Wideband Matched Feed Design Employing Conjugate Field Radiated from a Square Choke Excited by Two Slots on a Diagonal Waveguide

Rajib Jana^{1, *} and Ratnajit Bhattacharjee²

Abstract—A simple and compact diagonal matched feed structure is proposed for offset reflector antenna, which includes a square choke to radiate the desired conjugate mode (TE₄: a higher order rectangular coaxial cable mode) for suppressing the cross-polar power of an offset reflector antenna when the reflector is illuminated by the dominant diagonal mode (TE_V): a linear combination of rectangular TE₀₁ and TE₁₀ modes) radiated from the aperture of a central diagonal waveguide. Square choke is excited by two identical slots on the central diagonal waveguide using the longitudinal magnetic field of main operating mode TE_V. Wideband conjugate matching as well as impedance matching for broadband operation can be achieved by such radiating main mode and conjugate mode from apertures which are spatially separated. Based on the above configuration, a J-band matched feed structure is designed using HFSS software for a given offset reflector geometry. The proposed matched feed structure is fabricated and measured. The measured results are compared with simulated ones, and close agreements are found.

1. INTRODUCTION

Wideband conjugate multi-mode matched feed for offset reflector has been drawing considerable attention due to its demands in the application of satellite communication, mono-pulse tracking and telemetry [1]. Various matched feed structures are recently reported for this purpose. For example, an adaptive dual-mode dual-polarized $(TE_{11}^1 + TE_{21}^1, TE_{11}^2 + TE_{21}^2)$ circular matched feed structure is reported as a wideband conjugate matched performer for an offset reflector antenna in [2], and six SMA connectors are attached with it to control the polarization as well as excited power of desired modes of the matched feed. A new kind of aperture, which is an intersection of circle and rectangle, is used for wideband matched feed designed employing the two operating modes having close cutoff wavenumber where x-polarized TE mode as a main operating mode and conjugate mode as a TM mode are reported by the present authors in [3]. In [4], a new type of symmetrical cascaded waveguide discontinuities, which act like irises, has been placed in an oversize circular waveguide supporting TE_{11}^1 and TE_{21}^1 modes. Each discontinuity consists of intersection of three off-centered junctions of circular waveguides, placed symmetrically with angular spacing of 120°. The amplitude and phase flatness of TE_{21}^1 mode relative to TE_{11}^1 mode required for broadband operation are achieved using cascaded iris sections.

Also, it is found in the literature that the presence of choke in front portion of the horn can improve the radiation characteristic of the reflector. However, very limited works have been published on the design of matched feed using the choke. In an interesting work, a multi-ring structure in front of a coaxial feed employing appropriate multi-modes has been designed by Koch [5] to improve the aperture

Received 4 September 2017, Accepted 13 November 2017, Scheduled 1 December 2017

^{*} Corresponding author: Rajib Jana (itsrajibjana@gmail.com).

¹ Department of Electrical Engineering, NIT Arunachal Pradesh, Yupia 791112, India. ² Department of Electronics and Electrical Engineering, IIT Guwahati, Guwahati 781039, India

Jana and Bhattacharjee

efficiency and spill-over of parabolic reflector. In another recent work [6], a matched feed for a wide-band system is proposed where a ring choke is used to radiate the conjugate TE_{21}^1 mode, and central circular waveguide emits TE_{11}^1 mode.

The proposed matched feed is based on diagonal horn, is a variant of rectangular waveguide. In this communication, a square choke is proposed for the design of a diagonal matched feed as illustrated in Fig. 1. For the matched feed proposed in this paper, a semi-analytical technique as found in [1, 3] is used to investigate the performance of a given offset reflector. It may be noted that Physical Optics (PO) is used in this framework for the observation of reflector pattern. Analytical far-field patterns of main operating modes of diagonal horn $(TE_H^D, TE_V^D;$ both are the linear combination of rectangular TE_{01} and TE_{10} modes) are used for this semi-analytical study. The modes associated with the square choke ring are calculated using 2-D FEM technique. Normalized numerical far-field patterns of conjugate modes present at the square choke as TE_3 and TE_4 (rectangular higher order coaxial modes) are calculated based on the treatment as specified in [3]. Conjugate mode TE_3 for TE_H^D main operating mode and TE_4 for TE_V^D as the main diagonal mode are identified to obtain the low cross-polar pattern of reflector. The mode pairs are chosen based on our study using the semi-analytical technique developed for this purpose. The required relative phases and mode coefficients for the chosen pair of modes at the design frequency are calculated based on the study using the probabilistic approach like particle swarm optimization technique (PSO) to ensure the low cross-polar pattern for the considered reflector. Further, finer adjustments in matched feed design is carried out through the parametric study in HFSS and using a proper junction. Our design is implemented based on the operating modes TE_V^D and TE_4 . Using two identical longitudinal slots, TE_4 mode is generated inside the square choke which is shown in Fig. 1. Such kind of design has the advantage that it is very easy to maintain the wideband conjugate matching as well as impedance bandwidth. Further, the proposed matched feed structure is fabricated, and its measured results such as return loss and far-field patterns are compared with simulated results of HFSS. Close match is obtained between simulated and measured results.



Figure 1. (a) Aperture: $\{a', a, b, b'\} = \{26, 29, 35, 43\}$ mm. (b) Proposed matched feed structure: L = 22 mm. (C) Slot: $\{S_L, S_W\} = \{20, 12\}$ mm. (d) Junction between standard circular waveguide (diameter 32.512 mm) and diagonal waveguide.

As discussed, a prototype matched feed is proposed based on the operating modes TE_V^D and TE_4 . On the other hand, TE_H^D and TE_3 mode combination is studied only using our developed semi-analytical technique. As per our observation, generation of the TE_3 mode from TE_H^D using the proper junction is a challenging task which can be considered as a future work.

Progress In Electromagnetics Research M, Vol. 63, 2018

To the best of our knowledge, such kind of matched feed structure and associated modes of such feed aperture have not been studied in literature. Also, the proposed matched feed structure is simple, and it is expected that it will find widespread application in future considering its ability to maintain the wideband conjugate matching and impedance bandwidth.

The rest of the article is organized as follows: The details of proposed feed geometry and its associated modes are explained in Section 2. The study using a semi-analytical technique with proper combination modes to obtain the low cross-polar pattern of reflector is discussed in Section 3. Measured and simulated results are shown in Section 4, and conclusions are drawn in Section 5. Also, an appendix is added as Appendix A to show the derivation of far-field pattern for the dominant modes of diagonal horn.

2. FEED GEOMETRY AND OPERATING MODES

The design of matched feed is carried out for J-band (5.85-8.2 GHz). The feed structure, which is considered for study, is shown in Fig. 1. It mainly consists of three parts. The first part is J-band standard circular waveguide WC-128 which is operated in circular TE_{11}^1 mode to supply the power into the second part. The second part is an inner diagonal waveguide; its one side is connected with standard circular waveguide, and the other side is open ended which is used to radiate the TE_V^D mode. Side a' of inner diagonal waveguide, as specified in Fig. 1, is chosen based on the cutoff wavenumber $\frac{\pi}{a'}$ of TE_V^D operating mode. Side a' is chosen to be 26 mm so that the guide operates only in TE_V^D mode approximately over the desired band of 5.85 to 8.2 GHz. Two identical longitudinal slots are used to excite the third part which is basically another outer diagonal waveguide forming a choke as shown in Fig. 1. Such longitudinal slots are utilized to generate a higher order rectangular coaxial mode (TE₄) inside the choke. Moreover, it is found through semi-analytical study that the use of such a higher order rectangular coaxial mode has the ability to suppress the cross-polar level of offset reflector antenna. Section 3 presents this study in detail. Main advantage of the proposed scheme is that such kind of junction generates the relative phase of conjugate mode as required for conjugate matching, and also the desired mode coefficient of TE_4 can be easily adjusted through the parametric study by varying the slots length (S_L) and width (S_W) . It may be noted that the identical slots are only used to generate TE_4 mode in the square choke. Otherwise, other higher order modes will appear with TE_4 mode which may degrade the performance of matched feed.

As discussed earlier, total radiated far field of matched feed is a summation of the fields of two modes which are excited from two separate apertures (illustrative drawing of aperture is shown in Fig. 1(a)). It is well known that associated modes of diagonal horn are analogous to rectangular modes. If we consider first four modes in the analytical solution of Helmholtz equation, two groups of degenerated modes are found. First group is TE_V^D , TE_H^D having cutoff wavenumber $\frac{\pi}{a'}$, and its field distributions are linear combination of rectangular TE_{01} and TE_{10} as shown in Fig. 2. Second group is similar to rectangular TE₁₁ and TM₁₁ modes having cutoff wavenumber $\frac{\pi\sqrt{2}}{a'}$. The other aperture specified as a square choke has the analogous modes of rectangular coaxial cable in the solution of Helmholtz equation. Generally, exact analytical solution is not available for such aperture. However, some approximated solutions are found in [7,8]. For the sake of accuracy, solution of this aperture is carried out using 2-D FEM technique using the same treatment as specified by authors in [3]. The first mode is a TEM mode, and after that four TE modes appear according to cutoff wavenumber in this solution. The second and third modes are odd mode having the similar pattern of rectangular TE_{01} and TE_{10} mode, respectively. Our interest is only conjugate matching modes which appear in the fourth (TE_3) and fifth (TE_4) positions, and their field distributions are shown in Fig. 2. Calculated cutoff wavenumbers using the mentioned numerical technique for such modes with respect to $\frac{b}{a}$ ratios (related to dimensions of such geometry) are plotted in Fig. 3. Some approximate equations are derived using cubic interpolation technique employing the values of cutoff wavenumber of FEM solution for such modes. The cubic interpolated equations are given below.

$$\mathbf{k}_{c}^{\text{TE}_{3}} = -0.12 \frac{b^{3}}{a^{4}} + 0.88 \frac{b^{2}}{a^{3}} - 2.9 \frac{b}{a^{2}} + \frac{5.28}{a}$$
(1)





Figure 2. Operating modes pattern of proposed geometry $(x \rightarrow \text{vertical polarization}, y \rightarrow \text{horizontal polarization}).$

Figure 3. Cutoff wavenumber of modes in the choke vs $\frac{b}{a}$ for a = 1 m is plotted.

$$\mathbf{k}_{c}^{\mathrm{TE}_{4}} = 0.045 \frac{b^{3}}{a^{4}} - 0.064 \frac{b^{2}}{a^{3}} - 0.84 \frac{b}{a^{2}} + \frac{4.02}{a}$$
(2)

To observe the accuracy of our derived equations, they are compared with exact solution based on FEM technique, and comparisons are shown in Fig. 3. The derived equations are very useful to predict cutoff wavenumber of those modes.

The dimensions of our considered feed aperture are decided based on the cutoff wavenumber of operating modes. As mentioned, the arm's length (a') of inner diagonal waveguide is based on the cutoff wavenumber of main operating mode. On the other hand, the dimensions (a, b) of square choke are chosen based on the physical reliability as well as having the cutoff wavenumber of TE₄ close to TE_V^D mode cutoff wavenumber $(\frac{\pi}{a'})$. As we discussed, TE₄ mode is generated inside the square choke by slots, and it provides the required conjugate phase in the vicinity of junction within the design band. It may be noted that a extension length of waveguides (L-SL; as shown in Fig. 1) from the end point of junctions (longitudinal slots) is required for the stability of the generated TE₄ mode. To avoid the change of the relative phase of TE₄ mode for this extended length, cutoff wavenumber of TE₄ is fixed close to TE_V^D mode by the choice of dimensions b and a. On this basis, a', a, and b are decided for J-band matched feed design, and its cutoff wavenumbers of operating modes are 120.83 for TE_V^D and 103.17 for TE₄ mode.

3. SEMI-ANALYTICAL STUDY TO REDUCE THE CROSS-POLAR LEVEL OF OFFSET REFLECTOR

As mentioned in Section 1, the performances of the associated modes of proposed feed are investigated using a semi analytical technique. This technique is developed based on the PO to investigate the reflector pattern, and multi-mode approximated feed pattern is used as a source for the reflector. Here, feed pattern is considered as the combination of analytical and numerical far-field patterns of the associated modes of specified apertures. The closed-form expressions for the analytical mode field patterns of main operating modes TE_V^D and TE_H^D of diagonal waveguide are given in Section A. The conjugate mode patterns of specified TE_3 and TE_4 modes which get radiated from square choke are calculated based on the numerical technique as given in [3].

Generally, most crucial planes to observe the cross polar level of secondary pattern are at $\phi = \{45^\circ, 90^\circ\}$ for the matched and unmatched cases, respectively. However, the cross-polar levels

Progress In Electromagnetics Research M, Vol. 63, 2018

have been investigated for $\phi = \{0^{\circ}, 5^{\circ}, 10^{\circ}, \dots, 175^{\circ}\}$. It may be noted that power balance of far-field pattern of feed is maintained, and 1 W power is considered while investigating the reflector pattern in this semi-analytical study.

The cross-polar performance of reflector for main operating mode TE_{H}^{D} for a considered offset reflector having diameter 1.3 m, focal length 1 m and offset height 0.12 m is investigated at 7.5 GHz using our developed semi-analytical technique, and its cross-polar level is specified in Table 1.

Table 1. Secondary cross-polar level of y polarized unmatched and matched feed (using semi-analytical technique) and its mode coefficients at 7.5 GHz.

Mode	$\mathrm{TE}_\mathrm{H}^\mathrm{D}$	TE_3	Cross-polar power		
			45°	90°	
Case 1:	$1 \angle -$	-	$-26.12\mathrm{dB}$	$-22.85\mathrm{dB}$	
Case 2:	0.9417∠0°	$0.3365 \angle -89.33^{\circ}$	$-38.62\mathrm{dB}$	$-43.12\mathrm{dB}$	

To suppress the high level of cross-polar power, TE_3 mode is used with the specified main operating mode, and PSO optimization technique is used to estimate the required mode coefficient and relative phase of conjugate mode. Significant improvements of cross-polar levels are found, and their values and required mode coefficients are presented in Table 1.

Similarly, x polarized main operating TE_V^D mode is studied for same reflector using the developed semi-analytical technique as well as HFSS-15. The obtained patterns for both cases are presented in Fig. 4, and close agreements are found. Also, the cross-polar levels are presented in Table 2. As mentioned, TE_4 mode is used to cancel the cross-polar power of considered reflector for the main operating TE_V^D mode. The estimation of required conjugate mode coefficient and its relative phase are given in Table 2 which are calculated by PSO technique. For this case, the optimum cross-polar levels are specified in Table 2. The obtained cross-polar patterns are shown in Fig. 5.



Figure 4. Secondary cross-polar level for x polarized unmatched feed at 7.5 GHz.



Figure 5. Secondary optimum cross-polar level for ideal matched feed (analytical) and our proposed structure (HFSS) at 7.5 GHz.

As observed, the cross-polar performances at secondary pattern of both cases $TE_H^D + TE_3$ and $TE_V^D + TE_4$ are quite similar. However, for practical realization of matched feed, the TE_4 mode at the square choke can be easily generated using the primary source of diagonal TE_V^D mode. Moreover, our proposed structure as discussed in Section 2 has the ability to maintain the required relative phase as well as impedance matching for wideband of frequencies.

4. FINAL SIMULATION AND MEASURED RESULTS

The preliminary design of the feed is carried out using semi-analytical technique described in the previous section while fine tuning of the feed parameters is carried out performing parametric study in HFSS-15. As discussed, two symmetric longitudinal slots as illustrated in Fig. 1 are used to generate TE_4 mode. Slot length (SL) and width (SW) are fixed on the basis of adjusting conjugate mode coefficient as specified in Table 2 at the design frequency. Simultaneously, the waveguide length (L) is decided to adjust the relative phase of conjugate mode of our proposed matched feed. Finally, a fine tuning on crucial parameters (L, SL, SW) of feed is carried out using HFSS-15 to obtain proper matching with the reflector. The process of fine tuning involves varying the crucial dimensions which affect the cross-polar performance of the reflector so as to obtain the lowest cross-polar power at the design frequency while carrying out the fine tuning using HFSS-15. Feed pattern is evaluated using 3D FEM and reflector pattern using the PO module.

Table 2. Secondary cross-polar level of x polarized unmatched and matched feed (using semi-analytical technique) and its mode coefficients at 7.5 GHz.

Mode	$\mathrm{TE}_\mathrm{V}^\mathrm{D}$	${ m TE}_4$	Cross-polar power		
			45°	90°	
Case 3:	$1 \angle -$	-	$-26.53\mathrm{dB}$	$-23.24\mathrm{dB}$	
Case 4:	0.8735∠0°	$0.4868 \angle -91.9^{\circ}$	$-42.68\mathrm{dB}$	$-44.24\mathrm{dB}$	

After fine tuning, the adjusted feed dimensions are shown in Fig. 1. The cross-polar performance of our proposed matched feed structure is shown in Fig. 5. It can be seen that the patterns computed using our semi-analytical technique match the patterns obtained using HFSS. The adjusted mode ratio and relative phase of TE_4 mode at the feed aperture are given in Fig. 6.



Figure 6. Adjusted mode ratio and relative phase of TE_4 mode of our proposed matched feed.



Figure 7. A figure of prototype matched feed structure during the measurement of far field patterns at anechoic chamber.

A prototype matched feed structure is fabricated, and it is shown in Fig. 7. It may be noted that an adapter (coaxial cable to WR-137) and one transition (WR-137 to WC-128) are used to feed the power to the prototype for the verification of its performances.

Figure 8 presents the simulated and measured reflection coefficient magnitudes of the matched feed. A close match is observed between them. The simulated cross-polar performances of the reflector antenna system for unmatched and matched conditions are shown in Fig. 8.



Figure 8. Impedance bandwidth of our proposed matched feed and its cross-polar performance at secondary pattern.



Figure 9. Normalized simulated and measured co-pol and cross-pol level of feed antenna at 7 GHz.

The measured result of co-polar and cross-polar patterns of the matched feed at the selected planes are compared with simulated results of HFSS at the frequencies 7, 7.5 and 8 GHz, respectively as shown in Figs. 9, 10 and 11. As expected, simulated and measured far field patterns agree quite satisfactorily with each other for the considered testing frequencies. Also, the measured and simulated gains of our proposed matched feed for several frequencies are compared in Table 3, and close agreements are obtained.

Table 3. Simulated and measured gain of our proposed matched feed.

Frequency in GHz	7	7.25	7.5	7.75	8
Simulated gain	$8.7\mathrm{dBi}$	$9.1\mathrm{dBi}$	$8.1\mathrm{dBi}$	$9.7\mathrm{dBi}$	$9.7\mathrm{dBi}$
Measured gain	$7.5\mathrm{dBi}$	$9.4\mathrm{dBi}$	$8.5\mathrm{dBi}$	$9.1\mathrm{dBi}$	$9.6\mathrm{dBi}$



- Measured co pol at $\phi = 90^{\circ}$ Simulated co pol at $\phi = 0^{\circ}$ ■ Simulated cross pol at \$\phi=90⁶ Measured co pol at $\phi = 0^0$ Measured cross pol at $\phi = 90^{\circ}$ Simulated co pol at $\phi = 90^{\circ}$ 10 -10 đB -20 9311111 -40 -100 -50 0 50 100 Theta

Figure 10. Normalized simulated and measured co-pol and cross-pol level of feed antenna at 7.5 GHz.

Figure 11. Normalized simulated and measured co-pol and cross-pol level of feed antenna at 8 GHz.

5. CONCLUSION

In this communication, a novel dual-mode matched feed structure has been reported for an offset reflector antenna to achieve the wide-band impedance and cross-polar bandwidth. Presented conjugate modes (TE_3, TE_4) associated with square choke of the proposed matched feed to reduce the cross-polar power of an offset reflector antenna induced by main diagonal modes TE_H^D and TE_V^D , respectively, have not been reported in any earlier study. Further, a practical feed design has been carried out on the basis of operating modes TE_V^D and TE_4 for a given dimensions of an offset reflector antenna. Moreover, the main mode and conjugate mode have been kept spatially separated so that they can be adjusted individually to achieve overall wide-band system design. It has been observed that required relative phase of TE_4 mode of our proposed matched feed could be easily maintained for wide-band conjugate matching. On the other hand, the required amount of TE_4 mode could be easily adjusted by varying the length and width of slots. Moreover, generation amount of TE_4 mode does not have effect on impedance matching. Such characteristics are always desirable for a good matched feed.

APPENDIX A.

The closed form expressions of power balanced far-field radiation pattern of multi-mode rectangular horn is found in [9], based on Chu's model. Similarly, the power balanced far-field patterns for $TE_{\rm H}^{\rm D}$ and $TE_{\rm V}^{\rm D}$ modes are formulated based on the concept of linear combination of rectangular TE_{01} and TE_{10} degenerated modes, considering $\frac{\pi}{4}$ geometrical azimuth angle difference between rectangular and diagonal waveguides and using the far-field expressions as specified of associated modes for rectangular waveguide. The far-field patterns as considered for different modes are calculated as

$$E_{\theta}^{TE_{H}^{D}} = j \frac{V_{TE_{H}^{D}}}{\sqrt{\beta_{H}}} \left(1 + \frac{\beta_{H}}{k} \cos\theta\right) \left(\psi_{10} \sin^{2}\phi' - \psi_{01} \cos^{2}\phi'\right)$$

$$E_{\phi}^{TE_{H}^{D}} = j \frac{V_{TE_{H}^{D}}}{2\sqrt{\beta_{H}}} \left(\cos\theta + \frac{\beta_{H}}{k}\right) \left(\psi_{10} + \psi_{01}\right) \sin 2\phi'$$

$$E_{\theta}^{TE_{V}^{D}} = j \frac{V_{TE_{V}^{D}}}{\sqrt{\beta_{V}}} \left(1 + \frac{\beta_{V}}{k} \cos\theta\right) \left(-\psi_{10} \sin^{2}\phi' - \psi_{01} \cos^{2}\phi'\right)$$

$$E_{\phi}^{TE_{V}^{D}} = j \frac{V_{TE_{V}^{D}}}{2\sqrt{\beta_{V}}} \left(\cos\theta + \frac{\beta_{V}}{k}\right) \left(-\psi_{10} + \psi_{01}\right) \sin 2\phi'$$

where $\psi_{mn} = G \frac{\sin c(\frac{ka' \sin \theta \cos \phi' + m\pi}{2}) \sin c(\frac{ka' \sin \theta \sin \phi' + n\pi}{2})}{(ka' \sin \theta \cos \phi' - m\pi)(ka' \sin \theta \sin \phi' - n\pi)}$, $G = \frac{a'^2 k^2 e^{-jkR}}{2R} \sqrt{\frac{k\eta}{2}} \sin \theta$ and $\phi' = \phi - \frac{\pi}{4}$. Here, R, θ and ϕ are the observation points of far-field. $V_{TE_H}^D$ and $V_{TE_V}^D$ are respectively complex mode coefficient of TE_H^D and TE_V^D modes. β_H and β_V represent propagation constant of TE_H^D and TE_V^D modes. k is free space wave number, and (a') represents the edge length of diagonal feed as mentioned.

ACKNOWLEDGMENT

The authors gratefully acknowledge the help of Mr. Satyajit Chakraborty of SAMEER Kolkata center in carrying out the measurements.

REFERENCES

- Rudge, A. W. and N. A. Adaita, "Offset-parabolic-reflector antennas: A review," *IEEE Proceedings*, Vol. 66, No. 12, 1592–1618, Dec. 1978.
- Pour, Z. and L. Shafai, "A novel dual-mode dual-polarized circular waveguide feed excited by concentrically shorted ring patches," *IEEE Transactions on Antennas and Propagation*, Vol. 61, No. 10, 4917–4925, Oct. 2013.

Progress In Electromagnetics Research M, Vol. 63, 2018

- 3. Jana, R. and R. Bhattacharjee, "A novel matched feed structure for achieving wide crosspolar bandwidth for an offset parabolic reflector antenna system," *IEEE Antennas and Wireless Propagation Letters*, Vol. 14, 1590–1593, 2015.
- 4. Dey, R. and S. B. Chakrabarty, "Broadband conjugate matched feed horn a novel concept," *IEEE Antennas and Wireless Propagation Letters*, Vol. 15, 164–169, 2016.
- 5. Koch, G. F., "Coaxial feeds for high aperture efficiency and low spillover of paraboloidal reflector antennas," *IEEE Transactions on Antennas and Propagation*, Vol. 21, No. 2, 4917–4925, Mar. 1973.
- Pour, Z. and L. Shafai, "A ring choke excited compact dual-mode circular waveguide feed for offset reflector antennas," *IEEE Transactions on Antennas and Propagation*, Vol. 60, No. 6, 3011–3015, Jun. 2012.
- Gruner, L., "Higher order modes in rectangular coaxial waveguides," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 15, No. 8, 483–485, Aug. 1967.
- 8. Cruzan, O. and R. Garver, "Characteristic impedance of rectangular coaxial transmission lines," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 12, No. 5, 488–495, Sep. 1964.
- 9. Silver, S., Microwave Antenna Theory and Design, 3rd edition, McGraw Hill, New York, 1949.