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New Features of the "Double-Humped Effect" in the Magnetized Plasma

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Abstract—Statistical moments of a scattered field are calculated in the first and second approximations using modified smooth perturbation method. Analytical expressions of both the variance and correlation function are obtained in the principle plane containing wave vector of an incident wave and external magnetic field. Observation points are spaced apart at small distances taking into account diffraction effects. Numerical calculations are carried out for the anisotropic Gaussian spectral function containing both anisotropic factor and the angle of inclination of elongated anisotropic plasma irregularities using the experimental data. It was shown that 3D surface of the correlation function of the phase fluctuation oscillate and these variations are decreased increasing characteristic spatial scale of plasma irregularities. New peculiarities of the "Double-humped Effect" are revealed in the collisionless magnetized plasma. It was shown that spatial scale and the inclination angle of elongated anisotropic plasma irregularities play important role in formation of a gap in the spatial power spectrum. Varying the magneto-ionospheric plasma parameters and values of characteristic spatial scales of anisotropic irregularities the depth of a dip increases and oscillates.

1. INTRODUCTION

Fluctuations in amplitude and phase (scintillation) of radio waves passing through the ionosphere are caused by spatial irregularities in the electron density. The sizes of ionospheric irregularities have been obtained by several techniques, including topside sounding, radio-satellite scintillations and direct measurements by satellite probes. The irregularities have a variety of sizes and usually are elongated in the magnetic field direction [1]. Electromagnetic waves propagation in the turbulent atmosphere is discussed for the case when the scattering is weak. Geomagnetic field leads to the birefringence and anisotropy.

The connection between irregularities of random media, phase and amplitude fluctuations has been studied in [2, 3].

The features of the spatial power spectrum (SPS) of scattered radiation in magnetized anisotropic plasma in the complex geometrical optics approximation using the perturbation method have been investigated in [4, 5]. The power spectra of scintillation signals can yield valuable information about wavy processes in plasma and the structure of the irregularities. Evolution of the SPS of EM waves propagating and scattering in magnetized ionospheric plasma with elongated plasma irregularities has been investigated in [6–8].

The present paper reports second order statistical moments of the SPS and new peculiarities of the "Double-humped effect" of multiple scattered electromagnetic waves in randomly inhomogeneous anisotropic collisionless magnetized plasma with electron density fluctuations. The evaluation of a

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double-peak shape in the SPS of multiple scattered field is analyzed under oblique illumination of turbulent plasma by mono-directed incident radiation taking into account diffraction effects in the principle plane containing wave vector of an incident wave and external magnetic field. Numerical calculations have been carried out for the Gaussian anisotropic plasma irregularities using experimental data for the ionospheric F-region.

2. FORMULATION OF THE PROBLEM

One of the important research objectives is the relationship between scintillation and ionospheric plasma-density irregularities in the context of space weather and scintillation models. The ionospheric scintillation study comes from its significant impact on satellite radio communications which can degrade the performance of navigation systems and generate errors in received messages. The irregularities distort the original wavefront, giving rise to a randomly phase-modulated wave.

Electric field in the turbulent collisionless magnetized plasma satisfies the wave equation:

$$\left(\frac{\partial^2}{\partial x_i \partial x_j} - \Delta\delta_{ij} - k_0^2 \varepsilon_{ij}(\mathbf{r})\right)\mathbf{E_j}(\mathbf{r}) = 0, \tag{1}$$

with the components of the dielectric permittivity [9]:

$$\begin{aligned}\varepsilon_{xx} &= 1 - \frac{v}{1-u}, \quad \varepsilon_{xy} = -\varepsilon_{yx} = i\frac{v\sqrt{u}\cos\alpha}{1-u}, \quad \varepsilon_{xz} = -\varepsilon_{zx} = -i\frac{v\sqrt{u}\sin\alpha}{1-u},\\ \varepsilon_{yy} &= 1 - \frac{v\,(1-u\sin^2\alpha)}{1-u}, \quad \varepsilon_{yz} = \varepsilon_{zy} = \frac{uv\sin\alpha\cos\alpha}{1-u}, \quad \varepsilon_{zz} = 1 - \frac{v(1-u\cos^2\alpha)}{1-u},\end{aligned} \tag{2}$$

where: Δ is the Laplacian; δ_{ij} is the Kronecker symbol; α is the angle between the Z-axis (the direction of the wave propagation) and the static external magnetic field $\mathbf{H_0}$ lying in the YZ principle plane; $\omega_p(\mathbf{r}) = [4\pi N(\mathbf{r})e^2/m]^{1/2}$ is the plasma frequency; $N(\mathbf{r})$ is the electron concentration; $u(\mathbf{r}) = (e\,H_0(\mathbf{r})/mc\omega)^2$ and $v(\mathbf{r}) = \omega_p^2(\mathbf{r})/\omega^2$ are the magneto-ionic parameters. At high frequency the effect of ions can be neglected.

Consider a plane wave propagating in the Z direction. Wave field we introduce as [7] $E_j(\mathbf{r}) = E_{0j}\exp(\varphi_1 + \varphi_2 + ik_\perp y + ik_0 z)$ $(k_\perp \ll k_0)$, $k_\perp$ is the wavenumber normal to the principle plane and k_0 the wavenumber of an incident wave. Electron density $v(\mathbf{r}) = v_0[1 + n_1(\mathbf{r})]$ is a random function of position: $\varepsilon_{ij}(\mathbf{r}) = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)}(\mathbf{r})$, $|\varepsilon_{ij}^{(1)}(\mathbf{r})| \ll 1$. First component represents zero-order approximation; fluctuations of complex phase are of the order $\varphi_1 \sim \varepsilon_{ij}^{(1)}$, $\varphi_2 \sim \varepsilon_{ij}^{(1)2}$ The parameter $\mu = k_\perp/k_0$ describing diffraction effects is calculated in zero-order approximation.

Taking into account inequalities characterizing modify smooth perturbation method [2, 10]:

$$\left|\frac{\partial\varphi_1}{\partial z}\right| \ll k_0|\varphi_1|, \qquad \left|\frac{\partial^2\varphi_1}{\partial z^2}\right| \ll k_0\left|\frac{\partial\varphi_1}{\partial z}\right|, \qquad \left|\frac{\partial\varphi_2}{\partial z}\right| \ll k_0|\varphi_2|, \qquad \left|\frac{\partial^2\varphi_2}{\partial z^2}\right| \ll k_0\left|\frac{\partial\varphi_2}{\partial z}\right|,$$

in the first approximation we obtain:

$$\left[\frac{\partial^2\varphi_1}{\partial x_i \partial x_j} + \frac{\partial\varphi_0}{\partial x_i}\frac{\partial\varphi_1}{\partial x_j} + \frac{\partial\varphi_1}{\partial x_i}\frac{\partial\varphi_0}{\partial x_j} - \delta_{ij}\left(\Delta_\perp + 2ik_\perp\frac{\partial\varphi_1}{\partial y} + 2ik_0\frac{\partial\varphi_1}{\partial z}\right) - k_0^2\varepsilon_{ij}^{(0)}\right]E_{0j} = 0. \tag{3}$$

where $\Delta_\perp = (\partial^2\varphi_1/\partial x^2) + (\partial^2\varphi_1/\partial y^2)$ is the transversal Laplasian.

Fourier transform of the phase fluctuations is

$$\varphi_1(x, y, z) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} d\,k_y \psi(k_x, k_y, z)\exp(i\,k_x x + i\,k_y y).$$

2D spectral function of the phase fluctuation in the principle plane (Y-component) satisfies differential equation:

$$\begin{aligned}&\frac{\partial\psi}{\partial z} + \frac{i}{(k_y + k_\perp)(E_{0z}/E_{0x}) - 2k_0(E_{0y}/E_{0x})}\left[k_x(k_y + k_\perp) + k_0 k_y\frac{E_{0z}}{E_{0x}} - k_x^2\frac{E_{0y}}{E_{0x}}\right]\psi\\ &= -i\frac{i}{(k_y + k_\perp)(E_{0z}/E_{0x}) - 2k_0(E_{0y}/E_{0x})}\left(-i\varepsilon_{xy}^{(1)} + \varepsilon_{yy}^{(1)}\frac{E_{0y}}{E_{0x}} + \varepsilon_{yz}^{(1)}\frac{E_{0z}}{E_{0x}}\right),\end{aligned}$$

Polarization coefficients are defined as [9]:

$$\frac{E_{0y}}{E_{0x}} = i\mathrm{P}_j, \qquad \frac{E_{0z}}{E_{0x}} = i\Gamma_j, \tag{4}$$

here: $\mathrm{P}_j = \frac{2\sqrt{u}(1-\mathrm{V})\cos\alpha}{u\sin^2\alpha \pm \sqrt{u^2\sin^4\alpha + 4u(1-\mathrm{v})^2\cos^2\alpha}}$, $\Gamma_j = -\frac{\mathrm{v}\sqrt{u}\sin\alpha + \mathrm{P}_j u\mathrm{v}\sin\alpha\cos\alpha}{1-u-\mathrm{v}+u\mathrm{v}\cos^2\alpha}$, upper sign ($j = 1$) before square root corresponds to the ordinary wave, lower sign to the extraordinary wave ($j = 2$). External magnetic field changes electromagnetic properties of plasma make it magnetized (birefringence,gyrotropic and anisotropic) medium. Gyrotropy of plasma is revealed in elliptic polarization of normal waves; anisotropy is appeared in the direction of propagation depending of their characteristics (polarization, refractive index and absorption).

Using the Fourier transformation for the spectral component we obtain:

$$\frac{\partial\psi}{\partial z} + \frac{id_1 + d_2}{2k_0\mathrm{P}_j}\psi(æ, z) = \frac{ik_0}{2\mathrm{P}_j}\left[-\varepsilon_{xy}^{(1)}(æ, z) + \mathrm{P}_j\varepsilon_{yy}^{(1)} + \Gamma_j\varepsilon_{yz}^{(1)}\right] n_1(æ, z), \tag{5}$$

Solving Equation (5) using the boundary condition $\psi(æ, z = 0) = 0$ we calculate statistical characteristics of scattered electromagnetic waves.

3. SECOND ORDER STATISTICAL MOMENTS OF SCATTERED ELECTROMAGNETIC FIELD

Using Equation (5) taking into account $\langle\Theta_{\alpha\beta}(æ, z')\Theta_{\gamma\delta}(æ', z'')\rangle == W_{\alpha\beta,\gamma\delta}(æ, z' - z'')\delta(æ + æ')$ and new variables: $z' - z'' = \rho_z$, $z' + z'' = 2\eta$, for the variance of the phase fluctuation we obtain:

$$\left\langle\varphi_1^2(\mathbf{r})\right\rangle = -\frac{\pi}{2}\frac{k_0 L}{\mathrm{P}_j^2}\int\limits_{-\infty}^{\infty} dk_x \int\limits_{-\infty}^{\infty} dk_y Q_1(k_x, k_y), \tag{6}$$

where: $Q_1(k_x, k_y) = V_{xy,xy} + \mathrm{P}_j^2 V_{yy,yy} + \Gamma_j^2 V_{yz,yz} - 2(\mathrm{P}_j V_{xy,yy} + \Gamma_j V_{xy,yz} + \mathrm{P}_j\Gamma_j V_{yy,yz})$ contains arbitrary 3D spectral correlation function of electron density fluctuation, $æ = \{k_x, k_y\}$, $V_{\alpha\beta,\gamma\delta}(æ, iG_3 - G_4)$, L is the distance travelling by electromagnetic wave in the ionospheric plasma, and the angular brackets indicate the statistical average: $G_3 = k_x k_\perp / 2k_0\mathrm{P}_j$, $G_4 = k_0 k_y \Gamma_j / 2k_0\mathrm{P}_j$.

Correlation function of the phase fluctuations is:

$$W_\varphi(\rho) >\equiv \langle\varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r} + \rho)\rangle = \frac{\pi}{2}\frac{k_0^2 L}{\mathrm{P}_j^2}\int\limits_{-\infty}^{\infty} dk_x \int\limits_{-\infty}^{\infty} dk_y Q_2(k_x, k_y)\exp(-ik_x\rho_x - i\,k_y\rho_y), \tag{7}$$

where: $Q_2(k_x, k_y)$ function contains $V_{\alpha\beta,\gamma\delta}(æ, G_5)$, $G_5 = k_0 k_x(\mathrm{P}_j k_x + ik_\perp)/2\mathrm{P}_j k_0^2$, and ρ_y and ρ_x are distances between observation points spaced apart in the principle and perpendicular planes, respectively. Phase fluctuations at different observation points are not independent and they correlate. The asterisk indicates the complex conjugate.

Phase fluctuations in the second order approximation satisfies stochastic differential equation:

$$\frac{\partial\langle\varphi_2\rangle}{\partial z} - k_0(\Lambda_1 + i\Lambda_2)\langle\varphi_2(æ, z)\rangle = \frac{1}{2\mathrm{P}_j k_0}\langle F_1(æ, z)\rangle, \tag{8}$$

where: $\Lambda_1 = \frac{k_x}{2\mathrm{P}_j k_0^2}(k_\perp + k_y)$, $\Lambda_2 = \frac{1}{2\mathrm{P}_j k_0^2}(\Gamma_j k_0 k_y - \mathrm{P}_j k_x^2)$,

$$\begin{gathered}\langle F_1(æ, z)\rangle = -\left\langle\frac{\partial\varphi_1}{\partial x}\frac{\partial\varphi_1}{\partial y}\right\rangle + i\mathrm{P}_j\left\langle\left(\frac{\partial\varphi_1}{\partial x}\right)^2\right\rangle, \\ \left\langle\frac{\partial\varphi_1}{\partial x}\frac{\partial\varphi_1}{\partial y}\right\rangle = -\frac{\pi}{2}\frac{k_0^2}{\mathrm{P}_j^2}\int\limits_{-\infty}^{\infty} dk_x \int\limits_{-\infty}^{\infty} dk_y k_x k_y \frac{G_1 + iG_2}{G_1^2 + G_2^2}\left\{1 - \exp\left[G_1 - iG_2)z\right] V_{\alpha\beta,\gamma\delta}(æ, iG_3 - G_4)\right\}.\end{gathered} \tag{9}$$

Solution of the Equation (8) is:

$$\langle\varphi_2(x,y,L)\rangle = -\frac{\pi}{4}\frac{k_0^4}{\mathrm{P}_j^3}\int\limits_{-\infty}^{\infty}dx\int\limits_{-\infty}^{\infty}dy\left(-xy+i\mathrm{P}_jx^2\right)\frac{\tilde{G}_1+i\tilde{G}_2}{\tilde{G}_1^2+\tilde{G}_2^2}V_{\alpha\beta,\gamma\delta}\left(x,y,i\tilde{G}_3-\tilde{G}_4\right)$$
$$\cdot\exp\left[(\Lambda_1+i\Lambda_2)k_0L\right]\int\limits_0^L dz\left\{1-\exp\left[(\tilde{G}_1-i\tilde{G}_2)k_0z\right]\right\}\exp\left[-(\Lambda_1+i\Lambda_2)k_0z\right], \quad (10)$$

where: $\tilde{G}_i = k_0G_i$, $x = k_x/k_0$, $y = k_y/k_0$ are non-dimension wave parameters.

The transverse correlation function of a scattered field $W_{EE^*}(\rho) = \langle E(\mathbf{r})E^*(\mathbf{r}+\rho)\rangle$ is expressed via the correlation function and the variances of the phase fluctuations in the first and second approximations [11–13]:

$$W_{EE^*}(\rho,k_\perp) = E_0^2\exp\left[\frac{1}{2}\left(\left\langle\varphi_1^2(\mathbf{r})\right\rangle+\left\langle\varphi_1^{2*}(\mathbf{r}+\rho)\right\rangle\right)+\langle\varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r}+\rho)\rangle+2\mathrm{Re}\,\langle\varphi_2\rangle\right]\cdot\exp(-i\rho_yk_\perp), \quad (11)$$

where: E_0^2 is the intensity of an incident radiation. In the ray-(optics) approximation describing multiple scattering in the random media, the condition $\sqrt{\lambda L} \ll l_\parallel$ is fulfilled, but it neglects the diffraction effects. If a distance L travelled by the wave in a turbulent magnetized plasma is substantially big, $L \gg (l_\parallel/\lambda)$, diffraction effects become essential. The smooth perturbation method is more general method for the solution of diffraction effects if the parameter $\lambda/l_\parallel$ is small. SPS of the scattered field in case of an incident plane wave is easily calculated by Fourier transform of the transversal correlation function $W(k',k_\perp)$ of a scattered field [2, 14]:

$$W(k,k_\perp) = \int\limits_{-\infty}^{\infty}d\rho_yW_{EE^*}(\rho_y,k_\perp)\exp(ik\rho_y). \quad (12)$$

If the angular power spectrum (APS) of an incident wave has a finite width and its maximum coincide with z axis, APS of scattered radiation is given:

$$I(k) = \int\limits_{-\infty}^{\infty}dk_\perp W(k,k_\perp)\exp\left(-k_\perp^2\beta^2\right), \quad (13)$$

where β characterizes the dispersal of an incident radiation (disorder of an incident radiation), k is a transverse component of the wave vector of scattered field.

4. NUMERICAL RESULTS

The incident electromagnetic wave has the frequency of 3 MHz ($k_0 = 6.28\cdot10^{-2}\,\mathrm{m}^{-1}$). Plasma parameters at the altitude of 300 km are: $u = 0.22$, $\mathrm{v} = 0.28$. Numerical calculations are carried out for the angle $\alpha = 20°$ between incident wave propagation and external magnetic field. $\mathrm{v} = 0.28$.

An RH-560 rocket flight was conducted from Sriharikota rocket range (SHAR), India (14°N, 80°E, dip latitude 5.5°N) to study electron density irregularities during spread F. The rocket was launched an apogee of 348 km. It was found that the irregularities were present continuously between 150 and 257 km. Large-scale structures, with vertical scale sizes up to a few tens of km, are also seen in this region. Vertical structures, in the scale size range of 5–10 km, are also quite prominent. The most intense irregularities occurred in three patches at 165–178 km, 210–257 km and 290–330 km [15, 16]. Irregularities of a range of scale sizes starting from a few hundred meters to a few ten of kilometers are observed in this patch. Studying the equatorial spread F irregularities using RH-560 rocket instrumented with Langmuir probes launched from SHAR it was established [15, 16] that the relationship between the spectral index, p and the mean integrated spectral power (in 20 m to 200 m scale size range) could be represented by a Gaussian function.

A knowledge of the SPS of ionospheric refractive index fluctuations can lead to an understanding of the physical processes that characterize the region of the ionosphere under study. The anisotropic 3D Gaussian autocorrelation function describing narrow-band process has the following form [4]:

$$V_n(k_x,k_y,k_z)=\sigma_n^2\frac{l_\parallel^3}{8\pi^{3/2}\chi^2}\exp\left(-\frac{k_x^2l_\perp^2}{4}-p_1\frac{k_y^2l_\parallel^2}{4}-p_2\frac{k_z^2l_\parallel^2}{4}+p_3k_yk_zl_\parallel^2\right), \tag{14}$$

where: $p_1=(\sin^2\gamma_0+\chi^2\cos^2\gamma_0)^{-1}\left[1+(\chi^2-1)^2\sin^2\gamma_0\cos^2\gamma_0/\chi^2\right]$, $p_2=(\sin^2\gamma_0+\chi^2\cos^2\gamma_0)/\chi^2$, $p_3=(\chi^2-1)\sin\gamma_0\cos\gamma_0/2\chi^2$, σ_n^2 is the mean-square fractional deviation of electron density. Diffusion processes along and across directions with respect to the geomagnetic lines of force leads to the anisotropy of plasma irregularities in the ionospheric F-region. The shape of electron density irregularities has a spheroidal form with its axis along the magnetic field line. Elongated spheroid characterizes by anisotropy coefficient $\chi=l_\parallel/l_\perp$, the ratio of longitudinal and transverse linear sizes of plasma irregularities, and the slope angle γ_0 of its axis to the geomagnetic lines of force. The structure, the generation and the evolution of plasma irregularities are dependent on the condition in the ionospheric F-region.

It was established that small ($\leq 10\,\text{km}$) F-region irregularities are highly elongated along the direction of the magnetic field, and large anisotropy may also be expected in the case of certain types of plane waves, propagating in the ionosphere. On the other hand, large irregularities ($\geq 10\,\text{km}$) may be only weakly anisotropic.

For small-scale ionospheric irregularities having characteristic linear scale $\sim 1\,\text{km}$ the ratio of the diffusion coefficients along and transverse directions with respect to the geomagnetic field on the altitude 300 km is $D_\parallel/D_\perp\sim(\nu_{in}^2+\Omega_i^2)/\nu_{in}^2\sim 10^4-10^5$, taking into account that the diffusion coefficients are proportional to the corresponding mobility and the diffusion spread of the ionospheric irregularities are determined by ions mobility, not by electrons mobility. Solving the diffusion equation and taking into account the initial conditions it is easy to show that for creation of elongated ionospheric irregularities along the external magnetic field having anisotropy factor ≥ 10 it is necessary time ≥ 2 minutes. We should also point out that lifetime of ionospheric irregularities is determined by turbulence varying substantially along latitude but not by diffusion.

Substituting the spectral function in Equation (14) into Equation (6) and applying the saddle-point method ($\xi=k_0l_\parallel\gg 1$) in the saddle point $x_0=0$ the variance of the phase fluctuations caused by the electron density fluctuations is:

$$\begin{aligned}\langle\varphi_1^2(\mathbf{r})\rangle &= -\frac{\pi}{4}\sigma_n^2\xi k_0L\frac{A_0}{\mathrm{P}_j^2\chi^2}\left(\frac{1}{\chi^2}-\frac{p_2\mu^2}{4\mathrm{P}_j^2}\right)^{-1/2}\left[p_1+\frac{p_2}{4}\frac{\Gamma_j^2}{\mathrm{P}_j^2}-2p_3\frac{\Gamma_j}{\mathrm{P}_j}-\frac{\mu^2}{\mathrm{P}_j^2}\left(p_3-\frac{p_2}{4}\frac{\Gamma_j}{\mathrm{P}_j}\right)^2\right.\\ &\quad\left.\cdot\left(\frac{1}{\chi^2-\frac{p_2\mu^2}{4\mathrm{P}_j^2}}\right)^{-1}\right]^{-1/2},\end{aligned} \tag{15}$$

where: $A_0=\frac{\mathrm{v}_0^2}{(1-u_0)^2}[u_0\cos^2\alpha+\mathrm{P}_j^2(1-u_0\sin^2\alpha)^2+\Gamma_j^2u_0^2\sin^2\alpha\cos^2\alpha]$.

$$-2\frac{\mathrm{v}_0^2\sqrt{u_0}\cos\alpha}{(1-u_0)^2}\left[\mathrm{P}_j(1-u_0\sin^2\alpha)+\Gamma_ju_0\sin\alpha\cos\alpha+\mathrm{P}_j\Gamma_j\sqrt{u_0}(1-u_0\sin^2\alpha)\sin\alpha\right].$$

Correlation function of the phase fluctuations in Eq. (7) for the Gaussian correlation function yields:

$$\begin{aligned}W_\varphi(\eta_x,\eta_y,L) &= \sigma_n^2\cdot\frac{\xi^2k_0L}{16\sqrt{\pi}}\cdot\frac{A_0}{\mathrm{P}^2\chi^2}\int\limits_{-\infty}^{\infty}dx\int\limits_{-\infty}^{\infty}dy\exp\left(-\frac{\xi^2}{4}p_1y^2-i\eta_yy\right)\\ &\quad\cdot\exp\left\{-\frac{\xi^2}{4}\left[\frac{p_2}{4}x^4+\left(\frac{1}{\chi^2}-\frac{p_2\mu^2}{4\mathrm{P}^2}+2p_3y\right)x^2\right]\right\}\cdot\exp\left\{-i\frac{\xi^2}{4}\left[\frac{\mu}{2\mathrm{P}}(x^3+4p_3xy)\right]-i\eta_xx\right\}.\end{aligned} \tag{16}$$

In isotropic case ($\chi=1$) neglecting diffraction effects we obtain the well-known result [9] $W_\varphi(L)=\sqrt{\pi}\sigma_n^2\mathrm{v}_0^2\xi k_0L$.

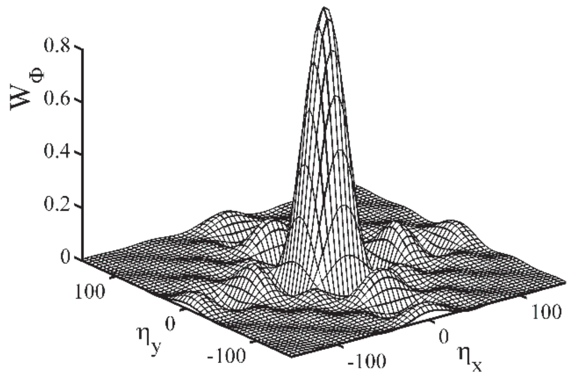

Figure 1. Correlation function of the phase fluctuation at $\xi = 200$, $\chi = 20$, $\gamma_0 = 0°$.

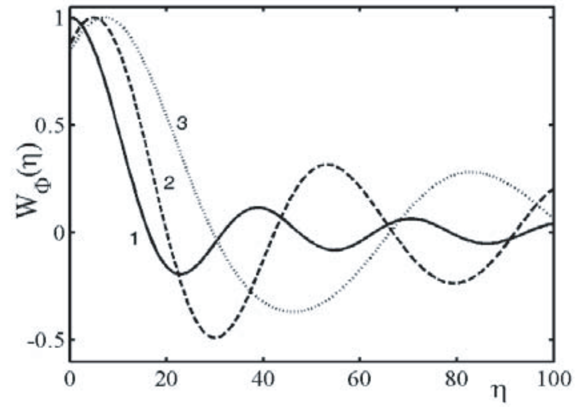

Figure 2. Correlation function as a function of distance between observation points in the principle plane for different angle γ_0.

Figure 1 illustrates oscillating 3D surface of the correlation function of the phase fluctuations versus distances between observation points in the principle and perpendicular planes for large scale plasma irregularities ($l_{\parallel} \approx 3\,\mathrm{km}$) at the diffraction parameter $\mu = 0.06$, anisotropy factor of elongated plasma irregularities $\chi = 20$.

Figure 2 depicts the behaviour of the second order statistical moment for different distances between observation points in the principle plane at the diffraction parameter $\mu = 1$ and $\chi = 5$. Analyses show that for small scale ionospheric plasma irregularities $l_{\parallel} = 300\,\mathrm{m}$ correlation function oscillates having maximums at $\eta_y = 0$ and $\eta_y = 39$ (distance between observation points is equal 600 m) and $\gamma_0 = 0°$ (curve 1); $\eta_y = 5$ and $\eta_y = 53$ (observation points space apart at 200 m and 800 m, respectively) for $\gamma_0 = 20°$ (curve 2); at $\eta_y = 7$ and $\eta_y = 83$ (100 m and 1300 m, respectively) for $\gamma_0 = 40°$. Hence, increasing inclination angle γ_0 between observation points oscillations of the correlation function become smooth.

Figure 3 illustrates the behaviour of the second order statistical moment of the phase fluctuations for different characteristic spatial scale of anisotropic plasma irregularities $l_{\parallel} = 320\,\mathrm{m}$ (curve 1), 1.6 km (curve 2), 3.2 km (curve 3) and 8 km (curve 4) having anisotropy factor $\chi = 10$ and elongating along the line of forces of geomagnetic field ($\gamma_0 = 0°$). Increasing characteristic spatial scale of plasma irregularities from $\xi = 100$ oscillations are disappeared.

Figure 4 represents curves of correlation function of the phase fluctuations for elongated irregularities $l_{\parallel} = 32\,\mathrm{km}$ for different inclination angle $\gamma_0 = 0°,\ 10°,\ 20°,\ 30°$.

Substituting Equation (14) into Equation (10), complex statistical moment can be easily calculated in the second order approximation:

$$\langle \varphi_2(L) \rangle = -\sigma_n^2 \frac{\xi^3 k_0 L}{32\sqrt{\pi}} \frac{A_0}{\mathrm{P}^2 \chi^2} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \left\{ 1 + \frac{\tilde{G}_1 + i\tilde{G}_2}{k_0 L(\tilde{G}_1^2 + \tilde{G}_2^2)} \left[1 - \exp(\tilde{G}_1 - i\tilde{G}_2) k_0 L \right] \right\} \cdot \exp\left[-\frac{\xi^2}{4}(a x^2 + b\, y^2) \right] \exp(-igxy), \tag{17}$$

where $a = \frac{1}{\chi^2} - \frac{p_2 \mu^2}{4\mathrm{P}_j^2}$, $b = p_1 + \frac{p_2}{4}\frac{\Gamma_j^2}{\mathrm{P}_j^2} - 2p_3 \frac{\Gamma_j}{\mathrm{P}_j}$, $g = \frac{\xi^2}{2}\frac{\mu}{\mathrm{P}_j}(p_3 - \frac{p_2}{4}\frac{\mathrm{P}_j}{\Gamma_j})$.

Figure 5 represents the SPS of a scattered field for different characteristic spatial scales of ionospheric plasma irregularities using Equation (12). Increasing parameter ξ from 100 ($l_{\parallel} = 1.6\,\mathrm{km}$) to 150 the gap appears in the SPS (curve 2). Curve 3 corresponds to $l_{\parallel} = 3.2\,\mathrm{km}$. Growing spatial scale of anisotropic irregularities from up to $l_{\parallel} = 6.4\,\mathrm{km}$ deep of a gap increases and oscillates (curve 4).

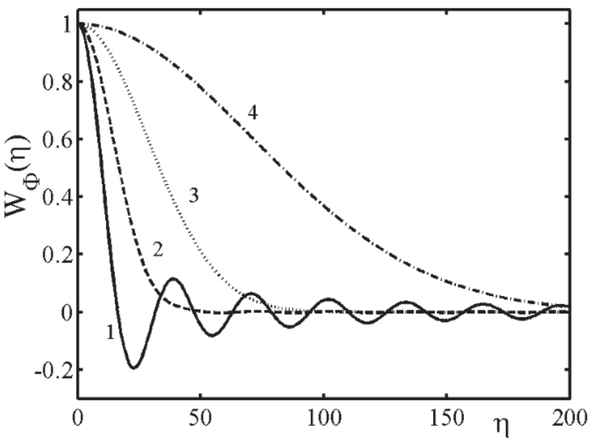

Figure 3. Correlation function as a function of distance between observation points in the principle plane for different characteristic special scale of plasma irregularities $l_{\|}$.

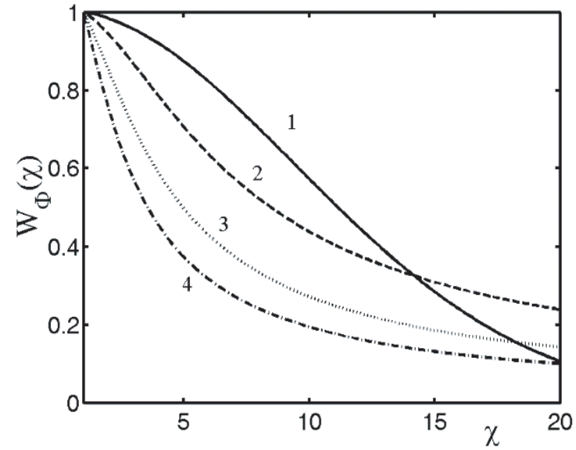

Figure 4. Correlation function of the phase fluctuations versus anisotropy factor χ.

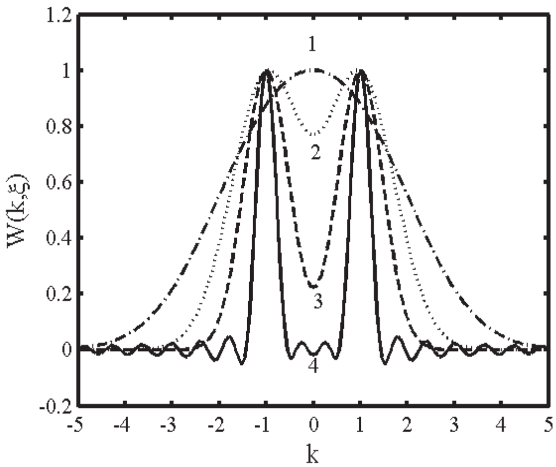

Figure 5. Spatial power spectrum of a scattered field.

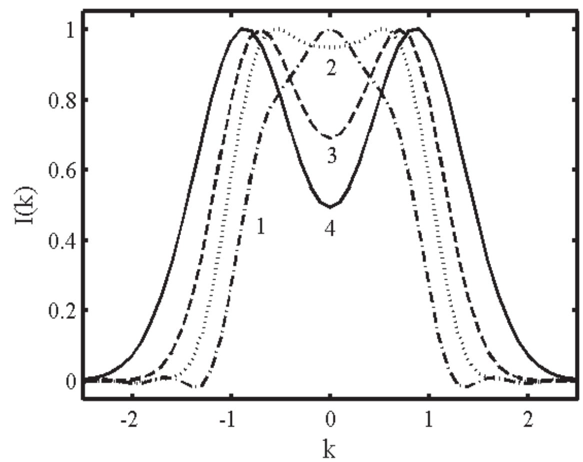

Figure 6. "Double-humped Effect" in the magnetized collisionless plasma.

Figure 6 depicts the "Double-humped Effect" in the collisionless magnetized turbulent plasma in the principle plane containing wave vector of an incident wave and external magnetic field. This effect appears if anisotropic plasma irregularities are strongly elongated along the line of forces of geomagnetic field ($\gamma_0 = 0^\circ$). In this case $\xi = 200$ (or $l_{\|} = 3.2\,\text{km}$), $\beta = 1$, $\chi = 5$ and parameter $B \equiv \sigma_n^2 \cdot \frac{\xi^2 k_0 L}{8} \cdot \frac{A_0}{\mathrm{P}^2 \chi^2}$ varies: $B = 50$ (curve 1), $B = 100$ (curve 2), $B = 200$ (curve 3), $B = 400$ (curve 4). This parameter includes both magneto-ionic plasma parameters and parameters characterizing anisotropic ionospheric plasma irregularities. Numerical calculations show that increasing B the gap becomes deep, SPS broadens and maximums are displaced symmetrically to the direction of wave propagation coinciding with the direction of elongated ionospheric irregularities.

5. CONCLUSION

Second order statistical moments of a scattered electromagnetic waves are calculated in the first and second approximations using modify smooth perturbation method (narrow-angle scattering). Analytical expressions of both the variance and correlation function are obtained in the principle plane containing wave vector of an incident wave and external magnetic field taking into account diffraction effects. Numerical calculations are carried out for the anisotropic Gaussian spectral function containing both anisotropic factor and the angle of inclination of elongated anisotropic plasma irregularities using the experimental data. Analyses are carried out for both small and a very elongated inhomogeneities. Correlation function of the phase fluctuation oscillates for small-scale irregularities and these variations are decreased increasing dip angle and characteristic spatial scale of plasma irregularities. New peculiarities of the "Double-Humped Effect" are revealed in the collisionless magnetized turbulent plasma. It was shown that the spatial scale and the inclination angle of elongated anisotropic plasma irregularities play important role in formation of a gap in the SPS. Varying the magneto-ionospheric plasma parameters and values of anisotropic irregularities the depth of a dip increases and oscillates.

The results could find practical application in optics and be useful in development of principles of remote sensing of random media.

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REFERENCES

1. Kravtsov, Yu. A., Z. I. Feizulin, and A. G. Vinogradov, *Radiowaves Propagation through the Earth Atmosphere, Moscow, Radio and Communication*, Russia, 1983 (in Russian).
2. Ishimaru, A., *Wave Propagation and Scattering in Random Media, Vol. 2, Multiple Scattering, Turbulence, Rough Surfaces and Remote Sensing*, IEEE Press, Piscataway, New Jersey, USA, 1997.
3. Rytov, S. M., Yu. A. Kravtsov, and V. I. Tatarskii, *Principles of Statistical Radiophysics. Vol. 4. Waves Propagation through Random Media*, Springer, Berlin, New York, 1989.
4. Jandieri, G. V., A. Ishimaru, V. G. Jandieri, A. G. Khantadze, and Zh. M. Diasamidze, "Model computations of angular power spectra for anisotropic absorptive turbulent magnetized plasma," *Progress In Electromagnetics Research*, Vol. 70, 307–328, 2007.
5. Jandieri, G. V., A. Ishimaru, B. S. Rawat, and N. K. Tugushi, "Peculiarities of the spatial spectrum of scattered electromagnetic waves in the turbulent collision magnetized plasma," *Progress In Electromagnetics Research*, Vol. 152, 137–149, 2015.
6. Jandieri, G. V., A. Ishimaru, N. F. Mchedlishvili, and I. G. Takidze, "Spatial power spectrum of multiple scattered ordinary and extraordinary waves in magnetized plasma with electron density fluctuations," *Progress In Electromagnetics Research M*, Vol. 25, 87–100, 2012.
7. Jandieri, G. V. and A. Ishimaru, "Some peculiarities of the spatial power spectrum of scattered electromagnetic waves in randomly inhomogeneous magnetized plasma with electron density and external magnetic field fluctuations," *Progress In Electromagnetics Research B*, Vol. 50, 77–95, 2013.
8. Jandieri, G. V., ""Double-humped effect" in the turbulent collision magnetized plasma," *Progress In Electromagnetics Research M*, Vol. 48, 95–102, 2016.
9. Ginzburg, V. L., *Propagation of Electromagnetic Waves in Plasma, Gordon and Beach*, New York, 1961.
10. Gershman, B. N., L. M. Eruxhimov, and Yu. Ya. Yashin, *Wavy Phenomena in the Ionosphere and Cosmic Plasma*, Moscow, Nauka, 1984 (in Russian).
11. Jandieri, G. V., V. G. Gavrilenko, and A. V. Aistov, "Some peculiarities of wave multiple scattering in a statistically anisotropic medium," *Waves Random Media*, Vol. 10, 435–445, 2000.

12. Jandieri, G. V., V. G. Gavrilenko, A. V. Sarokin, and V. G. Jandieri, "Some properties of the angular power distribution of electromagnetic waves multiply scattered in a collisional magnetized turbulent plasma," *Plasma Physics Report*, Vol. 31, 604–615, 2005.
13. Jandieri, G. V., A. Ishimaru, N. N. Zhukova, T. N. Bzhalava, and M. R. Diasamidze, "On the influence of fluctuations of the direction of an external magnetic field an phase and amplitude correlation functions of scattered radiation by magnetized plasma slab," *Progress In Electromagnetics Research B*, Vol. 22, 121–143, 2010.
14. Tatarskii, V. I., *Wave Propagation in a Turbulent Medium*, McGraw-Hill, New York, USA, 1961.
15. Prakash, S. S., S. Pal, and H. Chandra, "In-situ studies of equatorial spread F over SHAR-steep gradients in the bottomside F-region and transitional wavelength results," *J. Atmos. Terr. Phys.*, Vol. 53, 977–986, 1991.
16. Raizada, S. and H. S. S. Sinha, "Some new features of electron density irregularities over SHAR during strong spread F," *Ann. Geophysicae*, Vol. 18, 141–151, 2000.