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Effect of Surface Impedance on Radiation Fields of Spherical Antennas

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Abstract—Influence of surface impedance on radiation fields of spherical antennas excited by radially oriented electric dipole is investigated by using a Green's function for a space outside a spherical scatterer. This approach allows us to obtain analytical expressions for radiation fields of an impedance spherical antenna in the wave zone. The spherical antenna with the scatterer coated with a metamaterial layer is also considered. The surface impedance required for radiomasking of the spherical scatterer of resonant dimensions was estimated by mathematical modeling.

1. INTRODUCTION

Vibrator antennas of meter and decimeter wavelengths are now widely used on mobile objects including aerial and space vehicles [1], whose body or its component part can be approximated by spherical surfaces with diffraction dimensions lying in the resonant region. Asymmetrical radially oriented vibrator radiators (monopoles) are applied most frequently due to simplicity of their excitation. The radiation fields of such spherical antennas were studied earlier both in the dipole approximation (see, e.g., [1–7]) and for the resonant perfectly conducting [3] and impedance [8] monopoles. In all these cases, however, the surface of the spherical scatterer was considered to be perfectly conductive. In the present article, the influence of the impedance layer coating the sphere upon the radiation field of the antenna excited by a radially oriented electric dipole is investigated within the framework of the impedance approach. A special case of the sphere coated by a layer of metamaterial is also considered.

2. FIELDS OF A SPHERICAL ANTENNA IN THE WAVE ZONE

A rigorous analysis of radiation fields induced by a radial electric dipole located on an impedance sphere is performed using the Green's function for the Hertz vector potential. Using the Green's function representation required for the problem solution [9], we can write the radial component of the electric Green's function $G^e_{\rho\rho'}(\rho,\theta,\varphi;\rho',\theta',\varphi')$ in a system of spherical coordinates (ρ,θ,φ) for a homogeneous space outside the sphere with material parameters (ε_1,μ_1) as

$$G^e_{\rho\rho'}\left(\rho,\theta,\varphi;\rho',\theta',\varphi'\right) = -\sum_{n=0}^{\infty}\frac{n+1/2}{2\pi}h_n(\rho,\rho')P_n\left(\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi-\varphi')\right). \quad (1)$$

Here $P_n(\cos\theta)$ are the Legendre polynomials, and the functions $h_n(\rho,\rho')$ can be determined from the inhomogeneous Bessel differential equation with the delta-function $\delta(\rho-\rho')$ in the right-hand side. Taking into account the impedance boundary condition $\left[\vec{n},\vec{E}\right]\Big|_S = -z_i\left[\vec{n},\left[\vec{n},\vec{H}\right]\right]\Big|_S$ on the sphere

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surface S (z_i is the constant intrinsic impedance, $\vec{n}$ is the external normal vector) and the radiation condition at infinity, we can write

$$h_n(\rho,\rho') = \begin{cases} k_1 h_n^{(2)}(k_1\rho')\left[h_n^{(2)}(k_1\rho)\overline{Q}_n\left(y_n\left(k_1\tilde{R}\right)\right) - y_n(k_1\rho)\right], & \tilde{R} \le \rho < \rho', \\ k_1 h_n^{(2)}(k_1\rho)\left[h_n^{(2)}(k_1\rho')\overline{Q}_n\left(y_n\left(k_1\tilde{R}\right)\right) - y_n(k_1\rho')\right], & \rho > \rho', \end{cases} \quad (2)$$

where $\overline{Q}_n(y(k_1\tilde{R})) = \frac{k_1\tilde{R}y_{n-1}(k_1\tilde{R})+(i\omega\varepsilon z_i\tilde{R}-n)y_n(k_1\tilde{R})}{k_1\tilde{R}h_{n-1}^{(2)}(k_1\tilde{R})+(i\omega\varepsilon z_i\tilde{R}-n)h_n^{(2)}(k_1\tilde{R})}$, $h_n^{(2)}(k_1\rho) = j_n(k_1\rho) - iy_n(k_1\rho) = \sqrt{\frac{\pi}{2k_1\rho}}H_{n+1/2}^{(2)}(k_1\rho)$ are the spherical Hankel functions of the second kind; $j_n(k_1\rho) = \sqrt{\frac{\pi}{2k_1\rho}}J_{n+1/2}(k_1\rho)$ and $y_n(k_1\rho) = \sqrt{\frac{\pi}{2k_1\rho}}N_{n+1/2}(k_1\rho)$ are the spherical Bessel and Neumann functions, respectively; $J_{n+1/2}(k_1\rho)$ are Bessel functions; $N_{n+1/2}(k_1\rho)$ and $H_{n+1/2}^{(2)}(k_1\rho)$ are Neumann and Hankel functions of the second kind and half-integral order [10]; $k_1 = k\sqrt{\varepsilon_1\mu_1}$ and $k = 2\pi/\lambda$ are wave numbers in the medium and free space, respectively; λ is free space wavelength; $\tilde{R}$ is the sphere radius; ω is the circular frequency. The impedance boundary conditions on the sphere $E_\theta = z_i H_\varphi$ and $E_\varphi = -z_i H_\theta$ are equivalent to the requirement [8]

$$\frac{\partial\left(k_1\rho h_n(\rho,\rho')\right)}{\partial\rho} = -i\omega\varepsilon z_i\left(k_1\rho h_n(\rho,\rho')\right)\Big|_{\rho=\tilde{R}}. \quad (3)$$

Expressions (1) and (2) can be converted into the formulas [8] for the perfectly conducting sphere if $z_i = 0$.

Let us define a coordinate system related to the spherical antenna, as shown in Fig. 1, and assume that the dipole is located at the point ($\rho' = \tilde{R}$, $\theta' = \theta_0$; $\varphi' = \varphi_0$). Without losing the problem generality, we use the model of a point harmonic oscillator with a constant complex amplitude of the electric current J_0 as a model of a monochromatic dipole radiator. The electromagnetic fields depend on time t as $e^{i\omega t}$.

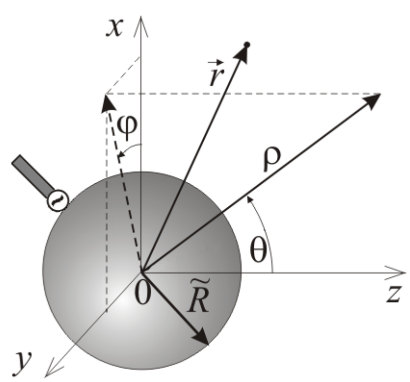


Figure 1. The problem geometry and accepted notations.

The components of the total radiation field can be found by using the following relations [8]:

$$\begin{aligned} E_\rho(\vec{r}) &= \frac{\partial^2\left(k_1\rho J_0 G^e_{\rho\rho'}(\vec{r})\right)}{\partial\rho^2} + k^2\varepsilon_1\mu_1\left(k_1\rho J_0 G^e_{\rho\rho'}(\vec{r})\right), \quad H_\rho(\vec{r}) = 0, \\ E_\theta(\vec{r}) &= \frac{1}{\rho}\frac{\partial^2\left(k_1\rho J_0 G^e_{\rho\rho'}(\vec{r})\right)}{\partial\rho\partial\theta}, \quad H_\theta(\vec{r}) = \frac{ik\varepsilon_1 k_1 J_0}{\sin\theta}\frac{\partial G^e_{\rho\rho'}(\vec{r})}{\partial\varphi}, \\ E_\varphi(\vec{r}) &= \frac{1}{\rho\sin\theta}\frac{\partial^2\left(k_1\rho J_0 G^e_{\rho\rho'}(\vec{r})\right)}{\partial\rho\partial\varphi}, \quad H_\varphi(\vec{r}) = -ik\varepsilon_1 k_1 J_0\frac{\partial G^e_{\rho\rho'}(\vec{r})}{\partial\theta}, \end{aligned} \quad (4)$$

where $\vec{r}$ is the radius vector of the observation point. Expressions (4) allow us to find electromagnetic radiation fields at any distance from the antenna satisfying the relation $\rho \geq \tilde{R}$.

If the external medium is lossless and ε_1 is real, formulas (4) in the wave zone ($\rho \gg \lambda$) can be simplified, since the terms proportional to $1/\rho^2$ can be omitted. In the wave zone, expressions (4) can be easily transformed. If $k_1\rho \to \infty$ and $|k_1\rho| \gg n$, the spherical Hankel functions of the second kind have the well-known asymptotic representation [10]

$$h_n^{(2)}(k_1\rho) \approx (i)^{n+1}\frac{e^{-ik_1\rho}}{k_1\rho}. \tag{5}$$

If $\varepsilon_1 = \mu_1 = 1$ and, hence, $k_1 = k$, the wave zone magnetic field in the equatorial plane, $\varphi' = 0$ and $\theta' = \pi/2$, can be written in the form

$$\begin{aligned} &H_\rho(\vec{r}) = 0, \quad H_\varphi(\vec{r}) = 0, \\ &H_\theta(\vec{r}) = -\frac{k^2 J_0}{\omega \sin\varphi}\frac{e^{-ik\rho}}{\rho k\tilde{R}}\sum_{n=1}^{\infty}\frac{n+1/2}{2\pi}\left[P_{n+1}(\cos\varphi) - P_n(\cos\varphi)\cos\varphi\right] \\ &\qquad\frac{(i)^{n+1}(n+1)}{k\tilde{R}h_{n-1}^{(2)}(k\tilde{R}) + \left(i\bar{Z}_{sp}k\tilde{R} - n\right)h_n^{(2)}(k\tilde{R})}. \end{aligned} \tag{6}$$

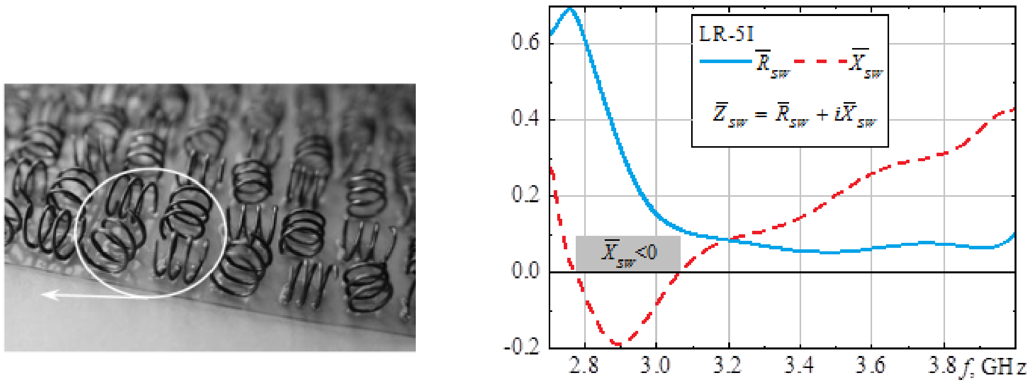
Here $\bar{Z}_{sp} = \frac{z_i}{120\pi}$ is surface impedance normalized to the resistance of free space. Since only one component of the magnetic field is nonzero in Eq. (6), this formula can be conveniently used for calculation of the radiation pattern of the spherical antenna. Of course, for the problem under consideration, the magnetic field component $H_\theta(\vec{r})$ in the wave zone is an alternative for a single component electric field. Note that the summation in Eq. (6) was limited by a number of terms $N_{\max}$ that provided the determination of quantities with an error not exceeding 0.5%. For example, for spheres with $k\tilde{R} = 1$ the $N_{\max} = 10...15$ is sufficient, while for spheres with $k\tilde{R} = 10$ the larger number of terms should be taken into account, and $N_{\max} = 50...60$.

3. SURFACE IMPEDANCE OF A METAMATERIAL LAYER

The normalized surface impedance of a natural magneto-dielectric layer coating a perfectly conducting plane is determined by the expression [11]

$$\bar{Z}_{\mathrm{SW}} = i\sqrt{\frac{\mu}{\varepsilon}}tg(k_d h_d), \tag{7}$$

where h_d is the layer thickness, and $\varepsilon = \varepsilon' - i\varepsilon''$ and $\mu = \mu' - i\mu''$ are material parameters of the layer. Formula (7) is transfered to $\bar{Z}_{\mathrm{SW}} \approx ik\mu h_d$ if the inequality $|k_d h_d| \ll 1$ holds ($k_d = k\sqrt{\varepsilon\mu}$). One can see that the surface impedance of the electrically thin layer is inductive and does not depend on the permittivity ε of the dielectric layer. The surface impedance of the metamaterial layer can be calculated by the formula [12] $\bar{Z}_{\mathrm{SW}} = \bar{R}_{\mathrm{SW}} + i\bar{X}_{\mathrm{SW}} = \pm i\sqrt{\frac{\mu}{\varepsilon}}tg(k_d h_d)$, where the plus or minus signs are used if $\varepsilon' > 0$ or $\varepsilon' < 0$. If $\mu' < 0$, the surface impedance is capacitive ($\bar{X}_{\mathrm{SW}} < 0$). As an example, we now calculate the surface impedance of the metamaterial LR-5I [13]. This metamaterial was developed to provide resonant absorption of electromagnetic waves in the vicinity of the frequency $f \approx 2.8$ GHz. The metamaterial cell consists of four three-coil spirals made of nichrome wire. The wire diameter is 0.4 mm, the spiral outer diameter 5.0 mm and spiral pitch 1.0 mm. The spirals are arranged in a special way on 0.2 mm polyurethane substrate as shown Fig. 2(a). Fig. 2(b) shows the plots $\bar{R}_{\mathrm{SW}}(f)$ and $\bar{X}_{\mathrm{SW}}(f)$ for the LR-5I layer with a total thickness $h_d = 5.2$ mm [12]. The plots were built by using formula (7) and the experimental parameters of the metamaterial measured in the frequency range $f = 2.7 \div 4.0$ GHz [13]. As can be seen from Fig. 2(b), the imaginary part of the surface impedance becomes negative in some frequency range. In the subsequent calculations, we assume that $\bar{Z}_{sp} = \bar{Z}_{\mathrm{SW}}$ for the metamaterial layer on the sphere.



(a) (b)

Figure 2. The metamaterial LR-5I: (a) fragment of metamaterial; (b) the plot $\bar{R}_{\text{SW}}(f)$ and $\bar{X}_{\text{SW}}(f)$.

4. SIMULATION RESULTS

Due to angular symmetry of formula (6), Fig. 3 shows the angular radiation pattern (RP), $|\bar{H}_\theta|$ as function of the angular coordinate φ, only for the first two quadrants of the Cartesian plane ($0 < \varphi < \pi$). The RPs, normalized to maximum values, are presented in Fig. 3 for spherical antennas with various diffraction radii and impedances. As can be seen, if the sphere diffraction radius $k\tilde{R}$ increases, the antenna RP becomes multilobed both for impedance and perfectly conducting spheres. The oscillations of the radiation field amplitudes which define multilobed RP, are mainly observed in the geometric shadow region ($\varphi > \pi/2$). These oscillations can be explained by interference of waves propagating along the spherical scatterer surface along the meridians in the forward and backward directions [1, 8]. The larger is $k\tilde{R}$, the greater is the number of standing waves on the sphere surface and the more side lobes are in the RP. Therefore, inductive surface impedance $\bar{Z}_{sp} = 0.25i$ increases and capacitive impedance $\bar{Z}_{sp} = -0.25i$ decreases the numbers of the side lobes. One can also observe an insignificant dependence of the RP shape upon the impedance for the spheres of small $k\tilde{R} \leq 1.0$ and large $k\tilde{R} \geq 20.0$ radii. Influence of the impedance on the RP shape becomes significant for the spheres of resonant dimensions $2.0 \leq k\tilde{R} \leq 10.0$. Thus, if $k\tilde{R} = 10.0$ and the sphere impedance varies in the range $-0.25i \leq \bar{Z}_{sp} \leq 0.25i$, the main lobe maximum of the antenna RP can be scanned in the sector [70°, 170°] (Fig. 3(e)). However, one should take into account that the half power width of the antenna RP varies almost four times at the ends of the impedance variation range.

The normalized angular RP for the sphere coated with the metamaterial layer is presented in Fig. 4 and Fig. 5, where the RP of a isolated dipole located in the center of the spherical coordinate system is also shown for comparison.

The RP of the antenna with sphere of resonant dimensions covered with a layer of metamaterial LR-5I at frequencies $f = 2.9$ GHz ($\bar{Z}_{sp} = 0.35 - 0.18i$) and $f = 2.75$ GHz ($\bar{Z}_{sp} = 0.7 + 0.08i$) are shown in Fig. 4. As can be seen from Fig. 4(a), the coating layer can eliminate the effect of the spherical scatterer with the diffraction radius $k\tilde{R} = 1.5$ on the formation of the dipole radiation field if $\bar{Z}_{sp} = 0.7 + 0.08i$. The coating significantly reduce this effect for $k\tilde{R} = 2.0$ (Fig. 4(b)). Since surface currents induced on the sphere by a dipole radiator are defined by the real part of the surface impedance, the influence of spheres with dimensions $k\tilde{R} \leq 1.5$ on the antenna RP is reduced practically to zero if $\bar{R}_{sp} = 0.7$.

To ensure a similar effect for the large spheres, a further increase of the $\bar{R}_{sp}$ is required. Fig. 5 shows the RP of the antenna with the diffraction radius $k\tilde{R} = 10.0$ and the surface impedance $\bar{Z}_{sp} = 1.0$ and $\bar{Z}_{sp} = 2.0$. The simulation have shown that further increase of the impedance $\bar{Z}_{sp}$ does not lead to a significant change of the antenna RP and to expected convergence of curves 1 and 4. The effect can be explained by two reasons. Firstly, by the dipole displacement from the center of the spherical coordinate system, since it is placed on the sphere with the diffraction radius $k\tilde{R} = 10.0$ as defined by the problem formulation. Secondly, the RP becomes symmetric in the angular sector of the forward half-space.

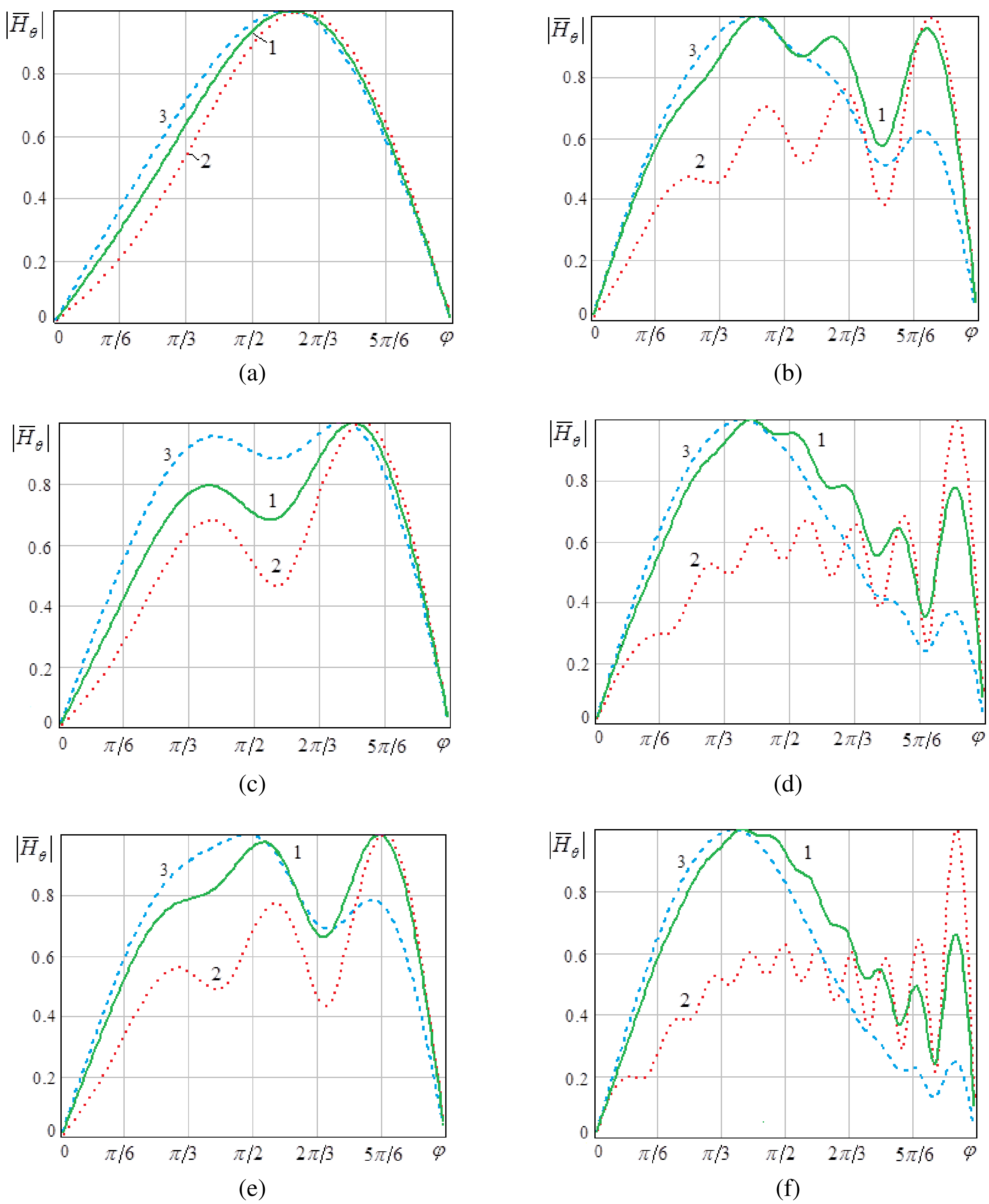
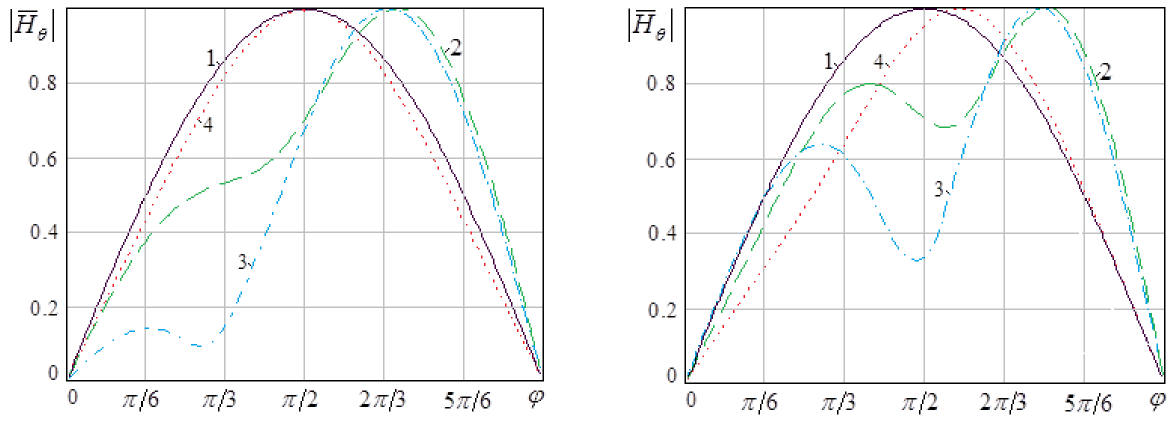


Figure 3. The RP of the spherical antenna calculated for various values of the surface impedance: **1** — $\bar{Z}_{sp} = 0.0$; **2** — $\bar{Z}_{sp} = -i0.25$; **3** — $\bar{Z}_{sp} = i0.25$. (a) $k\tilde{R} = 1.0$, (b) $k\tilde{R} = 2.0$, (c) $k\tilde{R} = 3.0$, (d) $k\tilde{R} = 4.0$, (e) $k\tilde{R} = 7.0$, (f) $k\tilde{R} = 10.0$.



(a) (b)

Figure 4. The spherical antenna RP whose sphere is covered with the metamaterial layer: **1** — isolated dipole; **2** — $\bar{Z}_{sp} = 0.0$; **3** — $\bar{Z}_{sp} = 0.35 - 0.18i$; **4** — $\bar{Z}_{sp} = 0.7 + 0.08i$. (a) $k\tilde{R} = 1.5$, (b) $k\tilde{R} = 2.0$.

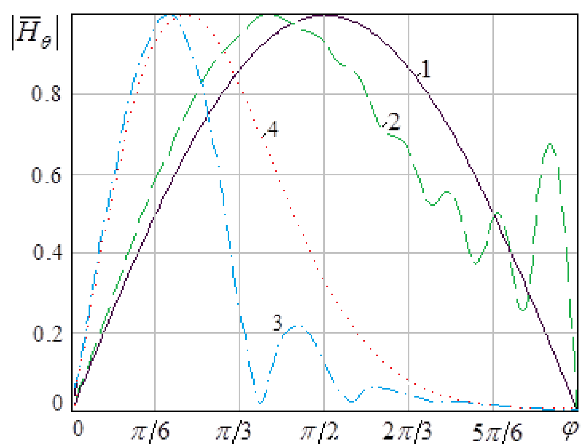


Figure 5. The spherical antenna RP with the diffraction radius $k\tilde{R} = 10.0$: **1** — isolated dipole; **2** — $\bar{Z}_{sp} = 0.0$; **3** — $\bar{Z}_{sp} = 1.0$; **4** — $\bar{Z}_{sp} = 2.0$.

5. CONCLUSION

Analytical expressions for the simulation of radiation fields created by the radially oriented dipole located on the impedance sphere have been derived. The simulation results have shown that the surface impedance of the antenna with resonant spheres $2.0 \leq k\tilde{R} \leq 10.0$ influences significantly the antenna RP in the wave zone. The inductive surface impedance increases the sphere effective radius, while capacitive impedance reduces it. As one would expect, such an effect becomes insignificant for very small or large spherical scatterers.

Numerical estimates of the impedance real part required to minimize the effect of spherical scatterers of resonant dimensions upon the spherical antenna RP were obtained. Thus, for example, the conditions $\bar{R}_{sp} \geq 0.4$ and $\bar{R}_{sp} \geq 0.8$ should be fulfilled for spheres whose dimensions are in the ranges $0 < k\tilde{R} \leq 1.0$ and $1.0 \leq k\tilde{R} \leq 2.0$, respectively. If the sphere dimensions are in the range $2.0 \leq k\tilde{R} \leq 10.0$, the impedance real part should satisfy the inequality $\bar{R}_{sp} \geq 2.0$. The obtained results can be directly used for development of antennas for mobile objects of spherical shape.

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