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Joint DOA and Polarization Estimation Using a UCA of Single-Polarized Antennas

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Abstract—In this paper, the direction of arrival (DOA) and polarization parameters are estimated by a uniform circular array (UCA) with several single-polarized sensors. An efficient and improved polarization MUSIC algorithm for estimating the DOA and polarization parameters is presented. This method uses information on the amplitude to reduce the computational complexity. When the source is linearly polarized, the proposed algorithm is more accurate at a low signal-to-noise ratio (SNR). Monte Carlo simulations verify the efficacy of the proposed method.

1. INTRODUCTION

Polarization-sensitive arrays (PSAs) have received considerable attention in many disciplines, including radar, sonar and mobile communications. The direction of arrival (DOA) plays an important role in the processing of signals from polarization-sensitive arrays. An electric vector sensor consists of three spatially co-located nonidentical anisotropic antennas that measure the incident wavefield's three electric-field components separately [1] and [2]. Most antenna arrays are of the dual-polarized type. In a real environment, a dual-polarized sensor may be inapplicable, and discussing the single-polarized sensor becomes meaningful.

Estimating multiple parameters of a signal is an important task that focuses primarily on improving the accuracy and resolution. Therefore, some conventional techniques for DOA estimation are used in PSAs. The estimation of signal parameters by using techniques based on rotational invariance (ESPRIT) has been presented [3–5]. The algorithm provides closed-form estimates of the DOA and polarization. In [6] and [8], the multiple signal classification (MUSIC) algorithm and other subspace approaches are extended to polarization sensitive arrays. The maximum likelihood (ML) algorithm [9, 10] is an optimal algorithm for the DOA and is much more complex than the other two algorithms.

Over the last decade, as vector-sensors have become increasingly reliable, polarization has been added to the DOA as an essential attribute for the characterization of sources in estimation processes. In [7], a quaternion-based method for data covariance is proposed. The Q-MUSIC algorithm decreases the computational effort required. In [11], a method that can reduces the computation time by using a ratio of amplitudes is proposed. In [12], a high-resolution algorithm for estimating the DOA and polarization using real-world arrays with imperfections is proposed. Based on this array, a novel polarization MUSIC algorithm that uses information on the amplitude of the impinging source is proposed. A uniform circular array (UCA) provides 360° azimuthal coverage and information on the source's elevation angle. The amplitude of the signal contains information on its DOA, polarization and power. Therefore, the goal of this paper is to arrive at a method for reducing the computational complexity of the algorithm for estimating the DOA and polarization when the source impinges on a PSA. To this end, an improved MUSIC algorithm for estimating the DOA and polarization is proposed.

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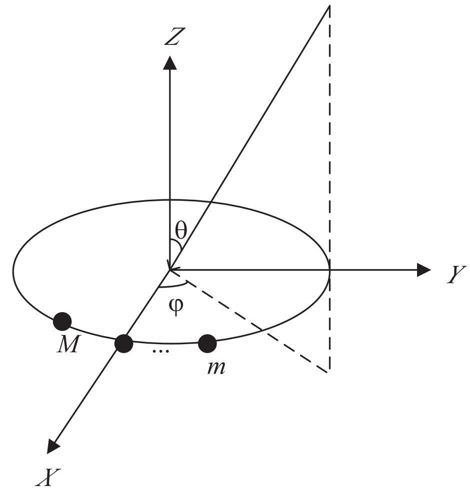

Figure 1. Uniform circular array.

2. RECEIVED SIGNALS

We consider an array with M identical antenna as shown in Fig. 1. These elements are uniformly distributed over a circle with radius r. We assume that the phase center of each antenna element is located in the xy-plane. The identical antenna element is the single-polarization antenna. For a completely polarized transverse electromagnetic (TEM) wave propagating into the array, the x- and y-components of the electric field are defined as follows [3]:

$$\begin{aligned}\begin{bmatrix} e_{\mathrm{x}} \\ e_{\mathrm{y}} \end{bmatrix} &= \begin{bmatrix} -\sin\varphi & \cos\theta\cos\varphi \\ \cos\varphi & \cos\theta\sin\varphi \end{bmatrix} \begin{bmatrix} \cos\gamma \\ \sin\gamma \mathrm{e}^{\mathrm{j}\eta} \end{bmatrix} \\ &= \mathbf{\Xi}(\theta,\varphi)\mathbf{h}(\gamma,\eta) \end{aligned} \tag{1}$$

where $0 \le \theta \le \pi/2$ denotes the signal's elevation angle, $0 \le \varphi \le 2\pi$ the azimuth angle, $0 \le \gamma \le \pi/2$ the auxiliary polarization angle, and $-\pi \le \eta \le \pi$ the polarization phase difference. However, the electric component parallel to the antenna can be received completely, whereas the vertical electric component cannot be received. Therefore, the received electric component of the mth element of the UCA can be expressed as follows:

$$e_m = e_x \sin\alpha_m + e_y \cos\alpha_m, \tag{2}$$

where $m = 1, \dots, M$ and $0 \le \alpha_m < \pi$ is the angle between the mth element and the y-axis.

Consider the scenario in which D uncorrelated narrow-band signals with elevation angles θ_d, azimuth angles ϕ_d, auxiliary polarization angle parameters γ_d and polarization phase differences η_d, where $d = 1, \dots, D$. These signals impinge on the single-polarization sensor array. The output of the mth sensor can be modeled as

$$x_m(t) = \sum_{d=1}^{D} s_d(t) e^{j\tau_{md}} e_m + n_m(t), \tag{3}$$

where $s_d(t)$ denotes the waveform of the dth signal, and $n_m(t)$ denote the additive complex-valued zero-mean white noise, which are presumed to be independent of the signals. τ_{md} is the phase shift associated with the dth signal's propagation time delay between the mth sensor and the phase reference point. They can be expressed as

$$s_d(t) \overset{def}{=} \sqrt{p_d} e^{j2\pi(c/\lambda)t} \tag{4}$$

$$\tau_{md} = \frac{2\pi r}{\lambda} \cos(\varphi_d - \frac{2\pi(m-1)}{M}) \sin\theta_d, \tag{5}$$

where p_d denotes the dth signal's power, λ the signal's wavelength, and c the propagation speed. From Equation (3), the data received by the single-polarized antenna can be written in matrix form as

$$\mathbf{X}(\mathbf{t}) = \Re\mathbf{S}(\mathbf{t}) + \mathbf{N}(\mathbf{t}), \tag{6}$$

where

- $\mathbf{X}(\mathbf{t}) = [\mathbf{x_1}(\mathbf{t}), \ldots, \mathbf{x_m}(\mathbf{t}), \ldots, \mathbf{x_M}(\mathbf{t})]^{\mathbf{T}}$,
- $\mathbf{S}(\mathbf{t}) = [\mathbf{s_1}(\mathbf{t}), \ldots, \mathbf{s_d}(\mathbf{t}), \ldots, \mathbf{s_D}(\mathbf{t})]^{\mathbf{T}}$,
- $\mathbf{N}(\mathbf{t}) = [\mathbf{n_1}(\mathbf{t}), \ldots, \mathbf{n_m}(\mathbf{t}), \ldots, \mathbf{n_M}(\mathbf{t})]^{\mathbf{T}}$ and
- $\Re = [\mathbf{a_1}, \ldots, \mathbf{a_d}, \ldots, \mathbf{a_D}]$,

where

$$\begin{aligned}\mathbf{a_d} &= \boldsymbol{\Upsilon}(\theta_\mathbf{d}, \varphi_\mathbf{d})\mathrm{B}\boldsymbol{\Xi}(\theta_\mathbf{d}, \varphi_\mathbf{d})\mathbf{h}(\gamma_\mathbf{d}, \eta_\mathbf{d}) \\ &= \mathbf{D}(\theta_\mathbf{d}, \varphi_\mathbf{d})\mathbf{h}(\gamma_\mathbf{d}, \eta_\mathbf{d})\end{aligned} \tag{7}$$

is the steering vector of the dth source. Matrix B is associated with the location of the antenna and can be expressed as

$$B = \begin{bmatrix} \sin\alpha_1, \ldots, \sin\alpha_m, \ldots, \sin\alpha_M \\ \cos\alpha_1, \ldots, \cos\alpha_m, \ldots, \cos\alpha_M \end{bmatrix}^T \tag{8}$$

and

$$\boldsymbol{\Upsilon}(\theta_\mathbf{d}, \varphi_\mathbf{d}) = \mathrm{diag}\{\mathbf{e}^{\mathbf{j}\tau_{\mathbf{1d}}}, \ldots, \mathbf{e}^{\mathbf{j}\tau_{\mathbf{md}}}, \ldots .\mathbf{e}^{\mathbf{j}\tau_{\mathbf{Md}}}\}. \tag{9}$$

The superscript T denotes the transpose.

2.1. DOA and Polarization Estimation

We assume that t_l is the sample time and that the random noise vectors, $n(t_l)$, at different sample times are independent. The array output sample covariance matrix is

$$\mathbf{R_x} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{X}(\mathbf{t_l})\mathbf{X}(\mathbf{t_l})^{\mathbf{H}}, \tag{10}$$

where L denotes the number of snapshots. Its eigenvalue decomposition is

$$\mathbf{R_x} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^{\mathbf{H}} = \sum_{\mathbf{i=1}}^{\mathbf{M}}\lambda_\mathbf{i}\mathbf{u_i}\mathbf{u_i^H}, \tag{11}$$

where $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_D, \lambda_{D+1}, \lambda_M\}$, $\lambda_1 > \lambda_2 > \ldots > \lambda_D > \lambda_{D+1} = \ldots = \lambda_M = \sigma^2$. The eigenvectors corresponding to the $M - D$ smallest eigenvalues comprise a noise subspace $\langle\mathbf{N}\rangle = \mathbf{span}\{\mathbf{U_N}\}$. The D largest eigenvalues comprise a signal subspace $\langle\mathbf{S}\rangle = \mathbf{span}\{\mathbf{U_S}\}$. It can be shown that the signal subspace is orthogonal to the noise subspace, that is,

$$span\{\mathbf{U_N}\}\perp span\{\mathbf{U_s}\} \tag{12}$$

Then, an estimate based on the MUSIC algorithm is as follows:

$$(\hat{\theta}, \hat{\varphi}, \hat{\gamma}, \hat{\eta}) = \arg\max_{\theta,\varphi,\gamma,\eta}\frac{1}{\mathbf{a^H}(\theta, \varphi, \gamma, \eta)\mathbf{U_n}\mathbf{U_n^H}\mathbf{a}(\theta, \varphi, \gamma, \eta)} \tag{13}$$

Then, the estimate of the signal parameters $(\theta, \varphi, \gamma, \eta)$ is the location of the extreme value of Equation (13).

3. ARRAY STRUCTURE

In this section, we propose a 2-D search method for 4-D estimation using information on the amplitude. An 8-element uniform circular array of single-polarized antennas is depicted in Fig. 2. We set $[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8] = [90°, 45°, 0°, 135°, 90°, 45°, 0°, 135°]$. Then, the output of this array is given by $\mathbf{X(t)} = [\mathbf{x_1(t)}, \ldots, \mathbf{x_8(t)}]^{\mathbf{T}}$.

When a source impinges on this single-polarization sensor array and the effects of noise are neglected, x_m can be calculated by substituting Equation (2) into Equation (3).

$$\mathbf{x_m} = (\tilde{a}_m(\varphi, \gamma) + \tilde{b}_m(\varphi, \theta, \gamma) e^{j\eta}) e^{j\tau_m} \sqrt{p} e^{j2\pi(c/\lambda)t}, \tag{14}$$

where

$$\tilde{a}_m(\varphi, \gamma) = \cos\alpha_m \cos\varphi \cos\gamma - \sin\alpha_m \sin\varphi \cos\gamma \tag{15}$$

$$\tilde{b}_m(\theta, \varphi, \gamma) = \cos\alpha_m \cos\theta \sin\varphi \cos\gamma + \sin\alpha_m \cos\theta \cos\varphi \sin\gamma \tag{16}$$

and $m = 1, 2, \ldots, 8$. Therefore, $\tilde{\mathbf{a}}_{\mathbf{m}}(\varphi, \gamma)$ and $\tilde{\mathbf{b}}_{\mathbf{m}}(\varphi, \theta, \gamma)$ are real numbers. The amplitude of x_m can be expressed as

$$|x_m| = \sqrt{p} \cdot \sqrt{\tilde{a}_m^2(\varphi, \gamma) + \tilde{b}_m^2(\varphi, \theta, \gamma) + 2\tilde{a}_m(\varphi, \gamma)\tilde{b}_m(\varphi, \theta, \gamma)\cos\eta}, \tag{17}$$

where $|\bullet|$ represents the absolute value.

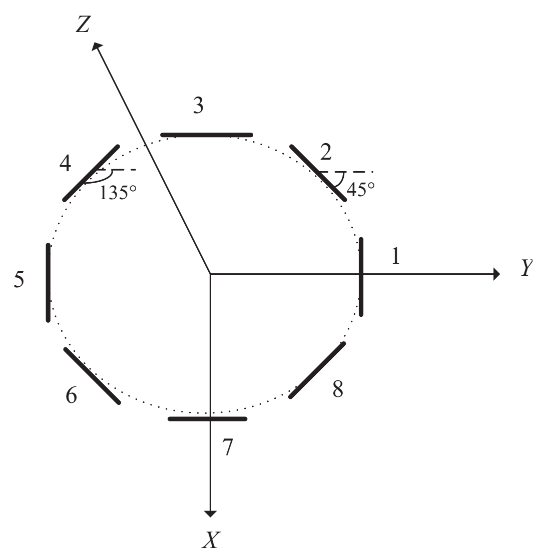

Figure 2. Structure of a UCA with single-polarized antennas.

3.1. Estimating the DOA of an Arbitrary Polarization Signal

If the power of a signal is a constant a priori, we can determine the relation between its amplitude and polarization angle. For simplicity, we set $p = 1$. Then, Equations (18) and (19) can be obtained from Equation (17) easily;

$$|x_1|^2 = (-\sin\varphi\cos\gamma)^2 + (\cos\theta\cos\varphi\sin\gamma)^2 + 2(-\sin\varphi\cos\gamma\cos\theta\cos\varphi\sin\gamma)\cos\eta \tag{18}$$

$$|x_1|^2 + |x_3|^2 = 1 - \sin^2\gamma \sin^2\theta. \tag{19}$$

From Equations (18) and (19), the relation between γ and η can be easily derived;

$$\hat{\gamma} = \sin^{-1}\sqrt{\frac{1 - (|x_1|^2 + |x_3|^2)}{\sin^2\hat{\theta}}} \tag{20}$$

$$\hat{\eta} = \cos^{-1}\frac{|x_1|^2 - (\sin\hat{\varphi}\cos\hat{\gamma})^2 - (\cos\hat{\theta}\sin\hat{\gamma})^2}{-2\sin\hat{\varphi}\cos\hat{\gamma}\cos\hat{\theta}\sin\hat{\gamma}\cos\hat{\varphi}} \tag{21}$$

From Equations (7) and (12), we know that the signal and noise subspaces are orthogonal. From [13], we obtain

$$\begin{aligned} \mathbf{h^H}(\gamma,\eta)\mathbf{D^H}(\theta,\varphi)\mathbf{U_n}\mathbf{U_n^H}\mathbf{D}(\theta,\varphi)\mathbf{h}(\gamma,\eta) = 0 \\ \Rightarrow \mathbf{D^H}(\theta,\varphi)\mathbf{U_n}\mathbf{U_n^H}\mathbf{D}(\theta,\varphi) = 0. \end{aligned} \tag{22}$$

Therefore, Equation (13) can be simplified to

$$\{\hat{\theta},\hat{\varphi}\} = \arg\max_{\theta,\varphi} \frac{1}{\det\{\mathbf{D^H}(\theta,\varphi)\mathbf{U_n}\mathbf{U_n^H}\mathbf{D}(\theta,\varphi)\}} \tag{23}$$

In the conventional polarization MUSIC (P-MUSIC) algorithm, we need to insert the estimated DOA into Equation (13) to obtain γ and η. The proposed method enables the polarization angle to be determined by substituting $\hat{\theta}$ and $\hat{\varphi}$ into Equations (20) and (21). Hence the computational complexity is changed from Δ_s^4 to Δ_s^2 (Δ_s denotes the number of searching points). The algorithm is summarized in Table 1.

Table 1. Algorithm for estimating signals with arbitrary polarization.

Steps in the Proposed Method
1. Signal modeling following Equation (6)
2. Compute $\lvert x_1\rvert$ and $\lvert x_1\rvert^2 + \lvert x_3\rvert^2$
3. Compute R_X using x and Equation (10)
4. Compute the noise subspace, U_n, by eigendecomposition of R_X
5. Find $\hat{\varphi}$ and $\hat{\theta}$ using Equation (23): $0^\circ \leq \varphi \leq 360^\circ$ and $0^\circ \leq \theta \leq 90^\circ$
6. Compute $\hat{\gamma}$ and $\hat{\eta}$ using $\hat{\varphi}$ and $\hat{\theta}$ with Equations (20) and (21)

3.2. Estimating the DOA of a Linearly Polarized Signal

When $\eta = 0$, the signal is linearly polarized. Equation (27) can be simplified to

$$|x_m| = \sqrt{p}\cdot[\tilde{a}_m(\varphi,\gamma) + \tilde{b}_m(\varphi,\theta,\gamma)] \tag{24}$$

Therefore, the ratio of the amplitudes of x_1 and x_2 can be written as

$$T = \frac{|x_1|}{|x_3|} = \frac{-\sin\varphi\cos\gamma + \cos\theta\cos\varphi\sin\gamma}{\cos\varphi\cos\gamma + \cos\theta\sin\varphi\sin\gamma} \tag{25}$$

$$\Rightarrow \theta = \cos^{-1}\frac{\sin\varphi\cos\gamma + T\cos\varphi\cos\gamma}{\cos\varphi\sin\gamma - T\sin\varphi\sin\gamma} \tag{26}$$

Equation (26) expresses angle θ in terms of (T), angle φ and γ. Effectively, from Equation (26), angle θ in the search vector $\mathbf{a}(\theta,\varphi,\gamma)$ of Eq. (13) can be represented only in terms of φ and γ. Therefore, the search dimension of the steering vector $\mathbf{a}(\theta,\varphi,\gamma)$ in the estimation process is reduced to two dimensions, and the power of the signal can be arbitrary in this method. Hence the computational complexity is changed from Δ_s^3 to Δ_s^2. The algorithm is summarized in Table 2.

4. SIMULATION

This section illustrates the feasibility of the proposed method based on an 8-element uniform circular array composed of single-polarized sensors. A far-field source impinges on the array with the following parameter values: $\theta = 40^\circ$, $\varphi = 25^\circ$, $\gamma = 23^\circ$, $\eta = 0^\circ$. That is, the source is linearly polarized. The central frequency of the source is $f_0 = 8\,\text{GHz}$, and $L = 200$ represents the number of snapshots. The search grid used for the two methods is 0.1°. The performance of proposed algorithm is compared to that of the conventional P-MUSIC algorithm. The Cramer-Rao lower bound (CRB) is also used as a benchmark.

Table 2. Algorithm for estimating a linearly polarized signal.

Steps in the Proposed Method
1. Signal modeling following Equation (6)
2. Compute the ratio $T = \frac{\lvert x_1 \rvert}{\lvert x_3 \rvert}$
3. Compute R_X using x and Equation (10)
4. Compute the noise subspace, U_n, by eigendecomposition of R_X
5. Compute $\mathbf{a}(\theta, \varphi, \gamma)$ by substituting Equation (26) into Equation (13)
6. Find $\hat{\varphi}$ and $\hat{\gamma}$ using Equation (13): $0° \leq \varphi \leq 360°$, $0° \leq \gamma \leq 90°$
7. Compute $\hat{\theta}$ using the $\hat{\phi}$ and $\hat{\gamma}$ found using Equation (26)

For comparison, we use the same radius and wave length in two methods and $r/\lambda = 1.5$. Fig. 3 and Fig. 4 show the root mean square error of the DOA estimated using both methods. The results were obtained from 200 independent trials. It is evident from Fig. 3 and Fig. 4 that the proposed method outperforms the P-MUSIC when the SNR is low.

Table 3. Comparison of two methods for accuracy analysis.

Method	**Actual Polarization angle** $(\gamma°, \eta°)$	**Estimated Polarization angle** $(\gamma°, \eta°)$
Proposed	$(23°, 81°)$	$(23.17°, 81.52°)$
P-MUSIC	$(23°, 81°)$	$(23°, 81°)$

Table 4. Comparison of two methods for time-based analysis.

Method	$1°$ **Search interval** (seconds)	$0.1°$ **Search interval** (seconds)
Proposed	0.51	45.47
P-MUSIC	1.11	77.44

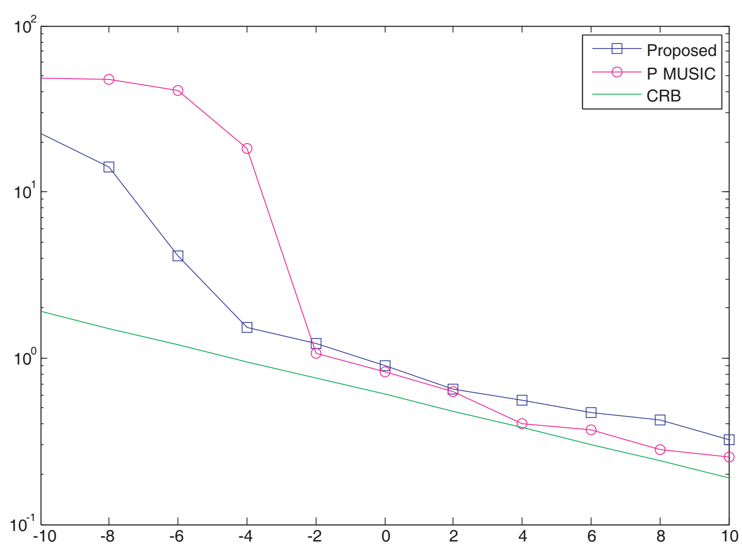

Figure 3. RMSE of the azimuth angle.

When the signal has an arbitrary polarization, we set $\eta = 81°$. Tables 3 and 4 show the the accuracy of the estimate and the computation time, respectively, when the signal has an arbitrary polarization. Noise is considered through an SNR of 30 dB in the modeled data. In Table 4, the decrease in the computation time required by the proposed method is outstanding. Additionally, the proposed method has a lower accuracy, which is presented in Table 3.

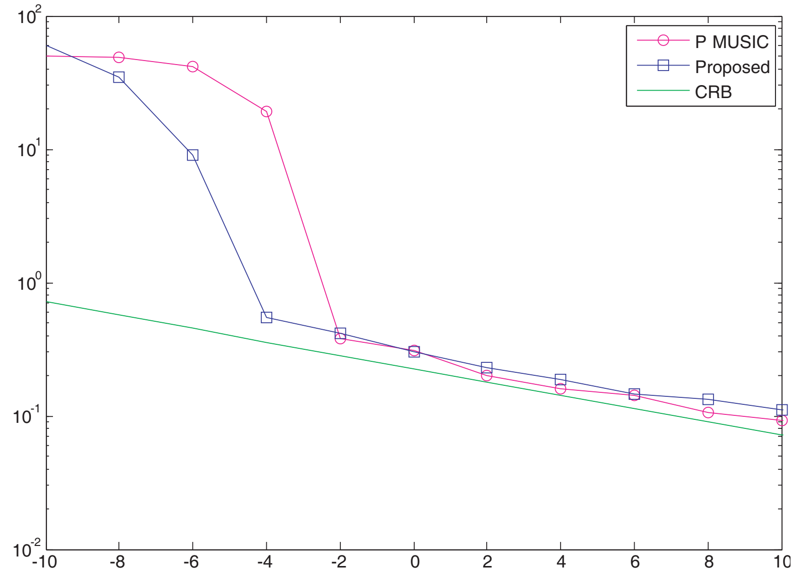

Figure 4. RMSE of the elevation angle.

5. CONCLUSION

This paper has proposed a realistic model for single-polarized arrays. When the source is linearly polarized, the proposed method can estimate the DOA and auxiliary polarization angle based on the amplitude ratio. This method includes an elegant 2-D search technique for 3-D estimation that involves the elevation, azimuth and polarization angles. If the source has an arbitrary polarization, we need to estimate the DOA using the P-MUSIC algorithm first. Then, the polarization angle can be calculated using information on the amplitude. The simulation results show that this method requires less computation time than the P-MUSIC algorithm does for the same number of sensors.

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