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Synthesis of a Broadband Rat-Race Hybrid Using Transmission Lines and Lumped-Element Components

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Abstract—This letter presents the synthesis of a broadband rat-race consisting of a miniaturized broadband rat-race hybrid and transmission line cascades. This broadband technique involves connecting a cascade of transmission lines with lengths equal to a quarter of the wavelength at the design frequency to each port of a previously proposed rat-race hybrid. Butterworth and Chebyshev performances of the broadband rat-race hybrid are also reported. The broadband rat-race hybrid was implemented on an FR4 substrate using spiral inductors and chip capacitors. For the frequency range of 420–800 MHz, which corresponds to a relative bandwidth of more than 62%, the broadband rat-race hybrid exhibited power splits of -3.8 ± 1.0 dB, return losses of greater than 19 dB, and isolation between output ports of greater than 20 dB. The phase difference between S_{21} and S_{41} was $180^\circ \pm 3^\circ$.

1. INTRODUCTION

Rat-race hybrids have a wide range of applications in microwave-balanced mixers, amplifiers, and antenna arrays [1–7]. Because wireless systems must be of substantially low mass and volume, the miniaturization of such hybrids is of great interest. Fig. 1(a) shows a conventional rat-race hybrid consisting of three 90° transmission lines and a 270° transmission line with a characteristic impedance $\sqrt{2}Z_0$ at a design frequency f_0, where Z_0 is the reference characteristic impedance. The reduction methods depend on the equivalent circuits of these transmission lines, which include T-type, Π-type, and lumped-element circuits. Most rat-race hybrids miniaturized by conventional methods are not sufficiently compact, because three 90° equivalent circuits are substituted for a 270° transmission line. To overcome this problem, we adopted the miniaturized broadband rat-race hybrid shown in Fig. 1(b) [8] and made the following substitutions.

1. The long 270° high-frequency transmission line was replaced by a 90° transmission line.
2. The 90° transmission line was replaced by a lumped-element circuit with a 90° phase lead using inductors and capacitors.

Compared with a conventional rat-race hybrid, the phase lead and lag of the proposed rat-race hybrid are completely reversed. In a previous study, we reported simulation and measurement results for a miniaturized broadband rat-race hybrid for biomedical applications and television white space systems [8].

Figure 2 shows a conventional transmission line and its equivalent T-type circuit consisting of two series capacitors and a shunt inductor. By applying a matrix formulation, the $ABCD$ parameters of the equivalent circuit shown in Fig. 2(b) can be deduced from Fig. 2(a). Furthermore, equating the parameters of each circuit yields the relation

$$\begin{bmatrix} \cos(-\theta) & jZ\sin(-\theta) \\ j\dfrac{\sin(-\theta)}{Z} & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{1}{j\omega_0 C} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{j\omega_0 L} & 1 \end{bmatrix} \begin{bmatrix} 1 & \dfrac{1}{j\omega_0 C} \\ 0 & 1 \end{bmatrix}. \tag{1}$$

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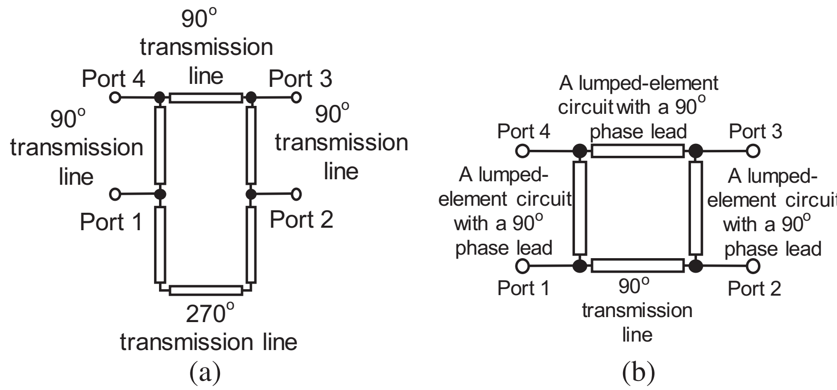

Figure 1. (a) Conventional rat-race hybrid. (b) Miniaturized broadband rat-race hybrid [7].

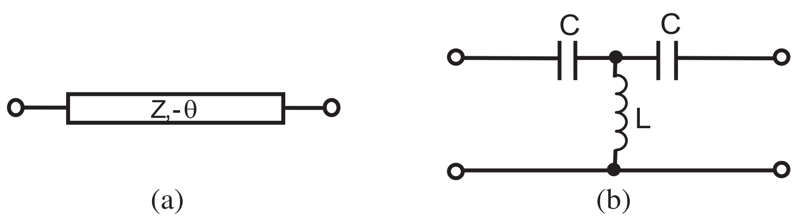

Figure 2. (a) Conventional transmission line and (b) its equivalent T-network, which consists of two capacitors and an inductor.

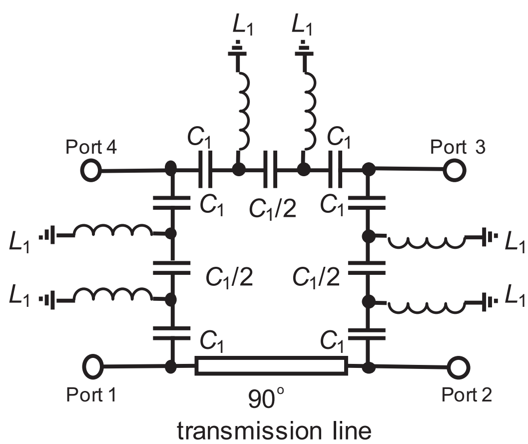

Figure 3. Miniaturized broadband rat-race hybrid using a transmission line and lumped-element components [8].

where ω_0 is the angular frequency corresponding to the design frequency f_0. Equation (1) yields the relation

$$L = \frac{Z}{2\pi f_0 \sin\theta}, \quad C = \frac{\sin\theta}{2\pi f_0 Z\,(1-\cos\theta)}. \tag{2}$$

Thus, a lumped-element circuit with a 90° phase lead can be constructed by connecting two stages of lumped-element circuits with a 45° phase lead, as shown in Fig. 3. The circuit parameters can be calculated as follows at the design frequency f_0:

$$L_1 = \frac{Z}{\sqrt{2}\pi f_0}, \quad C_1 = \frac{1}{2\pi f_0 Z(\sqrt{2}-1)}. \tag{3}$$

A miniaturized rat-race hybrid with a bandwidth wider than that of a conventional rat-race hybrid can be realized because the frequency characteristics of the phase slope of the 90° transmission line and those of the lumped-element circuit are almost the same near the design frequency f_0 [7].

2. SYNTHESIS OF BROADBAND RAT-RACE HYBRID

Figure 4 shows a broadband rat-race hybrid in which a cascade of transmission lines is connected to each port of the previously reported rat-race hybrid shown in Fig. 3. The length of each transmission line is a quarter of the wavelength at the design frequency f_0, and characteristic impedances $Z_1 - Z_{n+1}$ are normalized by Z_0. Rehnmark studied the Butterworth and Chebyshev performances of the wide-band hybrid consisting of a reversed-phase hybrid ring and transmission-line cascades [9]. Tables 1 and 2 give the normalized characteristic impedances calculated using the synthesis procedure proposed by Rehnmark. For example, with normalized characteristic impedances $Z_1 = 0.793$, $Z_2 = 0.932$, and $Z_3 = 0.932$ for $n = 2$ in Table 2, with input reflection coefficient S_{11} of less than -20 dB, a bandwidth of 0.838 will be obtained. Note that this relative bandwidth of the proposed circuit configuration does not increase in proportional to n, while that of the conventional wide-band hybrid consisting of a reversed-phase hybrid ring and transmission-line cascades is in proportional to n [9].

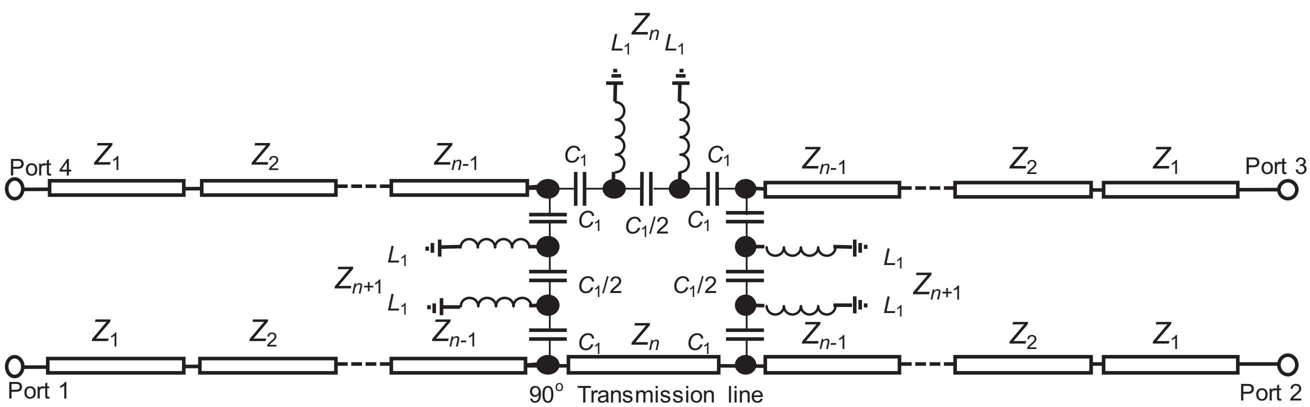

Figure 4. Broadband rat-race consisting of a miniaturized broadband rat-race hybrid and transmission line cascades.

Table 1. Normalized characteristic impedances of hybrid shown in Fig. 4 — Butterworth case (3 dB coupling).

$Z_1 - Z_{n-1}$				Z_{n+1}	Z_n	Coupling [dB]	Relative bandwidth of $S_{11} < -20$ dB	Relative bandwidth of $S_{11} < -15$ dB	Relative bandwidth of $S_{11} < -10$ dB	n
				1.4142	1.4142	3.01	0.5200	0.6679	0.8431	1
			0.9020	1.1506	1.1506	3.01	0.7155	0.8205	0.9300	2
		0.9706	0.8050	0.9730	0.9730	3.01	0.7995	0.8842	0.9685	3
	0.9915	0.9246	0.7180	0.8385	0.8385	3.01	0.8344	0.9178	0.9895	4
0.9976	0.9735	0.8704	0.6412	0.7310	0.7310	3.01	0.7744	0.9374	1.0021	5

Table 2. Normalized characteristic impedances of hybrid shown in Fig. 4 — Chebyshev case (3 dB coupling).

$Z_1 = Z_{n-1}$				Z_{n+1}	Z_n	Coupling [dB]	Relative bandwidth of $S_{11} < -20$ dB	Relative bandwidth of $S_{11} < -15$ dB	Relative bandwidth of $S_{11} < -10$ dB	n
				1.407	1.407	3.01	0.529	0.673	0.844	1
			0.871	1.078	1.078	3.01	0.765	0.853	0.947	2
		0.926	0.701	0.805	0.805	3.01	0.768	0.927	0.995	3
	0.949	0.787	0.532	0.585	0.585	3.01	0.750	0.837	1.014	4
0.960	0.836	0.622	0.387	0.413	0.413	3.01	0.765	0.840	0.923	5
				1.380	1.380	3.01	0.558	0.728	0.851	1
			0.828	0.993	0.993	3.01	0.812	0.887	0.967	2
		0.876	0.621	0.694	0.694	3.01	0.733	0.846	1.009	3
	0.898	0.691	0.439	0.472	0.472	3.01	0.757	0.826	0.918	4
0.910	0.734	0.504	0.297	0.312	0.312	3.01	0.785	0.851	0.918	5
				1.348	1.348	3.01	0.590	0.708	0.859	1
			0.793	0.932	0.932	3.01	0.838	0.909	0.980	2
		0.836	0.571	0.628	0.628	3.01	0.718	0.816	1.015	3
	0.857	0.631	0.388	0.413	0.413	3.01	0.765	0.824	0.903	4
0.870	0.669	0.441	0.253	0.263	0.263	3.01	0.800	0.862	0.920	5
				1.291	1.291	3.01		0.736	0.869	1
			0.742	0.852	0.852	3.01		0.934	0.996	2
		0.779	0.509	0.552	0.552	3.01		0.796	0.922	3
	0.798	0.558	0.332	0.349	0.349	3.01		0.824	0.890	4
0.809	0.589	0.371	0.208	0.215	0.215	3.01		0.876	0.925	5
				1.240	1.240	3.01		0.758	0.879	1
			0.702	0.796	0.796	3.01		0.797	1.007	2
		0.735	0.468	0.503	0.503	3.01		0.763	0.899	3
	0.753	0.510	0.298	0.311	0.311	3.01		0.824	0.882	4
				1.195	1.195	3.01		0.775	0.886	1
			0.670	0.751	0.751	3.01		0.770	1.014	2
		0.700	0.437	0.467	0.467	3.01		0.714	0.892	3
	0.716	0.474	0.273	0.284	0.284	3.01			0.877	4
				1.155	1.155	3.01			0.891	1
			0.642	0.715	0.715	3.01			1.020	2
		0.670	0.413	0.438	0.438	3.01			0.886	3
	0.685	0.446	0.254	0.263	0.263	3.01			0.874	4

3. SIMULATION AND MEASUREMENT RESULTS FOR BROADBAND RAT-RACE HYBRID

The validity of the proposed circuit configuration was investigated by simulating its frequency characteristics using radio frequency (RF) and microwave design software (Microwave Office and ADS). Fig. 5 shows the simulation results for the ideal broadband rat-race hybrid shown in Fig. 4 with normalized characteristic impedances $Z_1 = 0.793$, $Z_2 = 0.932$, and $Z_3 = 0.932$ for $n = 2$ in Table 2. Assuming a design frequency f_0 is 590 MHz and a reference characteristic impedance Z_0 is $50\,\Omega$, a

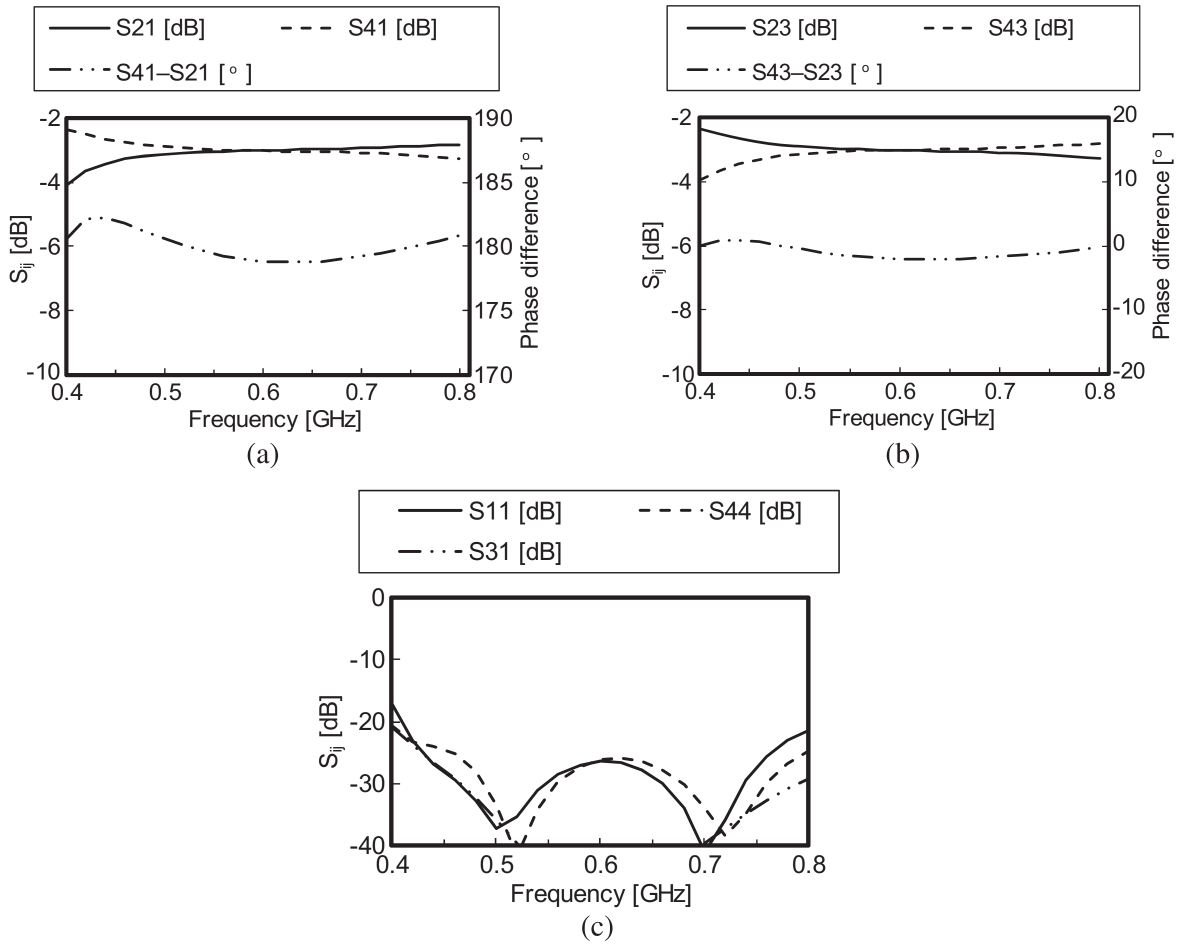

Figure 5. Simulated results for ideal broadband rat-race hybrid shown in Fig. 4 ($n = 2$). (a) S_{21}, S_{41}, and $S_{41} - S_{21}$. (b) S_{23}, S_{43}, and $S_{43} - S_{23}$. (c) S_{11}, S_{44}, and S_{31}.

capacitance C_1 is 13.98 pF and an inductance L_1 is 17.78 nH, respectively. For the frequency range of 420–800 MHz, which corresponds to a relative bandwidth of more than 62%, the broadband rat-race hybrid exhibits power splits of -3.1 ± 0.6 dB, return losses of greater than 21 dB, isolation between output ports of greater than 23 dB, and errors in the desired relative phase differences between output ports of less than 3°.

The feasibility of operating the circuit was experimentally investigated by fabricating the proposed broadband rat-race hybrid on a commercially available 1.6-mm-thick 2-layer FR4 substrate (dimensions of 146 mm $\times$ 50 mm $=$ 73 cm^2) [10]. We selected a 2-layer substrate for low-cost fabrication. A photograph of the fabricated broadband rat-race hybrid is shown in Fig. 6. The relative permittivity and loss tangents at 1 GHz typically range between 4.0 and 4.2 and between 0.012 and 0.014, respectively. The thickness of the outer copper was 35 µm. We used spiral inductors and chip capacitors (15 pF and 7 pF) which were soldered by hand.

Figure 7 shows the simulation and measurement results for the fabricated broadband rat-race hybrid shown in Fig. 6. The simulation used S-parameters of the commercial chip capacitors and three-dimensional electromagnetic analysis of the vias and the spiral inductors. For the frequency range of 420–800 MHz, which corresponds to a relative bandwidth of more than 62%, the rat-race hybrid exhibited power splits of -3.8 ± 1.0 dB, return losses of greater than 19 dB, and isolation between output ports of greater than 20 dB. The phase difference between S_{21} and S_{41} was $180^\circ \pm 3^\circ$, and that between S_{23} and S_{43} was $-4^\circ \pm 3^\circ$. The discrepancies between the measurement and simulation results are caused by the parasitic effects of the spiral inductors and soldering. Table 3 compares the results of this study and

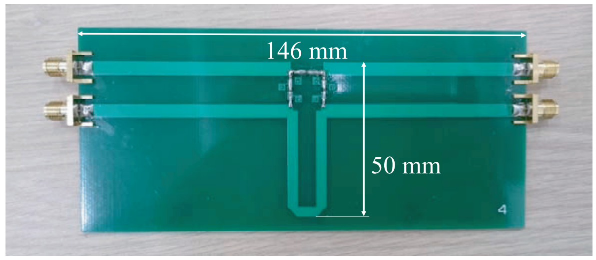

Figure 6. Photograph of fabricated rat-race hybrid.

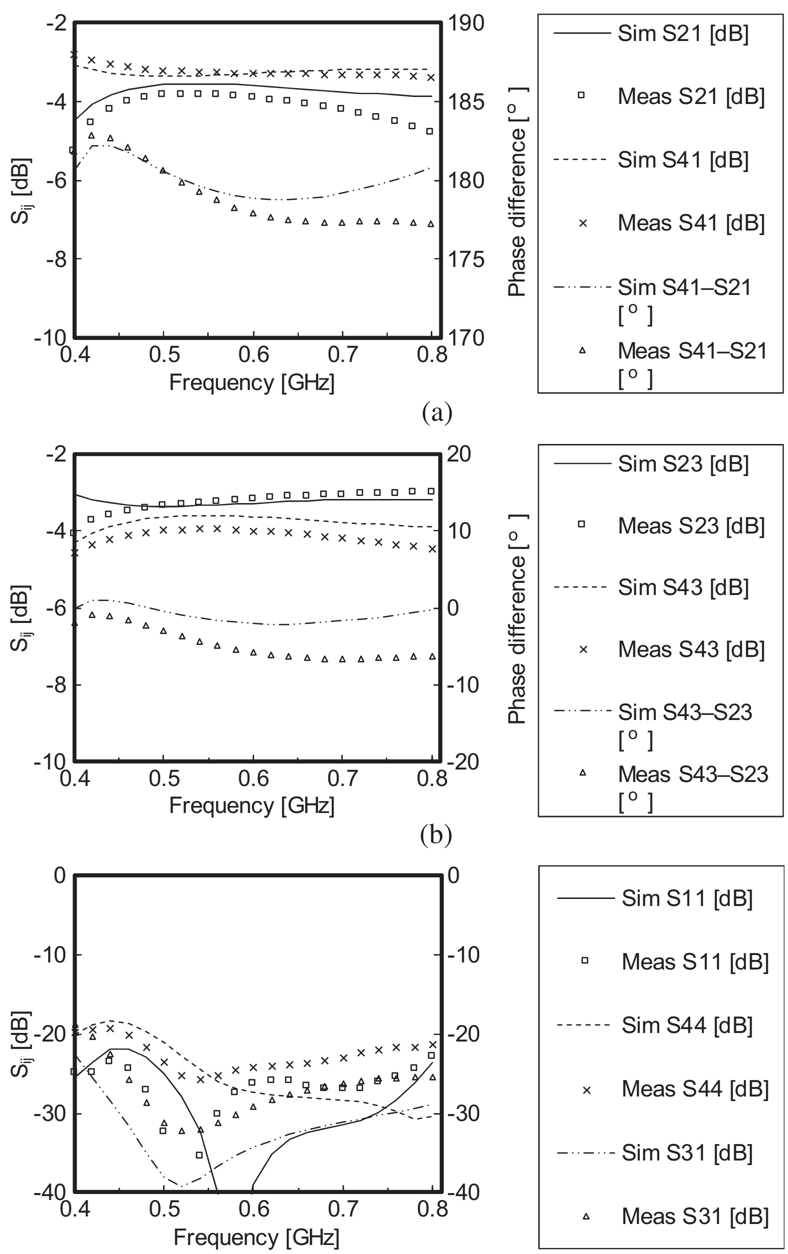

Figure 7. EM Simulated and measured results for fabricated broadband rat-race hybrid shown in Fig. 6. (a) S_{21}, S_{41}, and $S_{41} - S_{21}$. (b) S_{23}, S_{43}, and $S_{43} - S_{23}$. (c) S_{11}, S_{44}, and S_{31}.

Table 3. Performance comparison of proposed and previously reported rat-race hybrid.

	Size [mm]	Relative Bandwidth	Return losses [dB]	Design frequency [GHz]
Proposed rat race hybrid	146×50	62%	20	0.59
[8]	28×60	49%	14	0.59
[11]	-	36%	20	4
[12]	120×70	111%	20	1.8

various reported rat-race hybrid characteristics. Compared to the rat-race hybrid utilizing composite right/left-handed transmission lines [11], of which the relative bandwidth is approximately 36%, the miniaturized broadband rat-race hybrid using a transmission line and lumped-element components [8] is very simple while achieving both miniaturization and broadband characteristics sufficiently. This comparison demonstrates that the proposed device achieved broadband characteristics and the overall size of the circuit is comparable with similar design [12].

4. CONCLUSION

We reported the synthesis of a broadband rat-race consisting of a miniaturized broadband rat-race hybrid and transmission line cascades. For the frequency range of 420–800 MHz, which corresponds to a relative bandwidth of more than 62%, the broadband rat-race hybrid exhibited power splits of -3.8 ± 1.0 dB, return losses of greater than 19 dB, and isolation between output ports of greater than 20 dB. The phase difference between S_{21} and S_{41} was $180° \pm 3°$.

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