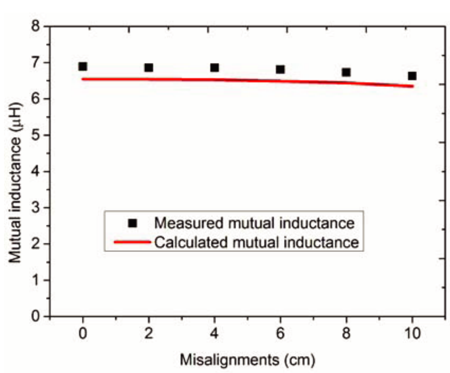


Figure 9. Measured and calculate mutual inductance versus misalignments.

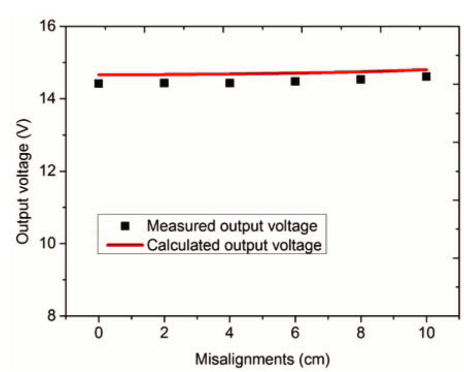


Figure 10. Measured and calculated output voltage versus misalignments.

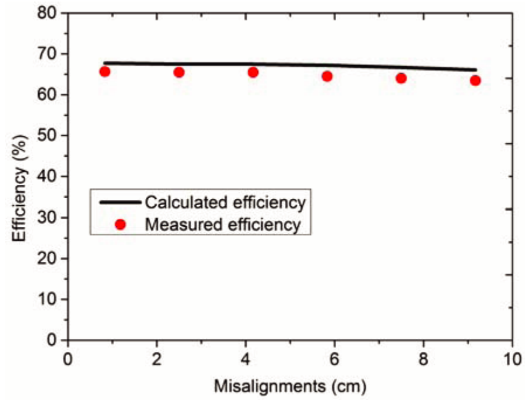


Figure 11. Measured and calculated efficiency versus misalignments.

6. CONCLUSIONS

In this paper, an asymmetric two-coil WPT system is presented. The mutual inductance between Tx and Rx is changed with the variations of lateral misalignments and the parameters of each coil. In order to decrease the variation of the mutual inductance, the turns of Rx and the diameter of Tx should be increased. And the turns of Tx and the diameter of Rx should be decreased. In addition, a method of parameters optimization is proposed. The optimum parameters of each coil can be obtained by using the proposed method. The experimental results show that the mutual inductance is changed from 6.88 μH to 6.63 μH, and the output voltage is varied from 14.4 V to 14.6 V as the lateral misalignments is varied from 0 cm to 10 cm. The optimum ε is only 3.7%, and the output voltage is nearly constant. In the future, we will further decrease the variations of mutual inductance and improve the efficiency as the lateral misalignments increase in order to meet the complex environment requirements.

ACKNOWLEDGMENT

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REFERENCES

1. Musavi, F. and W. Eberle, "Overview of wireless power transfer technologies for electric vehicle battery charging," *IET Power Electronics*, Vol. 7, No. 1, 60–66, Jan. 2014.
2. Chen, J., Z. Ding, and Z. Hu, "Metamaterial-based high-efficiency wireless power transfer system at 13.56 MHz for low power applications," *Progress In Electromagnetics Research B*, Vol. 72, No. 1, 17–30, 2017.
3. Shaw, T., A. Roy, and D. Mitra, "Efficiency enhancement of wireless power transfer system using MNZ metamaterials," *Progress In Electromagnetics Research C*, Vol. 68, No. 1, 11–19, 2016.
4. Choi, S. Y., B. W. Gu, S. Y. Jeong, et al., "Advances in wireless power transfer systems for roadway-powered electric vehicles," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, Vol. 3, No. 1, 18–36, Aug. 2015.
5. Li, Z., C. Zhu, J. Jiang, et al., "A 3-kW wireless power transfer system for sightseeing car supercapacitor charge," *IEEE Transactions on Power Electronics*, Vol. 32, No. 5, 3301–3316, Jun. 2017.

6. Li, H., J. Li, K. Wang, et al., "A maximum efficiency point tracking control scheme for wireless power transfer systems using magnetic resonant coupling," *IEEE Transactions on Power Electronics*, Vol. 30, No. 7, 3998–4008, Aug. 2015.
7. Huh, J., S. W. Lee, W. Y. Lee, et al., "Narrow-width inductive power transfer system for online electrical vehicles," *IEEE Transactions on Power Electronics*, Vol. 26, No. 12, 3666–3679, Jun. 2011.
8. Choi, S. Y., S. Y. Jeong, B. W. Gu, et al., "Ultraslim S-type power supply rails for roadway-powered electric vehicles," *IEEE Transactions on Power Electronics*, Vol. 30, No. 11, 6456–6468, Jun. 2015.
9. Waffenschmidt, E., "Homogeneous magnetic coupling for free positioning in an inductive wireless power system," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, Vol. 3, No. 1, 226–233, Jun. 2015.
10. Zhang, Z. and K. T. Chau, "Homogeneous wireless power transfer for move-and-charge," *IEEE Transactions on Power Electronics*, Vol. 30, No. 11, 6213–622, Mar. 2015.
11. Su, Y. C., H. Jin, W. Y. Lee, et al., "Asymmetric coil sets for wireless stationary EV chargers with large lateral tolerance by dominant field analysis," *IEEE Transactions on Power Electronics*, Vol. 29, No. 12, 6406–6420, Feb. 2014.
12. RamRakhyani, A. K., S. Mirabbasi, and M. Chiao, "Design and optimization of resonance-based efficient wireless power delivery systems for biomedical implants," *IEEE Transactions on Biomedical Circuits and Systems*, Vol. 5, No. 1, 48–63, Oct. 2011.
13. Soma, M., D. C. Galbraith, and R. L. White, "Radio-frequency coils in implantable devices: Misalignment analysis and design procedure," *IEEE Transactions on Biomedical Engineering*, Vol. 34, No. 4, 276–282, Apr. 1987.
14. Zierhofer, C. M. and E. S. Hochmair, "Geometric approach for coupling enhancement of magnetically coupled coils," *IEEE Transactions on Biomedical Engineering*, Vol. 43, No. 7, 708–714, Jul. 1996.