

3D Computation of Electric Field by a Stochastic Method

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Abstract—This paper deals with the calculation of electric field in a copper piece of cubic shape which is submitted to a sinusoidal magnetic field. This 3D problem is set into equation and solved by means of two different approaches. A stochastic method for 3-D electric field computations is presented and compared to a finite element method. The main goal of this paper is to compare these two methods on a classical problem putting forward the advantages of the chosen method. First of all, we present the problem modelling. Then, the Monte-Carlo method used to solve 3D time dependent problem is described and is compared to the finite element method, in the last part.

1. INTRODUCTION

In this paper, we propose a novel technique for a 3-D computation of electro-magnetic field, which is based on a stochastic method that has several advantages, such as calculating only a single or several arbitrarily selected points in the studied domain. In practice, we do not need to have an exhaustive information about the analyzed process at all space points and all time moments. Moreover, frequently only a single particular point of the domain is of interest. Unfortunately, a process described by parabolic or elliptic differential equations is highly correlated in space. Because of these correlations, for all the deterministic methods (finite elements, volume elements, finite differences) the solution reduces to a matrix equation, which determines it at all points together. In other words, the solution may be obtained only in the whole domain. Contrarily to them, random walk methods are capable of calculating the solution only at a single point of the spatio-temporal domain, despite the existing high spatial correlations [1–3].

As the computation of the entire domain is no longer necessary, a random walk method is much less time-consuming than any other deterministic method.

The method is tested on the example of a problem of 3-D electric field computation of a copper cube surrounded by air immersed in a time-dependent sinusoidal magnetic field.

We choose to solve this problem using an electric field formulation, indeed, it is not a usual way; however, some studies can be found [4–6].

Before solving it numerically, we have solved two theoretical problems that enabled us to reduce the system of governing equations and boundary conditions.

First of all, we have reduced the system of vector Maxwell equations for three vectors \mathbf{J} , \mathbf{E} , \mathbf{B} (electrical current, electrical field and magnetic flux density respectively) to a scalar parabolic equation for a single component of the electric field vector E_x . This became possible, due to some symmetry that is proper to the solution. Namely, we have proven mathematically that the components E_x and E_z are anti-symmetric with respect to the operation of permutation of the arguments.

Secondly, we have found the appropriate boundary conditions within a dielectric formulated in terms of the electric field \mathbf{E} . Within the traditional approach, the governing equations and boundary conditions are formulated in terms of the electric current \mathbf{J} . Such a formulation may be easily transformed to the

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formulation in terms of \mathbf{E} within a conductor, since \mathbf{E} and \mathbf{J} are related through the Ohm's law (3). However in a dielectric, the reconstruction of \mathbf{E} from \mathbf{J} is impossible, as $\mathbf{J} \equiv 0$. We have found the boundary conditions in terms of \mathbf{E} from the analytical solution of the problem in a dielectric immersed in magnetic field and additional orthogonality conditions.

The obtained diffusion problem for E_x has been solved by the random walk method. The efficiency of the stochastic method is illustrated by comparing the results obtained with the finite element method.

2. PROBLEM FORMULATION. REDUCTION TO A DIFFUSION EQUATION

We analyse the problem of the variation in time of the magnetic field induced in a three-dimensional domain occupied by air perturbed by a small conductor placed in the domain centre. First of all, we reduce the equations that govern electrical and magnetic fields to a single vector equation for the vector of electric field \mathbf{E} . To formulate the boundary conditions in terms of the electric field, we derive the analytical formulae that are the solution of the problem of a steady-state non-perturbed field in a dielectric immersed in magnetic field. Then, we prove that the solution is characterized by a symmetry, which enables us to reduce the problem for \mathbf{E} to a single scalar equation for one component of the electrical vector E_x .

2.1. Governing Equations

We consider a sinusoidal magnetic flux density, characterized by the vector of magnetic tension $\mathbf{B} = \{B_x, B_y, B_z\}$, in the infinite space \mathbf{R}^3 . According to Faraday's law of induction, it generates an electric field denoted $\mathbf{E} = \{E_x, E_y, E_z\}$ in the overall space. We analyse how the field \mathbf{E} can be perturbed by a bounded conducting domain Ω_c , for instance a piece of copper that is placed in this field as depicted in Fig. 1. Therefore, in our study, we solve this problem by the calculation of electric field [1, 2]. Parameters of the problem are given in Table 1.

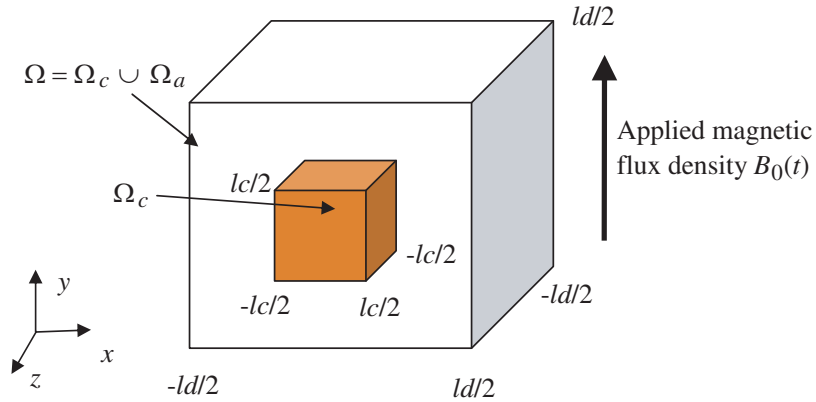


Figure 1. Geometric description of the problem.

Table 1. Problem parameters.

Symbol	Quantity	Value
Ω_c	Conducting subdomain (copper)	
Ω_a	Air subdomain	
$\partial\Omega_a$	Interface between the copper piece and air	
$\Omega = \Omega_a \cup \Omega_c$	Whole domain	
$\partial\Omega$	Boundary of the whole domain	
L	Side dimension of the whole cubic domain	1 cm
L_c	Side dimension of the copper cube	0.5 cm

The process is governed by Maxwell's equations:

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \tag{1}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{2}$$

$$\mathbf{J} = \sigma \mathbf{E} \tag{3}$$

$$\nabla \cdot \mathbf{J} = 0 \tag{4}$$

where σ is the electrical conductivity of the copper cube [S/m], assumed to be constant; μ_0 is the magnetic permeability of the free space [H/m], and $\mathbf{J} = \{J_x, J_y, J_z\}$ is the induced electrical current density [A/m²]. Eq. (3) is the Ohm's law, while Eq. (4) is the conservation of the electrical current.

System in Eqs. (1)–(4) may be converted into a single vector equation for the electrical field \mathbf{E} :

$$\Delta \mathbf{E} = \mu_0 \sigma \partial_t \mathbf{E}, \quad x, y, z \in \Omega; \quad t > 0 \tag{5}$$

Indeed, the curl of Eq. (1) yields: $\nabla \times \nabla \times \mathbf{E} = -\partial_t (\nabla \times \mathbf{B})$, or $\nabla (\nabla \cdot \mathbf{E}) - \Delta \mathbf{E} = -\partial_t (\nabla \times \mathbf{B})$ according to the property of vector operators. Substituting Eq. (2) we obtain $\nabla (\nabla \cdot \mathbf{E}) - \Delta \mathbf{E} = -\mu_0 \partial_t \mathbf{J}$, which yields $\Delta \mathbf{E} = \mu_0 \partial_t \mathbf{J}$, since $\nabla \cdot \mathbf{E} = 0$, which follows from Eqs. (3) and (4). Substituting Eq. (3) we obtain Eq. (5).

Equation (5) is considered in the domain $\Omega \subseteq \mathbb{R}^3$, which consists of the cubic subdomain of copper Ω_c surrounded by the non-conducting subdomain Ω_a (air), as depicted in Fig. 1.

The magnetic flux density consists of the source field \mathbf{B}_s and the field \mathbf{B}' induced by the electrical current:

$$\mathbf{B} = \mathbf{B}_s + \mathbf{B}' \tag{6}$$

The source field \mathbf{B}_s is assumed to be mono-dimensional and oriented along the axis y . It fluctuates periodically in time and is uniform in space, so that:

$$\mathbf{B}_s = B_0(t) \mathbf{e}_y \quad B_0(t) = B_{\max} \cos(\omega t) \tag{7}$$

where ω is the angular frequency.

The initial condition is

$$\mathbf{E} = 0, \quad t = 0 \tag{8}$$

2.2. Boundary Conditions

We assume that the boundary of the domain Ω is far away enough from the copper cube and therefore is not disturbed by it. Such a non-disturbed field is described by Equations (1)–(4) in which $\sigma = 0$ in the overall space. The induced magnetic flux density \mathbf{B}' is then zero, so $\mathbf{B} = \mathbf{B}_s$ in Eq. (6). Substituting Eqs. (6) and (7) into Eq. (1), one obtains:

$$\nabla \times \mathbf{E} = -d_t B_0 \mathbf{e}_y, \quad \text{in } \mathbb{R}^3 \tag{9}$$

Instead of boundary conditions for this equation we should use some orthogonality conditions. Indeed, as the source magnetic flux density in Eq. (7) is directed along the axis y and is uniform along x and z , then the induced electric field is expected to be orthogonal to y , and its isolines should be circular in the plane (x, z) around the axis y . Such conditions are satisfied for the following analytical solution of Eq. (9).

$$E_x = -\frac{z}{2} d_t B_0 + C_1, \quad E_y \equiv C_2, \quad E_z = \frac{x}{2} d_t B_0 + C_3 \tag{10}$$

To be consisted to the initial condition in Eq. (8), the constant values C_1 , C_2 and C_3 must be zero.

Indeed, solution of Eq. (10) determines a field orthogonal to y and gives the circles in the plane (x, z) for any constant value of E : $E^2 = E_x^2 + E_z^2 = \frac{1}{4} d_t B_0 (x^2 + z^2)$.

Therefore, we apply relationships in Eq. (10) as the boundary conditions for problem in Eq. (5) on each face of the domain Ω . In summary, we obtain the following problem to solve:

$$\begin{cases} \Delta \mathbf{E} = \lambda \partial_t \mathbf{E}, & x, y, z \in \Omega = \Omega_a \cup \Omega_c \\ E_x = -\frac{z}{2} \partial_t B_0(t), \quad E_y = 0, \quad E_z = \frac{x}{2} \partial_t B_0(t) & \text{on the boundary } \partial\Omega \\ \mathbf{E}|_{t=0} = 0 \end{cases} \tag{11}$$

with $\lambda = \begin{cases} \sigma\mu_0, & \text{in } \Omega_c \\ 0, & \text{in } \Omega_a \end{cases}$.

This problem should be completed with the conditions of the continuity through the interface Γ between Ω_a and Ω_c :

$$[\mathbf{E}] = 0, \quad \left[\frac{\partial \mathbf{E}}{\partial n} \right] = 0 \quad ([\cdot] \text{ means a jump}) \quad (12)$$

Remark:

Instead of the boundary conditions in Eq. (11), one can use more general conditions that result directly from Eq. (9):

$$\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right)_{\partial\Omega} = 0, \quad \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right)_{\partial\Omega} = 0, \quad \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right)_{\partial\Omega} = -\partial_t B_0 \quad (13)$$

Conditions in Eq. (10) are a particular case of Eq. (13).

2.3. Symmetry of the Solution — Reduction to a Single Scalar Equation

The solution of problem in Eq. (11) satisfies the following conditions of symmetry, for any moment t :

$$E_z(x, y, z) = -E_x(z, y, x) \quad (14)$$

$$E_y \equiv 0 \quad (15)$$

So, it is unnecessary to calculate all the components of the field \mathbf{E} , but only one component E_x is sufficient.

Proof:

1. Let us consider components $E_x(x, y, z, t)$ and $E_z(x, y, z, t)$ of vector \mathbf{E} and permute the arguments x and z . Let us designate the functions with permuted arguments x and z as: $\bar{E}_x = E_x(z, y, x, t)$, $\bar{E}_z = E_z(z, y, x, t)$ and $\bar{E}_y = E_y(z, y, x, t)$. Problem in Eq. (11) for E_x and E_z is:

$$\left\{ \begin{array}{l} \partial_{xx}^2 E_x + \partial_{yy}^2 E_x + \partial_{zz}^2 E_x = \lambda \partial_t E_x, \quad \text{in } \Omega \\ E_x|_{\partial\Omega} = -\frac{z}{2} d_t B_0, \quad E_x|_{t=0} = 0 \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \partial_{xx}^2 E_z + \partial_{yy}^2 E_z + \partial_{zz}^2 E_z = \lambda \partial_t E_z, \quad \text{in } \Omega \\ E_z|_{\partial\Omega} = \frac{x}{2} d_t B_0, \quad E_z|_{t=0} = 0 \end{array} \right. \quad (*)$$

For functions \bar{E}_x and \bar{E}_z we obtain from Eq. (11) by permuting the arguments $x \leftrightarrow z$:

$$\left\{ \begin{array}{l} \partial_{zz}^2 \bar{E}_x + \partial_{yy}^2 \bar{E}_x + \partial_{xx}^2 \bar{E}_x = \lambda \partial_t \bar{E}_x, \quad \text{in } \Omega \\ \bar{E}_x|_{\partial\Omega} = -\frac{x}{2} d_t B_0, \quad \bar{E}_x|_{t=0} = 0 \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \partial_{zz}^2 \bar{E}_z + \partial_{yy}^2 \bar{E}_z + \partial_{xx}^2 \bar{E}_z = \lambda \partial_t \bar{E}_z, \quad \text{in } \Omega \\ \bar{E}_z|_{\partial\Omega} = \frac{z}{2} d_t B_0, \quad \bar{E}_z|_{t=0} = 0 \end{array} \right. \quad (**)$$

Comparing (*) and (**), we obtain: $\bar{E}_z = -E_x$ and $\bar{E}_x = -E_z$, which proves Eq. (14).

2. To prove Eq. (15), first of all we prove the following property:

$$E_y = E_y(y, t) \quad (16)$$

Indeed, the following results from conditions in Eq. (13) at the domain boundary and Eq. (14):

$$\partial_x E_y = \partial_y E_x \quad \text{and} \quad \partial_z E_y = \partial_y E_z = -\partial_y \bar{E}_x \quad (17)$$

Let us apply the operation of permutation $x \leftrightarrow z$ to the first relationship in Eq. (17), then we obtain: $\partial_z \bar{E}_y = \partial_y \bar{E}_x$. Comparing this with the second relationship in (17), we deduce that $\partial_z \bar{E}_y = -\partial_z E_y$, or $\partial_z (\bar{E}_y + E_y) = 0$, which means that $\bar{E}_y + E_y$ does not depend on z . But the function $f = \bar{E}_y + E_y$ is invariant with respect to the permutation $x \leftrightarrow z$ (indeed, $\bar{f} = E_y + \bar{E}_y = f$). Then it does not depend on x anymore. We then obtain Eq. (16).

3. For E_y , we obtain then the following problem from Eq. (11) and (16):

$$\begin{aligned} \partial_{yy}^2 E_y &= \lambda d_t E_y, \quad \text{in } \Omega \\ E_y|_{\tau=0} &= 0, \quad E_y|_{\partial\Omega} = 0 \end{aligned}$$

which is a homogeneous problem for a homogeneous equation. Its solution is zero, which proves Eq. (15).

Therefore, the resolution of the differential system of three equations (11) is reduced to a single scalar equation for E_x :

$$\begin{cases} \Delta E_x = \lambda \partial_t E_x, & \text{in } \Omega \\ E_x|_{\partial\Omega} = -\frac{z}{2} d_t B_0(t), \\ E_x|_{t=0} = 0 \end{cases} \quad (18)$$

Once E_x is computed then E_z can be deduced by the permutation of the arguments in Eq. (14). Thus, the 3D vector problem can be reduced to a 3D scalar problem, which is a great advantage when solving numerically.

3. SOLUTION BY A STOCHASTIC METHOD

Stochastic methods are applied to solve various kinds of physical problems but are very rarely used in the electrical engineering [1–3]. However, the diffusive origin of Equations (11) or (18) makes in evidence the opportunity to apply the Monte-Carlo random walk methods.

3.1. Stochastic Resolution of PDE

The principle of a Monte Carlo method is based on the analogy between a parabolic differential equation like Eq. (18) and the random walk process. Let us consider the Dirichlet problem:

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} &= \frac{\partial u}{\partial t}, & x \in \Omega_T &= (0 < x < L) \times (0 < t) \\ u|_{x=0, x=L} &= f_1(t), & u|_{t=0} &= f_2(x) \end{aligned} \quad (19)$$

Boundary and initial conditions can be rewritten in the form:

$$u|_{\Gamma} = f(x, t) \quad (19')$$

where Γ is the boundary of the spatio-temporal domain Ω_T ; $f = f_1$ in $x = 0, x = L$ and $f = f_2$ in $t = 0$. Functions f_1 and f_2 are given. The discretisation yields:

$$\frac{u_i^j - u_i^{j-1}}{\Delta t} = \frac{u_{i+1}^j - 2u_i^j + u_{i-1}^j}{\Delta x^2}$$

where Δx and Δt are the space and time steps, and i and j are the numbers of discrete points in space and time. This leads to an algebraic formula for u_i^j :

$$u_i^j = p_{x+} u_{i+1}^j + p_{x-} u_{i-1}^j + p_{t-} u_i^{j-1} \quad (20)$$

where

$$p_{x+} = p_{x-} = \frac{1}{2 + \alpha}, \quad p_{t-} = \frac{\alpha}{2 + \alpha}, \quad \alpha = \frac{\Delta x^2}{\Delta t}. \quad (21)$$

As $p_{x-} + p_{x+} + p_{t-} = 1$, they may be interpreted as the probabilities of some random events.

Such a random process may be defined as random walking of a virtual particle in the spatio-temporal domain Ω_T from an arbitrary point $M_i^j = (x_i, t^j)$. When it reaches the boundary Γ , it stops. It can move to the left, to the right and to the past with the probabilities p_{x-} , p_{x+} and p_{t-} respectively. A transition to the future is prohibited. Let ξ be a random function that takes the value $f(Q)$ once the walker reaches point Q on the boundary and is zero if it reaches any other point. Then the solution of the problem in Eq. (19) is the average value of the function ξ over the boundary Γ .

To prove this, let us introduce the probability $P(M_i^j, Q)$ of reaching point $Q \in \Gamma$ from an arbitrary point M_i^j . Then the average value of ξ over the boundary is:

$$v(M_i^j) = \sum_{Q \in \Gamma} f(Q) P(M_i^j, Q) \quad (22)$$

The walker can reach point Q from M_i^j in three ways: it displaces to one of three neighbouring points $M_{i-1}^j = (x_{i-1}, t^j)$, $M_{i+1}^j = (x_{i+1}, t^j)$, or $M_i^{j-1} = (x_i, t^{j-1})$ and after this reaches point Q . Then $P(M_i^j, Q)$ is calculated as:

$$P(M_i^j, Q) = p_{x-}P(M_{i-1}^j, Q) + p_{x+}P(M_{i+1}^j, Q) + p_{t-}P(M_i^{j-1}, Q) \quad (23)$$

At the boundary, for two points Q and Q' :

$$P(Q', Q) = \begin{cases} 1, & Q' = Q \\ 0, & Q' \neq Q \end{cases} \quad (24)$$

where the latter relationship means that the particle does not move longer if it is already on the boundary.

Using Eq. (23) we deduce that $v(M_i^j)$ satisfies the following relationship in the domain Ω_T :

$$v(M_i^j) = \sum_{Q \in \Gamma} \left[p_{x-}f(Q)P(M_{i-1}^j, Q) + p_{x+}f(Q)P(M_{i+1}^j, Q) + p_{t-}f(Q)P(M_i^{j-1}, Q) \right]$$

or

$$v(M_i^j) = p_{x-}v(M_{i-1}^j) + p_{x+}v(M_{i+1}^j) + p_{t-}v(M_i^{j-1}) \quad (25)$$

which is the same as Eq. (20).

Using Eq. (22), we deduce that $v(M_i^j)$ satisfies the condition: $v(Q) = f(Q)$, for any point Q on the boundary Γ , which is the boundary condition in Eq. (19)'.

Thus, $v(M_i^j)$ is the solution u of the problem (19).

In practice, the probability $P(M_i^j, Q)$ may be calculated as follows. If N particles are launched from point M_i^j and N_Q of them reach point Q on the boundary Γ , then the probability $P(M_i^j, Q)$ may be calculated as:

$$P(M_i^j, Q) \rightarrow \frac{N_Q}{N}, \quad \text{if } N \rightarrow \infty \quad (26)$$

Using Eqs. (24) and (23), we obtain the definite formula for U_i^j :

$$u_i^j = \sum_{Q \in \Gamma} \frac{f(Q)N_Q}{N} \quad (27)$$

The Monte Carlo method consists of generating a set of random walks starting from an arbitrary point (i, j) and calculating u_i^j using Eq. (27). This is illustrated in Fig. 2, where three particles are reaching the boundary Γ .

According to the theory of numerical schemes, the stability conditions require that parameter

$$\alpha < 1$$

The high advantage of this method is that it enables to calculate the solution only at some arbitrarily selected points both in space as in time, whereas most of the widely used numerical methods in electrical engineering (such as finite differences, finite volumes, or finite elements) require solving in the entire domain and at each time step.

3.2. How Many Random Walks Should We Retain?

The solution obtained is the average of a high number of random walks. The question is how many Brownian particles should we run? Let U_n be the solution associated with the n^{th} walk, and \hat{U}_N be the average for N random walks:

$$\hat{U}_N = \frac{1}{N} \sum_{n=1}^N u_n$$

The central limit theorem indicates that \hat{U}_N follows a Gaussian stochastic law with u_N the mathematical expectation and $\sqrt{\sigma^2/N}$ the standard deviation (σ^2 the variance), if N is sufficiently high. The

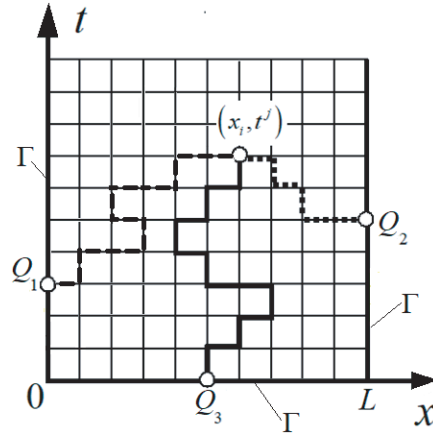


Figure 2. Example of a spatio-temporal random walk of three particles in the domain $\partial\Omega_T$ from point (x_i, t^j) to points Q_k at the boundary Γ .

probability that u_N belongs to the interval $[\hat{U}_N - 2\sqrt{\sigma^2/N}, \hat{U}_N + 2\sqrt{\sigma^2/N}]$ is 95%. As N increases, the range of this 95% confidence interval tends to zero. Therefore, we obtain not only an estimate of the solution but also an estimate of the error by using the value of the standard deviation.

3.3. Application to the 3D Problem in Eq. (18)

In this part, we apply the Monte Carlo method described above to solve the problem in Eq. (18). The numerical scheme for Eq. (18) is

$$E_{ijk}^n = \frac{\alpha}{6\alpha + \lambda} \left(E_{i+1,jk}^n + E_{i-1,jk}^n + E_{i,j+1,k}^n + E_{i,j-1,k}^n + E_{ij,k+1}^n + E_{ij,k-1}^n + E_{ijk}^{n-1} \frac{\lambda}{\alpha} \right) \quad (28)$$

We assume that $\Delta x = \Delta y = \Delta z$, then $\alpha = \frac{\Delta x^2}{\Delta t}$. The subscripts i, j and k correspond to the spatial coordinates and n to the time coordinate. This system defines seven probabilities of random walk in 3D space, which is summarized in Table 2.

Table 2. Probabilities for the Monte Carlo Method.

Probabilities	Magnetic problem	
	Air	Cooper
p_{x+}	1/6	$\frac{\alpha}{6\alpha + \mu_0\sigma}$
p_{x-}	1/6	$\frac{\alpha}{6\alpha + \mu_0\sigma}$
p_{y+}	1/6	$\frac{\alpha}{6\alpha + \mu_0\sigma}$
p_{y-}	1/6	$\frac{\alpha}{6\alpha + \mu_0\sigma}$
p_{z+}	1/6	$\frac{\alpha}{6\alpha + \mu_0\sigma}$
p_{z-}	1/6	$\frac{\alpha}{6\alpha + \mu_0\sigma}$
p_{t-}	0	$\frac{\mu_0\sigma}{6\alpha + \mu_0\sigma}$

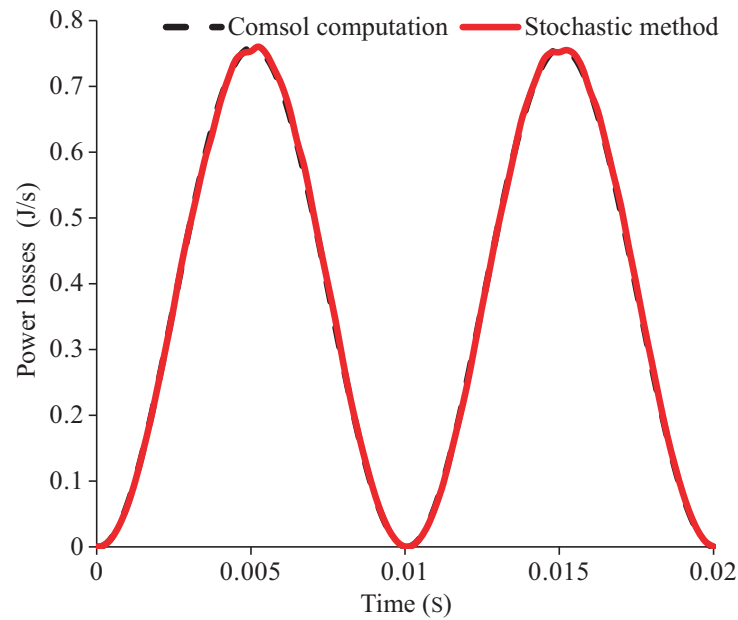
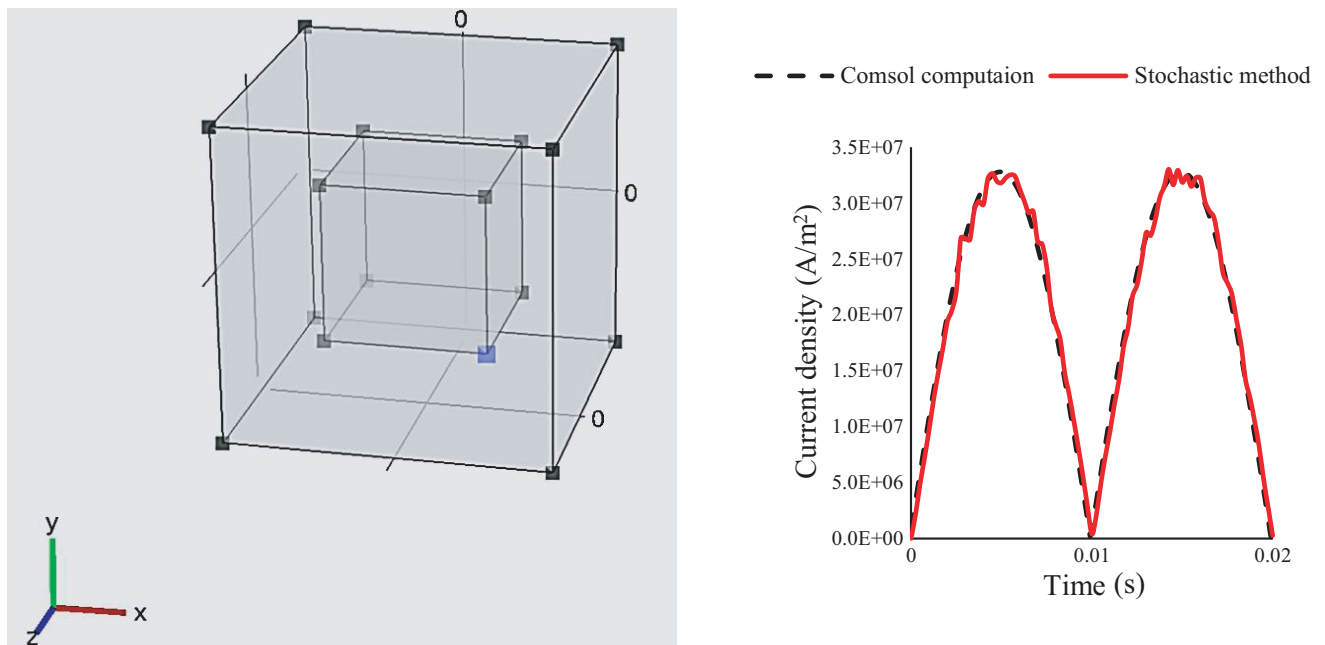
At the interface between air and copper we use the continuity of the tangential electric field component in Eq. (12). Thus, for the direction perpendicular to the interfaces, we have the same probabilities equal to 1/2: $p_{x+} = p_{x-}, p_{y+} = p_{y-}, p_{z+} = p_{z-}$.

4. STUDY OF A COPPER CUBE IN A TIME-DEPENDENT MAGNETIC FIELD

The main characteristics of the studied problem are summarized in Table 3.

Table 3. Characteristics of the problem.

Parameters	Value
Electric conductivity σ (S/m)	$59 \cdot 10^6$
Frequency f (Hz)	50
Maximum flux density B_{\max} (T)	1.0

**Figure 3.** Calculated losses in the cooper piece versus time.**Figure 4.** Norm of induced current density at a point (blue) of the cooper piece versus time.

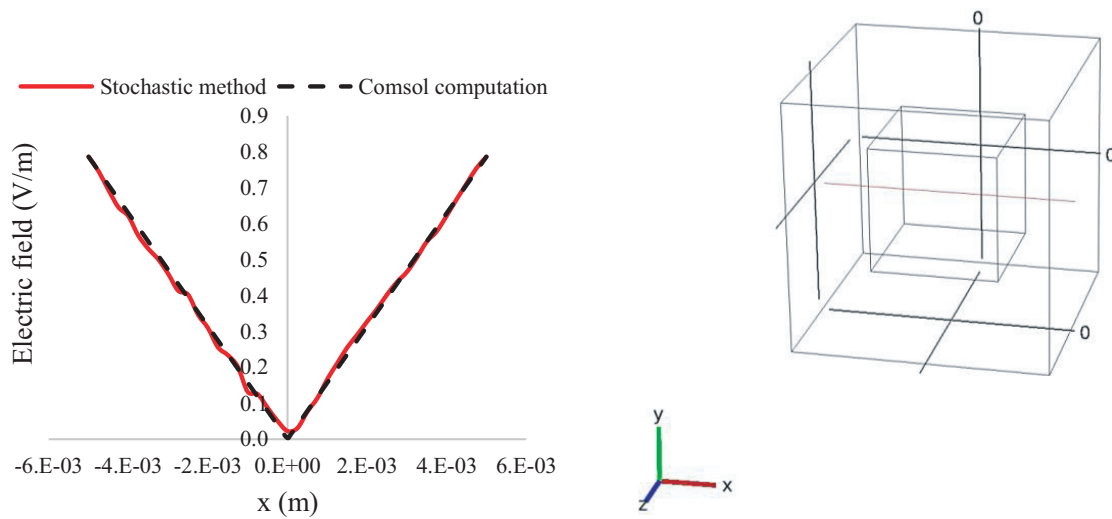


Figure 5. Norm of electric field along the red line, at $t = 0.005$ s.

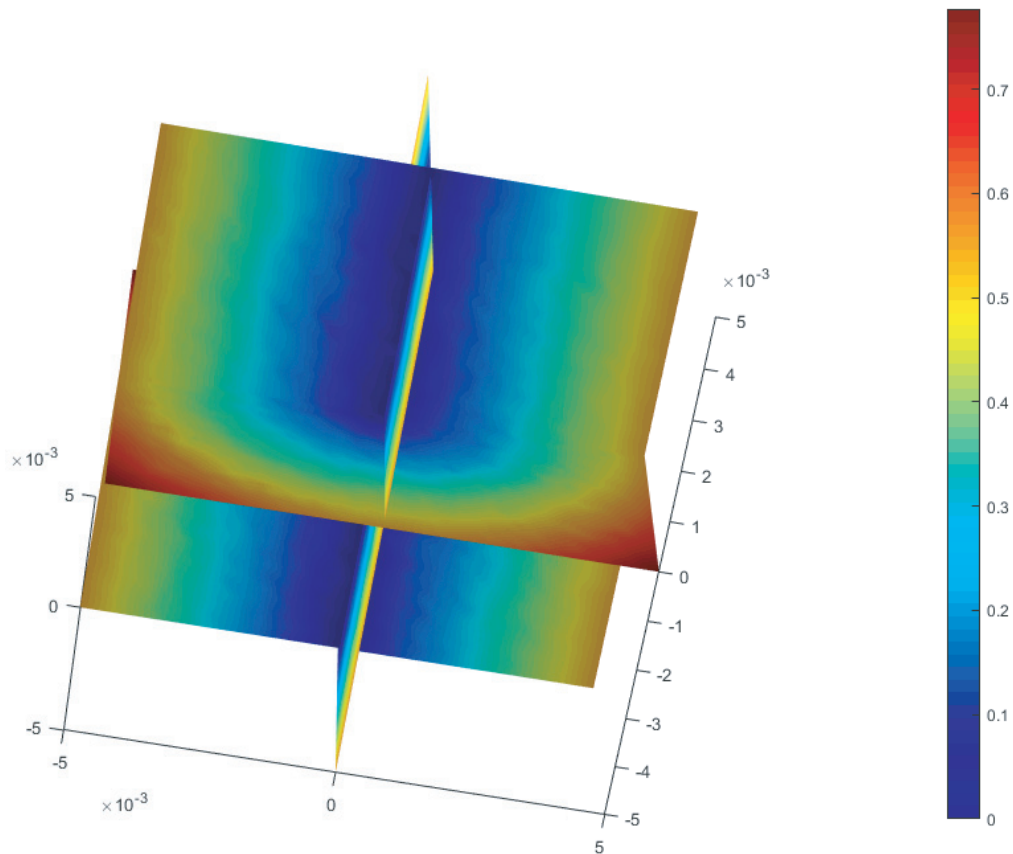


Figure 6. Norm of the magnetic flux density in Ω at 0.005 s.

In order to study the efficiency of the stochastic method, the results have been compared with those obtained from 3D simulations with COMSOL[®] software. The numerical method used in COMSOL[®] is the finite elements with a fully implicit discretization in time, which is well adapted to parabolic PDE. We calculated the problem in Eq. (18).

The first result that we present is the calculus of the joule power in the copper piece. The power losses are computed using the following relationship:

$$P = \iiint \mathbf{E} \cdot \mathbf{J} \, d\vartheta \quad (29)$$

Fig. 3 highlights a good agreement between the two methods.

As shown above, the stochastic method allows calculating the electric field at a single point of the domain. This ensures high economy of computing time compared to other numerical methods, in which the solution must be obtained in the whole domain. Fig. 4 presents the computed results of the induced current density in a corner at point $(x, y, z) = (L_c, L_c, L_c)$. The calculus time is a couple of seconds. It is one of the great advantages of the stochastic method. But, this calculus on a single point implies that all the properties of the material are linear.

Then, the norm of electric field is computed along a line as shown in Fig. 5. As seen, once again the stochastic method gives the same results as the finite element method.

At last, Fig. 6 presents the magnetic flux density computed by Monte Carlo method in the whole domain at a given time.

5. CONCLUSION

In this paper, we have solved a 3-D electric field problem in a bounded conducting domain submitted to a time dependent magnetic field using stochastic method. To highlight the advantages of this resolution method, we have compared the obtained results with those obtained by solving the same problem of a copper cube submitted to a sinusoidal magnetic field by finite element method. We have thus put forward the accuracy and the speed of calculation of the Monte-Carlo method as well as the possibility of computing only the desired point of space. This method can be more appropriate in case of solving equation in several domains of different sizes than finite element method that will implies very fine meshing in some area and thus might lead to numerical instability.

Among the advantages of stochastic method to solve partial differential equation, we underline that:

- It is very easy to program it.
- It is not necessary to mesh all space and, so, to use a lot of computer memory.
- It is possible to calculate solution in a single point and not in the whole space.
- It is also possible to calculate solution at one time and not at each space time.

Thanks to these advantages, the method can be easily implemented in a larger design program to study problems of eddy currents.

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