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DOA Estimation of Quasi-Stationary Signals Using a Partly-Calibrated Uniform Linear Array with Fewer Sensors than Sources

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Abstract—A two-step method is proposed to estimate the direction-of-arrivals (DOAs) of quasi-stationary source signals, with a partly-calibrated uniform linear array (PC-ULA). The special structure of Toeplitz matrix is utilized to estimate the sensors' uncertainties. Then, a Khatri-Rao (KR) based multiple signal classification (MUSIC) algorithm is proposed to estimate the DOAs of source signals. Simulation results show that the proposed method renders lower root-mean-square-error (RMSE) than existing KR-based ESPRIT algorithms, especially under low signal-to-noise-ratio (SNR) and small angle separation between DOAs. It is also shown that the proposed method increases the degree-of-freedom (DOF) by one, as compared to the counterpart ESPRIT methods.

1. INTRODUCTION

Direction-of-arrivals (DOAs) are important information for applications in radars, wireless communications, radio telescopes, etc. Different estimation methods have been proposed, including those based on multiple signal classification (MUSIC) algorithm [1] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [2]. However, these algorithms may be compromised by uncertainties in gain and phase of some sensors in the array. In [3], an ESPRIT-like algorithm was proposed for joint estimation of sensor uncertainties and DOAs, which can resolve $N-2$ signal sources with N sensors. In [4], a refinement on [3] was proposed by iterating ESPRIT-like algorithm to reach a near-optimal estimation of uncertainties and DOAs. Some limitation of methods was also reported, such as the maximum sources detectable [4].

In [5], two enhancements on the method of [3] were proposed to tackle nonuniform noise powers at different sensors. Given uncorrelated signals, specific structure of covariance matrix was exploited to calibrate uncertainties and deal with nonuniform noise, followed by a MUSIC algorithm to estimate the DOAs. Given correlated signals, an iterative approach was also proposed to determine the signal subspace, then an ESPRIT-like method was adopted to estimate the DOAs. The above methods require at least two calibrated sensors to work properly. In [6], a least squares method was proposed to jointly estimate the phase error of a sensor, and the DOA of source signals, by using only one calibrated sensor.

Recently, a Khatri-Rao (KR) subspace was proposed to increase the degrees-of-freedom (DOFs) of an array [7]. A KR-based MUSIC algorithm was proposed to increase the DOF of an N-element sensor array from $N-1$ to $2N-2$. A reduced-dimension method was also proposed prior to applying the singular value decomposition (SVD) technique. However, the performance of KR-MUSIC algorithm significantly deteriorates when uncertainties exist in gain and phase of sensors. Motivated by [3] and [7], a KR-ESPRIT-like method [8] was proposed to jointly estimate the uncertainties of sensors and the DOAs of source signals. The noise-removal in [7] were adopted before applying the subspace processing.

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A reduced-dimension version was also proposed by first using KR-ESPRIT-like method to estimate uncertainties and then applying reduced-dimension method [7] prior to subspace processing.

The KR-ESPRIT-like algorithm can detect up to $2N-3$ source signals with a partly-calibrated uniform linear array (PC-ULA) of N sensors, two of which are calibrated [8]. In this work, a two-step KR-MUSIC algorithm is proposed to first estimate the uncertainties in gain and phase of some sensors, then to estimate the DOAs of source signals. The proposed method can solve up to $2N-2$ source signals by using N sensors, with two of them calibrated. This work is organized as follows. The signal model in KR-subspace with PC-ULA is presented in Section 2, the proposed KR-MUSIC-like algorithm is presented in Section 3, simulation results are discussed in Section 4. Finally, some conclusions are drawn in Section 5.

2. SIGNAL MODEL IN KR-SUBSPACE WITH PC-ULA

Figure 1 shows a partly calibrated uniform linear array (PC-ULA), which is composed of N omnidirectional sensors at spacing d. Without loss of generality, assume that the first N_c sensors are calibrated while the last $N-N_c$ sensors bear amplitude and phase uncertainties. Assume there are M uncorrelated source signals. The mth signal s_m is incident in the direction-of-arrival (DOA) θ_m, with $1 \le m \le M$. The complex-valued baseband signal received at the ℓth time interval can be represented as

$$\bar{x}[\ell] = \bar{\bar{A}}' \cdot \bar{s}[\ell] + \bar{n}[\ell] \tag{1}$$

where $\bar{\bar{A}}'$ is the actual array-gain matrix; $\bar{s}[\ell]$ is the source signal vector; $\bar{n}[\ell]$ is an additive white Gaussian noise vector with zero mean and covariance matrix $\sigma_n^2 \bar{\bar{I}}_N$; $\bar{\bar{I}}_N$ is an $N \times N$ identity matrix. The noise is assumed to be uncorrelated to the source signals.

Matrix $\bar{\bar{A}}'$ can be further decomposed into

$$\bar{\bar{A}}' = \text{diag}\left\{\gamma_1, \gamma_2, \ldots, \gamma_N\right\} \cdot \bar{\bar{A}} \tag{2}$$

where $\bar{\bar{A}}$ is the reconstructed array-gain matrix, and γ_n is the gain and phase uncertainties of the nth sensor. The γ_n's are compiles into an uncertainty vector

$$\bar{\gamma} = \left[1, 1, \ldots, 1, \rho_1 e^{j\phi_1}, \rho_2 e^{j\phi_2}, \rho_{N-N_c} e^{j\phi_{N-N_c}}\right]^t$$

in which the first N_c elements are unity, and ρ_n and ϕ_n, with $1 \le n \le N - Nc$, are uncertainties to be calibrated. The mth column of $\bar{\bar{A}}$ is a steering vector associated with the source signal incident at θ_m, namely,

$$\bar{a}(\theta_m) = \left[1, e^{jkd\sin\theta_m}, e^{j2kd\sin\theta_m}, \ldots, e^{j(N-1)kd\sin\theta_m}\right]^t$$

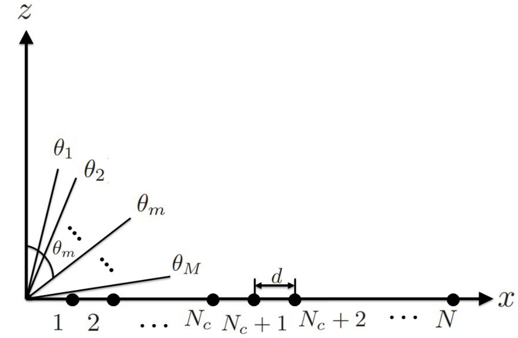

Figure 1. A partly-calibrated uniform linear array (PC-ULA) of N sensors, in which the first N_c sensors are calibrated.

where $k = 2\pi/\lambda$ is the wavenumber.

The source signals are assumed to be quasi-stationary and will be observed over Q non-overlapped time frames, with each time frame containing L time intervals. The power of the mth signal in the qth time frame can be represented as

$$P_{qm} = \mathrm{E}\left\{|s_m[\ell]|^2\right\} \tag{3}$$

with $(q-1)L \leq \ell \leq qL-1$ and $1 \leq q \leq Q$.

The covariance matrix in the qth time frame can be represented as

$$\bar{\bar{R}}_q = \mathrm{E}\{\bar{x}[\ell]\bar{x}^\dagger[\ell]\} = \bar{\bar{A}}' \cdot \bar{\bar{D}}_q \cdot \bar{\bar{A}}'^\dagger + \sigma_n^2 \bar{\bar{I}}_N \tag{4}$$

where $\bar{\bar{D}}_q = \mathrm{diag}\{P_{q1}, P_{q2}, \ldots, P_{qM}\}$. In practice, $\bar{\bar{R}}_q$ is estimated as

$$\tilde{\bar{\bar{R}}}_q = \frac{1}{L} \sum_{\ell=(q-1)L}^{qL-1} \bar{x}[\ell]\bar{x}^\dagger[\ell] \tag{5}$$

By concatenating all the columns of $\bar{\bar{R}}_q$ into an $N^2 \times 1$ vector as

$$\bar{y}_q = \mathrm{vec}\left\{\bar{\bar{R}}_q\right\} = \bar{\bar{A}}'_{\mathrm{KR}} \cdot \bar{P}_q + \sigma_n^2 \bar{\mu}_N \tag{6}$$

where $\bar{P}_q = [P_{q1}, P_{q2}, \ldots, P_{qM}]^t$, $\bar{\bar{A}}'_{\mathrm{KR}} = \bar{\bar{A}}'^* \odot \bar{\bar{A}}'$ is called a generalized manifold matrix, $\odot$ stands for column-wise Kronecker product, and $\bar{\mu}_N = \mathrm{vec}\{\bar{\bar{I}}_N\}$. Next, by stacking the quasi-stationary signals $\bar{y}_q$ over all Q time frames, we have

$$\bar{\bar{Y}} = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_Q] = \bar{\bar{A}}'_{\mathrm{KR}} \cdot \bar{\bar{\Psi}} + \sigma_n^2 \bar{\mu}_N \bar{1}_Q^t \tag{7}$$

where $\bar{\bar{\Psi}} = [\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_Q]$ and $\bar{1}_Q$ is a $Q \times 1$ vector with all entries equal to unity. By applying an orthogonal complement matrix $\bar{\bar{\Gamma}} = \bar{\bar{I}}_Q - (1/Q)\bar{1}_Q\bar{1}_Q^t$ to $\bar{\bar{Y}}$, the noise covariance term $\sigma_n^2 \bar{\mu}_N \bar{1}_Q^t$ can be eliminated to have [7]

$$\bar{\bar{Y}}^\perp = \bar{\bar{A}}'_{\mathrm{KR}} \cdot \bar{\bar{\Psi}} \cdot \bar{\bar{\Gamma}} \tag{8}$$

up which the singular value decomposition (SVD) technique is applied to obtain

$$\bar{\bar{Y}}^\perp = \begin{bmatrix} \bar{\bar{E}}_s & \bar{\bar{E}}_n \end{bmatrix} \begin{bmatrix} \bar{\bar{\Sigma}}_s & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\bar{V}}_s^\dagger \\ \bar{\bar{V}}_n^\dagger \end{bmatrix} \tag{9}$$

where $\bar{\bar{E}}_s$ and $\bar{\bar{E}}_n$ are the left singular matrices associated with the nonzero singular values contained in the diagonal matrix $\bar{\bar{\Sigma}}_s$ and zero singular values, respectively; $\bar{\bar{V}}_s$ and $\bar{\bar{V}}_n$ are the corresponding right singular matrices. By the subspace theory, matrix $\bar{\bar{E}}_s$ spans the same subspace as matrix $\bar{\bar{A}}'_{\mathrm{KR}}$ does, which is labeled as the signal subspace. Formally, these two matrices are related by an $M \times M$ nonsingular matrix $\bar{\bar{T}}$ as

$$\bar{\bar{E}}_s = \bar{\bar{A}}'_{\mathrm{KR}} \cdot \bar{\bar{T}} \tag{10}$$

3. PROPOSED KR-MUSIC-LIKE ALGORITHM

The special structure of Toeplitz matrix is utilized to estimate the uncertainties in gain and phase of sensors, then a KR-MUSIC algorithm is proposed to estimate the DOAs of source signals. A PC-ULA of N sensors can be used to estimate up to $2N-2$ source signals, which is the upper bound a KR-MUSIC algorithm can achieve with a fully-calibrated ULA [7].

3.1. Estimation of Uncertainties

First, Eq. (4) is rewritten as

$$\bar{\bar{R}}_q = \mathrm{diag}\left\{\bar{\gamma}\right\} \cdot \bar{\bar{R}}'_q \cdot \mathrm{diag}\left\{\bar{\gamma}\right\}^\dagger + \sigma^2 \bar{\bar{I}}_N \tag{11}$$

where $\bar{\bar{R}}'_q = \bar{\bar{A}} \cdot \bar{\bar{D}}_q \cdot \bar{\bar{A}}^\dagger$ and $\mathrm{diag}\left\{\bar{\gamma}\right\}$ is a diagonal matrix with its nth diagonal entry being the nth entry of $\bar{\gamma}$. The nth row of $\bar{\bar{A}}$ is

$$\bar{\alpha}_n = \left[e^{j(n-1)kd\sin\theta_1}, e^{j(n-1)kd\sin\theta_2}, \ldots, e^{j(n-1)kd\sin\theta_M}\right]$$

Hence, the uvth entry of $\bar{\bar{R}}'_q$ can be expressed as

$$R'_{q,uv} = \bar{\alpha}_u \cdot \bar{\bar{D}}_q \cdot \bar{\alpha}_v^\dagger = \sum_{m=1}^{M} e^{jkd(u-1)\sin\theta_m} P_{qm} e^{jkd(1-v)\sin\theta_m} = \sum_{m=1}^{M} P_{qm} e^{jkd(u-v)\sin\theta_m} \tag{12}$$

leading to

$$\begin{aligned} R'_{q,uu} &= \sum_{m=1}^{M} P_{qm} \\ R'_{q,uv} &= R'_{q,rs} \text{ if } u-v = r-s \end{aligned} \tag{13}$$

which implies that $\bar{\bar{R}}'_q$ is a Hermitian Toeplitz matrix.

By substituting Eq. (12) into Eq. (11), we have

$$R_{q,uv} = \begin{cases} |\gamma_u|^2 \sum_{m=1}^{M} P_{qm} + \sigma^2, & u = v \\ \gamma_u \gamma_v^* R'_{q,uv}, & u \neq v \end{cases} \tag{14}$$

From Eqs. (13) and (14), the uncertainties in each time frame can be estimated as

$$\tilde{\gamma}_{q(n+1)} = \left(\frac{R_{q,n(n+1)}}{\tilde{\gamma}_{qn} R_{q,12}}\right)^*, \qquad n = N_c, N_c+1, \ldots, N-1 \tag{15}$$

assuming that

$$\tilde{\gamma}_{q1} = \tilde{\gamma}_{q2} = \ldots = \tilde{\gamma}_{qN_c} = 1, \quad q = 1, 2, \ldots, Q$$

Thus, the uncertainties are estimated as

$$\tilde{\gamma}_n = \frac{1}{Q} \sum_{q=1}^{Q} \tilde{\gamma}_{qn}, \quad n = N_c+1, N_c+2, \ldots, N \tag{16}$$

By substituting Eq. (16) into Eq. (11), we have

$$\bar{\bar{R}}''_q = \mathrm{diag}\left\{\tilde{\bar{\gamma}}^{-1}\right\} \cdot \bar{\bar{R}}_q \cdot \mathrm{diag}\left\{\tilde{\bar{\gamma}}^{-1}\right\}^\dagger = \bar{\bar{R}}'_q + \mathrm{diag}\left\{\sigma_1'^2, \sigma_2'^2, \ldots, \sigma_N'^2\right\} \tag{17}$$

where $\mathrm{diag}\left\{\tilde{\bar{\gamma}}^{-1}\right\} = \mathrm{diag}\left\{1, \ldots, 1, \tilde{\gamma}_{N_c+1}^{-1}, \ldots, \tilde{\gamma}_N^{-1}\right\}$ and $\sigma_n'^2 = \sigma^2 / |\tilde{\gamma}_n|^2$. To maintain a Toeplitz structure of the matrix, the diagonal entries of $\bar{\bar{R}}''_q$ are averaged to derive

$$\bar{\bar{R}}'''_q = \bar{\bar{R}}'_q + \sigma'^2 \bar{\bar{I}}_N \tag{18}$$

where $\sigma'^2 = \dfrac{1}{N} \sum_{n=1}^{N} \sigma_n'^2$.

3.2. Estimation of DOAs

Similar to the procedure in Eqs. (6)–(8), a KR-subspace is constructed in the following order:

$$\begin{aligned}\bar{y}'_q &= \text{vec}\left\{\bar{\bar{R}}'''_q\right\},\ q = 1, 2, \ldots, Q \\ \bar{\bar{Y}}' &= \left[\bar{y}'_1, \bar{y}'_2, \ldots, \bar{y}'_Q\right] = \bar{\bar{A}}_{\text{KR}} \cdot \bar{\bar{\Psi}} + \sigma'^2_n \bar{\mu}_N \bar{1}^t_Q \\ \bar{\bar{Y}}'^{\perp} &= \bar{\bar{Y}}' \cdot \bar{\bar{\Gamma}} = \bar{\bar{A}}_{\text{KR}} \cdot \bar{\bar{\Psi}} \cdot \bar{\bar{\Gamma}}\end{aligned} \tag{19}$$

where $\bar{\bar{A}}_{\text{KR}} = \bar{\bar{A}}^* \odot \bar{\bar{A}}$. A reduced-dimension KR-subspace can also be constructed as

$$\bar{\bar{Y}}''^{\perp} = \bar{\bar{C}}^{-1/2} \cdot \bar{\bar{G}}^t \cdot \bar{\bar{Y}}'^{\perp} = \bar{\bar{C}}^{-1/2} \cdot \bar{\bar{G}}^t \cdot \bar{\bar{A}}_{\text{KR}} \cdot \bar{\bar{\Psi}} \cdot \bar{\bar{\Gamma}} = \bar{\bar{C}}^{-1/2} \cdot \bar{\bar{G}}^t \cdot \bar{\bar{G}} \cdot \bar{\bar{B}} \cdot \bar{\bar{\Psi}} \cdot \bar{\bar{\Gamma}} = \bar{\bar{C}}^{1/2} \cdot \bar{\bar{B}} \cdot \bar{\bar{\Psi}} \cdot \bar{\bar{\Gamma}} \tag{20}$$

where $\bar{\bar{C}} = \bar{\bar{G}}^t \cdot \bar{\bar{G}}$,

$$\bar{\bar{G}}_{N^2 \times (2N-1)} = \begin{bmatrix} \bar{\bar{G}}_0 \\ \vdots \\ \bar{\bar{G}}_{N-1} \end{bmatrix}, \quad \bar{\bar{B}} = \left[\bar{b}(\theta_1), \bar{b}(\theta_2), \ldots, \bar{b}(\theta_M)\right]$$

with

$$\begin{aligned}\bar{\bar{G}}_n &= \left[\bar{\bar{0}}_{N\times(N-1-n)} \quad \bar{\bar{I}}_N \quad \bar{\bar{0}}_{N\times n}\right], \qquad n = 0, 1, \ldots, N-1 \\ \bar{b}(\theta) &= \left[e^{-j(N-1)kd\sin\theta}, e^{-j(N-2)kd\sin\theta}, \ldots, e^{-j2\pi\sin\theta}, 1, e^{jkd\sin\theta}, e^{j2kd\sin\theta}, \ldots, e^{j(N-1)kd\sin\theta}\right]^t\end{aligned}$$

By applying the SVD technique on $\bar{\bar{Y}}''^{\perp}$, we have

$$\bar{\bar{Y}}''^{\perp} = \left[\bar{\bar{E}}''_s \quad \bar{\bar{E}}''_n\right] \begin{bmatrix} \bar{\bar{\Sigma}}''_s & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\bar{V}}''^{\dagger}_s \\ \bar{\bar{V}}''^{\dagger}_n \end{bmatrix} \tag{21}$$

Then, apply the MUSIC algorithm to obtain a DOA spectrum as

$$P_{\text{KR-MUSIC}}(\theta) = \left\| \bar{\bar{E}}''^{\dagger}_n \cdot \bar{\bar{C}}^{1/2} \cdot \bar{b}(\theta) \right\|_F^{-2} \tag{22}$$

where $-\pi/2 \leq \theta \leq \pi/2$. Fig. 2 shows the flow-chart of estimating uncertainties and DOAs with the proposed method.

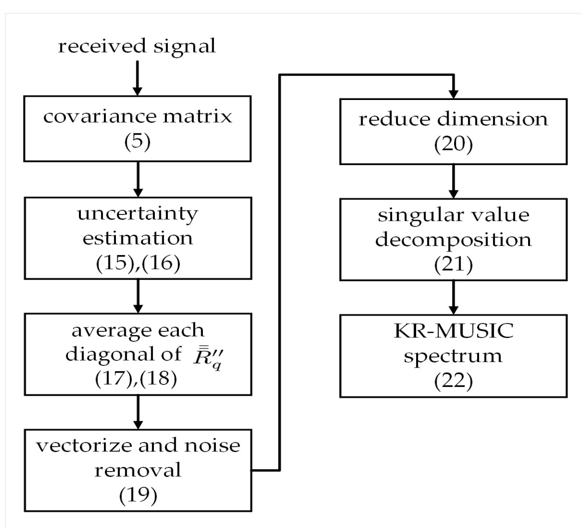

Figure 2. Flow-chart of estimating uncertainties and DOAs with proposed method.

4. SIMULATIONS AND DISCUSSIONS

A partly calibrated ULA consisted of five sensors ($N = 5$) will be used in the simulations. The sensor spacing is half a wavelength. The first two sensors are calibrated, and the last three sensors bear uncertainties in gain and phase, namely, $\bar{\gamma} = [1, 1, 0.8e^{j\pi/5}, 1.2e^{j\pi/10}, 0.6e^{-j\pi/3}]^t$. The effect of noise is characterized by the signal-to-noise ratio (SNR), which is defined as [7]

$$\text{SNR} = \frac{\frac{1}{T}\sum_{t=1}^{T} \text{E}\left\{\left\|\bar{\bar{A}}' \cdot \bar{s}(t)\right\|^2\right\}}{\text{E}\left\{\|\bar{n}(t)\|^2\right\}} \tag{23}$$

where $T = LQ$.

In the first scenario, $M = 4$ source signals are incident from the directions of $\bar{\theta} = [-25^\circ, 10^\circ, 20^\circ, 30^\circ]^t$, under SNR = 10 dB. A total of $Q = 200$ time frames are processed, each with a length of $L = 512$; and $K = 500$ Monte Carlo trials will be implemented in each scenario. The estimated uncertainties by using the proposed method are listed in Table 1. It is observed that the variance of estimation is pretty low.

Table 1. Estimation of gain and phase uncertainties.

true value (ρ, ϕ)	mean	variance
(0.8000, 0.6283)	(0.8013, 0.6264)	(0.00003, 0.00003)
(1.2000, 0.3142)	(1.1992, 0.3115)	(0.00008, 0.00008)
(0.6000, −1.0472)	(0.6016, −1.0492)	(0.00005, 0.00017)

The unit of ϕ is rad, $M = 4$, $Q = 200$, $L = 512$, SNR = 10 dB, 500 Monte-Carlo trials.

To compare the performance of different methods, a root-mean-square-error (RMSE) of DOA estimation is defined as

$$\text{RMSE} = \sqrt{\frac{1}{KM}\sum_{k=1}^{K}\sum_{m=1}^{M} |\tilde{\theta}_{km} - \theta_m|^2} \tag{24}$$

where θ_m and $\tilde{\theta}_{km}$ are the actual DOA and the estimated DOA, respectively, of the mth source signal in the kth trial. Fig. 3 shows the RMSE of DOA estimation under different SNRs, with the proposed method, an KR-ESPRIT-like method [8] and its reduced-dimension (RD) version. It is observed that the proposed method renders lower RMSE than the other two methods, under all SNRs of consideration.

Figure 4 shows the effect of frame length on the RMSE of DOA estimation. The data size is fixed at $T = LQ = 102,400$ while L is varied from 64 to 2,048, at SNR = 10 dB. It is observed that the proposed method renders the lowest RMSE under all frame lengths of consideration. The RMSE increases when the frame length is either large or small, possibly attributed to the estimation error in computing the covariance matrix of the simulated source signal, of which the stationary interval lies between 300 and 700. However, the proposed method is less affected by the frame length than the other two methods.

Figure 5 shows the RMSE of DOA estimation versus different numbers of frames, with the frame length fixed to 512 and SNR = 10 dB. It is observed that the proposed method renders the lowest RMSE at different numbers of frames, and degrades more smoothly than the other two methods when the number of frames becomes too small.

Next, consider scenarios with small angle separation between different source signals. Assume that there are $M = 3$ source signals, the DOAs of first two are $\theta = -5^\circ$ and 15°, and the angle separation between the second and the third ones is $\Delta\theta = 2^\circ$ and 3°, respectively. The frame length is $L = 512$ and the number of frames is $Q = 200$. Fig. 6 shows the success probability of DOA estimation under different SNRs. The success probability is the ratio between the number of trials with RMSE of DOA lower than 0.6° and the total number of trials. It is observed that the proposed method renders the

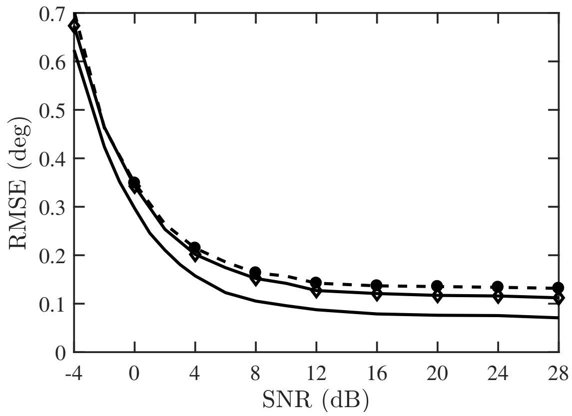

Figure 3. RMSE of DOA estimation, $M = 4$, $Q = 200$, $L = 512$, 500 Monte-Carlo trials at each SNR. ——: proposed method, $- \bullet -$: KR-ESPRIT-like method, —◇—: KR-ESPRIT-like method with RD.

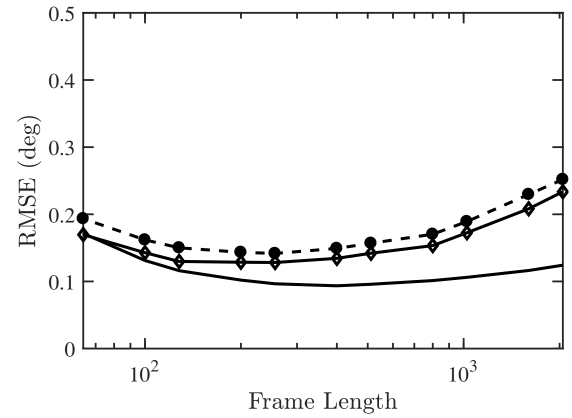

Figure 4. RMSE of DOA estimation versus frame length, $M = 4$, $T = 102,400$, SNR = 10 dB, 500 Monte-Carlo trials at each frame length. ——: proposed method, $- \bullet -$: KR-ESPRIT-like method, —◇—: KR-ESPRIT-like method with RD.

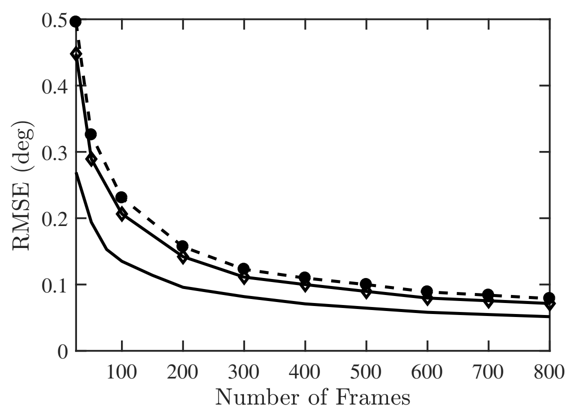

Figure 5. RMSE of DOA estimates, $M = 4$, $L = 512$, SNR = 10 dB, 500 Monte-Carlo trials at each number of frames. ——: proposed method; $- \bullet -$: KR-ESPRIT-like method; $- \diamond -$: KR-ESPRIT-like method with RD.

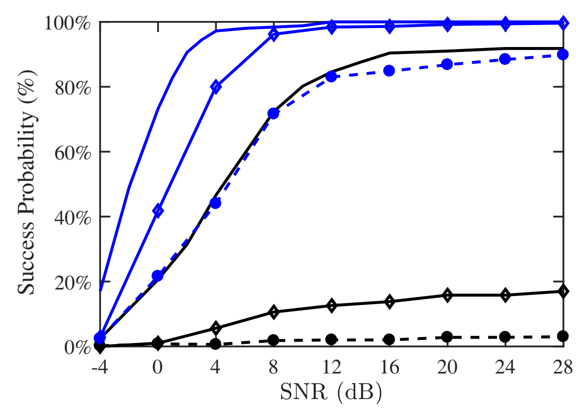

Figure 6. Success probability of DOA estimation, $M = 3$, $Q = 200$, $L = 512$, 500 Monte-Carlo trials at each SNR. ——: proposed method ($\Delta\theta = 2°$); $- \bullet -$: KR-ESPRIT-like method ($\Delta\theta = 2°$); $- \diamond -$: KR-ESPRIT-like method with RD ($\Delta\theta = 2°$); ——: proposed method ($\Delta\theta = 3°$); $- \bullet -$: KR-ESPRIT-like method ($\Delta\theta = 3°$); $- \diamond -$: KR-ESPRIT-like method with RD ($\Delta\theta = 3°$).

highest success probability at both angle separations. The KR-ESPRIT-like method works fine when the angle separation is 3° and the SNR is high. At the angle separation of 2°, the success probability of the other two methods is lower than 20% at the SNRs of consideration. In comparison, the proposed method can achieve about 70% of success probability at SNR = 8 dB.

Figure 7 shows the RMSE of DOA at different angle separations ($\Delta\theta$), with SNR = 10 dB and the other parameters are same as in Fig. 6. It is observed that when the angle separation is large, the proposed method is slightly better than the other two methods. However, when the angle separation is less than 3°, the proposed method outperforms the other two more significantly.

Next, consider scenarios with more source signals than sensors. Assume $M = 8$, $\bar{\theta} =$

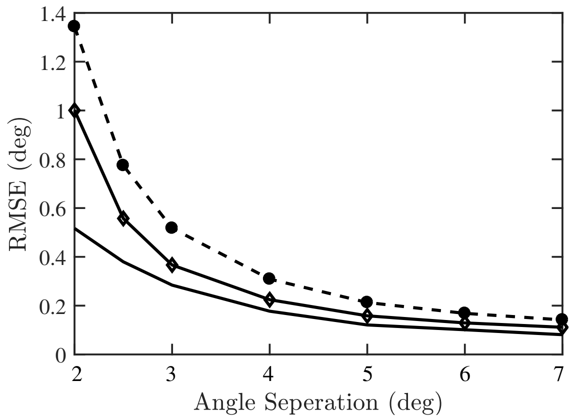

Figure 7. RMSE of DOA estimation, $M = 3$, $Q = 200$, $L = 512$, $\mathrm{SNR} = 10\,\mathrm{dB}$, 500 Monte-Carlo trials at each angle separation. ——: proposed method; $-\bullet-$: KR-ESPRIT-like method; $-\diamond-$: KR-ESPRIT-like method with RD.

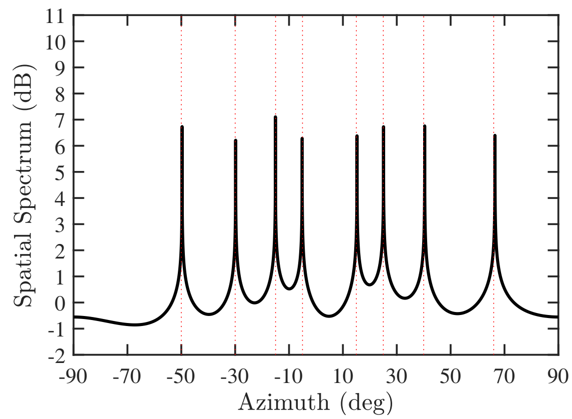

Figure 8. Spatial spectrum, $M = 8$, $Q = 200$, $L = 512$, $\mathrm{SNR} = 10\,\mathrm{dB}$. ——: proposed method ; ...: actual DOAs of source signals.

Table 2. Estimation of gain and phase uncertainties.

true value (ρ, ϕ)	mean	variance
$(0.8000, 0.6283)$	$(0.7768,\ 0.6296)$	$(0.0008,\ 0.0015)$
$(1.2000, 0.3142)$	$(1.2076,\ 0.3173)$	$(0.0026,\ 0.0039)$
$(0.6000, -1.0472)$	$(0.5822,\ -1.0429)$	$(0.0011,\ 0.0079)$

The unit of ϕ is rad, $M = 8$, $Q = 200$, $L = 512$, $\mathrm{SNR} = 10\,\mathrm{dB}$, 500 Monte-Carlo trials.

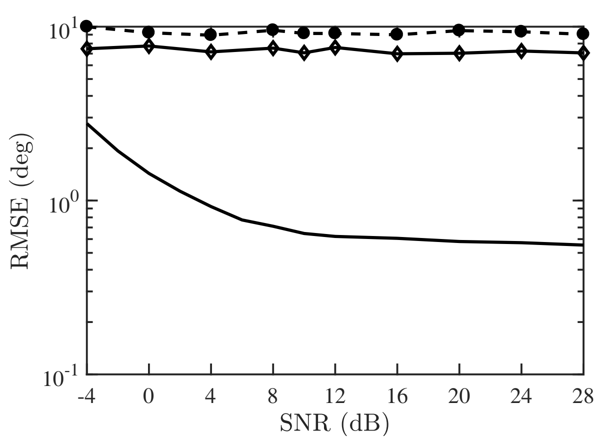

Figure 9. RMSE of DOA estimation, $M = 8$, $Q = 200$, $L = 512$, 500 Monte-Carlo trials at each SNR. ——: proposed method; $-\bullet-$: KR-ESPRIT-like method; $-\diamond-$: KR-ESPRIT-like method with RD.

$[-50^\circ, -30^\circ, -15^\circ, -5^\circ, 15^\circ, 25^\circ, 40^\circ, 66^\circ]^t$, $\mathrm{SNR} = 10\,\mathrm{dB}$, $Q = 200$ and $L = 512$. The estimation of gain and phase uncertainties with the proposed method is listed in Table 2. It is observed that the mean value is sufficiently accurate, but the variances are relatively larger than those in Table 1, as the number of source signals is equal to the maximum degree-of-freedom achievable.

Figure 8 shows a realization of DOA spectrum obtained with the proposed method, where the spectrum is defined as $\log\{P_{\mathrm{KR-MUSIC}}(\theta)\}$, in terms of Eq. (22). It is verified that the proposed method can reach the upper bound of the KR-MUSIC algorithm with a fully-calibrated ULA, namely, resolving $2N - 2$ sources with N sensors.

Finally, the RMSEs of DOA estimation with the proposed method as well as two other methods, under different SNRs, are shown in Fig. 9. The frame length is 512 and the number of frames is 200. It is observed that the proposed method achieves more accurate estimation than the other methods.

5. CONCLUSION

A KR-subspace MUSIC-like algorithm is proposed to estimate the DOAs of quasi-stationary signals with a partly-calibrated ULA. The special structure of Toeplitz matrix is utilized to estimate the uncertainties of gain and phase. The KR-subspace MUSIC algorithm is then applied to derive the DOA spectrum of source signals to estimate their DOAs. Simulation scenarios are designed to verify that the proposed method performs better than two ESPRIT-like methods at low SNR, short frame length, small number of frames and small angle separations, respectively. It is also verified that the proposed method can resolve up to $2N-2$ source signals with N sensors, which is the maximum achievable degree-of freedom.

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