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Research on Space-Time Adaptive Processing with Respect to the Signal Powers

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Abstract—Diverse array processing methods with higher order statistics (HOS) have been developed in the last three decades. One of the main interests in using HOS relies on the increase of effective aperture and the number of sensors of the considered array. In this work, we further exploit space-time adaptive processing (STAP) using HOS based on the phased array radar with uniform linear array (ULA). We implement STAP with respect to signal powers instead of with respect to signal amplitudes as the convention. The purpose of this paper is to provide some important insights into the STAP with respect to signal powers (SP-STAP), such as the output response, output signal-to-interference-plus-noise ratio (SINR), minimum detectable velocity (MDV) performance and effect of internal clutter motion (ICM). Compared with the conventional STAP under the condition of the same number of array elements and transmitting pulses, the simulation results show that SP-STAP can gain narrower target main beam, lower side-lobe levels, better MDV performance and less deleterious effect of ICM.

1. INTRODUCTION

Space-time adaptive processing (STAP) [1–4] is a crucial technique of clutter suppression and target detection for airborne/spaceborne moving target indicator (MTI) radar. It can adaptively adjust a two-dimensional space-time filter response to maximize the output signal-to-interference-plus-noise ratio (SINR), and thus provide a better target detection performance in strong clutter environments. The performance of STAP is relevant to the effective system degrees of freedom (DOFs). In particular, better minimum detectable velocity (MDV) performance can be obtained with more effective system DOFs. For instance, compared with the phased array radar, MIMO radar can achieve more effective system DOFs due to its orthogonal transmitting waveforms. However, the waveforms are generated by those extra waveform control modules at the expense of design and control complexities [5, 6]. Moreover, it is well known that a sparse array increases the virtual aperture dimension for a given physical aperture area, which can be regarded as effective system DOFs extension. (Virtual aperture dimension refers to the total length of the array from one end to the other, whereas, physical aperture area refers to the are a actually covered by aperture hardware.) [7]. A reference for an effective sparse array and the design procedure can be found in [8]. In comparison with the dense array, the sparse array can reduce the antenna pattern clutter Doppler spread and the Doppler width of blind zones, which results in better MDV performance [7, 9]. However, the sparse array causes grating lobes and may introduce additional Doppler blind zones in the case of high sparsity [7]. In practice, the phased array radar with uniform linear array (ULA) is widely used in the civil or military field. Though its system DOFs can be simply increased by adding the number of array elements or transmitting pulses, the equipment weight and size or the duration of coherent processing interval (CPI) will increase at the same time. However, the load of airborne/spaceborne platform is limited, and we commonly wish to decrease the weight or

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size as possible as we can. In addition, too long CPI duration will result in serious range-walk, which largely affects the STAP performance [2]. Therefore, we expect to increase the effective system DOFs without changing system hardware frame work or just increasing transmitting pulses for the purpose of saving hardware resource, reducing upgrade difficulty of hardware and avoiding some other performance declines.

On the other hand, for about three decades, higher order statistics (HOS) have been extensively used in many diverse application fields [10]. For example, many HOS-typed methods which make use of fourth order statistics of non-Gaussian signals have been developed for both direction finding [11, 12] and blind identification [13, 14]. Moreover, some other algorithms that exploit data statistics at an arbitrary even order have been developed in [15–18] for array processing. Note that those aforementioned HOS-typed methods can increase even more spatial resolution, robustness to modeling errors, and number of sources to be processed than traditional second-order statistics (SOS) methods based on an eigen structure [18]. This fact is reasonable when one considers that using SOS, commonly represented as a covariance matrix, is a data reduction technique [19]. According to information theory, it is well known that we always lose information by data reduction. Using HOS, which is usually represented as a cumulant matrix, can be considered as an alternate data reduction technique. However, it is a much better one since it is possible to recover more information from the cumulant matrix about the sources illuminating the array [19]. Additionally, the virtual array (VA) concept has been presented as a powerful tool for performance evaluation of HOS-typed array processing methods [19–21]. In summary, according to the information theory and VA concept, we can consider that one of the main reasons for the improvements of HOS-typed array processing methods relies on the increase of the effective aperture and the number of sensors of the considered array, i.e., the increase of effective system DOFs [21].

Recently, a nonlinear adaptive spatial beamforming technique using HOS performed on nested array has been described in [22]. This proposed scheme implemented beamforming with respect to signal powers instead of with respect to signal amplitudes as is the convention. This approach needs to take the autocorrelation of the received signal vector and enables us to perform beamforming with the full DOFs of the difference co-array and gain $O(N^2)$ DOFs using only N physical sensors. It can be regarded as a HOS-typed algorithm. Enlightened by [22], a novel nonlinear STAP method using HOS has been applied to nested array, but the performances of the new method have not been exhaustively discussed in [23].

In this work, we continue to exploit STAP using HOS and further study a novel HOS-typed STAP method, which is referred to as SP-STAP, based on the phased array radar with ULA. In this method, signal powers are taken as the inputs of the space-time filters for the purpose of obtaining more effective system DOFs without altering system hardware framework or adding transmitting pulses. The aim of this paper is to make some important investigations into SP-STAP, such as the output response, the output signal-to-interference-plus-noise ratio (SINR), the minimum detectable velocity (MDV) performance and the effect of internal clutter motion (ICM). Compared with the conventional STAP under the condition of the same number of array elements and transmitting pulses, the simulation results show that SP-STAP can get narrower target main beam, lower side-lobe levels, better MDV performance and less deleterious effect of ICM. The manuscript is organized as follows. The rationale of SP-STAP based on ULA is presented in Section 2. Comparative simulations and results are addressed in Section 3. Conclusions are finally summarized in Section 4.

2. RATIONALE OF SP-STAP BASED ON ULA

Consider a N elements ULA with inter-element spacing $d = \lambda/2$, where λ indicates the radar wavelength, and the spatial steering vector can be given by

$$\mathbf{a}(f_s) = \left[1,\, e^{j2\pi f_s},\, \ldots,\, e^{j2\pi f_s n},\, \ldots,\, e^{j2\pi f_s (N-1)}\right]^T \tag{1}$$

where $n = 0, 1, \ldots, N-1$, $[\cdot]^T$ denotes the transpose operation, and f_s is the spatial frequency which is given by

$$f_s = \frac{d \sin(\theta)}{\lambda} \tag{2}$$

where θ is the angle of arrival (AOA). In addition, given M transmitting pulses with pulse repetition interval (PRI) T in a CPI for a fixed radar range bin, the Doppler steering vector $\mathbf{b}(f_d)$ can be written as

$$\mathbf{b}(f_d) = \left[1,\, e^{j2\pi f_d},\, \ldots,\, e^{j2\pi f_d m},\, \ldots,\, e^{j2\pi f_d (M-1)}\right]^T \tag{3}$$

where $m = 0, 1, \ldots, M-1$ and f_d is the normalized Doppler frequency which is given by

$$f_d = \frac{2v\sin(\theta)T}{\lambda} = \beta f_s \tag{4}$$

where $\beta = \frac{2vT}{d}$ and v is the relative velocity between the moving radar platform and the signal source from direction θ. Note that to avoid temporal aliasing, T should be chosen such that $-0.5 \leq f_d \leq 0.5$. Then, the space-time steering vector $\mathbf{v}$ can be expressed as

$$\mathbf{v} = \mathbf{b}(f_d) \otimes \mathbf{a}(f_s) \tag{5}$$

where $\otimes$ denotes the Kronecker product.

Suppose that a target is located at spatial frequency f_{st} with normalized Doppler frequency f_{dt}, and the target signal $\mathbf{s}$ can be written as

$$\mathbf{s} = \alpha_t \mathbf{b}(f_{dt}) \otimes \mathbf{a}(f_{st}) = \alpha_t \mathbf{v}_t \tag{6}$$

where α_t is the complex target amplitude, and $\mathbf{v}_t$ denotes the target space-time steering vector. Remarkably, we mainly consider the clutter scenarios without jamming in this paper, and N_c clutter patches are supposed to be uniformly distributed among the angle 0° to 360° for a fixed range bin. The whole clutter signals can be given by

$$\mathbf{c} = \sum_{i=1}^{N_c} \alpha_i \mathbf{b}\left(f_{dc}^i\right) \otimes \mathbf{a}\left(f_{sc}^i\right) = \sum_{i=1}^{N_c} \alpha_i \mathbf{v}_i = \sum_{i=1}^{N_c} \mathbf{c}_i \tag{7}$$

where α_i, f_{sc}^i, f_{dc}^i, $\mathbf{v}_i$ and $\mathbf{c}_i$ denote the complex amplitude, spatial frequency, normalized Doppler frequency, space-time steering vector and return signal of the ith clutter patch, respectively. Accordingly, the received data snapshot $\mathbf{x}$, which consists of target, clutter and noise, can be expressed by

$$\mathbf{x} = \mathbf{s} + \mathbf{c} + \mathbf{n} \tag{8}$$

where $\mathbf{n}$ denotes the noise vector.

It is assumed that the clutter patches are mutually uncorrelated, and then the clutter covariance matrix (CCM) $\mathbf{R}_c$ can be given by [2]

$$\mathbf{R}_c = \sum_{i=1}^{N_c} E\left[\mathbf{c}_i \mathbf{c}_i^H\right] \tag{9}$$

where $[\cdot]^H$ is the conjugate transpose operation, and $E[\cdot]$ indicates the expectation operator. The noises are also assumed to be mutually uncorrelated, and then the noise covariance matrix $\mathbf{R}_n$ can be described as

$$\mathbf{R}_n = \sigma_n^2 \mathbf{I}_{MN} \tag{10}$$

where σ_n^2 is the noise power, and $\mathbf{I}_{MN}$ is an $MN \times MN$ identity matrix.

For the conventional STAP method, one tries to find a weight vector $\mathbf{w}$ for direct filtering the received data snapshot $\mathbf{x}$. The filter output y can be obtained by

$$y = \mathbf{w}^H \mathbf{x}. \tag{11}$$

The output SINR can also be given by

$$\text{SINR} = \frac{\left|\mathbf{w}^H \mathbf{s}\right|^2}{\mathbf{w}^H \mathbf{R}_I \mathbf{w}} \tag{12}$$

where $\mathbf{R}_I = \mathbf{R}_c + \mathbf{R}_n$ is the covariance matrix of the clutters plus noises in the assumption that the clutters are independent of the noises. To achieve an upper bound of performance in the maximum SINR sense, the optimum space-time weight vector $\mathbf{w}_{opt}$ can be given by [1]

$$\mathbf{w}_{opt} = \kappa \mathbf{R}_I^{-1} \mathbf{s} \tag{13}$$

where κ is an arbitrary scalar that does not alter the output SINR, and $\mathbf{R}_I^{-1}$ denotes the inverse matrix of $\mathbf{R}_I$. Then the upper bound of SINR, which is called as the optimum SINR, can be given by [1]

$$\mathrm{SINR}_{opt} = \mathbf{s}^H \mathbf{R}_I^{-1} \mathbf{s}. \tag{14}$$

Note that the conventional STAP method, which employs the covariance matrix of signal amplitudes to construct the filter weights and make them work on the received signal vector, is regarded as an SOS-type algorithm. Furthermore, in order to make use of more information contained in HOS, a new SP-STAP method takes signal powers as the inputs of the space-time filter. Then we will completely deduce the data model and the output SINR expression of SP-STAP in the follows.

Firstly, a new vector $\mathbf{z}$, which is named as power space-time steering vector, is defined by

$$\mathbf{z} = \mathbf{v}^* \otimes \mathbf{v} \tag{15}$$

where $[\cdot]^*$ denotes the conjugate operation. Accordingly, a new power vector of the target signal can be constructed as

$$\mathbf{s}_p = \mathrm{vec}\left(\mathbf{s}\mathbf{s}^H\right) = \sigma_t^2 \left(\mathbf{v}_t^* \otimes \mathbf{v}_t\right) = \sigma_t^2 \mathbf{z}_t \tag{16}$$

where $\mathrm{vec}(\cdot)$ denotes vectorization operation, $\mathbf{z}_t$ the power space-time steering vector of target, and $\sigma_t^2 = E[|\alpha_t|^2]$ the target signal power. Similarly, a new power vector corresponding to N_c clutter patches can also be written as

$$\mathbf{c}_p = \mathrm{vec}\left(\mathbf{R}_c\right) = \sum_{i=1}^{N_c} \sigma_i^2 \left(\mathbf{v}_i^* \otimes \mathbf{v}_i\right) = \sum_{i=1}^{N_c} \sigma_i^2 \mathbf{z}_i = \sum_{i=1}^{N_c} \mathbf{c}_{p,i} \tag{17}$$

where $\sigma_i^2 = E[|\alpha_i|^2]$ denotes the power of the ith clutter patch, and $\mathbf{z}_i$ and $\mathbf{c}_{p,i}$ are the power space-time steering vector and power vector of the ith clutter patch. Thus, the clutter power covariance matrix (CPCM) can be obtained by

$$\mathbf{R}_{cp} = \sum_{i=1}^{N_c} E\left[\mathbf{c}_{p,i}\mathbf{c}_{p,i}^H\right]. \tag{18}$$

Moreover, a new noise power vector is given by

$$\mathbf{n}_p = \mathrm{vec}\left(\mathbf{R}_n\right) = \sigma_n^2 \mathbf{1}_n \tag{19}$$

where $\mathbf{1}_n = [\mathbf{e}_1^T, \mathbf{e}_2^T, \ldots, \mathbf{e}_{MN}^T]^T$ and $\mathbf{e}_j$ $(j = 1, \ldots, MN)$ is a column vector of all zeros except for a one in the jth position. Meanwhile, the new noise power covariance matrix can also be acquired by

$$\mathbf{R}_{np} = E\left[\mathbf{n}_p\mathbf{n}_p^H\right]. \tag{20}$$

Consequently, the power snapshot of the received data for SP-STAP can be formulated as

$$\mathbf{x}_p = \mathbf{s}_p + \mathbf{c}_p + \mathbf{n}_p. \tag{21}$$

The SP-STAP filter output with respect to $\mathbf{x}_p$ is certainly given by

$$y_p = \mathbf{w}_p^H \mathbf{x}_p \tag{22}$$

where $\mathbf{w}_p$ is a new weight vector corresponding to the signal powers. The corresponding output SINR can be computed by

$$\mathrm{SINR}_p = \frac{\left|\mathbf{w}_p^H \mathbf{s}_p\right|^2}{\mathbf{w}_p^H \mathbf{R}_{Ip} \mathbf{w}_p} \tag{23}$$

where $\mathbf{R}_{Ip} = \mathbf{R}_{cp} + \mathbf{R}_{np}$. Then, the expression (23) can achieve its upper bound when $\mathbf{w}_p$ is substituted by the optimum weight vector $\mathbf{w}_{p\text{-}opt}$, which is given by

$$\mathbf{w}_{p\text{-}opt} = \gamma \mathbf{R}_{Ip}^{-1} \mathbf{s}_p \tag{24}$$

where γ is an arbitrary scalar which does not alter the output SINR, and $\mathbf{R}_{Ip}^{-1}$ is the inverse matrix of $\mathbf{R}_{Ip}$. Then the optimum output SINR of SP-STAP can be given by

$$\mathrm{SINR}_{p\text{-}opt} = \mathbf{s}_p^H \mathbf{R}_{Ip}^{-1} \mathbf{s}_p. \tag{25}$$

As is evident, SP-STAP depends on the new power vectors $\mathbf{s}_p$ and $\mathbf{c}_p$, which are obtained by the nonlinear preprocessing of taking the Kronecker product of the corresponding space-time steering vectors. Therefore, SP-STAP should be regarded as a HOS-type algorithm and also belong to a nonlinear processing technique.

3. NUMERICAL SIMULATION

In this section, a set of comparative simulations between SP-STAP and the conventional STAP are carried out. The clairvoyant SINR-loss, which is generally defined by comparing the clutter-limited performance to the noise-limited capability, is employed as one of the primary evaluated criteria in this work. The clutter-limited performance is represented by the optimum SINR, while the noise-limited capability is represented by the signal-to-noise ratio (SNR). Then, the clairvoyant SINR-loss of the conventional STAP can be written by

$$\mathrm{SINR}_{\mathrm{loss}} = \frac{\mathrm{SINR}_{opt}}{\mathrm{SNR}} = \frac{\mathbf{s}^H \mathbf{R}_I^{-1} \mathbf{s}}{MN\frac{\sigma_t^2}{\sigma_n^2}}. \tag{26}$$

On the other hand, the clairvoyant SINR-loss of the SP-STAP can be given by

$$\mathrm{SINR}_{p\text{-loss}} = \frac{\mathrm{SINR}_{p\text{-}opt}}{\mathrm{SNR}_p} = \frac{\mathbf{s}_p^H \mathbf{R}_{Ip}^{-1} \mathbf{s}_p}{(MN)^2\frac{\sigma_t^4}{\sigma_n^4}}. \tag{27}$$

In the next simulations, the number of array elements is $N = 6$, and the number of transmitting pulses in a CPI is $M = 6$. The carrier frequency is 10 GHz, and the pulse repetition frequency (PRF) is 600 Hz, i.e., $T = 1/\mathrm{PRF}$. To avoid temporal aliasing, β is equivalent to 1. A hypothetic target is located at angle 33.5 degrees, and its normalized Doppler frequency is assumed to be -0.21. The noise power is normalized to 0 dB, and the clutter-to-noise ratio (CNR) is 40 dB for all clutter patches.

Figures 1(a) and 1(b) show the normalized space-time filter output responses of the conventional STAP and SP-STAP, respectively. It can be seen from the two pictures that both of the methods can form a deep notch at the clutter ridge and make a maximum peak at the target position. Nevertheless, the side-lobe levels of SP-STAP are lower than those of the conventional STAP. For the sake of explicit display, the spatial and normalized Doppler frequency principal plane cuts at the target location are illustrated in Figures 2(a) and 2(b), respectively. It is shown that the target main beam of SP-STAP is even narrower than that of the conventional STAP.

Next, fixing the angle at $\theta = 0°$ the clairvoyant SINR-loss curves of SP-STAP and the conventional STAP are shown in Figure 3, respectively. It is demonstrated that the clutter notch width of SP-STAP is smaller than that of the conventional STAP, which means that the MDV performance of SP-STAP is better than that of the conventional STAP. Therefore, SP-STAP has a greater potential to detect lower velocity target.

As aforementioned before, the SP-STAP method can increase effective system DOFs to improve the MDV performance without changing hardware architecture or adding transmitting pulses. To verify this standpoint strictly, we compare the SINR-loss of SP-STAP with that of the conventional STAP indifferent cases of array elements and transmitting pulses. The SINR-loss curve of SP-STAP with ($N = 6$, $M = 6$), and the SINR-loss curves of the conventional STAP with ($N = 6$, $M = 6$), ($N = 7$, $M = 7$), ($N = 8$, $M = 8$) and ($N = 9$, $M = 9$) are all presented in Figure 4. Figure 4(b) is the closeup

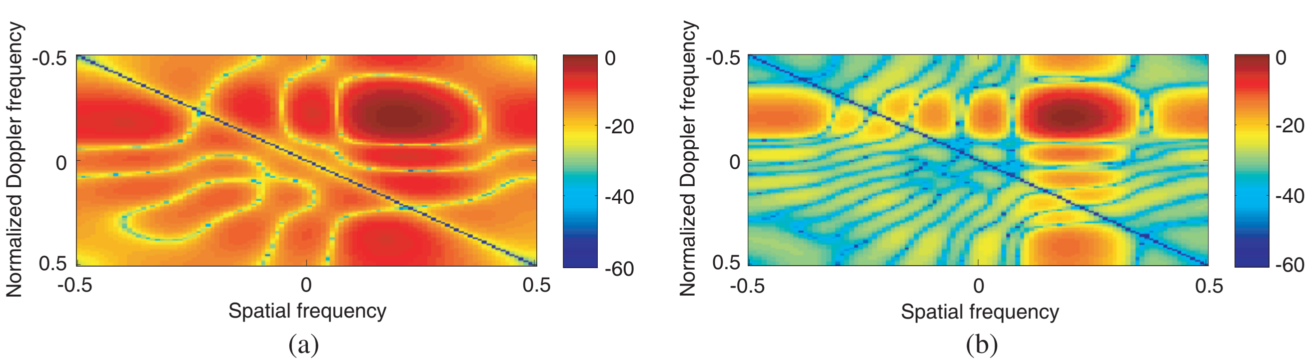

Figure 1. Normalized space-time filter output responses. (a) The conventional STAP, (b) SP-STAP.

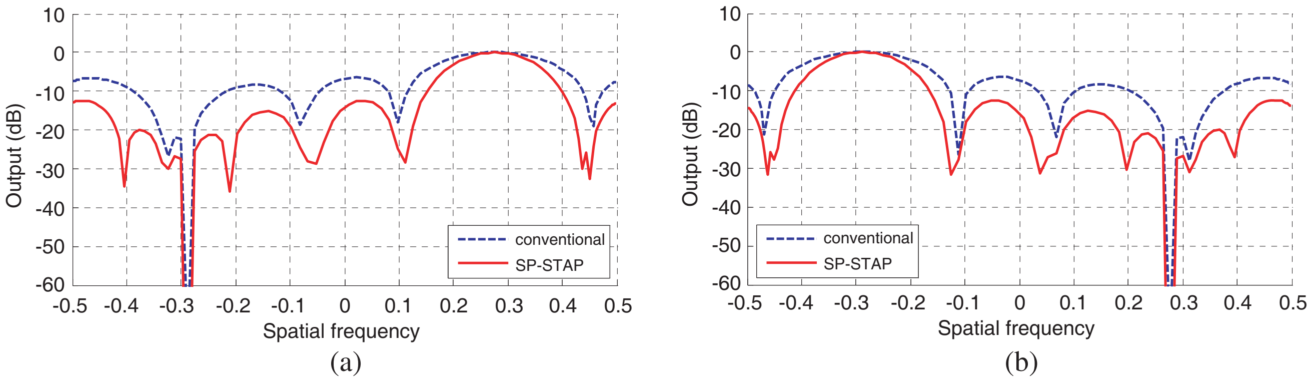

Figure 2. Principal plane cuts at the target location. (a) Spatial cuts, (b) Doppler cuts.

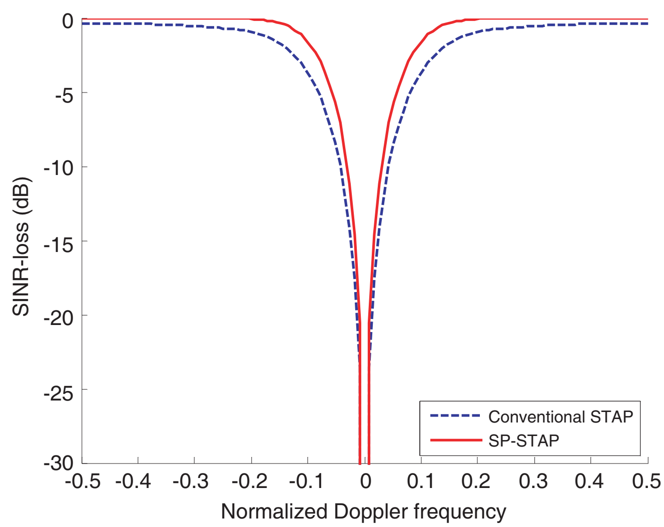

Figure 3. The clairvoyant SINR-loss.

of Figure 4(a). It is well known that the apparent system DOFs of STAP can be represented by the number of array elements and transmitting pulses. Therefore, as shown in Figure 4, the clutter notch width of the conventional STAP decreases with the increase of array elements and transmitting pulses. In other words, the MDV performance improves with the increase of DOFs. On the other hand, we can see that the clutter notch width of SP-STAP with ($N = 6$, $M = 6$) is still smaller than that of the conventional STAP with ($N = 6$, $M = 6$). Meanwhile, it is almost the same as the clutter notch width of the conventional STAP with ($N = 9$, $M = 9$). The result can be owing to the increasing effective system DOFs caused by the HOS characteristics of the SP-STAP method. Consequently, it has been demonstrated again that the SP-STAP method is able to get higher effective system DOFs and better MDV performance than the conventional STAP method under the condition of the same number of array elements and transmitting pulses.

Finally, we explore the SINR-loss performance in ICM case, and the results are presented in Figure 5. It is shown that, because of the deleterious effect of ICM, the output SINR of the conventional STAP deteriorates more seriously than that of SP-STAP. The extended clutter notch of the conventional STAP is still wider than that of SP-STAP. Obviously, we can see that, under the influence of ICM, SP-STAP can keep lower SINR loss and better MDV performance than the conventional STAP.

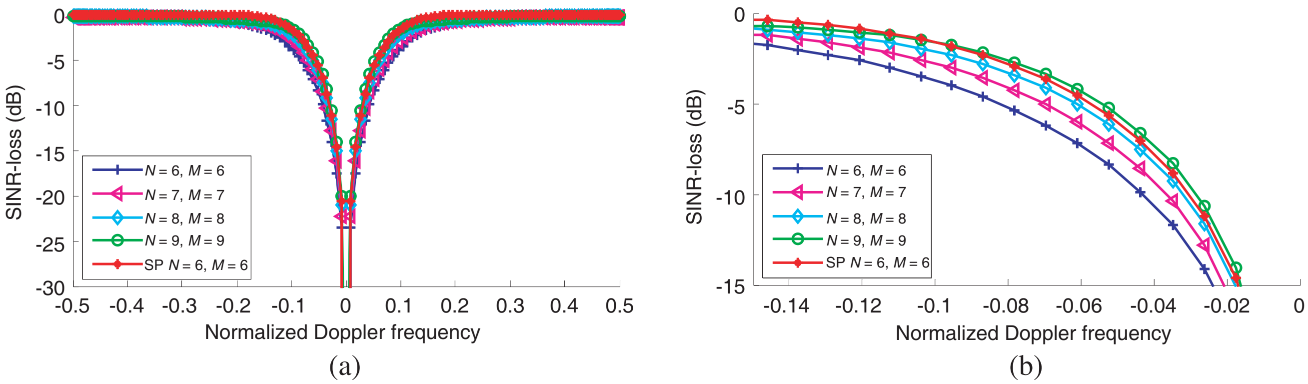

Figure 4. The clairvoyant SINR-loss for different system DOFs. (a) Original curves, (b) close up of original curves.

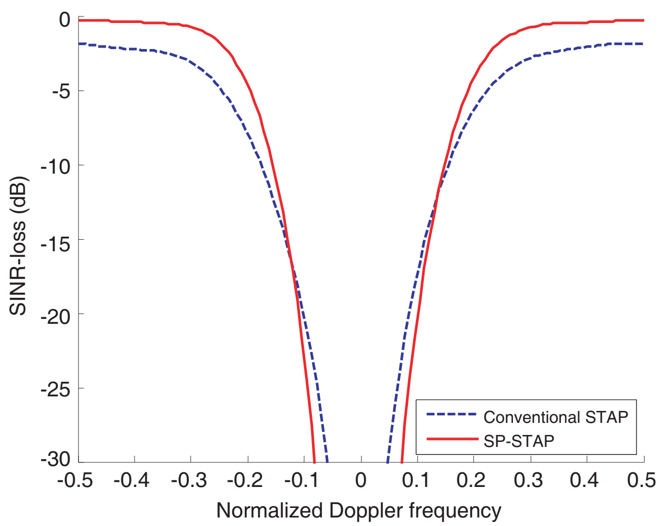

Figure 5. The output SINR-loss with ICM.

4. CONCLUSION

In this work, a further research on space-time adaptive processing with respect to signal powers (SP-STAP) based on the phased array radar with ULA has been carried out. After deducing the general data model and output SINR expression of SP-STAP, the comparative simulations between the conventional STAP and SP-STAP have been implemented. We mainly focus on the aspects of the output response, the output SINR, the MDV performance and the effect of ICM.The simulation results demonstrate that, under the condition of the same number of array elements and transmitting pulses, SP-STAP can increase the effective system DOFs without changing hardware framework. Furthermore, it can also achieve narrower target main beam, lower sidelobe levels, better MDV performance and less deleterious effect of ICM than the conventional STAP. The performance of SP-STAP in the non-homogenous clutter environments is still a topic for the future studies. Additionally, considering that the grating lobes are possible to be avoided or suppressed using some particular algorithms [8, 24], the SP-STAP method can also be applied to sparse array to further increase the effective system DOFs. Particularly, for the airborne or spaceborne application sensitive to the weight of load, we think that it is valuable to further analyze the performance of SP-STAP based on the sparse array which is constructed through a compressive sensing (CS) strategy with minimum possible number of elements [24].

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