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Multitarget Tracking Based on PHD Smoother with Unknown Clutter Spatial Density

Ran Zhu*, Yunli Long, and Wei An

Abstract—Conventional multitarget tracking techniques assume that clutter density is known a priori and use it directly in the recursive processing. However, in practical surveillance systems, the spatial distribution density of measurements generated by clutter is unknown and time-variant. Therefore, in order to achieve better tracking performance as well as the ability to evaluate the surveillance environment, we propose a fully forward-backward probability hypothesis density (PHD) smoother integrated with clutter spatial density estimator in this paper. Details on the sequential Monte Carlo (SMC) implementation method are presented as well. Simulation results of tracking performance evaluation verify the effectiveness of the proposed PHD smoother.

1. INTRODUCTION

Multitarget tracking requires simultaneous detection and estimation of multiple targets from a sequence of measurements involving false alarm, miss-detection and target birth/disappear. Typical mutlitarget tracking technique includes joint probabilistic data association (JPDA), multiple hypothesis tracking (MHT), particle filter (PF), etc. A thorough study and analysis on multitarget tracking methods for practical surveillance systems is well studied in [1, 2].

Random finite set (RFS) is a natural extension of the concept of random vector and provides a natural way of modeling such multitarget sensing phenomenologies. Based on RFS theory, finite set statistics (FISST) provides a generic formulation of the recursive multitarget states and generates numerous multi-sensor multitarget filters in recent years. The developments of probability hypothesis density (PHD) filter [3], cardinalized PHD (CPHD) filter [4, 5] and multi-Bernoulli filter [6] provide systematic solutions to multiple targets tracking problem based on RFS modeling with different prior probability distribution assumptions. On the other hand, techniques such as sequential Monte Carlo (SMC) implementations [7–9] and Gaussian mixture (GM) implementations [10, 11] are widely applied to fulfill novel extensions.

In order to yield better estimates by delaying the decision time and using the data till a later time, a forward-backward PHD smoother involving forward filtering followed by a backward smoother with particle implementation was proposed in [12, 13]. Closed-form Gaussian sum smoother under linear Gaussian multitarget assumptions is well studied in [14, 15]. Nadarajah et al. provide a smoothing algorithm combined with multiple model approaches, which is a natural extension of the PHD smoothing algorithm for tracking multiple maneuvering targets [16]. In addition, the authors provide a fast SMC implementation approach to fulfill an efficient PHD smoothing in [17]. Image observation Bernoulli smoother incorporating the sensor model with multi-scan information is investigated in [18].

Note that in the multitarget filters/smoothers mentioned above, the clutter spatial distribution model and its parameters are assumed to be known a priori. However, in a practical multitarget tracking

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* Corresponding author: Ran Zhu (zhuran@nudt.edu.cn).
The authors are with the College of Electronic Science and Engineering, National University of Defense Technology, Changsha, Hunan 410073, China.

system, the clutter parameters are not time-invariant due to challenging surveillance environment. Therefore, there is a need to both track the multitarget states and evaluate the current clutter density. A PHD filter with clutter estimation is derived in [19] based on the Poisson point process assumption. Later on, the authors further proposed an online clutter estimation method under the Gaussian kernel density estimation framework in [20, 21]. In particular, the work [22] focuses on the extended object tracking with unknown parameters in the measurement process. Furthermore, PHD filtering using multiple scans to estimate non-stationary clutter density is proposed in [23]. In this paper, we follow the previous work and present a forward-backward PHD smoother integrated with clutter density estimation.

The organization of the paper is as follows. Conventional PHD filter with a straightforward unknown clutter density estimator is introduced in Section 2. Theoretical formulations of the forward-backward smoothing recursion of both real target states and clutter states simultaneously are introduced in Section 3. Sequential Monte Carlo implementation of the proposed PHD smoother with clutter parameter estimator is discussed in Section 4. Performance evaluation results are presented in Section 5, and concluding remarks are given in Section 6.

2. PHD FILTER WITH CLUTTER PARAMETER ESTIMATOR

Following the FISST theory proposed in [1], the multitarget state X_k in a standard PHD filter is modeled as an RFS. We let the multitarget state density of time step k' be denoted as $f_{k'|k}(X_{k'}|Z_{1:k})$ given the measurements Z_k up to time step k. Multitarget state transition function is $f_{k|k-1}(X_k|X_{k-1})$, and $g_k(Z_k|X_k)$ is the multitarget likelihood function. Under the generalized Bayesian recursion framework, multitarget state prediction and update can be formulated as

$$f_{k|k-1}(X_k|Z_{1:k-1}) = \int f_{k|k-1}(X_k|X_{k-1}) f_{k-1|k-1}(X_{k-1}|Z_{1:k-1}) \delta X_{k-1} \tag{1}$$

and

$$f_{k|k}(X_k|Z_{1:k}) = \frac{g_k(Z_k|X_k) f_{k|k-1}(X_k|Z_{1:k-1})}{f_k(Z_k|Z_{1:k-1})} \tag{2}$$

where

$$f_k(Z_k|Z_{1:k-1}) = \int g_k(Z_k|X_k) f_{k|k-1}(X_k|Z_{1:k-1}) \delta X_k \tag{3}$$

is the normalization term.

Based on the FISST theory, probability hypothesis density function (or PHD intensity) is the first-order moment of the multitarget state density [3]. In this way, the recursion in the mutlitarget state space is simplified to the recursion of PHD intensity in the single target state space. Let $D_{k'|k}(x_{k'}|Z_{1:k})$ denote PHD function at time step k' given the measurements up to time k, then the prediction step of the PHD filter is shown as

$$D_{k|k-1}(x_k|Z_{1:k-1}) = b_k(x_k) + \int p_s(x_{k-1}) f_{k|k-1}(x_k|x_{k-1}) D_{k-1|k-1}(x_{k-1}|Z_{1:k-1}) dx_{k-1} \tag{4}$$

where $b_k(x_k)$ is the PHD intensity of new born targets, and $p_s(x_k)$ is the surviving probability of target of state x_k to the next time step. $f_{k|k-1}(x_k|x_{k-1})$ denotes the state transition equation of a single target.

According to the study [1, 3], update step of the PHD intensity is

$$\begin{aligned} D_{k|k}(x_k|Z_{1:k}) \;=\; & \left[1 - p_d(x_k) + p_d(x_k) \sum_{z \in Z_k} \frac{h(z|x_k)}{\lambda c(z) + \int p_d(x_k) h(z|x_k) D_{k|k-1}(x_k|Z_{1:k-1}) dx_k} \right] \\ & \times D_{k|k-1}(x_k|Z_{1:k-1}) \end{aligned} \tag{5}$$

where $p_d(x_k)$ is the detection probability of target x_k, and $h(z|x_k)$ is the likelihood function in the observe model. λ and $c(z)$ are, respectively, the prior average of number of measurements per scan and the spatial distribution of false alarms generated by the clutters. However, in the practical applications, it is impossible to determine the parameter set for the PHD filter in advance. Therefore, the ability to simultaneously estimate unknown clutter density is required to improve the capability and robustness of the PHD filter [24].

In the previous work [25] and [26], the authors propose an intuitive way to estimate the clutter density along with the filtering process. Assume that measurement set observed at time step k is denoted as $Z_k = \{z_1, \dots, z_{m_k}\}$, where $m_k = |Z_k|$ is the number of measurements. $M = \{m_1, \dots, m_L\}$ is the collection of m_k over a sliding window of length L. Then the likelihood of M given λ is

$$f(M|\lambda) = \prod_{k=1}^{L} f(m_k|\lambda) = e^{-\lambda L} \cdot \lambda^{\sum_{k=1}^{L} m_k} / \prod_{k=1}^{L} m_k! \tag{6}$$

Take the maximum likelihood estimation (MLE) of λ by solving the equation $\partial \log f(M|\lambda)/\partial\lambda = 0$. Then we get

$$\hat{\lambda} = \arg\max_{\lambda} \{\log f(M|\lambda)\} = \frac{1}{L}\sum_{k=1}^{L} M_k \tag{7}$$

The spatial distribution of measurement z can be approximated by the finite mixture models (FMM) technique as follows.

$$c(z) = f(z|\theta) = \sum_{i=1}^{N} \omega_i f_i(z|\theta_j) \tag{8}$$

where θ_j and ω_j are the parameter and portion weight of approximation, and $\sum_{i=1}^{N} \omega_j = 1$. If we assume that the elements in the measurement Z_k are independent, then the likelihood of Z_k given θ is

$$f(Z_k|\theta) = \prod_{i=1}^{m_k} f(z_i|\theta) = \prod_{i=1}^{m_k}\sum_{j=1}^{N} \omega_j f_j(z_i|\theta_j) \tag{9}$$

The MLE of parameter θ can be calculated by

$$\hat{\theta} = \arg\max_{\theta} \{\log f(Z|\theta)\} \tag{10}$$

Note that the above estimation is more complicated than the estimation of λ. Therefore, expectation maximum (EM) or Markov chain Monte Carlo (MCMC) technique is required. For implementation details such as how to select distribution functions f_j, please refer to [25].

3. PHD SMOOTHING WITH CLUTTER PARAMETER ESTIMATOR

As for the target state smoothing, a single-target tracking algorithm dealing with uncertain clutter measurements is well studied in [27]. In this paper, we focus on the PHD smoothing of multi-target states with adaptive clutter density estimation. At first, we present theoretical multitarget state smoothing when the Poisson clutter model has unknown parameters following the coupled modeling of both target state and clutter state as proposed in [19].

In this paper, we simultaneously model the moving targets and relatively static clutters as RFSs to be estimated. Let $D^{(X)}_{k'|k}(x_{k'}|Z_{1:k})$ and $D^{(C)}_{k'|k}(c_{k'}|Z_{1:k})$ denote the PHD intensity of the target states and the clutter states at time step k' given the measurements up to time k, respectively. If we assume that the state evolution of targets does not affect the clutter states, the predict step contains 2 separate operations. Note that, for simplicity, no spawn new targets are considered in this paper.

$$D^{(X)}_{k|k-1}(x_k|Z_{1:k-1}) = b^{(X)}_k(x_k) + \int p^{(X)}_s(x_{k-1}) f^{(X)}_{k|k-1}(x_k|x_{k-1}) D^{(X)}_{k-1|k-1}(x_{k-1}|Z_{1:k-1}) dx_{k-1} \tag{11}$$

$$D^{(C)}_{k|k-1}(c_k|Z_{1:k-1}) = b^{(C)}_k(c_k) + \int p^{(C)}_s(c_{k-1}) f^{(C)}_{k|k-1}(c_k|c_{k-1}) D^{(C)}_{k-1|k-1}(c_{k-1}|Z_{1:k-1}) dc_{k-1} \tag{12}$$

Since we cannot determine whether a measurement is generated by a target or by the clutter, the update step in the PHD intensity recursion has to be performed in a coupled way [19].

$$D^{(X)}_{k|k}(x_k|Z_{1:k}) = \left[1 - p_d(x_k) + p_d(x_k) \sum_{z \in Z_k} \frac{h^{(X)}(z|x_k)}{L(z)}\right] D^{(X)}_{k|k-1}(x_k|Z_{1:k-1}) \tag{13}$$

$$D^{(C)}_{k|k}(c_k|Z_{1:k}) = \left[\sum_{z \in Z_k} \frac{h^{(C)}(z|c_k)}{L(z)}\right] D^{(C)}_{k|k-1}(c_k|Z_{1:k-1}) \tag{14}$$

In the above, the coupled likelihood function is denoted as

$$L(z) = \int p_d(x_k) h^{(X)}(z|x_k) D^{(X)}_{k|k-1}(x_k|Z_{1:k-1}) dx_k + \int h^{(C)}(z|c_k) D^{(C)}_{k|k-1}(c_k|Z_{1:k-1}) dc_k \tag{15}$$

where $h^{(X)}(z|x_k)$ and $h^{(C)}(z|c_k)$ are the likelihood function of target generated measurement and clutter generated measurement, respectively. For simplicity, we assume that $p_d(c_k) \equiv 1$, which means that a clutter generated measurement can always be detected.

Target states smoothing is a straightforward way to improve the tracking performance of a surveillance system. Given the measurements collected up to current time, the smoothed multitarget state of a previous time step can be calculated as

$$f_{t|k}(X_t|Z_{1:k}) = f_{t|t}(X_t|Z_{1:t}) \int \frac{f_{t+1|k}(X_{t+1}|Z_{1:k}) f_{t+1|t}(X_{t+1}|X_t)}{f_{t+1|t}(X_{t+1}|Z_{1:t})} \delta X_{t+1} \tag{16}$$

In addition, clutter states smoothing is required as well for some specific applications, which have the need to evaluate the spatial-temporal distribution of the clutters. Following the assumption on the state evolution independency in the prediction step, the smoothing step should be performed separately as well. $p_s^{(X)}(x_t)$ and $p_s^{(C)}(c_t)$ are the surviving probability of a target/clutter state persist to the next time step, respectively [13, 16].

$$D^{(X)}_{t|k}(x_t|Z_{1:k}) = \left[1 - p_s^{(X)}(x_t) + p_s^{(X)}(x_t) \int \frac{D^{(X)}_{t+1|k}(x_{t+1}|Z_{1:k}) f^{(X)}_{t+1|t}(x_{t+1}|x_t)}{D^{(X)}_{t+1|t}(x_{t+1}|Z_{1:t})} dx_{t+1}\right] D^{(X)}_{t|t}(x_t|Z_{1:t}) \tag{17}$$

$$D^{(C)}_{t|k}(c_t|Z_{1:k}) = \left[1 - p_s^{(C)}(c_t) + p_s^{(C)}(c_t) \int \frac{D^{(C)}_{t+1|k}(c_{t+1}|Z_{1:k}) f^{(C)}_{t+1|t}(c_{t+1}|c_t)}{D^{(C)}_{t+1|t}(c_{t+1}|Z_{1:t})} dc_{t+1}\right] D^{(C)}_{t|t}(c_t|Z_{1:t}) \tag{18}$$

Finally, we present the overall recursion of the PHD intensity of both the target and clutters in Fig. 1 to illustrate the processing scheme of the proposed PHD smoother integrated with clutter estimator (R is the backward smoothing depth).

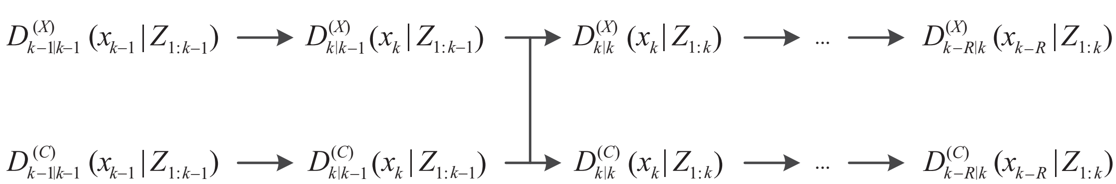

Figure 1. Recursive forward-backward smoothing process with clutter estimation.

4. SMC IMPLEMENTATION METHOD

In this part, a thorough SMC implementation of the above PHD smoothing algorithm is presented. The overall processing scheme consists of 5 recursive steps: *prediction*, *update*, *smoothing*, *resampling* and *state estimation*.

Step 1 Prediction

Given the prior target state particle set $\{x_{k-1}^{(p)}, \omega_{k-1}^{(p)}\}_{p=1}^{L_{k-1}^{(X)}}$ and the clutter state particle set $\{c_{k-1}^{(p)}, w_{k-1}^{(p)}\}_{p=1}^{L_{k-1}^{(C)}}$, the prediction step is performed separately. $q_k^{(X)}(\cdot|x_{k-1}^{(p)}, Z_k)$, $p_k^{(X)}(\cdot|Z_k)$, $q_k^{(C)}(\cdot|c_{k-1}^{(p)}, Z_k)$ and $p_k^{(C)}(\cdot|Z_k)$ are the proposal densities for applying importance sampling of predicted/birth targets and predicted/birth clutters.

For target state prediction, we sample the predicted target states

$$\tilde{x}_k^{(p)} \sim q_k^{(X)}\left(\cdot|x_{k-1}^{(p)}, Z_k\right), \quad p = 1, \ldots, L_{k-1}^{(X)} \tag{19}$$

and the predicted weights are

$$\tilde{\omega}_{k|k-1}^{(p)} = \frac{p_s^{(X)}\left(x_{k-1}^{(p)}\right) f_{k|k-1}^{(X)}\left(\tilde{x}_k^{(p)}|x_{k-1}^{(p)}\right)}{q_k^{(X)}\left(\tilde{x}_k^{(p)}|x_{k-1}^{(p)}, Z_k\right)} \omega_{k-1}^{(p)} \tag{20}$$

Then we sample the states of new born targets

$$\tilde{x}_k^{(p)} \sim p_k^{(X)}\left(\cdot|Z_k\right), \quad p = L_{k-1}^{(X)} + 1, \ldots, L_{k-1}^{(X)} + J_k^{(X)} \tag{21}$$

with corresponding weights

$$\tilde{\omega}_{k|k-1}^{(p)} = \frac{\gamma_k^{(X)}\left(\tilde{x}_k^{(p)}\right)}{p_k^{(X)}\left(\tilde{x}_k^{(p)}|Z_k\right)} \tag{22}$$

As we assume that the state transition of the clutter is statistically independent of that of the target, the same operations can be performed on the particle set of clutter states. Therefore, for clutter state prediction, we sample

$$\tilde{c}_k^{(p)} \sim q_k^{(C)}\left(\cdot|c_{k-1}^{(p)}, Z_k\right), \quad p = 1, \ldots, L_{k-1}^{(C)} \tag{23}$$

and the predicted weights are

$$\tilde{w}_{k|k-1}^{(p)} = \frac{p_s^{(C)}\left(c_{k-1}^{(p)}\right) f_{k|k-1}^{(C)}\left(\tilde{c}_k^{(p)}|c_{k-1}^{(p)}\right)}{q_k^{(C)}\left(\tilde{c}_k^{(p)}|c_{k-1}^{(p)}, Z_k\right)} w_{k-1}^{(p)} \tag{24}$$

Then we sample the states of new formed clutters

$$\tilde{c}_k^{(p)} \sim p_k^{(C)}\left(\cdot|Z_k\right), \quad p = L_{k-1}^{(C)} + 1, \ldots, L_{k-1}^{(C)} + J_k^{(C)} \tag{25}$$

with corresponding weights

$$\tilde{w}_{k|k-1}^{(p)} = \frac{\gamma_k^{(C)}\left(\tilde{c}_k^{(p)}\right)}{p_k^{(C)}\left(\tilde{c}_k^{(p)}|Z_k\right)} \tag{26}$$

Step 2 Update

We follow Eqs. (13), (14) and (15) to calculate the likelihood of a measurement to target states and clutter states as follows.

$$\psi_k^{(X)}\left(z_k^i\right) = \sum_{p=1}^{L_{k-1}^{(X)}+J_k^{(X)}} p_d\left(\tilde{x}_k^{(p)}\right) h^{(X)}\left(z_k^i|\tilde{x}_k^{(p)}\right) \tilde{\omega}_{k|k-1}^{(p)} \tag{27}$$

$$\psi_k^{(C)}\left(z_k^i\right) = \sum_{p=1}^{L_{k-1}^{(C)}+J_k^{(C)}} h^{(C)}\left(z_k^i|\tilde{c}_k^{(p)}\right) \tilde{w}_{k|k-1}^{(p)} \tag{28}$$

Then the coupled likelihood function is the sum of the above likelihoods.

$$\psi_k\left(z_k^i\right) = \psi_k^{(X)}\left(z_k^i\right) + \psi_k^{(C)}\left(z_k^i\right) \tag{29}$$

For target state update, we have

$$\tilde{\omega}_{k|k}^{(p)} = \left[\left(1 - p_d\left(\tilde{x}_k^{(p)}\right)\right) + \sum_{i=1}^{N_k^Z} \frac{p_d\left(\tilde{x}_k^{(p)}\right) h^{(X)}\left(z_k^i|\tilde{x}_k^{(p)}\right)}{\psi_k\left(z_k^i\right)}\right] \tilde{\omega}_{k|k-1}^{(p)} \tag{30}$$

and for clutter state update

$$\tilde{w}_{k|k}^{(p)} = \left[\sum_{i=1}^{N_k^Z} \frac{h^{(C)}\left(z_k^i|\tilde{c}_k^{(p)}\right)}{\psi_k\left(z_k^i\right)}\right] \tilde{w}_{k|k-1}^{(p)} \tag{31}$$

Step 3 Smoothing

For target state smoothing, based on the cached posterior PHD intensity of the recent time steps $\{\tilde{x}_t^{(p)}, \tilde{\omega}_{t|t}^{(p)}\}_{p=1}^{L_t^{(X)}}$, $t = k-R, \ldots, k$, the backward smoothing is performed as follows.

$$\mu_{t+1|t}^{(q)} = \gamma_{t+1}^{(X)}\left(\tilde{x}_{t+1}^{(q)}\right) + \sum_{p=1}^{L_t^{(X)}} p_s^{(X)}\left(\tilde{x}_t^{(p)}\right) f_{t+1|t}^{(X)}\left(\tilde{x}_{t+1}^{(q)}|\tilde{x}_t^{(p)}\right) \tilde{\omega}_{t|t}^{(p)} \tag{32}$$

$$\omega_{t|k}^{(p)} = \left[1 - p_s^{(X)}(\tilde{x}_t^{(p)}) + p_s^{(X)}\left(\tilde{x}_t^{(p)}\right) \sum_{q=1}^{L_{t+1}^{(X)}} \frac{f_{t+1|t}^{(X)}\left(\tilde{x}_{t+1}^{(q)}|\tilde{x}_t^{(p)}\right)}{\mu_{t+1|t}^{(q)}} \tilde{\omega}_{t+1|k}^{(q)}\right] \tilde{\omega}_{t|t}^{(p)}, \quad p = 1, \ldots, L_t^{(X)} \tag{33}$$

Given the updated particle set of recent several time steps $\{\tilde{c}_t^{(p)}, \tilde{w}_{t|t}^{(p)}\}_{p=1}^{L_t^{(C)}}$, $t = k-R, \ldots, k$, the smoothing is performed in a similar way.

$$\nu_{t+1|t}^{(q)} = \gamma_{t+1}^{(C)}\left(\tilde{c}_{t+1}^{(q)}\right) + \sum_{p=1}^{L_t^{(C)}} p_s^{(C)}\left(\tilde{c}_t^{(p)}\right) f_{t+1|t}^{(C)}\left(\tilde{c}_{t+1}^{(q)}|\tilde{c}_t^{(p)}\right) \tilde{w}_{t|t}^{(p)} \tag{34}$$

$$w_{t|k}^{(p)} = \left[1 - p_s^{(C)}\left(\tilde{c}_t^{(p)}\right) + p_s^{(C)}\left(\tilde{c}_t^{(p)}\right) \sum_{q=1}^{L_{t+1}^{(C)}} \frac{f_{t+1|t}^{(C)}\left(\tilde{c}_{t+1}^{(q)}|\tilde{c}_t^{(p)}\right)}{\nu_{t+1|t}^{(q)}} \tilde{w}_{t+1|k}^{(q)}\right] \tilde{w}_{t|t}^{(p)}, \quad p = 1, \ldots, L_t^{(C)} \tag{35}$$

Step 4 Resampling

Resampling algorithm in [13] can be applied directly on the particle set for target $\{\tilde{x}_k^{(p)}, \tilde{\omega}_k^{(p)}\}_{p=1}^{L_{k-1}^{(X)}+J_k^{(X)}}$ and the particle set for clutter $\{\tilde{c}_k^{(p)}, \tilde{w}_k^{(p)}\}_{p=1}^{L_{k-1}^{(C)}+J_k^{(C)}}$ to form the prior particle set $\{x_k^{(p)}, \omega_k^{(p)}\}_{p=1}^{L_k^{(X)}}$ and $\{c_k^{(p)}, w_k^{(p)}\}_{p=1}^{L_k^{(C)}}$ for the next time step.

Step 5 State Estimation

Postponed estimation is performed on $\{x_{k-R|k}^{(p)}, \omega_{k-R|k}^{(p)}\}_{p=1}^{L_{k-R|k}^{(X)}}$; therefore, the estimated number of targets is

$$\hat{N}_{k-R|k}^{(X)} = \sum_{p=1}^{L_{k-R|k}^{(X)}} \omega_{k-R|k}^{(p)} \tag{36}$$

The particles $x_{k-R|k}^{(p)}$ are clustered into several groups, and then the expected a posteriori (EAP) estimation is done as in the previous work [3, 7].

5. EXPERIMENT RESULTS

The single target state vector $x_k = [\tilde{x}_k^T, \theta_k]^T$ consists of motion state vector $\tilde{x}_k = [u_k, \dot{u}_k, v_k, \dot{v}_k]^T$ and the turn rate θ_k. On the other hand, the clutter state vector is similar to the target state and denoted as $c_k = [\tilde{c}_k^T, \vartheta_k]^T$. However, only positions of both the targets and clutters can be acquired in the measurements. We assume that the single target states and clutter states follow the same coordinated turn model with different parameters.

$$\begin{cases} \tilde{x}_k = F(\theta_{k-1})\tilde{x}_{k-1} + Gn_{k-1} \\ \theta_k = \theta_{k-1} + \Delta\theta_{k-1} \end{cases} \tag{37}$$

$$\begin{cases} \tilde{c}_k = F(\vartheta_{k-1})\tilde{c}_{k-1} + Gn_{k-1} \\ \vartheta_k = \vartheta_{k-1} + \Delta\vartheta_{k-1} \end{cases} \tag{38}$$

where

$$F(\omega) = \begin{bmatrix} 1 & \dfrac{\sin\omega\Delta t}{\omega} & 0 & -\dfrac{1-\cos\omega\Delta t}{\omega} \\ 0 & \cos\omega\Delta t & 0 & -\sin\omega\Delta t \\ 0 & \dfrac{1-\cos\omega\Delta t}{\omega} & 1 & \dfrac{\sin\omega\Delta t}{\omega} \\ 0 & \sin\omega\Delta t & 0 & \cos\omega\Delta t \end{bmatrix}, \quad G = \begin{bmatrix} \dfrac{\Delta t^2}{2} & 0 \\ \Delta t & \dfrac{\Delta t^2}{2} \\ 0 & \Delta t \end{bmatrix} \tag{39}$$

and Δt is the time interval between the consecutive frames. $n_{k-1} \sim \mathcal{N}(\cdot|0, \sigma_n^2 I)$ is the random generated process noise vector. $\Delta\theta_{k-1} \sim \mathcal{N}(\cdot|0, \sigma_\theta^2)$ and $\Delta\vartheta_{k-1} \sim \mathcal{N}(\cdot|0, \sigma_\vartheta^2)$ model the vibration on the turn rate of target state and clutter state, respectively.

Table 1 shows the details on the initial parameters set for real targets in the simulation. All the targets persist till the end of the 200 frames scenario. The track truth of all the targets are shown in Fig. 2. Moving objects that have the estimated velocity of $[1.0, 10.0]$ pixel/s are assumed to be targets of interest. Those potential objects with less than 1 pixel/s velocity are regarded as the static clutter. $\Delta t = 0.04\,\mathrm{s}$ is set for 25 Hz frame rate sensors. Probability of target detection is $p_d = 0.98$. Survival probabilities are $p_s^{(X)} = 0.99$ and $p_s^{(C)} = 0.99$. The motion models are set to $\sigma_\theta = \sigma_\vartheta = 0.1\pi/180\,\mathrm{rad/s}$, and $\sigma_n = 0.1$.

Table 1. Initial parameters of the targets in the test scenario.

target id	start frame	u_0	$\dot{u}_0$	v_0	$\dot{v}_0$	θ_0
1	10	20.0	7.5	20.0	6.0	0.025
2	15	110.0	−3.0	30.0	9.0	−0.05
3	20	40.0	6.0	50.0	7.0	−0.05
4	20	50.0	6.0	90.0	−7.0	−0.05
5	30	50.0	8.0	115.0	−5.0	−0.1

For tracking performance evaluation, 100 Monte Carlo trials are tested for the PHD filter with known clutter density, the PHD filter with clutter estimator, the PHD smoother with known clutter density and the proposed PHD smoother with clutter estimator respectively on the same scenario. The optimal subpattern assignment (OPSA) metric (with cut distance $c = 10$ and order $p = 2$) is adopted to measure the difference between estimates and real targets [28].

The average of OSPA values is presented in Fig. 3 for the tracking performance comparison. We see that the PHD filter with clutter estimator is slightly worse than the PHD filter with known clutter density for the miss of correct prior parameters. The same situation is observed in the smoother pairs. There is no doubt that the PHD smoother with known clutter density will reach the best tracking performance. However, we see that the performance degradation affected by the clutter modeling is significantly reduced in the PHD smoother version. In other words, we can achieve a better tracking

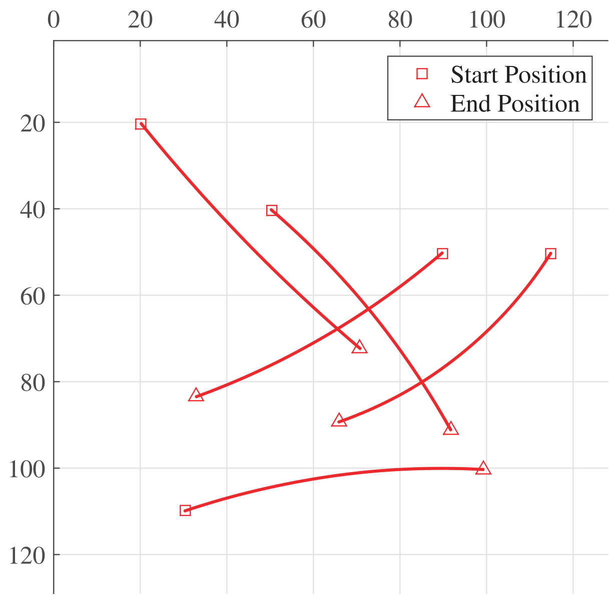

Figure 2. Track truth.

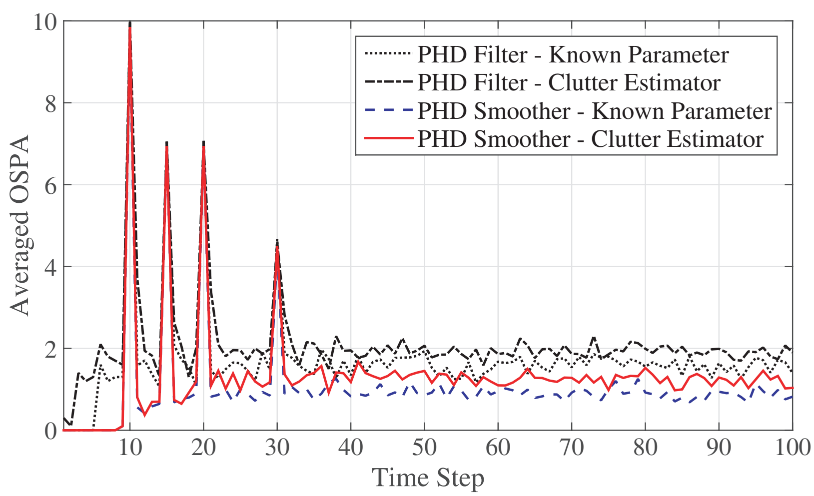

Figure 3. Tracking performance comparison between the PHD filters and the PHD smoothers.

performance by adding the backward smoothing step to compensate the degradation caused by the uncertain parameters in the clutter model.

The above experiment results are tested by the simulation with measurements generated by clutters following the ideal Poisson distribution. However, when we extract candidate measurements by performing segmentation on the raw data received from a real sensor (e.g., infrared imaging sensor), a portion of the measurements may always appear in several regions due to the heavy clutters. Therefore, in the following, we evaluate the tracking performance based on a more complicated scenario. About half of the simulated measurements are generated around fixed positions, and only the other half follows the ideal Poisson distribution. In this way, we generate a testing scenario approximating to the real measurement set observed in a practical tracking system.

Figure 4 shows the cumulative estimation results over the whole 200 frames. We see that the standard PHD smoother with known parameter gives false alarms around fixed clutters due to its sensitivity to the current measurement set. However, in Fig. 4(b), the fixed false alarm problem

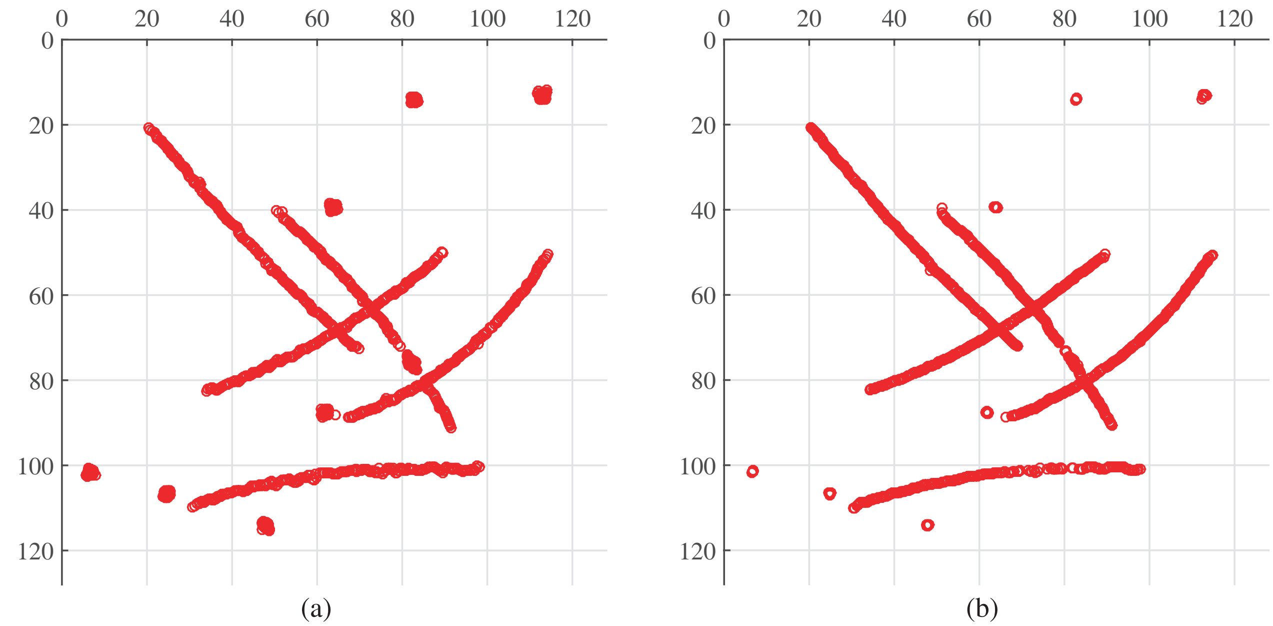

Figure 4. Cumulative estimate results illustration. (a) PHD smoother with known parameter. (b) PHD smoother with clutter estimator.

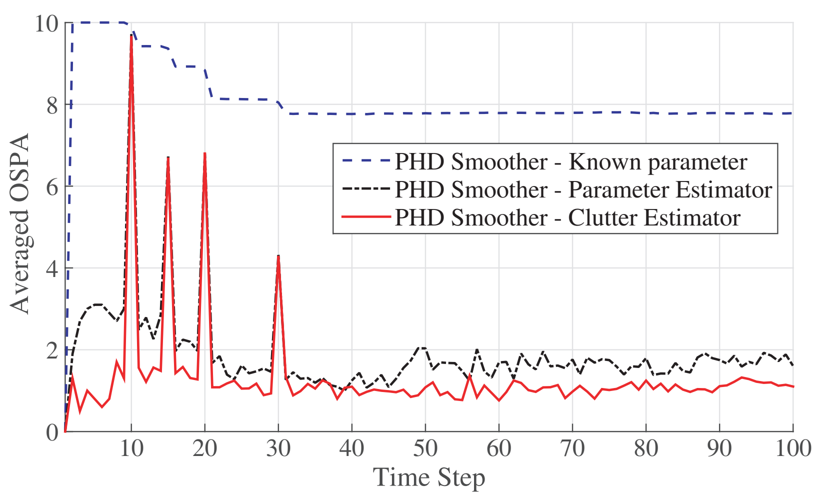

Figure 5. Tracking performance comparison between the PHD smoothers.

is apparently improved. Furthermore, we evaluate the tracking performance of an improved PHD smoother with the parameter estimation method described in Section 2. Length of the sliding window for estimating parameter λ and distribution $c(\cdot)$ is set as $L = 50$. The averaged OSPA error comparison is presented in Fig. 5. When the uniform spatial distribution rule of clutter generated measurements is violated, tracking performance of the standard PHD smoother with known parameter degrades significantly. Although the parameter estimation method can solve false alarm problem to a certain extent, the proposed PHD smoother with clutter estimator further improves the performance as well as the robustness of a practical tracking algorithm.

6. CONCLUSION

We propose a PHD smoothing scheme integrated with unknown clutter parameter estimation under prior clutter model. Except for the coupled update step, the prediction and smoothing of the target state and the clutter state are performed separately based on the assumption that the state transition processes are statistically independent of each other. Experiment results validate the effectiveness of the proposed PHD smoother under measurement model with unknown clutter parameters. A fully coupled PHD smoother with consideration for clutter spawn targets and unknown clutter parameter estimator will be studied in further research. Future work includes PHD type filter/smoother with clutter model shift detection ability, which will be compatible for both indeterminate clutter model with unknown parameters.

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