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Comparative Study of the Meissner and Skin Effects in Superconductors

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Abstract—The Meissner effect is studied by using an approach based on Newton and Maxwell's equations. The objective is to assess the relevance of London's equation and shed light on the connection between the Meissner and skin effects. The properties of a superconducting cylinder, cooled in a magnetic field, are accounted for within the same framework. The radial Hall effect is predicted. The energy, associated with the Meissner effect, is calculated and compared with the binding energy of the superconducting phase with respect to the normal one.

1. INTRODUCTION

The Meissner effect [1] highlights the rapid decay [2–4] of an applied magnetic field in bulk superconducting matter, provided that the field is lower than some critical field. Our current understanding is still based mainly on London's assumption [5]

$$B + \mu_0 \lambda_L^2 \operatorname{curl} j = 0, \quad \lambda_L = \sqrt{\frac{m}{\mu_0 \rho e^2}}, \tag{1}$$

where μ_0, j, λ_L stand for the magnetic permeability of vacuum, the persistent current, induced by the magnetic induction B, and London's length, whereas e, m, ρ refer to the charge, effective mass and concentration of superconducting electrons, respectively. Eq. (1), combined with the Ampère-Maxwell equation, entails [5] that the penetration depth of the magnetic field is equal to λ_L. The validity of Eq. (1) was questioned earlier by skin depth measurements [6] (see also [7] p. 37, 2nd paragraph, lines 7, 8), which has resulted in interesting but inconclusive debates:

- some authors [3, 8–10] have attempted to view the Meissner effect as a classical phenomenon, whereas another school claimed that the Meissner effect stemmed from some unknown quantum effect, possibly related to the BCS theory [11] and Cooper pairs [12];
- when a superconducting material is cooled in a magnetic field H, starting from its normal state, the latter is expelled [1, 6] from the bulk material, while crossing the critical temperature $T_c(H)$ at which superconductivity sets in. This manifestation of the Meissner effect has generated an inconclusive debate over the distinction between a real material superconductor and a fictitious perfect conductor [2, 5, 7, 13].

Furthermore, because the Meissner-Ochsenfeld experiment [1] yielded merely qualitative results, it was widely believed, till London's work [5], that H did not penetrate *at all* into the superconducting sample. Thus, since the Meissner-Ochsenfeld experiment failed to provide an accurate [3] assignment for the field penetration length, all of the experiments [6, 14, 15] have consisted of measuring the penetration depth of an electromagnetic wave into a superconductor, i.e., the skin depth [16, 17], at frequencies in the range [10 MHz, 100 GHz]. The penetration of the electromagnetic field into a conductor is

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impeded by the real part of the frequency dependent dielectric constant, associated with conduction electrons, being negative below the plasma frequency. The skin effect has been analyzed [18] previously in superconductors and found to have essentially the same properties, as those observed in a normal conductor.

The purpose of this work is then to show theoretically that the properties of the Meissner effect are conditioned by the limited time-duration t_0, needed in the experiment, for the applied magnetic field H to grow with time t from its starting value $H(t=0)=0$ up to its permanent one $H(t_0)$. The analysis relies entirely on Newton and Maxwell's equations. In particular, it will appear below that the spatial decay of the *static* field $H(t>t_0)$ inside the bulk superconductor, characterizing the Meissner effect, can be described as a sum involving $\delta(n\omega_0)$, $n=1,2,3...$, where $\delta(\omega)$ is the frequency dependent skin depth and $\omega_0=\frac{2\pi}{t_0}$.

The outline is as follows. Sections 2 and 3 deal with the skin effect in superconductors and with the Meissner effect, respectively, while establishing the connection between both effects. The validity of Eq. (1) is assessed in Section 4. The case of the field cooled superconductor is addressed in Section 5. The radial Hall effect is analyzed in Section 6. The energy, associated with the Meissner effect, is calculated in Section 7. The conclusions are given in Section 8.

Consider, as in Fig. 1, a superconducting material of cylindrical shape, characterized by its symmetry axis z and radius r_0 in a cylindrical frame with coordinates (r,θ,z). The superconducting sample is further inserted into a coil, producing $H(t)$. Both lengths of the superconducting sample and of the coil are taken $\gg r_0$, in order to get rid of any end effect, which will turn out to be a crucial requirement for the Hall effect experiment. The superconducting material contains electrons of charge e, effective mass m, and concentration ρ. The current $I(t)$, flowing through the coil, gives rise, thanks to the Faraday-Maxwell law, to an electric field $E_\theta(t,r)$, normal to the unit vectors along the r and z coordinates, such that $E_\theta(t,r)\neq 0$ for $t\in]0,t_0[$ only, which defines a transient regime $(0<t<t_0\Leftrightarrow E_\theta\neq 0)$ and a permanent one $(t>t_0\Leftrightarrow E_\theta=0)$.

2. TRANSIENT REGIME

E_θ induces a current $j_\theta(t,r)$ along the field direction, as given by Newton's law

$$\frac{dj_\theta}{dt}=\frac{\rho e^2}{m}E_\theta-\frac{j_\theta}{\tau}, \tag{2}$$

where $\frac{\rho e^2}{m}E_\theta$ and $-\frac{j_\theta}{\tau}$ are respectively proportional to the driving force accelerating the conduction electrons and a friction term. The friction force $\propto \frac{j_\theta}{\tau}$ in Eq. (2) ensues from the *finite* conductivity, observed in superconductors, carrying an *ac current*, as emphasized by Schrieffer [4] (see [4] p. 4, 2nd paragraph, lines 9, 10): *at finite temperature, there is a finite ac resistivity for all frequencies* >0. For example, for the superconducting phase of $BaFe_2(As_{1-x}P_x)_2$, the conductivity, measured in the microwave range, has been found (see [14] p. 1555, 3rd column, 2nd paragraph, line 11) to be $\approx 300\sigma_n$, where σ_n stands for the normal conductivity, measured just above the critical temperature T_c. Additional evidence is provided by commercial microwave cavity resonators, made up of superconducting materials, displaying a very high, albeit *finite* conductivity. As a last note, the finite conductivity, measured at $\omega\neq 0$ in superconductors, is consistent with the observation of persistent currents at *vanishing* electric field, even though a cogent explanation is still lacking [2–4].

The magnetic induction $B_z(r,t)$, parallel to the z axis [18], is related to E_θ through the Faraday-Maxwell equation as

$$-\frac{\partial B_z}{\partial t}=\frac{E_\theta}{r}+\frac{\partial E_\theta}{\partial r}, \tag{3}$$

while the magnetic field $H_z(t,r)$, parallel to the z axis, is given [18] by the Ampère-Maxwell equation as

$$-\frac{\partial H_z}{\partial r}=2j_\theta+\epsilon_0\frac{\partial E_\theta}{\partial t}. \tag{4}$$

ϵ_0 refers to the electric permittivity of vacuum.

$E_\theta(t,r), j_\theta(t,r), B_z(t,r), H_z(t,r)$ can be recast as Fourier series for $t \in]0, t_0[$

$$f(t,r) = \sum\nolimits_{n\in\mathbb{Z}} f(n,r)e^{in\omega_0 t}, \quad f(n,r) = \int_0^{t_0} e^{-in\omega_0 t} f(t,r)dt/t_0, \tag{5}$$

where $\omega_0 t_0 = 2\pi$, and $f(t,r), f(n,r)$ hold for $B_z(t,r)$, $H_z(t,r)$, $E_\theta(t,r)$, $j_\theta(t,r)$ and $B_z(n,r)$, $H_z(n,r)$, $E_\theta(n,r)$, $j_\theta(n,r)$, respectively. Replacing $E_\theta, j_\theta, B_z, H_z$ in Eqs. (2), (3), (4) by their expression in Eq. (5), while taking into account

$$B_z(n,r) = \mu\left(n\omega_0\right) H_z(n,r),$$

where $\mu(n\omega_0) = \mu_0(1+\chi_s(n\omega_0))$, and $\chi_s(\omega)$ is the magnetic susceptibility of superconducting electrons at frequency ω, yielding for $n \neq 0$

$$\begin{aligned} E_\theta\left(n,r\right) &= \frac{1+in\omega_0\tau}{\sigma} j_\theta\left(n,r\right) \\ in\omega_0 B_z\left(n,r\right) &= -\left(\frac{E_\theta\left(n,r\right)}{r} + \frac{\partial E_\theta\left(n,r\right)}{\partial r}\right) \\ \frac{\partial B_z\left(n,r\right)}{\partial r} &= -\mu\left(n\omega_0\right)\left(2j_\theta\left(n,r\right) + in\omega_0\epsilon_0 E_\theta\left(n,r\right)\right) \end{aligned} \tag{6}$$

where the conductivity [2] $\sigma = \frac{\rho e^2\tau}{m}$. Then eliminating $E_\theta(n,r), j_\theta(n,r)$ from Eq. (6) gives

$$\frac{\partial^2 B_z\left(n,r\right)}{\partial r^2} = \frac{B_z\left(n,r\right)}{\delta^2(n\omega_0)} - \frac{\partial B_z\left(n,r\right)}{r\partial r}. \tag{7}$$

$\delta(\omega) = \frac{\lambda_L}{\sqrt{(1+\chi_s(\omega))\left(\frac{2i\omega\tau}{1+i\omega\tau} - \frac{\omega^2}{\omega_p^2}\right)}}$, $\omega_p = \sqrt{\frac{\rho e^2}{\epsilon_0 m}}$ refer to skin depth and plasma frequency [2, 16, 17], respectively. The solution of Eq. (7) with $\frac{dB_z}{dr}(r=0) = 0$ is a Bessel function, such that $B_z(r \gg |\delta(n\omega_0)|) \approx e^{r/\delta(n\omega_0)}$.

3. PERMANENT REGIME

Because $E_\theta(t > t_0, r) = 0$ in the permanent regime, the friction force $\propto -\frac{j_\theta}{\tau}$ is no longer at work for $t > t_0$, so that the *transient* current $j_\theta(t < t_0)$ turns into the *persistent* one, $j_\theta(t > t_0, r) = j_\theta(t_0, r), \forall r$. Eq. (5) then yields

$$j_\theta(t > t_0, r) = 2\sum_{n\in\mathbb{Z},\, n\neq 0} j_\theta\left(n,r\right). \tag{8}$$

The Ampère-Maxwell equation reads now

$$-\frac{\partial H_z}{\partial r}(t > t_0, r) = j_\theta(t_0, r). \tag{9}$$

Comparing Eqs. (4), (9) reveals that $H_z(t_{0^-}, r) \neq H_z(t_{0^+}, r)$. The penetration depth λ_M of the static field $H_z(t_{0^+}, r)$ is defined as

$$\frac{1}{\lambda_M} = \frac{\partial \mathrm{Log} H_z(t_{0^+}, r_0)}{\partial r}.$$

Because of $r_0 \gg |\delta(n\omega_0)|$ under typical experimental conditions, there is $j_\theta(n, r \to r_0) \approx j_\theta(n, r_0)e^{\frac{r-r_0}{\delta(n\omega_0)}}$. Using Eq. (8) to integrate Eq. (9), we obtain

$$\frac{1}{\lambda_M} \approx \frac{\sum_n j_\theta(n,r_0)}{\sum_n \delta(n\omega_0) j_\theta(n,r_0)}, \tag{10}$$

where the sum is performed for $n \neq 0$ and $|n|\omega_0 < \omega_p$. Eq. (10) embodies the connection between the skin and Meissner effects. Due to $|\delta(n\omega_0)| = \lambda_L/\sqrt{2|n|\omega_0\tau} \gg \lambda_L$ for $|n|\omega_0\tau \ll 1$, it is ensued from Eq. (10) that $|\lambda_M| \gg \lambda_L$. Furthermore, by contrast with Eq. (1), it is obvious from the ω_0 dependence in Eq. (7) that there can be no one to one correspondence between $H_z(t > t_0, r)$ and $j_\theta(t > t_0, r)$, which is an irreversible consequence of the friction term $-\frac{j_\theta}{\tau}$ in Eq. (2). This is anyhow of little practical interest because λ_M cannot be measured, as already noted in Section 1.

4. VALIDITY OF LONDON'S EQUATION

Equation (1) was assumed [5], starting from the following version of Newton's equation

$$\frac{dj_\theta}{dt} = \frac{\rho e^2}{m} E_\theta, \tag{11}$$

which is identical to Eq. (2) in the case $\tau \to \infty$. Integrating both sides of Eq. (11) from $t = 0$ up to $t = t_0$ yields for for $r \in [0, r_0]$

$$j_\theta(t_0, r) = \frac{\rho e^2}{m} \int_0^{t_0} E_\theta(t, r)dt = -\frac{\rho e^2}{m} A_\theta(t_0, r), \tag{12}$$

by assuming $j_\theta(t = 0, r) = A_\theta(t = 0, r) = 0$ and taking advantage of $E_\theta = -\frac{\partial A_\theta}{\partial t}$, where the magnetic vector potential [16, 17] $A_\theta(t, r)$ is parallel to E_θ. Using furthermore $B_z = \mathrm{curl} A_\theta$, it is inferred from Eq. (12) for $r \in [0, r_0]$ in the permanent regime $t > t_0$

$$B_z + \mu_0 \lambda_L^2 \mathrm{curl} j_\theta = 0,$$

which is identical to Eq. (1). The validity of London's equation has thence been shown provided $\tau \to \infty$.

5. FIELD COOLED SAMPLE

As the susceptibility χ_s not being continuous at T_c (T_c refers to the critical temperature) will turn out to be *solely* responsible for the Meissner effect to occur in a superconductor, cooled inside a magnetic field, we set out to reckon it. Since no paramagnetic contribution is observed in the superconducting state [2–4], the latter is deemed to be in a macroscopic singlet spin state, so that the only contribution to χ_s can be calculated using Maxwell's equations. We begin with writing down the t-averaged density of kinetic energy

$$\mathcal{E}_K(r) = \frac{m}{2\rho} \left(\frac{j_\theta(r)}{e} \right)^2,$$

associated with the *ac* current $j_\theta(r)e^{i\omega t}$, flowing along the E_θ direction (this latter induces in turn a magnetic field $H_z(r)e^{i\omega t}$, parallel to the z axis). The Ampère-Maxwell equation simplifies into $\frac{\partial H_z}{\partial r} = -2j_\theta$ for practical $\omega \ll \omega_p$. As this discussion is limited to the case $r \to r_0$, both $H_z(r), j_\theta(r)$ are $\propto e^{r/\delta(\omega)}$, so that $\mathcal{E}_K(r)$ is recast into

$$\mathcal{E}_K(r) = \frac{\mu_0}{8} \left(\frac{\lambda_L}{|\delta(\omega)|} H_z(r) \right)^2. \tag{13}$$

Moreover, there is the identity $\frac{\partial \mathcal{E}_K}{\partial M} = -H_z$, where $M = \mu_0 \chi_s(\omega) H_z$ is the magnetization of superconducting electrons. Actually this identity reads in general $\frac{\partial F}{\partial M} = -H_z$, where F represents the Helmholz free energy [19]. However, the property that a superconducting state carries no entropy [2–4] entails that $F = \mathcal{E}_K$. Equating this expression of $\frac{\partial \mathcal{E}_K}{\partial M}$ with that inferred from Eq. (13) yields finally

$$\chi_s(\omega) = -\left(\frac{\lambda_L}{2|\delta(\omega)|} \right)^2.$$

As expected, χ_s is found diamagnetic ($\chi_s < 0$) and $|\chi_s(\omega)| \ll 1$ for $\omega \ll 1/\tau$. The calculation of $\chi_s(0)$ proceeds along the same lines, except for the Ampère-Maxwell equation reading $\frac{\partial H_z}{\partial r} = -j_\theta$ and λ_M showing up instead of $\delta(\omega)$, whence

$$\chi_s(0) = -\left(\frac{\lambda_L}{\lambda_M}\right)^2.$$

Note that our definition of $\chi_s = \frac{M(r)}{\mu_0 H_z(r)}$, where $H_z(r), M(r)$ refer to local field and magnetization at r, differs from the usual [2, 3, 5] one $\chi_s = \frac{M}{\mu_0 H_z(r_0)}$ with $H_z(r_0), M$ being external field and total magnetization.

While the sample is in its normal state at $T > T_c$, the applied magnetic field H_z penetrates fully into bulk matter and induces a magnetic induction

$$B_n = \mu_0 (1 + \chi_n) H_z,$$

where χ_n designates the magnetic susceptibility of conduction electrons. It comprises [2] the sum of a paramagnetic (Pauli) component and a diamagnetic (Landau) one and $\chi_n > 0$ in general. Moreover, the magnetic induction reads for $T < T_c(H_z)$

$$B_s = \mu_0 (1 + \chi_s(0)) H_z,$$

with $\chi_s(0) < 0$. Because of $\chi_s(0) \neq \chi_n$, the magnetic induction undergoes a finite step while crossing $T_c(H_z)$

$$\frac{\delta B}{\delta t} = \frac{B_s - B_n}{\delta t} = \mu_0 \frac{\chi_s(0) - \chi_n}{\delta t} H_z, \tag{14}$$

where δt refers to the time needed in the experimental procedure for T to cross $T_c(H_z)$. Due to the Faraday-Maxwell equation (see Eq. (3)), the finite $\delta B/\delta t$ induces an electric field E_θ such that $\mathrm{curl} E_\theta = -\frac{\delta B}{\delta t}$, giving rise to the persistent, H_z screening current.

It is noteworthy that though H_z remains *unaltered* during the cooling process, the magnetic induction B is indeed *modified* at T_c, as shown by Eq. (14). Then this B variation arouses E_θ via Eq. (3), giving rise to the screening current j_θ, and ultimately to H_z expulsion, as explained in Sections 2, 3. At last, an alternative explanation of the Meissner effect, based on Weber's force [10], is worth mentioning.

6. THE RADIAL HALL EFFECT

For $t < t_0$, the magnetic induction B_z exerts on the conduction electrons a radial Lorentz force $\frac{B_z j_\theta}{\rho}$, pushing the electrons inward, so that a charge distribution builds up, which in turn gives rise, via Poisson's law, to a radial electric field $E_r(r)$, characterizing the Hall effect. Meanwhile, E_r drives a transient radial current $j_r(t)$ ($j_r(t < t_0) \neq 0$, $j_r(t > t_0) = 0$), responsible for the charge distribution.

Moreover for $t > t_0$, equilibrium is secured by the radial centripetal force $-\frac{m}{r}\left(\frac{j_\theta(t_0,r)}{\rho e}\right)^2$, exerted on each electron of $j_\theta(t_0, r)$, being made up of the sum of the Lorentz force and an electrostatic one eE_r, with E_r given by

$$E_r = -\frac{j_\theta}{\rho e}\left(B_z + \frac{m}{\rho e^2 r} j_\theta\right). \tag{15}$$

Owing to the Ampère-Maxwell equation $j_\theta = -\frac{\partial H_z}{\partial r}$ and $B_z = \mu_0 H_z$, E_r can be recast as

$$E_r = \frac{\mu_0}{\rho e}\frac{\partial H_z}{\partial r}\left(H_z - \frac{\lambda_L^2}{r}\frac{\partial H_z}{\partial r}\right).$$

Because of $\frac{\partial H_z}{\partial r} \approx \frac{H_z}{\lambda_M}$, $r_0 \gg \lambda_L$ and $|\lambda_M| \gg \lambda_L$, the approximation $E_r \approx \frac{\mu_0}{2\rho e}\frac{\partial H_z^2}{\partial r}$ can be used for significant $r \gg \lambda_L$. For the Hall effect to be observed, a sample in shape of a hollow cylinder of inner and outer radii r_1, r_0, respectively, is needed (see Fig. 1). Furthermore, the length of the sample should

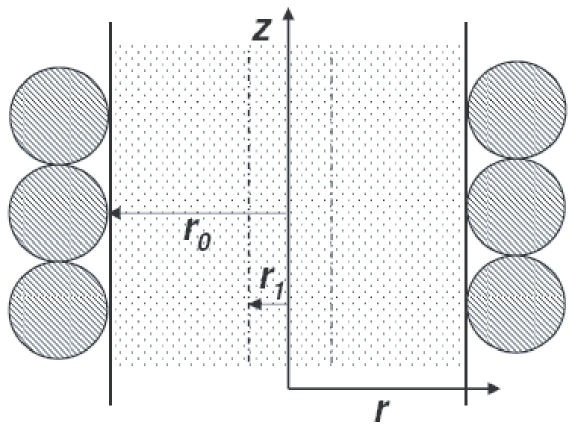

Figure 1. Cross-section of the superconducting sample (dotted) and the coil (hatched); E_θ, j_θ are both normal to the unit vectors along the r and z coordinates, whereas B_z, H_z and E_r, j_r are parallel to the unit vector along the z and r coordinates, respectively; the radius of the wire making up the coil has been magnified for the reader's convenience; the matter between the dashed lines should be carved out to carry out the radial Hall effect experiment.

be larger than that of the coil, and the measurement should be carried out in the middle of the sample to get rid of any end effect. Finally the Hall voltage reads, for $r_0 - r_1 \gg |\lambda_M|$

$$U_H = -\int_{r_1}^{r_0} E_r(r)dr \approx -\frac{\mu_0}{2\rho e} H_z^2\left(t_0^+, r_0\right).$$

As in normal metals [2], measuring U_H gives access to ρ. However, whereas H_z, j_θ are set independently from each other in the Hall effect observed in a normal conductor, which implies $U_H \propto H_z j_\theta$, they are both related by the Ampère-Maxwell equation in the experiment discussed above, which entails $U_H \propto H_z^2$. Note also that U_H is independent of r_1 provided $r_0 - r_1 \gg |\lambda_M|$. As λ_M cannot be measured, the Hall effect is likely to provide the only way to assess the validity of this work.

7. CALCULATION OF THE ENERGY

The whole energy, associated with the Meissner effect, comprises two contributions, i.e., the kinetic energy, carried by the persistent current and the electrostatic one, stemming from the Hall effect. Taking advantage of the the Ampère-Maxwell equation, the density of kinetic energy is inferred to read

$$\mathcal{E}_K(r) = \frac{m}{2\rho}\frac{j_\theta^2(t_0, r)}{e^2} = \frac{\mu_0}{2}\left(\frac{\lambda_L}{|\lambda_M|} H_z(t_0^-, r)\right)^2, \tag{16}$$

where $j_\theta(t_0, r)$ refers to the persistent current and $\frac{\lambda_L}{|\lambda_M|} \ll 1$.

The expression of the radial current j_r is needed to reckon the electrostatic energy. To that end we look for a solution of the Ampère-Maxwell equation with no magnetic field. Thus it reads:

$$j_r + \frac{\partial D_r}{\partial t} = 2j_r + \epsilon_0 \frac{\partial E_r}{\partial t} = 0,$$

where the electric displacement [18] D_r is parallel to the unit vector, along the r coordinate, and the time derivative of the space charge, stemming from the Hall effect, has been taken to vanish, so that $\frac{\partial P_r}{\partial t} = j_r \Rightarrow \frac{\partial D_r}{\partial t} = j_r + \epsilon_0 \frac{\partial E_r}{\partial t}$, with P_r being the radial polarisation [18]. This assumption is vindicated by the sought electrostatic energy, depending *only* on the permanent electric field $E_r(t_0, r)$, and accordingly being *independent* from the preceding transient behaviour $E_r(t < t_0, r)$. Multiplying then the above equation by E_r yields

$$E_r j_r + \frac{\epsilon_0}{4}\frac{\partial E_r^2}{\partial t} = 0.$$

Using Eq. (15), we obtain

$$j_r\left(\frac{j_\theta}{\rho e}B_z\right)dt = -E_r j_r dt = \frac{\epsilon_0}{4}\frac{\partial E_r^2}{\partial t}dt.$$

The left hand term is identified as the elementary work performed by the Lorentz force, so that the searched expression of the the density of electrostatic energy is obtained, thanks to the first law of thermodynamics, as

$$\mathcal{E}_e(r) = \frac{\epsilon_0}{4}\int_0^{t_0}\frac{\partial E_r^2}{\partial t}dt = \frac{\epsilon_0}{4}E_r^2(t_0,r) = \mu_0\frac{H_z^4(t_0,r)}{(2c\rho e|\lambda_M|)^2},$$

where c stands for the light velocity in vacuum. By replacing H_z by its upper bound H_c, it can be checked that $\mathcal{E}_e \ll \mathcal{E}_K$ in all cases.

In the mainstream treatment [3–5], the energy, pertaining to the Meissner effect, has rather been conjectured to read $\mathcal{E}_M = \frac{\mu_0}{2}H_z^2(r) \gg \mathcal{E}_K(H_z)$ in Eq. (16), due to $\lambda_L \ll |\lambda_M|$. Moreover, this expression of $\mathcal{E}_M$ turns out to be questionable from another standpoint, because its value for $H_z = H_c(T)$ is furthermore believed [2–5] to be equal to $\rho E_b(T)$, where $2E_b(T)$ designates the binding energy, needed to turn a pair of BCS electrons into two normal ones, at temperature T. However, the BCS theory [11] provides the estimate $\frac{E_b}{E_F} \approx (\frac{k_B T_c}{E_F})^2$, where E_F, k_B stand for the Fermi energy in the normal state and Boltzmann's constant, respectively. A numerical application with $E_b = \mathcal{E}_M(H_c(0))/\rho$ in the case of *Al* yields $T_c \approx 10^{-5}$ K, i.e., much less than the measured value $T_c = 1.19$ K.

Likewise, multiplication of both terms of Eq. (2) by j_θ and time-integration yield the following inequality

$$\frac{m}{\rho e^2}\int_0^{t_0} j_\theta\frac{dj_\theta}{dt}dt = \mathcal{E}_K(H_c(T)) \ll \int_0^{t_0} j_\theta(t)E_\theta(t)dt,$$

with $j_\theta(t)E_\theta(t)$ being the external power fed into the sample at t. Actually it ensues from Ohm's law, recast as

$$\frac{\sigma}{\tau}E_\theta = \frac{\rho e^2}{m}E_\theta = \frac{j_\theta}{\tau},$$

because the inertial force, $\propto |\frac{dj_\theta}{dt}|$ in Eq. (2), is negligible [18] with respect to the electric one $\propto \frac{\rho e^2}{m}|E_\theta|$, provided $|\frac{dj_\theta}{dt}\frac{\tau}{j_\theta}| \ll 1$, which always holds for the Meissner-Ochsenfeld experiment.

8. CONCLUSION

The applied, time-dependent magnetic field excites transient eddy currents according to Newton and Maxwell's equations, which turn to persistent ones, after the magnetic field stops varying, and the induced electric field thereby vanishes. Those eddy currents thwart the magnetic field penetration. Were the same experiment to be carried out in a normal metal, eddy currents would have built up the same way. However, once the electric field vanishes, they would have been destroyed quickly by Joule dissipation, and the magnetic field would have subsequently penetrated into bulk matter. As a matter of fact, the Meissner effect shows up as a *classical* phenomenon and a mere outcome of *persistent currents*, the very signature of superconductivity. The common physical significance of the Meissner and skin effects has been unveiled too. The radial Hall effect has been predicted. The energy, associated with the Meissner effect, has been calculated and compared with the binding energy of the superconducting phase.

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