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Bootstrap Based Sequential Detection in Non-Gaussian Correlated Clutter

Toufik Boukaba[1, *], Mohammed Nabil El Korso[2], Abdelhak M. Zoubir[3], and Daoud Berkani[4]

Abstract—In this paper, sequential parametric detection problem is addressed for non-Gaussian correlated clutter. It is well known that the assumption of normally distributed clutter leads, mostly, to analytical expressions of the threshold as well the distribution of detection statistic. Nevertheless, due to the resolution improvement of recent sensing instruments such as high resolution radar, the Gaussian assumption is unrealistic since the clutter is nonhomogeneous. As a consequence, using non-Gaussian assumption of the clutter prevents, mostly, of obtaining analytical expressions of the threshold and the distribution of detection statistics. In this work, we overcome this issue by use of the so-called bootstrap techniques for dependent data. Numerical simulations reveal that our proposed method outperforms the classical and sequential non-bootstrap based detection schemes in terms of probability of detection and selects the optimum sample size needed to achieve the required detection performances.

1. INTRODUCTION

Detection step is one of the major tasks of radar systems. It refers to the decision made by a radar receiver regarding the presence or absence of an object of interest, namely the target, based on the received echo signal. This decision is then used to estimate parameters of the target such as location, velocity or shape.

Specifically, there are two kinds of radar echoes: i) useful signal, which is the echo signal reflected from the targets, such as aircrafts, ships, missiles, etc. and ii) clutter, which represents echo reflected from unrelated bodies such as land, sea surface, clouds, and rain. Detection of targets in clutter has been a constantly evolving field of study. In this context, the main challenge is to study the complex statistical properties of the clutter and to derive solutions to achieve optimal performances.

Among characteristics that describe statistical properties of the clutter, its distribution is of importance [1]. Formerly, the Gaussian assumption is the most well known and widely used distribution in many fields such as engineering and statistics [2]. Nowadays, in radar applications, it turns out to be not appropriate to model observed clutter especially in consideration of recent technological advances and resolution increase [3–5].

Starting from this evidence, many efforts have been emphasized on the study of non-Gaussian clutter in order to work out a model which can be applied to analysis and synthesis purposes [6–8]. A general agreement has been reached about the validity of the general and elegant impulsive model called compound-Gaussian. In this case, the baseband equivalent of clutter returns is modelled as the product of two mutually independent quantities: a complex, zero-mean, possibly correlated Gaussian process, also referred to as speckle, times the square root of a non-negative random scalar, referred to

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* Corresponding author: Toufik Boukaba (tboukaba@spg.tu-darmstadt.de).

[1] Ecole Supérieure des Techniques de l'Aéronautique, Algiers, Algeria. [2] Université Paris X, France.[3] Signal Processing Group, Institute of Telecommunications, Technische Universität Darmstadt, Merckstr. 25, Darmstadt 64283, Germany. [4] Laboratoire Signal et Communication, Ecole Nationale Polytechnique, 10, Av. Hassen Badi, P. O. Box 182, El-Harrach 16200, Algeria.

as texture [9–11]. The clutter vector is then fully characterized by the unknown texture distribution and the speckle's covariance matrix.

Based on this model, sub-optimal solutions have been derived following a generalized likelihood ratio test (GLRT) approach [12–14]. In practice, adaptive techniques are used to estimate unknown and space-time varying clutter parameters, using secondary signal-free data taken from range cells, surrounding the cell under test (CUT) and assumed to share the same statistical properties with the former.

In all developed methods, it is clearly stated that one of the major challenging difficulties is to estimate the covariance matrix of the speckle [15–17]. To perform this, one can appeals to non-parametric methods, where no assumption is made on clutter. In this case, the number of used secondary cells must be at least equal to the sample size. In non-stationary scenarios, this condition can be relaxed by using parametric methods, where the clutter is assumed to belong to a certain model.

Notwithstanding used approach, the global estimation-detection solution is generally based on a fixed sample size without any real-time adaptation. A beforehand performance analysis is used to select a rough size value, which is not necessarily the optimum regarding to the non-stationary character of the clutter and to computation costs.

In spite of their ability to minimize the required sample size to achieve given performance, the literature on sequential methods for radar detection is very scarce, particularly in the case of non-Gaussian clutter. In this paper, we use a sequential approach to reduce the sample size in the case of non-Gaussian correlated clutter. A parametric version of the adaptive normalized matched filter (ANMF) is used, where the clutter is approximated by an autoregressif (AR) process. A bootstrap procedure is then proposed to update detection thresholds.

The remainder of the paper is organized as follows: Section 2 is dedicated to the model setup and general background about sequential tests and bootstrap technique. Section 3 is devoted to the proposed approach, while Section 4 presents some results obtained using real data sets. Finally, Section 5 concludes the study.

2. MODEL SETUP AND THEORITICAL BACKGROUND

2.1. Model Setup

In radar detection problems, random signal received after the transmission of N pulses is used to decide between two possible statistical hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$. These hypotheses correspond, respectively, to the absence and the presence of a target. This can be described as:

$$\begin{cases} \mathcal{H}_0 : \mathbf{x} = \mathbf{c} \\ \mathcal{H}_1 : \mathbf{x} = \delta\mathbf{d} + \mathbf{c} \end{cases} \tag{1}$$

where $\mathbf{x} = [x(0), x(1), \ldots, x(N-1)]^T$ is the received signal in the CUT, $\mathbf{c} = [c(0), c(1), \ldots, c(N-1)]^T$ the clutter vector, δ an unknown target's deterministic amplitude, and $\mathbf{d} = [d(0), d(1), \ldots, d(N-1)]^T$ the target steering vector, with components dependent on f_d, which is the assumed known Doppler frequency normalized with respect to the radar pulse repetition frequency (PRF).

The hypothesis test problem in Eq. (1) is solved by comparing a statistic $\Lambda(\mathbf{x})$ computed from available data to a given threshold γ.

The parametric adaptive normalized matched filter, based on AR model (AR-PANMF) derived in [18] is used in this work. A set of secondary data, taken from L range cells adjacent to the CUT, is used to estimate clutter unknown parameters.

The main assumption to be considered is that the clutter vector, in each secondary cell, is a zero mean compound Gaussian process approximated by an AR(p) process of assumed known order p, driven by a white noise process [19, 20]. In summary, the AR-PANMF detector is given by:

$$\Lambda_{\mathrm{AR-PANMF}}(\mathbf{x}) = \frac{\left|\mathbf{d}^H \widehat{\mathbf{A}}^H \widehat{\mathbf{A}}\mathbf{x}\right|^2}{\mathbf{d}^H \widehat{\mathbf{A}}^H \widehat{\mathbf{A}}\mathbf{d}\mathbf{x}^H \widehat{\mathbf{A}}^H \widehat{\mathbf{A}}\mathbf{x}} \tag{2}$$

where $\widehat{\mathbf{A}}$ is a lower triangular $N \times N$ matrix constructed with the estimated AR coefficients and is given by:

$$\widehat{\mathbf{A}} = \begin{bmatrix} 1 & 0 & . & . & . & . & . & 0 \\ \hat{a}_1 & 1 & 0 & . & . & . & . & . \\ . & . & . & 0 & . & . & . & . \\ \hat{a}_p & . & \hat{a}_1 & 1 & 0 & . & . & . \\ 0 & . & . & . & . & 0 & . & . \\ & 0 & . & . & . & . & 0 & . \\ . & & 0 & . & . & . & . & 0 \\ 0 & . & & 0 & \hat{a}_p & . & \hat{a}_1 & 1 \end{bmatrix} \tag{3}$$

where $\hat{\boldsymbol{a}} = [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_p]^T$ is the vector of estimated AR coefficients obtained by minimizing the global forward prediction error for L secondary cells [18]:

$$\hat{\boldsymbol{a}} = -\left(\sum_{k=1}^{L} \mathbf{C}_k^H \mathbf{C}_k\right)^{-1} \left(\sum_{k=1}^{L} \mathbf{C}_k^H \mathbf{c}_k\right) \tag{4}$$

in which, $\mathbf{c}_k$ is the clutter vector in the k-th secondary cell, and $\mathbf{C}_k$ is given by:

$$\mathbf{C}_k = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ c_k(0) & 0 & \ldots & 0 \\ c_k(1) & c_k(0) & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ c_k(p-1) & c_k(p-2) & \ldots & c_k(0) \\ c_k(p) & c_k(p-1) & \ldots & c_k(1) \\ \vdots & \vdots & \vdots & \vdots \\ c_k(N-2) & c_k(N-3) & \ldots & c_k(N-p-1) \end{bmatrix} \tag{5}$$

Using the whitening effect of the matrix $\widehat{\mathbf{A}}$, we obtain the following final expression:

$$\Lambda_{\text{AR-PANMF}}(\mathbf{x}) = \frac{\left|\hat{\mathbf{e}}_{\mathbf{d}}^H \hat{\mathbf{e}}_{\mathbf{x}}\right|^2}{\left(\hat{\mathbf{e}}_{\mathbf{d}}^H \hat{\mathbf{e}}_{\mathbf{d}}\right)\left(\hat{\mathbf{e}}_{\mathbf{x}}^H \hat{\mathbf{e}}_{\mathbf{x}}\right)} \tag{6}$$

in which $\hat{\mathbf{e}}_{\mathbf{x}} = \widehat{\mathbf{A}}\mathbf{x}$ represents the forward prediction error sequence, while $\hat{\mathbf{e}}_{\mathbf{d}} = \widehat{\mathbf{A}}\mathbf{d}$ is the steering residuals vector.

2.2. Background on Sequential Detection

In the test defined by Eq. (1), two decisions are possible: accept or reject $\mathcal{H}_0$. This is exactly what is done in the classical detection framework. In this case a fixed sample size and a single threshold are used. The threshold is fixed according to a given value of the first kind error, referred to as false alarm probability P_{fa}. In sequential tests, instead of the two obvious decisions, a third stage is introduced as follows [21]:

(i) Accept the hypothesis $\mathcal{H}_0$.

(ii) Accept the hypothesis $\mathcal{H}_1$.

(iii) Consider an additional observation.

Two thresholds, A_N (upper threshold) and B_N (lower threshold), are then used and determined by P_{fa} and the probability of miss detection P_{m}. These thresholds are used in order to ensure that the required number of observations is minimized under the constraint that both P_{fa} and P_{m} are bounded by pre-assigned values α and β:

$$P_{\text{fa}} = P\left(\Lambda(\mathbf{x}) \geq A_N | \mathcal{H}_0\right) \leq \alpha \tag{7}$$

$$P_{\text{m}} = P\left(\Lambda(\mathbf{x}) \leq B_N | \mathcal{H}_1\right) \leq \beta \tag{8}$$

Consequently, sequential test becomes:

- Decision 1 is made if $\Lambda(\mathbf{x})$ is less than or equal to B_N.
- Decision 2 is made if $\Lambda(\mathbf{x})$ is greater than or equal to A_N.
- Decision 3 is made if $\Lambda(\mathbf{x})$ is inside the interval $]A_N, B_N[$.

Precisely, any sequential test goes on as follows: based on the current observation, the test is terminated if the first or the second decision is made. Otherwise, an additional observation is considered and so on until a decision is made. Thresholds A_N and B_N are necessarily related to the distributions of the statistic $\Lambda(\mathbf{x})$ respectively under $\mathcal{H}_0$ and under $\mathcal{H}_1$. An exact knowledge of these distributions is then required to build a sequential test with performance set by bound values α and β. In practice, these distributions are seldom exactly known, and their parameters are space and time varying. In Section 3.2, we show how to come up with a sequential procedure based on thresholds update using bootstrap. Before that, we give a short overview about this technique.

2.3. Background on Bootstrap Technique

Nowadays, in most statistical signal processing applications, analytical results are pleasing and ensure the required rigour for parameters estimators. However, these solutions assume that the number of measurements, also called sample, is sufficiently large so that some assumptions are valid.

In many problems, these large sample methods are not usable. This is either due to processing time constraints or to the fact that the signal of interest is not stationary. In this case, analytical results cannot always be achieved and one may have to resort to Monte Carlo simulations. Unfortunately, this is not possible in practice because it requires the experiment to be repeated.

In this context, the bootstrap, which avoids the assumption of a large number of observations and does not need a new set of experiments, is an attractive solution [22].

Basically, the bootstrap is a computer-based method for assigning measures of accuracy to statistical estimates [22]. Meaning that, the bootstrap does with a computer what the experimenter would do in practice, if it were possible. Precisely simulate as much of the real world probability mechanism as possible, in which any unknowns are replaced with estimates from the observed data. The original observations are randomly rearranged, and reused to compute estimates. These reassignments and recomputations are done with a large number of times and treated as repeated experiments.

In the literature, diverse applications can be found in various fields [23–26]. Particularly, applications have been reported in radar and sonar signal processing [27–31].

In situations of unknown distributions and small samples, the bootstrap is highly appreciable as it substitutes estimates for unknowns and replaces mathematical analysis with computer simulations.

The basic principle of the bootstrap approach is summarized in Fig. 1.

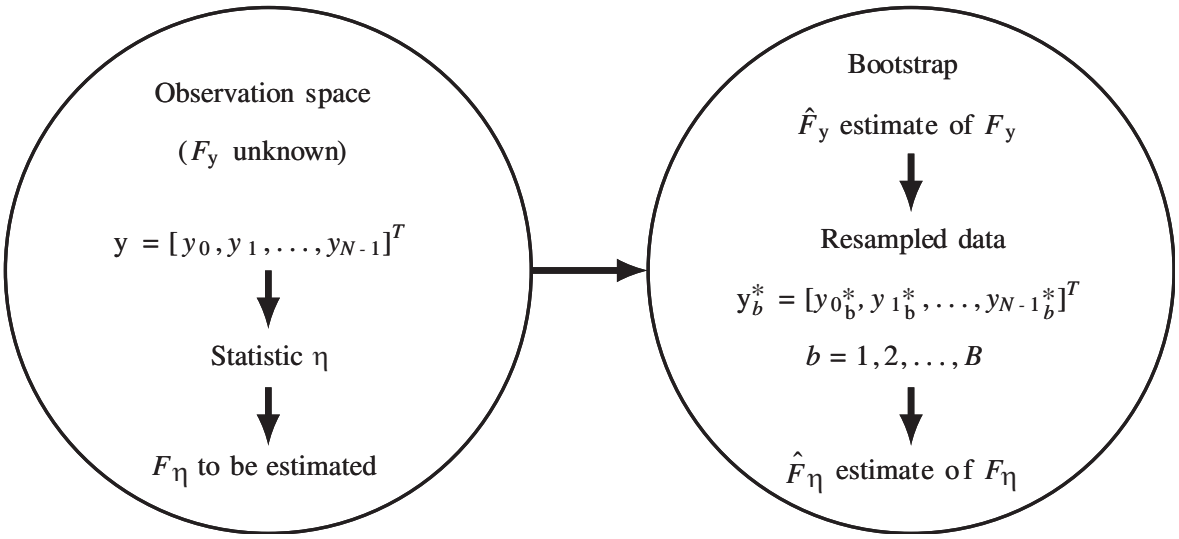


Figure 1. Principle of the bootstrap approach.

Assume that some statistic of interest η is calculated from a vector of random observations $\mathbf{y} = [y_0, y_1, \ldots, y_{N-1}]^T$, taken from an unknown distribution $F_{\mathbf{y}}$. In several applications, it is necessary to estimate the distribution F_η of the statistic η. If the experiment is repeatable, one can use the Monte Carlo recurrence. Unfortunately, this is not always available in practice.

Bootstrap approach allows to do this without need to repeat the experiment. We first use the observation vector $\mathbf{y}$ to obtain the estimate of $F_{\mathbf{y}}$, denoted $\hat{F}_{\mathbf{y}}$. Then, B bootstrap samples are generated from $\hat{F}_{\mathbf{y}}$. For each resample $\mathbf{y}^*$, the statistic η^* is calculated. At the end, the obtained B values of η are used to estimate F_η.

3. PROPOSED SEQUENTIAL DETECTION IN NON-GAUSSIAN CLUTTER

In the case of non-Gaussian clutter, it is convenient to consider that the distributions of the detection statistic belong to a given family, with unknown parameters. However, it is clear that any variation of these parameters will inevitably influence the performance of the detection scheme. In order to overcome this predictable performance degradation, it is appropriate to estimate these parameters and use theme to update thresholds.

In Section 3.1, we show that the distributions of $\Lambda_{\text{AR-PANMF}}$, in non-Gaussian clutter, are of the same type as those derived in [18], for Gaussian clutter. Recall that, in [18], performance evaluation has been handled under the assumption that these distributions where exactly the same. In Section 3.2, we use a bootstrap based method to estimate unknown parameters which characterize these distributions in non-Gaussian clutter and to update thresholds.

3.1. Analysis of the Distribution of the Parametric Detector AR-PANMF

Before the effective application of the bootstrap approach, recall that for Gaussian distributed clutter, the distribution of $\Lambda_{\text{AR-PANMF}}$ under $\mathcal{H}_0$ is a beta distribution, while it is a non-central beta distribution under $\mathcal{H}_1$ [18]. These distributions are given respectively by:

$$F_{0\Lambda,N}(\lambda) = I_\lambda(1, N-1) \tag{9}$$

$$F_{1\Lambda,N}(\lambda) = \sum_{j=0}^{+\infty} \left(\frac{1}{j!}\right) \left(\frac{\xi}{2}\right)^j \exp\left(\frac{-\xi}{2}\right) I_\lambda(1+j, N-1) \tag{10}$$

where $I_x(b, c)$ is the regularized incomplete beta function of x, with shape parameters b and c; N is the sample size; ξ is the non-centrality parameter.

The main goal of this section is to explore how, for non-Gaussian clutter, empirical distributions of $\Lambda_{\text{AR-PANMF}}$ fit with generic models of the same type as $F_{0\Lambda,N}$ and $F_{1\Lambda,N}$ given by (9) and (10). This statistical analysis is performed using simulated data, within different clutter distributions and under shape parameters fluctuations.

For each clutter distribution, Monte Carlo runs are used to obtain a set of $\Lambda_{\text{AR-PANMF}}$ values. Empirical cumulative distribution function (ECDF) is evaluated directly, while generic cumulative distribution function (CDF) is obtained using maximum likelihood (MLE) parameters estimation. Meaning that, for each vector of $\Lambda_{\text{AR-PANMF}}$ values, parameters of generic model are estimated using MLE estimators. Theoretical CDF is then generated and compared to the ECDF. Besides, the Gaussian CDF, generated in the same way is considered as a benchmark.

Four clutter distributions, consistent with the compound Gaussian model, are considered in this example. Namely lognormal, Weibull, gamma and k distribution [11]. Under $\mathcal{H}_0$, and according to (9), the beta CDF is considered as a generic model, whose parameters are estimated from $\Lambda_{\text{AR-PANMF}}$ values. Precisely, the plots of Fig. 2 show the ECDF of the statistic $\Lambda_{\text{AR-PANMF}}$ for different clutter distributions, under $\mathcal{H}_0$ compared to the generic Gaussian and beta CDF.

From these four graphs, it is clear that, under $\mathcal{H}_0$, the beta CDF provides a good fit to the distribution of $\Lambda_{\text{AR-PANMF}}$.

Under $\mathcal{H}_1$ and according to Eq. (10), the non-central beta CDF is considered as a generic model. The theoretical non-central beta CDF is approximated in Eq. (10) as a Poisson mixture of central beta distributions [32], where the Poisson mass functions $P(.)$ are evaluated at the non-centrality parameter

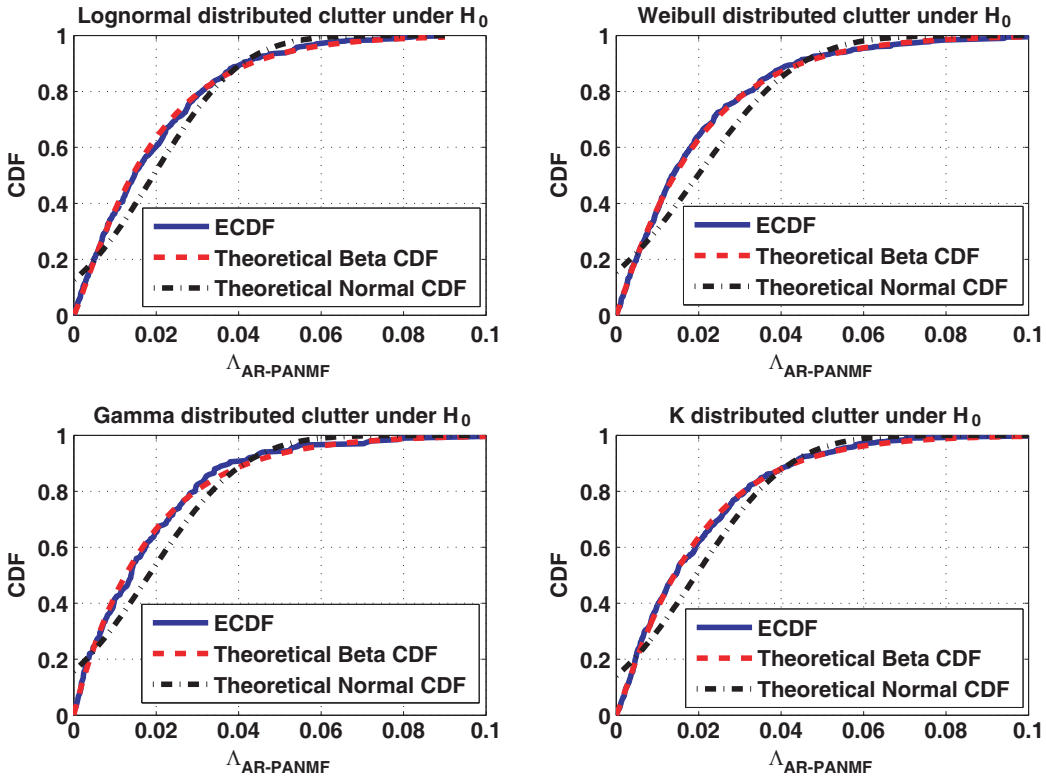


Figure 2. CDF analysis under $\mathcal{H}_0$.

ξ. The plots of Fig. 3 show the ECDF of the statistic $\Lambda_{\text{AR-PANMF}}$ for different clutter distributions, under $\mathcal{H}_1$ compared to generic Gaussian and non-central beta CDF. A good fitting between the ECDF and the non-central beta model is noticed.

In real situations, many parameters can affect this goodness of fit. In the sequel, we measure it under fluctuations of: i) clutter distribution shape parameters, ii) signal-to-clutter ratio (SCR) under $\mathcal{H}_1$ and iii) Doppler shift under $\mathcal{H}_0$. We investigate the variations of the root mean square error (RMSE), defined, between two functions F and G, as [33, 34]:

$$\text{RMSE}_{(\text{F,G})} = \sqrt{\frac{1}{\text{N}_\text{x}} \sum_{\text{i}=1}^{\text{N}_\text{x}} [\text{F}(\text{x}_\text{i}) - \text{G}(\text{x}_\text{i})]^2} \tag{11}$$

where N_x is the size of the observation vector and x_i the generic point of the X-axis in which both F and G are evaluated.

Using the same simulation procedure as above, we consider the RMSE between generic CDF whose parameters are estimated using MLE and the ECDF. We evaluate this metric for different values of the shape parameter of each clutter model.

Under $\mathcal{H}_0$, the maximum value of the RMSE is recorded for the Doppler shift varying from -0.5 to 0.5. Under $\mathcal{H}_1$, the maximum value is recorded for SCR varying from $-15\,\text{dB}$ to $15\,\text{dB}$. Fig. 4 and Fig. 5 show the variations of the RMSE respectively under $\mathcal{H}_0$ and under $\mathcal{H}_1$.

We note that, under parameters fluctuations, the RMSE exhibits better values for beta and non-central beta generic CDF, respectively under $\mathcal{H}_0$ and under $\mathcal{H}_1$.

3.2. Proposed Bootstrap Sequential-AR-PANMF (BS-AR-PANMF)

Based on the previous analysis, and instead of the challenging task, if feasable, which consists on deriving $F_{0\Lambda,N}$ and $F_{1\Lambda,N}$ for non-Gaussian clutter under parameters variations, we resort to a parametric

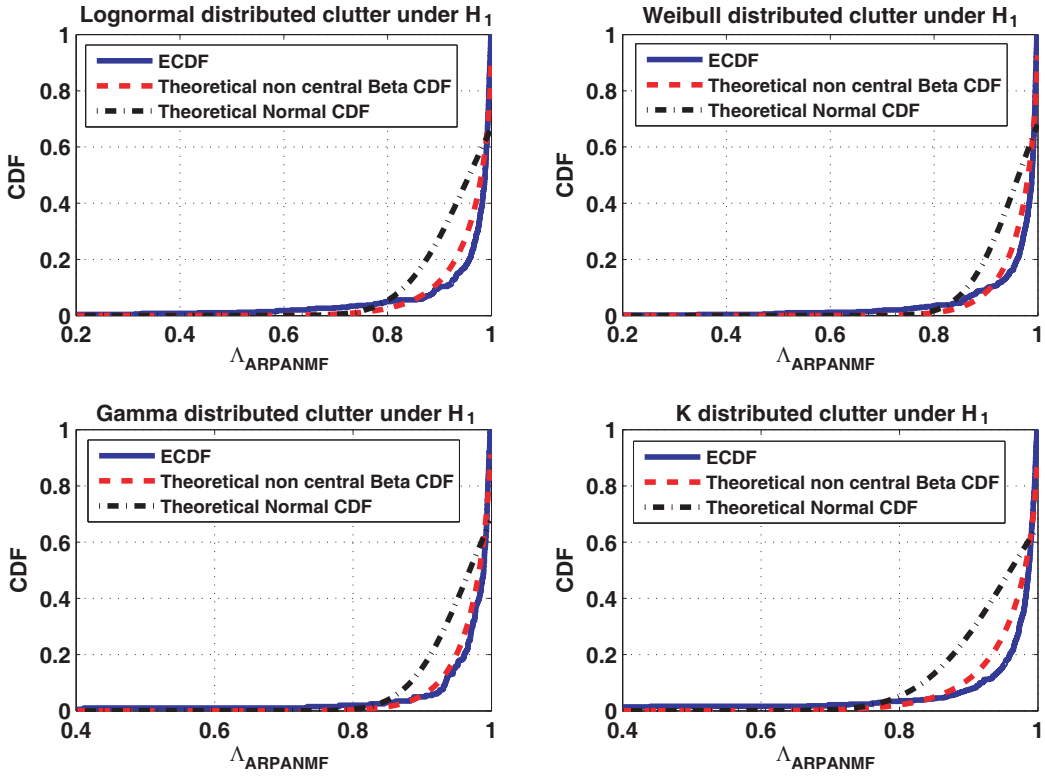


Figure 3. CDF analysis under $\mathcal{H}_1$.

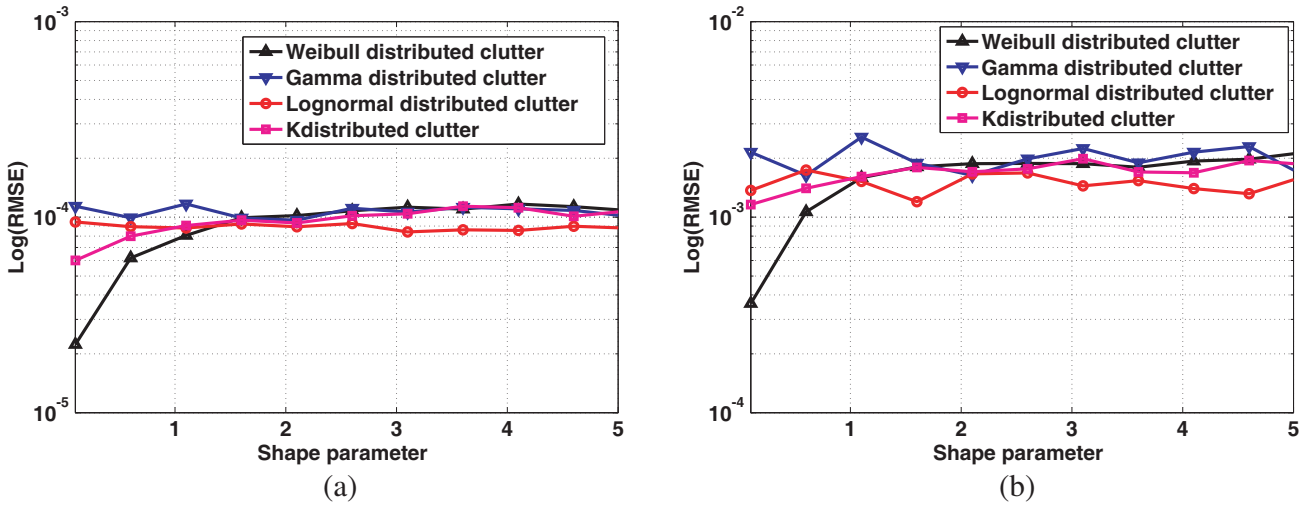


Figure 4. RMSE variations under $\mathcal{H}_0$: (a) Beta generic CDF, (b) Gaussian generic CDF.

approach. The analysis above suggests to assume $F_{0\Lambda,N}$ to be a beta distribution while $F_{1\Lambda,N}$ is assumed to be a non-central beta distribution, both with unknown parameters. A bootstrap method will be used to estimate these parameters.

In radar applications, the assumption of independent clutter samples breaks down [3]. Consequently, we propose in the sequel to use moving block bootstrap which is the appropriate method in this context [22, 35, 36].

In this approach, blocks of a given length l are randomly selected from the original data. The

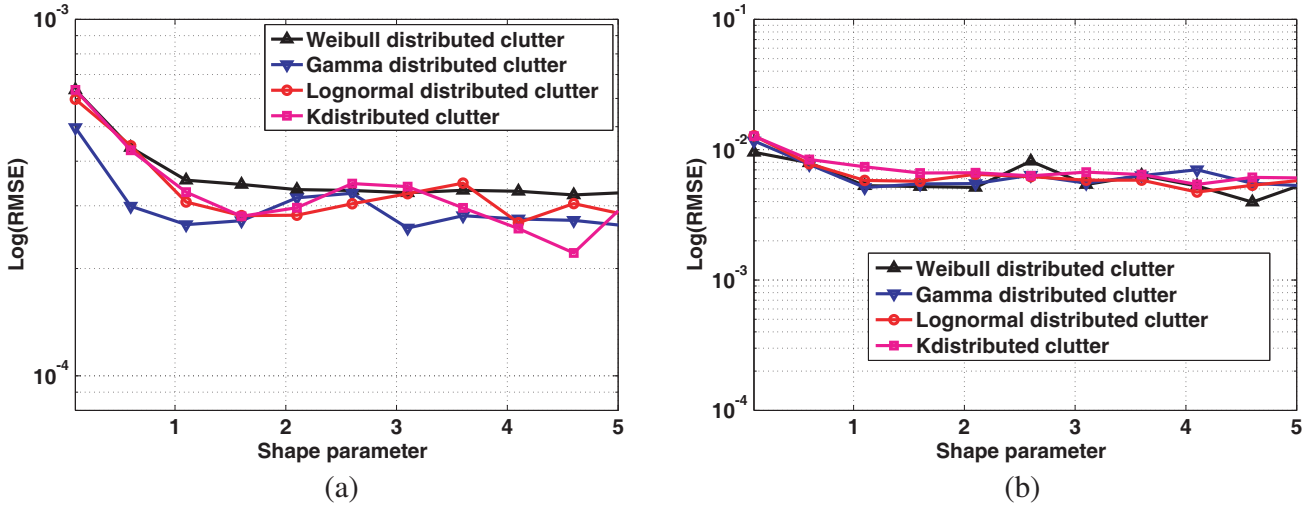


Figure 5. RMSE variations under $\mathcal{H}_1$: (a) Non-central beta generic CDF, (b) Gaussian generic CDF.

resample is then formed by concatenating these blocks. Before detailing the proposed approach, let us recall that the expressions of the lower threshold B_N and the higher threshold A_N to be used are given, respectively by [18]:

$$\sum_{k=1}^{N_{T_r}-p} \binom{N_{T_r}-p}{k} F_{1\Lambda,N}(B_N)^k \left[1 - F_{1\Lambda,N}(B_N)\right]^{N_{T_r}-p-k} - \beta = 0 \tag{12}$$

$$A_N = F_{0\Lambda,N}^{-1}\left[(1-\alpha)^{1/(N_{T_r}-p)}\right] \tag{13}$$

where N_{T_r} is the truncated sample number, which is chosen to be sufficiently large ($N_{T_r} \gg p+1$) to guarantee that the probability that the stopping time N_{stop} is higher than N_{T_r} is sufficiently small under each hypothesis.

Based on the analysis given in 3.1, we assume that $F_{0\Lambda,N}$ is a beta distribution function with unknown shape parameters a_0 and b_0 and that $F_{1\Lambda,N}$ is a non-central beta distribution function also with unknown shape and non-centrality parameters a_1, b_1 and ξ_1.

In the sequel, these unknown parameters are estimated using a moving block bootstrap procedure. They are then used to compute the thresholds B_N and A_N which are used to perform a sequential test.

In the sequel, we assume that the AR order p is known and that, at the beginning of the process, an analysis window of $(p+1)$ data, both in the CUT and in one secondary cell on each side of the CUT, is available. According to the general frame of sequential detection as described in Section 2.2, the proposed approach is performed in the following way:

- Using the first analysis window, the shape and non-centrality parameters of $F_{0\Lambda,N}$ and $F_{1\Lambda,N}$ are estimated using the vector of $\Lambda_{\text{AR-PANMF}}$ values obtained thanks to the moving block bootstrap. Thresholds A_N and B_N are then computed and memorized for all sample sizes until the decision is made. Assume that the decision is made for a sample size N_{stop_1}.
- For the next analysis window, the memorized thresholds are used. If the decision is made for a sample size less than or equal to N_{stop_1} there is no need to use bootstrap function. If the decision requires more samples, the shape and non-centrality parameters of $F_{0\Lambda,N}$ and $F_{1\Lambda,N}$ are estimated in the same way as above. Thresholds are then updated and stored for sizes $N_{stop_1}+1, \ldots, N_{stop_2}$.
- For the i-th analysis window, the bootstrap function is used only if the decision requires more than $N_{stop_{i-1}}$ samples.

The overall procedure is summarized in Tab. 1 and Tab. 2.

For the next analysis window, the procedure is summarized in Tab. 2.

Table 1. BS-AR-PANMF algorithm for the current analysis window.

Current analysis window

• **Step 1**: Generate resample using moving blocks bootstrap;
• **Step 2**: Estimate the AR coefficients using (4);
• **Step 3**: Compute $\Lambda_{\text{AR-PANMF}}$ given in (6) using $\hat{\mathbf{A}}$ given in (3); repeat Steps 1 until 3, B times;
• **Step 4**: Estimate the shape parameters a_0 and b_0 of $F_{0\Lambda,N}$ using the B values of $\Lambda_{\text{AR-PANMF}}$;
• **Step 5**: Estimate the shape and non centrality parameters a_1, b_1 and ξ_1 of $F_{1\Lambda,N}$ using the B values of $\Lambda_{\text{AR-PANMF}}$;
• **Step 6**: Compute thresholds using (12) and (13). $F_{0\Lambda,N}$ and $F_{1\Lambda,N}$ are computed using (9) and (10) where shape and non centrality parametrs are respectively a_0 and b_0 for $F_{0\Lambda,N}$ and a_1, b_1 and ξ for $F_{1\Lambda,N}$;
• **Step 7**: Estimate the AR coefficients using original data;
• **Step 8**: Compute $\Lambda_{\text{AR-PANMF}}$ as in step 3 using original data;
• **Step 9**: Compare $\Lambda_{\text{AR-PANMF}}$ to thresholds:
if $\Lambda_{\text{AR-PANMF}}(x) \leq B_N$ **then**
Accept $\mathcal{H}_0$;
else
if $\Lambda_{\text{AR-PANMF}}(x) \geq A_N$ **then**
Accept $\mathcal{H}_1$;
else
$n \rightarrow n+1$;
repeat Steps 1 until 9;
end
end
$N_{stop_1} \rightarrow n$;
• **Step 10**: Save thresholds A_i and B_i (i = 1 , ..., N_{stop_1});

4. RESULTS

Performance of the proposed approach is evaluated in terms of detection probability (P_{d}) and the required size to achieve it. A comparison of the two aspects is assessed between the proposed BS-AR-PANMF and the sequential approach, based on thresholds derived for Gaussian distributed clutter.

To carry out this, we use real sea clutter data sets collected by the X-band McMaster university, Canada, IPIX radar in February 1998.

After preprocessing, the data are stored in an $N_t \times N_c$ matrix.

Three data files are used, which correspond to range resolutions 30, 15 and 03 meters and to respective polarities HH, HV and VV. A summary description of the used data files, given in [18], is reminded in Tab. 3.

In order to give a general outline about computational cost carried out by the proposed approach, we evaluate the number of bootstrap function uses for analysis the 60000×34 data sets. Tab. 4 and Tab. 5 provide obtained values respectively, under $\mathcal{H}_0$, for the Doppler shift varying from -0.5 to 0.5 and under $\mathcal{H}_1$, for SCR ranging from -15 dB to 15 dB.

Table 2. BS-AR-PANMF algorithm for the next analysis window.

Next analysis window

- **Step 1**: Estimate the AR coefficients using original data;
- **Step 2**: Compute $\Lambda_{\text{AR-PANMF}}$ given in (6) using $\widehat{\mathbf{A}}$ given in (3);

```
if n ≤ N_stop1 then
    Compare Λ_AR-PANMF to saved thresholds;
        if Λ_AR-PANMF(x) ≤ B_N then
            Accept H_0;
        else
            if Λ_AR-PANMF(x) ≥ A_N then
                Accept H_1;
            else
                n → n + 1;
                repeat Steps 1 until 2;
            end
        end
else
    Repeat steps 1 until 10, in Tab. 1 using the second analysis
    window;
end
```

Table 3. Characteristics of the used real data files.

	File 1	File 2	File 3
File name	19980223_165836	19980223_171533	19980223_170435
Date	February 23 1998	February 23 1998	February 23 1998
Time	16 : 58 : 36	17 : 15 : 33	17 : 04 : 35
RF Fréquency (GHz)	9.39	9.39	9.39
Pulse width (ns)	200	20	100
PRF (Hz)	1000	1000	1000
Range resolution (m)	30	03	15
Polarization	Agility	Agility	Agility
Azimuth angle (°)	343.059	344.05	344.517
Grazing angle (°)	0.32	0.32	0.32
Range (m)	3989	3600	3996
Radar height (m)	20	20	20
Radar latitude (°)	43.21	43.21	43.21
Radar longitude (°)	79.60	79.60	79.60
Number of pulses N_t	60000	60000	60000
Number of cells N_c	34	34	34

4.1. Thresholds Variations

Based on the proposed approach, we come up, for each data file, with two thresholds computed using the procedure described in Tab. 1 and Tab. 2. The thresholds are computed until the maximum size required for the corresponding data set.

Table 4. Number of bootstrap function uses for analysis of all the 60000×34 data sets under $\mathcal{H}_0$, for the Doppler shift varying from -0.5 to 0.5.

Data set (resolution/polarity)	$30/HH$	$15/VV$	$03/VV$
Number of bootstrap function uses	39	40	47

Table 5. Number of bootstrap function uses for analysis of all the 60000×34 data sets under $\mathcal{H}_1$, for SCR ranging from -15 dB to 15 dB.

Data set (resolution/polarity)	$30/HH$	$15/VV$	$03/VV$
Number of bootstrap function uses	45	47	56

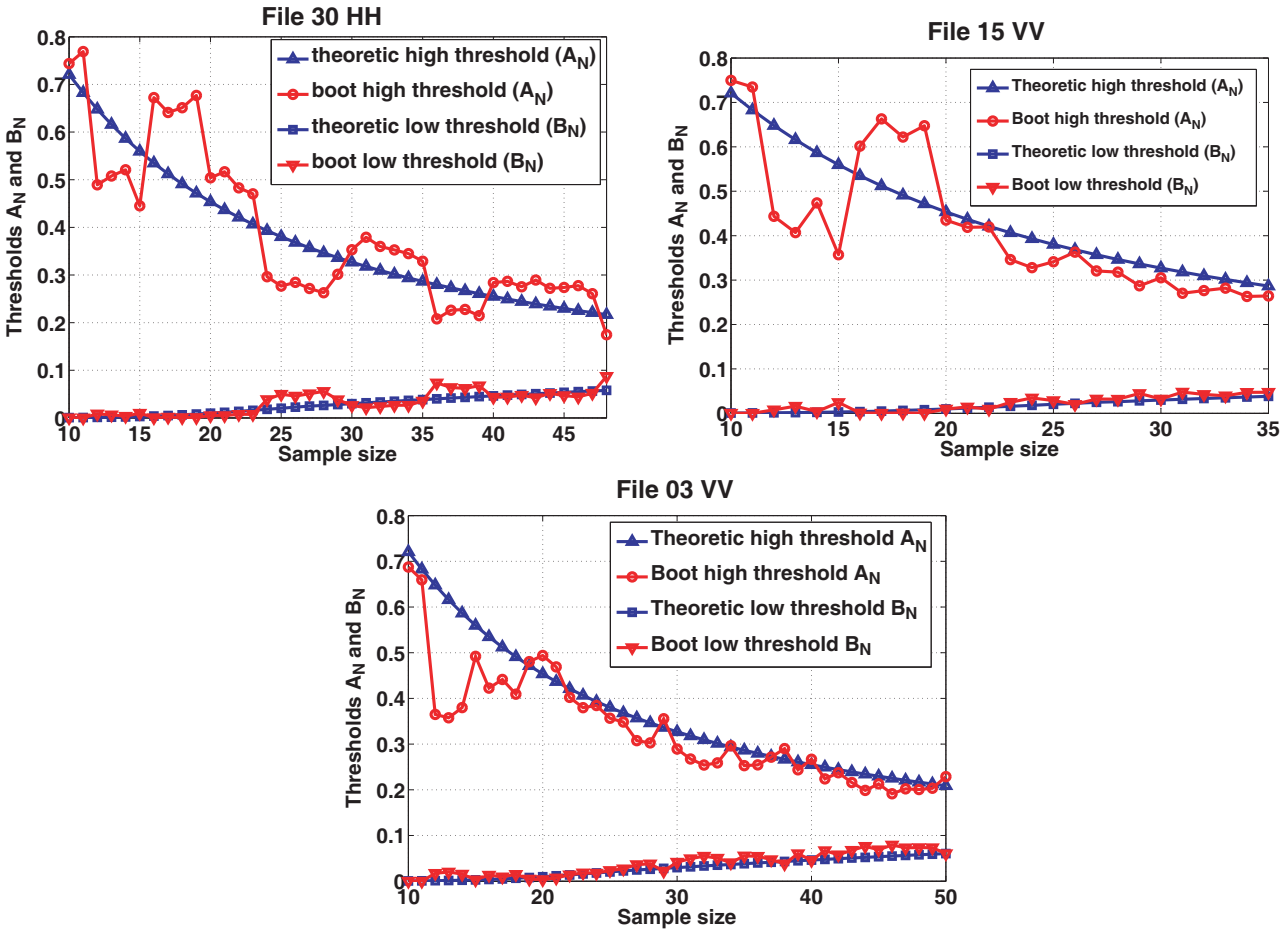


Figure 6. Thresholds A_N and B_N obtained using bootstrap (files $30HH$, $15VV$ and $03VV$).

Figure 6 represents the obtained thresholds. In each sub-figure, we also show, as a benchmark, the thresholds obtained for the Gaussian case, where shape parameters of $F_{0\Lambda,N}$ and $F_{1\Lambda,N}$ are 1 and $N-1$ [18].

4.2. Performance under $\mathcal{H}_1$

In each analysis window, the CUT is set in the middle of the window. One secondary cell is used on both sides of each CUT ($L = 2$). The bootstrap sequential test is run as described in Section 3.3. The probability of detection is evaluated by adding synthetic Swerling-I targets to the CUT.

Figure 7 represents the probabilities of detection obtained using bootstrap estimation of shape parameters, compared to ones obtained using the sequential approach based on theoretic thresholds which correspond to the Gaussian case and the ones obtained using the classical fixed size method. For the $30/HH$ and $15/VV$ data sets, the probability of detection is slightly improved. For the $03/VV$ data set the improvement is about 13%. Fig. 8 shows the size ratios N_f/N_s obtained using the sequential

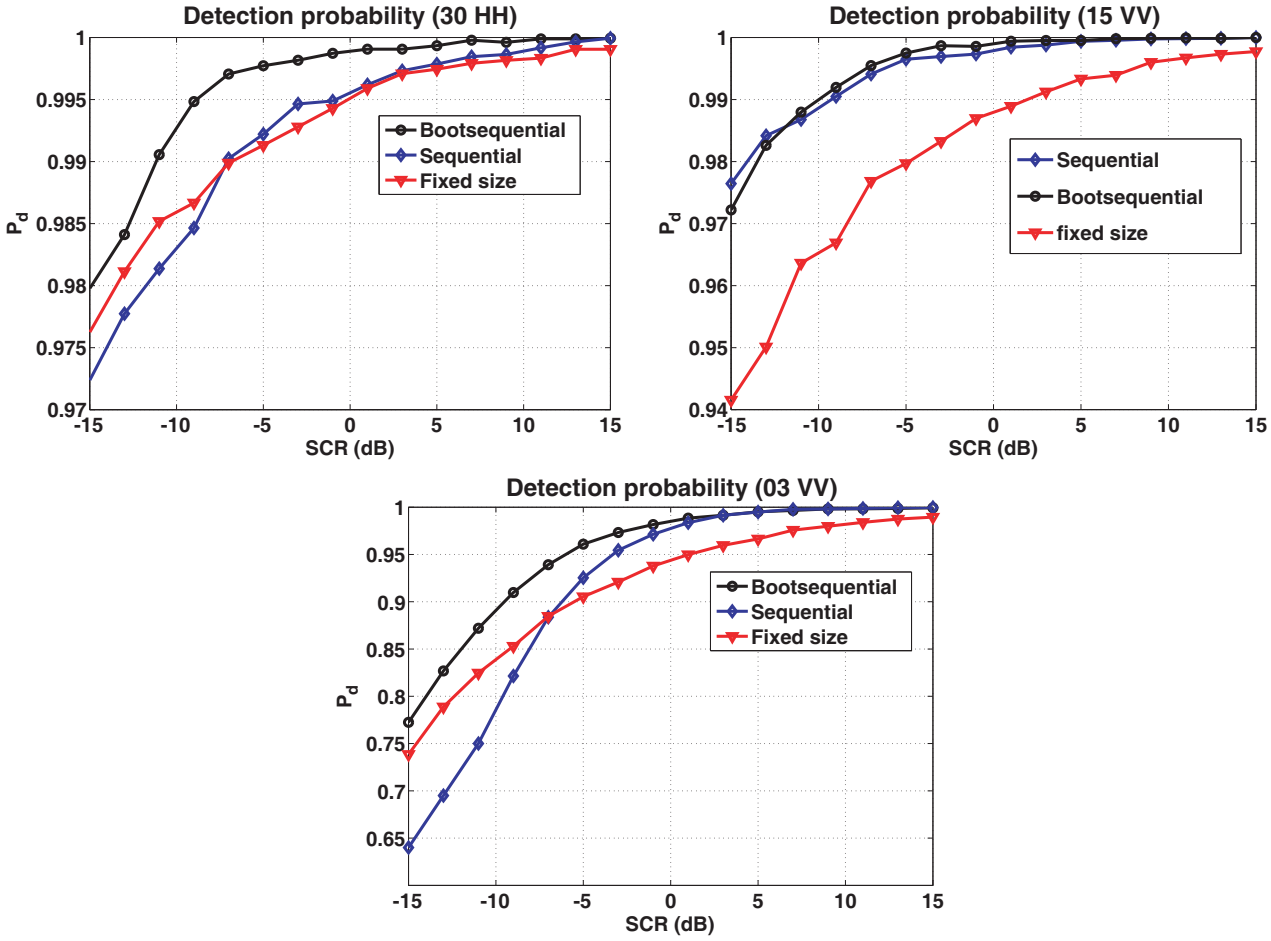
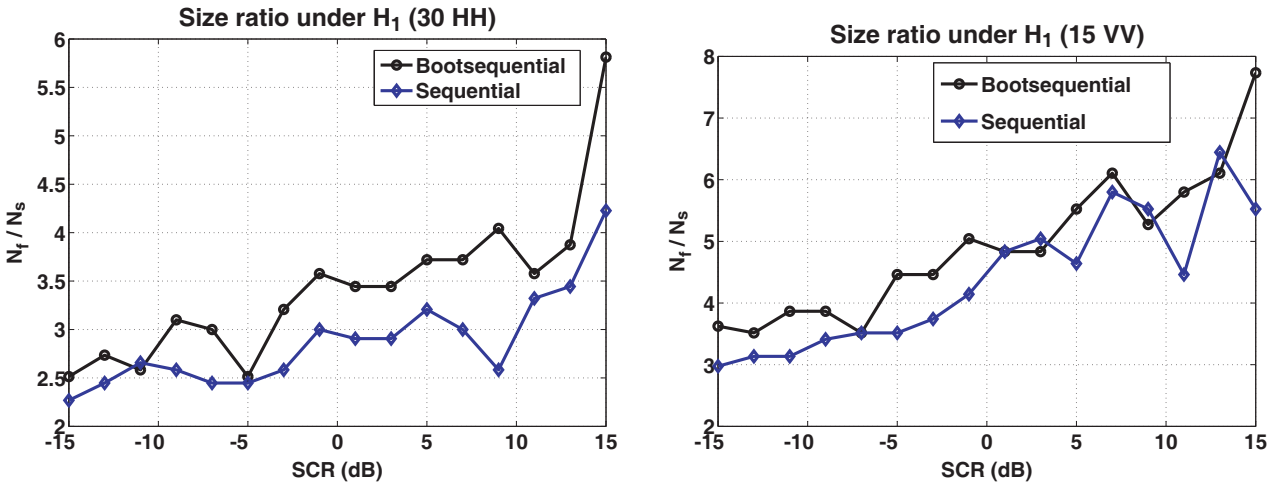


Figure 7. Probability of detection obtained using bootstrap (files $30HH$, $15VV$ and $03VV$).



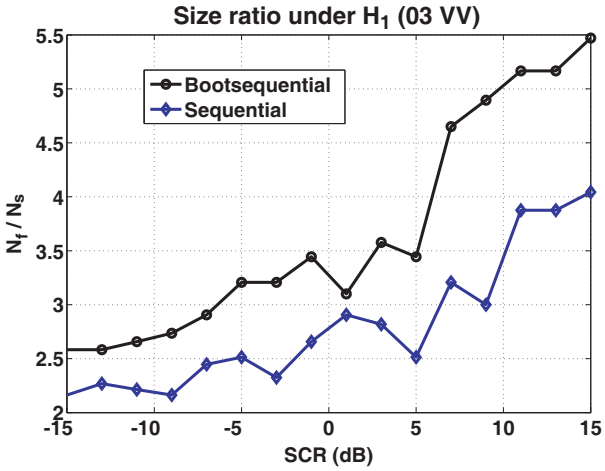


Figure 8. Size ratio, under $\mathcal{H}_1$, obtained using bootstrap (files $30HH$, $15VV$ and $03VV$).

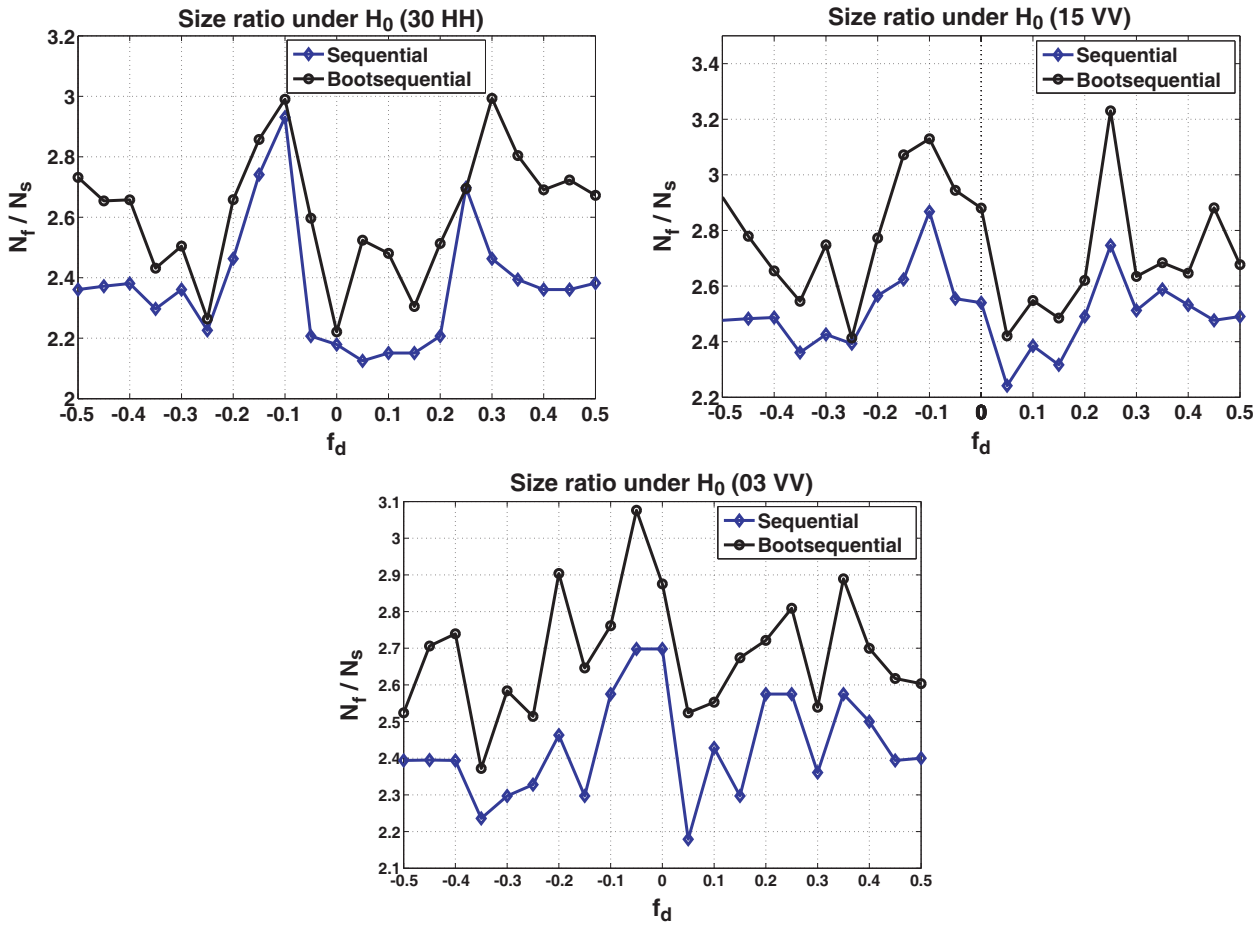


Figure 9. Size ratio, under $\mathcal{H}_0$, obtained using bootstrap (files $30HH$, $15VV$ and $03VV$).

approach, where the clutter is assumed to be Gaussian and the one corresponding to the proposed BS-AR-PANMF. For each approach, the ratio is obtained by dividing the required sample size for the classical method based on a fixed sample size N_f and the sample size N_s, required by each sequential approach. The improvement is noticeable for all data sets but more clear for the $03/VV$ one.

An intuitive reason for this high relative improvement for the $03/VV$ data set is the fact that as the resolution increases, the clutter distribution is farther from the Gaussian model.

4.3. Performance under $\mathcal{H}_0$

Under $\mathcal{H}_0$, the probability of false alarm P_{fa} has the same behaviour as in [18]. The results presented here concern the gain brought by the proposed BS-AR-PANMF in terms of sample size.

Figure 9 shows the size ratios obtained using the sequential approach and the BS-AR-PANMF. As under $\mathcal{H}_1$, we note that the higher is the resolution, the better is the sample size gain.

5. CONCLUSIONS

In this paper, we have proposed a new approach to apply the parametric bootstrap in sequential detection for composite hypothesis testing in non-Gaussian clutter. The detection statistic is based on the approximation of the clutter as an AR process.

Simulation study has shown that the distributions of this parametric detector, under under $\mathcal{H}_0$ and under $\mathcal{H}_1$ fit well, respectively, the beta and the non-central beta models, with unknown parameters. The moving block bootstrap is then used to estimate these parameters and to update the thresholds in an adaptive manner based on current observations. The general computation cost is reduced by saving updated thresholds and using theme as long as they allow to make a decision until given sample sizes.

Detection performance and required sample sizes are investigated using real data. For all used data files, the proposed BS-AR-PANMF provides detection performance better than that of the sequential AR-PANMF detector based on analytic thresholds derived for Gaussian clutter. In terms of required sample sizes, the BS-AR-PANMF gives acceptable gains.

Under the two hypothesis and regarding detection probabilities and required sample sizes, the performance is better for high resolution. This is because in this case the distribution of the clutter is far from the Gaussian assumption, and thresholds update is more suitable.

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