Progress In Electromagnetics Research B, Vol. 77, 71–84, 2017

Radiation Fields of a System of Two Impedance Crossed Vibrators Excited In-Phase and Placed over a Rectangular Screen

Nadezhda P. Yeliseyeva, Aleksey N. Gorobets, Victor A. Katrich, and Mikhail V. Nesterenko*

Abstract—An asymptotic solution of a 3D vector diffraction problem for a vibrator system placed over a rectangular perfectly conducting screen of finite dimensions is obtained in the framework of the uniform geometrical theory of diffraction (UGTD) using the asymptotic expressions for the impedance vibrator currents. The system consists of two orthogonally crossed vibrators with equal dimensions but different surface impedances. The vibrators are excited in-phase. An algorithm and respective software for computing the directional, power and polarization characteristics of the radiation field of this antennas system are developed. The conditions required to form a circularly polarized radiation with a maximally attainable directivity in the normal direction to the screen are determined depending on the screen dimensions and the distance between the vibrators and the screen.

1. INTRODUCTION

As is known, two mutually orthogonal symmetric perfectly conducting vibrators excited by currents with equal amplitudes and phases shifted at $\Delta\Phi = \pm\pi/2$, and a turnstile radiator, located above a perfectly conducting square screen of finite dimensions, form a circularly polarized wave in the forward and backward normal directions to the screen at any distance between the vibrators and the screen [1].

A new method of forming a circularly polarized radiation by a system of two crossed vibrators excited in-phase ($\Delta\Phi = 0$) with equal geometric dimensions but different surface impedances and placed parallel to a perfectly conducting infinite plane was proposed in [2]. The required phase shift of the currents was ensured by varying the vibrator impedances. It was shown that the mutually orthogonal crossed vibrators excited in-phase but with different surface reactive impedances allow to form elliptical (circular) polarized field. These antenna structures can be placed, for example, on a top of a car or on a board of an aircraft unit. High frequency techniques for antenna analysis have been developed on the base of the UGTD for an edge in a perfectly conducting surface in [3, 4].

The aim of this paper is to obtain an asymptotic solution of a 3D vector diffraction problem for two mutually orthogonal crossed impedance vibrators excited in-phase and placed over a rectangular perfectly conducting screen of finite dimensions and to find the required conditions of forming a circularly polarized radiation with the maximum achievable directivity in the normal direction to the screen. We will also study spatial amplitude and polarization characteristics, as well as a radiation resistance for the structure under consideration.

2. PROBLEM FORMULATION

Consider a radiating system consisting of two horizontal cylindrical vibrators **1** and **2** located at a distance h above a center O of an infinitesimally thin, perfectly conducting screen of rectangular

Received 25 May 2017, Accepted 12 July 2017, Scheduled 25 July 2017

* Corresponding author: Mikhail V. Nesterenko (mikhail.v.nesterenko@gmail.com).
The authors are with the Department of Radiophysics, Biomedical Electronics and Computer Systems, V. N. Karazin Kharkiv National University, 4, Svobody Sq., Kharkiv 61022, Ukraine.

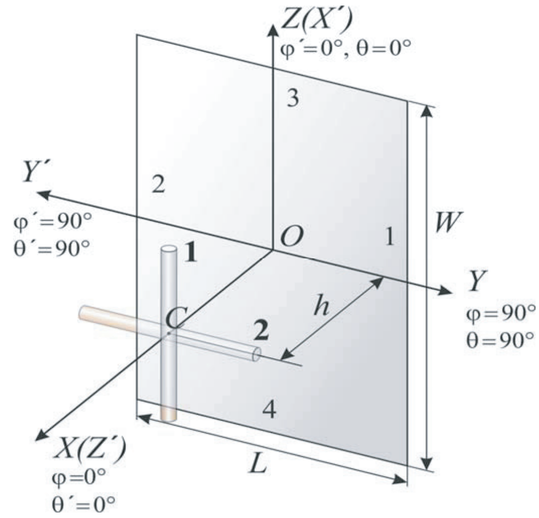

Figure 1. Geometry of the problem.

geometry with side lengths L and W (Fig. 1). The geometrical length of each vibrator is $2l$, and their radii are r. Let us introduce two interrelated the Cartesian frames (X, Y, Z) and (X', Y', Z'), and two spherical frames (R, θ, φ) and (R', θ', φ') with the origin at O.

The vibrators **1** and **2** are characterized by the constant distributed surface impedances Z_{S1} and Z_{S2}. Introduce the vibrator surface impedances, normalized by the wave resistance of the free space, $Z_0 = 120\pi\Omega$,

$$\bar{Z}_{S_{1,2}} = \bar{R}_{S_{1,2}} + j\bar{X}_{S_{1,2}} = 2\pi r z_{i_{1,2}}/Z_0, \tag{1}$$

where j is the imaginary unit, and $z_{i_{1,2}}$ are the intrinsic linear impedances of the vibrators, (Ohm/m). We consider that the vibrator impedance is purely reactive; therefore, we assume $\bar{R}_{S_{1,2}} = 0.001$, because it is necessary to bring small losses. It has been put for physically correct description of electrodynamics process under consideration, because real vibrators always have losses. Examples of concrete realizations of the surface impedances are given in [5, 6].

Consider the case when vibrators **1** and **2** are excited in-phase by a concentrated electromotive force with amplitude V_0 in the vibrator center C. The complex current amplitudes on the thin horizontal impedance vibrators placed over an infinite perfectly conducting plane, $\dot{I}^{Hs}_{0_{1,2}}(kr, \tilde{k}l, kh)$, (the superscript Hs is related to a half-space over the screen) can be obtained as solution of the integral equation by the asymptotic averaging method using a quasi-one-dimensional Green's function [5, 6]. If the current distributions on the vibrators are $\dot{I}_{1,2}(s) = \dot{I}^{Hs}_{0_{1,2}} \sin \tilde{k}_{1,2}(l - |s|)$ (s is a local coordinate along a vibrator), the asymptotic expressions with the accuracy to the first order in the small parameter $\alpha = \frac{1}{2\ln[r/2l]}$ ($|\alpha| \ll 1$) can be written as

$$\dot{I}^{Hs}_{0_{1,2}}\left(kr, \tilde{k}l, kh\right) = \frac{\left(1/\tilde{k}_{1,2}\right)}{\cos \tilde{k}_{1,2}l + \alpha\left\{P^{Fs}_l\left(kr, \tilde{k}_{1,2}l\right) + P^{Mir}_l\left[k(h+r), \tilde{k}_{1,2}l\right]\right\}}. \tag{2}$$

The complex current amplitudes $\dot{I}^{Hs}_{0_{1,2}}(kr, \tilde{k}l, kh)$ include the approximating analytical functions $\dot{P}^{Fs}_l(kr, \tilde{k}_{1,2}l)$ and $\dot{P}^{Mir}_l[k(h + r), \tilde{k}_{1,2}l]$, which determine the vibrator currents in the free space (superscript Fs) and the influence of the field reflected from an infinite perfectly conducting plane (superscript Mir) [5, 6]. The complex numbers $\tilde{k}_{1,2}$ in the above considered formulas can be written as

$$\tilde{k}_{1,2} = \tilde{k}'_{1,2} + j\tilde{k}''_{1,2} = k + j(\alpha/r)\bar{Z}_{S_{1,2}}, \tag{3}$$

where $k = 2\pi/\lambda$ and λ is the wavelength in the free space. Analyzing formula (3), we can state that the electrical lengths of the vibrators $\tilde{k}'_{1,2}l$ are varied by changing the reactive parts $\bar{X}_{S_{1,2}}$ of the vibrator surface impedances $\bar{Z}_{S_{1,2}}$ in Eq. (1), and hence, the amplitudes and phases of the currents of the vibrators **1** and **2** in Eq. (2) are varied too.

In a primary diffraction approximation of the UGTD [1, 6], the total radiation fields of each of the crossed vibrators **1** and **2**, $\dot{E}_{1,2}(\theta,\varphi)$, can be represented at any observation point (θ,φ) as a sum of two discontinuous geometric optical (GO) fields, $\dot{E}_i(\theta,\varphi)$, of the spherical waves incident ($i=1$) from the vibrators and reflected from the screen ($i=2$), plus the fields of the singly diffracted waves, $\dot{E}_{in}(\theta,\varphi)$, excited by the incident and reflected waves at each of the four screen edges ($n=1,2,3,4$) (Fig. 1), viz.

$$\begin{aligned}\dot{E}_{\theta_{1,2}}(\theta,\varphi)&=\sum_{i=1}^{2}\dot{E}_{\theta i}\chi_i+\sum_{n=1}^{4}\sum_{i=1}^{2}\dot{E}_{\theta in}\chi_{in},\\ \dot{E}_{\varphi_{1,2}}(\theta,\varphi)&=\sum_{i=1}^{2}\dot{E}_{\theta i}\chi_i+\sum_{n=1}^{4}\sum_{i=1}^{2}\dot{E}_{\varphi in}\chi_{in}.\end{aligned}\tag{4}$$

Here the coefficients χ_i are equal to unity in the illuminated area and turn to zero in the shadow zone of the respective GO wave ($i=1,2$). The coefficients χ_{in} are equal to unity in the illuminated area and become zeroes in the shadow zone of the wave diffracted at edge n. To conveniently represent the equations of the light-shadow boundary for the sources of GO waves and edge-diffracted waves, we will analyze the radiated field patterns on the surface of a sphere of infinite radius, with the center at the origin of a Cartesian frame XYZ (R, θ, φ) the point O placed (Fig. 1). The equations of the light-shadow boundaries in the angular space (θ, φ) can be found in [1, 7], and the expressions for the GO and diffracted fields of the impedance vibrator are given in [6]. The amplitudes of the orthogonal field components in Eq. (4) of the two crossed vibrators can be determined by summing the fields induced by vibrators **1** and **2** independently. Therefore, we can write

$$\dot{E}_\theta=\dot{E}_{\theta_1}+\dot{E}_{\theta_2},\quad \dot{E}_\varphi=\dot{E}_{\varphi_1}+\dot{E}_{\varphi_2}.\tag{5a}$$

Because mutual coupling between two mutually orthogonal vibrators is absent, the square modulus of the total electric field are defined as

$$\left|\dot{E}(\theta,\varphi)\right|^2=\left|\dot{E}_{\theta_1}(\theta,\varphi)\right|^2+\left|\dot{E}_{\theta_2}(\theta,\varphi)\right|^2+\left|\dot{E}_{\varphi_1}(\theta,\varphi)\right|^2+\left|\dot{E}_{\varphi_2}(\theta,\varphi)\right|^2.\tag{5b}$$

In the next section, we will obtain the expressions for the orthogonal field components of two orthogonal impedance vibrators excited in-phase in the free space.

3. RADIATION FIELDS OF ORTHOGONAL IMPEDANCE VIBRATORS IN FREE SPACE

In the frame R, θ, φ (Fig. 1) the complex amplitude of the electric field vector of the impedance vibrator **1** oriented along the Z axis in the far zone can be written [6] as

$$\dot{E}_{\theta 1}(\theta,\varphi)=\dot{E}_0\dot{I}_{0_1}^{Fs}\dot{F}_1(\theta).\tag{6}$$

The field of vibrator **2** oriented along the Y axis has two components:

$$\begin{aligned}\dot{E}_{\theta 2}(\theta,\varphi)&=-\dot{E}_0\dot{I}_{0_2}^{Fs}\dot{F}_2(\theta,\varphi)\sin\varphi\cos\theta,\\ \dot{E}_{\varphi 2}(\theta,\varphi)&=-\dot{E}_0\dot{I}_{0_2}^{Fs}\dot{F}_2(\theta,\varphi)\cos\varphi.\end{aligned}\tag{7}$$

In formulas (6) and (7) $\dot{E}_0=j\frac{Z_0}{2\pi}\frac{\exp(-jkR)}{R}$, $\dot{I}_{0_{1,2}}^{Fs}$ are the complex amplitudes of the currents on the impedance vibrators in the free space, derived [5, 6] as

$$\dot{I}_{0_{1,2}}^{Fs}\left(kr,\tilde{k}l\right)=\frac{\left(1/\tilde{k}_{1,2}\right)}{\cos\tilde{k}_{1,2}l+\alpha\dot{P}_l^{Fs}\left(kr,\tilde{k}_{1,2}l\right)}.\tag{8}$$

The functions $\dot{F}_1(\theta)$ and $\dot{F}_2(\theta,\varphi)$ are defined as

$$\dot{F}_1(\theta)=k\tilde{k}_1\sin\theta\frac{\cos\tilde{k}_1l-\cos(kl\cos\theta)}{(k\cos\theta)^2-\tilde{k}_1^2},\quad \dot{F}_2(\theta,\varphi)=k\tilde{k}_2\frac{\cos\tilde{k}_2l-\cos(kl\sin\theta\sin\varphi)}{(k\sin\theta\sin\varphi)^2-\tilde{k}_2^2}\tag{9}$$

and $\tilde{k}_1$ and $\tilde{k}_2$ are expressed by Eq. (3) after substitution of the vibrators impedances $\bar{X}_{S_1}$ and $\bar{X}_{S_2}$ in the free space.

The complex amplitudes of the orthogonal components of the field produced by the system of the crossed vibrators exited in-phase in the free space can be written using the expressions (5)–(9) as

$$\dot{E}_\theta^{Fs} = \dot{E}_0 \left(\dot{I}_{01}^{Fs} \dot{F}_1(\theta) - \dot{I}_{02}^{Fs} \dot{F}_2(\theta, \varphi) \sin\varphi \cos\theta \right), \quad \dot{E}_\varphi^{Fs} = -\dot{E}_0 \dot{I}_{02}^{Fs} \dot{F}_2(\theta, \varphi) \cos\varphi. \tag{10}$$

In the normal direction to the vibrators axes ($\varphi = 0°$ and $\varphi = 180°$, $\theta = 90°$, Fig. 1), the complex amplitudes of the orthogonal field components after formulas (10) are equal to

$$\dot{E}_\theta^{Fs} = \dot{E}_0 \dot{I}_{01}^{Fs} k \frac{1 - \cos\tilde{k}_1 l}{\tilde{k}_1}, \quad \dot{E}_\varphi^{Fs} = -\dot{E}_0 \dot{I}_{02}^{Fs} k \frac{1 - \cos\tilde{k}_2 l}{\tilde{k}_2}. \tag{11a}$$

As is known, the wave polarization is defined by the ratio of the orthogonal field components with the phase shift $\psi = \Phi_\varphi - \Phi_\theta$, viz.

$$\dot{p}^{Fs} = \left| \frac{\dot{E}_\varphi^{Fs}}{\dot{E}_\theta^{Fs}} \right| \exp\left(j \left(\Phi_\varphi^{Fs} - \Phi_\theta^{Fs} \right) \right). \tag{11b}$$

So, the circular polarization (the superscript cp) in the normal direction to the vibrators axes can be formed by choosing the impedances $\bar{X}_{S_1} = \bar{X}_{S_1}^{cp}$ and $\bar{X}_{S_2} = \bar{X}_{S_2}^{cp}$ of the respective vibrators, at which the phase shift between the orthogonal field components is equal to $\Phi_\varphi^{Fs} - \Phi_\theta^{Fs} \approx \pm 90^\circ$. In the other directions, an elliptically-polarized wave is formed, which is transformed into a linearly polarized wave in the vibrators plane ($\varphi = 90°$ and $\varphi = 270°$, Fig. 1).

The radiation patterns (RPs) for the field components presented in coordinates θ, φ are calculated in coordinates θ', φ' with account of the interrelation of the coordinates systems (Fig. 1). In coordinates θ', φ' the electric field vector can be represented as [1]

$$\begin{aligned} \vec{\dot{E}}_{\theta'}(\theta', \varphi') &= [\dot{E}_\theta(\theta, \varphi) C_{22} + \dot{E}_\varphi(\theta, \varphi) C_{23}] \vec{\theta}', \\ \vec{\dot{E}}_{\varphi'}(\theta', \varphi') &= [\dot{E}_\theta(\theta, \varphi) C_{32} + \dot{E}_\varphi(\theta, \varphi) C_{33}] \vec{\varphi}', \end{aligned} \tag{12}$$

where $C_{22} = C_{33} = -\cos\theta \cos\varphi / D$, $C_{23} = \sin\varphi / D$, $C_{32} = -C_{23}$, $D = \sqrt{1 - \sin^2\theta \cos^2\varphi}$. If $\varphi = 0°$, we have $C_{22} = -1$, $C_{23} = 0$. All numerical results in the paper are presented in coordinates θ', φ'.

As is known, the vector of the total electric field circumscribes an ellipse with axes $2a$ and $2b$; therefore, a vector position (polarization state) can be defined by the ellipse parameters. An equation for determining the slope angle β of the major ellipse axis is defined by the following formula [8]

$$\mathrm{tg} 2\beta = \frac{2|\dot{p}|}{1 - |\dot{p}|^2} \cos\Psi, \tag{13}$$

hence

$$\beta(\theta', \varphi') = \frac{1}{2} \mathrm{arctg} \frac{2|\dot{p}|}{1 - |\dot{p}|^2} \cos\Psi. \tag{14}$$

Let us define an auxiliary angle γ ($-\pi/4 < \gamma < \pi/4$) such that

$$\pm b/a = \mathrm{tg}\gamma \tag{15}$$

where a and b are the lengths of the polarization ellipse semi-axes. In this way, the magnitude of $\mathrm{tg}\gamma$ determines the ellipticity σ of the polarization ellipse, while its sign defines clockwise or counterclockwise rotation of the electric vector. The equation for γ is derived in [8]

$$\sin 2\gamma = (\sin 2\varepsilon) \sin\psi, \tag{16}$$

where the auxiliary angle ε is defined as

$$\mathrm{tg}\varepsilon = |\dot{p}|. \tag{17}$$

Using expression (15) and taking into account Eqs. (16)–(17), the ellipticity can be written as

$$\sigma(\theta', \varphi') = \mathrm{tg}\left[\frac{1}{2} \arcsin \frac{2|\dot{p}| \cdot \sin\Psi}{1 + |\dot{p}|^2} \right]. \tag{18}$$

If $\sin\Psi > 0$ $(0 < \gamma < \pi/4)$, the polarization is right-handed; otherwise, if $\sin\Psi < 0$ $(-\pi/4 < \gamma < 0)$ the polarization is left-handed.

Let us now determine the surface impedances $\bar{X}_{S_1}^{cp}$ and $\bar{X}_{S_2}^{cp}$ for the two crossed vibrators excited in-phase in the free space, at which the circularly polarized radiation can be formed in the normal direction to vibrators axes $(\theta' = 0,\ \varphi' = 0)$ at the wavelength $\lambda = 30\,\text{mm}$. The length and radius of the vibrators are $2l = 0.5\lambda$ and $r = 0.1\,\text{mm}$. The orthogonal field components $\dot{E}_{\theta}^{Fs}$ and $\dot{E}_{\varphi}^{Fs}$ are calculated after formulas (11)–(12) by varying the surface impedances $\bar{X}_{S_{1,2}}$ in the limits from -0.1 to 0.1 with a step of 0.001. Then the ellipticity $\sigma(0,0)$ and angle $\beta(0,0)$ of the polarization ellipse in the normal direction to vibrators axes $(\theta' = 0,\ \varphi' = 0)$ in the free space are calculated after formulas (14) and (18).

Figure 2(a) shows the equal-value contour of $\sigma_0(0,0)$ plotted in coordinates $\bar{X}_{S_1}$ and $\bar{X}_{S_2}$. From the data array of Fig. 2(a), we define the maximal values of $|\sigma_{cp}(0,0)|$ and the impedances $\bar{X}_{S_1}^{cp}$ and $\bar{X}_{S_2}^{cp}$ such that the maximum is attained. The extreme values σ_{cp} and corresponding impedances, electric lengths and complex current amplitudes of the crossed vibrators **1** and **2** in the free space are shown in Table 1.

Table 1. The extreme parameters of the vibrators **1** and **2** in the free space.

σ_{cp}	$\bar{X}_{S_1}^{cp}$	$\bar{X}_{S_2}^{cp}$	$\tilde{k}_1' l$	$\tilde{k}_2' l$	$\dot{I}_{0_1}^{Fs}$	$\dot{I}_{0_2}^{Fs}$
-0.99	0.012	-0.041	0.52π	0.4π	$(-19.4; -17.4)$	$(20.7; -23.06)$
0.99	-0.041	0.012	0.4π	0.52π	$(20.7; -23.06)$	$(-19.4; -17.4)$.

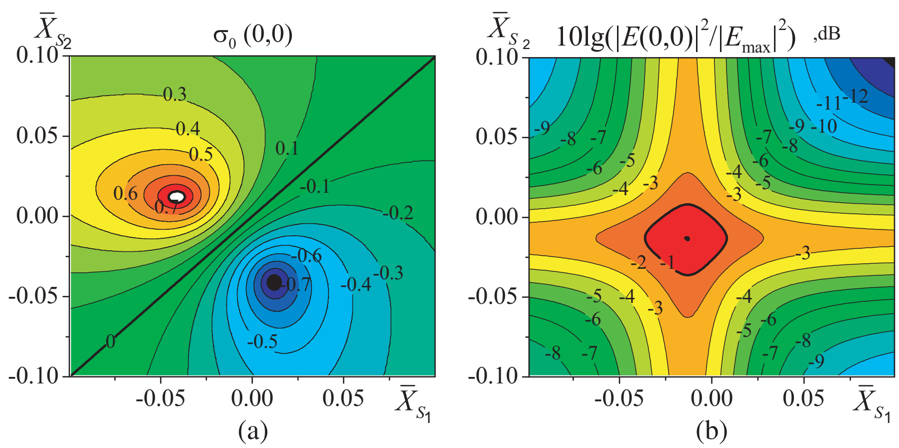

Figure 2. Equal-value contours of the radiation characteristics in the normal direction to the vibrator axes in the free space in coordinates $\bar{X}_{S_1}$ and $\bar{X}_{S_2}$.

As can be seen, the direction of the electric vector rotation can be changed by mutual interchange of the vibrator impedances.

Figure 2(b) shows the equal-value contour in decibels of the square modulus of the total electric field in the normal direction to the vibrator axes $|\dot{E}^{Fs}(0,0)|^2$ calculated after formula (5) and then normalized to maximum value $|\dot{E}_{\max}|^2 = |\dot{E}_{\max}^{Fs}(0,0)|^2$, viz. $10\lg|E(0,0)|^2/|E|^2_{\max}$.

The field maximum $|\dot{E}_{\max}^{Fs}(0,0)|^2$ is reached when the vibrator impedances are resonant. From the data array of Fig. 2(b) we define the resonant impedances $\bar{X}_{Sres1}^{Fs} = \bar{X}_{Sres2}^{Fs} = -0.013$. The same value is obtained for a single impedance vibrator in the free space (Fig. 4(a), curve 4).

The spatial RPs of the orthogonal field components $(20\lg(|\dot{E}_{\theta,\varphi}|/|\dot{E}_{\theta,\varphi\,\max}|)$ (Figs. 3(a), (b)) and polarization characteristics σ_0 and β_0 (Figs. 3(d), (e)) of the two crossed half-wave vibrators with the impedances $\bar{X}_{S_1}^{cp} = 0.012$ and $\bar{X}_{S_2}^{cp} = -0.041$ in the free space (Table 1) are plotted in coordinates θ', φ'. In this frame the forward and backward normal to the vibrators axes has respective coordinates

$\theta' = 0°$ and $\theta' = 180°$. As can be seen from Figs. 3(d), (e), the polarazation varies from the circular in the normal direction to the vibrators axes to the linear in the vibrator plane, $\theta' = 90°$, where the field has only the E'_φ-component. The direction of the electric vector rotation varies in the angular sector $85° < \theta' < 100°$.

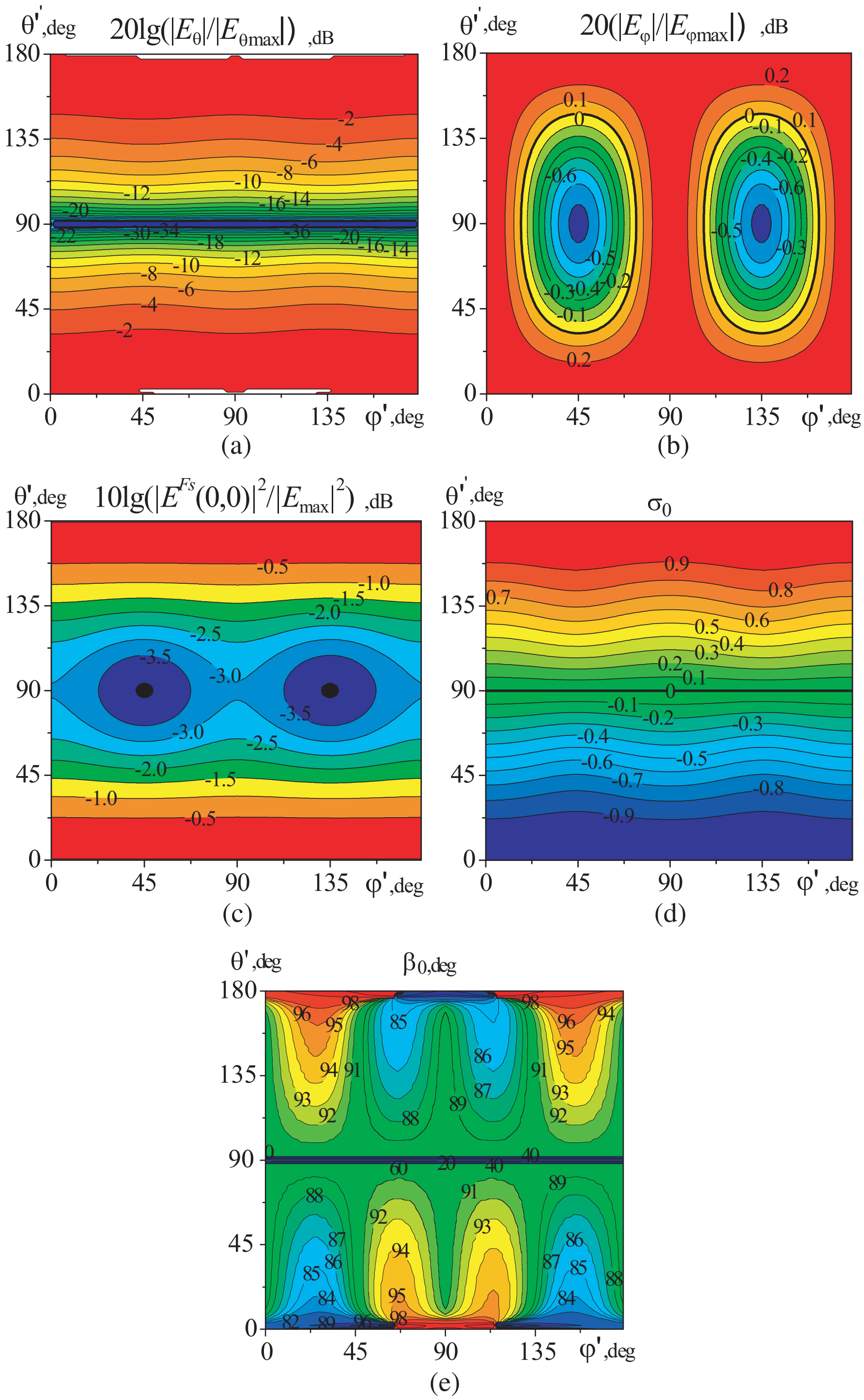

Figure 3. The spatial RPs of the two crossed vibrators ($\bar{X}^{cp}_{S_1} = 0.012$, $\bar{X}^{cp}_{S_2} = -0.041$) in the free space in coordinates θ', φ': (a), (b) field RPs, (c) power RP, (d), (e) polarization RPs.

The power RP $(10\lg(|\dot{E}^{Fs}|^2/|\dot{E}_{\max}|^2)$ (Fig. 3(c)) is defined by summing the squares of the E'_φ and $-E'_\theta$ field components in Eqs. (9)–(12) (Figs. 3(a), (b)), that is why zeros radiation levels are absent in the entire space (including vibrators plane $\theta' = 90°$, where the field is determined by the E'_φ-component).

4. SYNTHESIS OF CIRCULARLY POLARIZED RADIATION BY THE SYSTEM OF THE CROSSED VIBRATORS EXCITED IN-PHASE AND PLACED OVER A SQUARE SCREEN

In this section, we consider a system of crossed vibrators excited in-phase and placed at the different distances $h/\lambda = 0.1 \div 1$ over the middle of the square perfectly conducting screen with different side dimensions $L = W = (0.5 \div 2)\lambda$ (Fig. 1). For comparison, we also consider the vibrator system placed over an infinite perfectly conducting screen. If the vibrators are placed over an infinite perfectly conducting plane at the distance h, the RPs of the orthogonal field components, $\dot{f}_{\theta',\varphi'}$, in coordinates θ', φ' for the normal direction to the screen (defined by the angles $\theta' = 0$, $\varphi' = 0°$) can be written [9] as

$$\text{vibrator}\mathbf{1} \quad \dot{f}_{\varphi'1}(0,0) = 2j\sin(kh)\dot{F}_1, \quad \dot{f}_{\theta'1} = 0, \tag{19a}$$

$$\text{vibrator}\mathbf{2} \quad \dot{f}_{\theta'2}(0,0) = 2j\sin(kh)\dot{F}_2, \quad \dot{f}_{\varphi'2} = 0, \tag{19b}$$

where $\dot{F}_{1,2}=k\frac{1-\cos\tilde{k}_{1,2}l}{\tilde{k}_{1,2}}$. The orthogonal components of the electric field of the vibrator system placed over an infinite screen can be presented as

$$\dot{E}_{\varphi_1}(0,0) = \dot{E}_0\dot{f}_{\varphi'_1}(0,0)\dot{I}_{01}^{Hs}, \quad \dot{E}_{\theta_2}(0,0) = \dot{E}_0\dot{f}_{\theta'_2}(0,0)\dot{I}_{02}^{Hs}, \tag{20}$$

where the current amplitudes $\dot{I}_{0_{1,2}}^{Hs}$ are defined by expressions (2) and (3).

If the vibrator system is placed over the rectangular screen, the RPs of the orthogonal electric field components can be determined by taking into account the diffracted fields from the four screen edges using formulas (4), (5a) and (12). The calculations were carried out for the both configurations.

First of all, consider the resonant properties of the single impedance vibrator placed over the screen. As shown in [5, 6], the vibrator of fixed geometrical sizes located over the screen at the distance h can be tuned to resonance by using the resonant surface impedance $\bar{X}_{Sres}^{Hs}$. Note that there are several criteria for determination of the $\bar{X}_{Sres}^{Hs}$-values: from the condition that the reactive part of input impedance X_{in} equals zero, and from the definition of the resonant field square in the normal direction to the screen $|\dot{E}_{res}(0,0)|^2$ using formulas (4), (5), (12). Fig. 4(a) shows the plots of the resonant values of

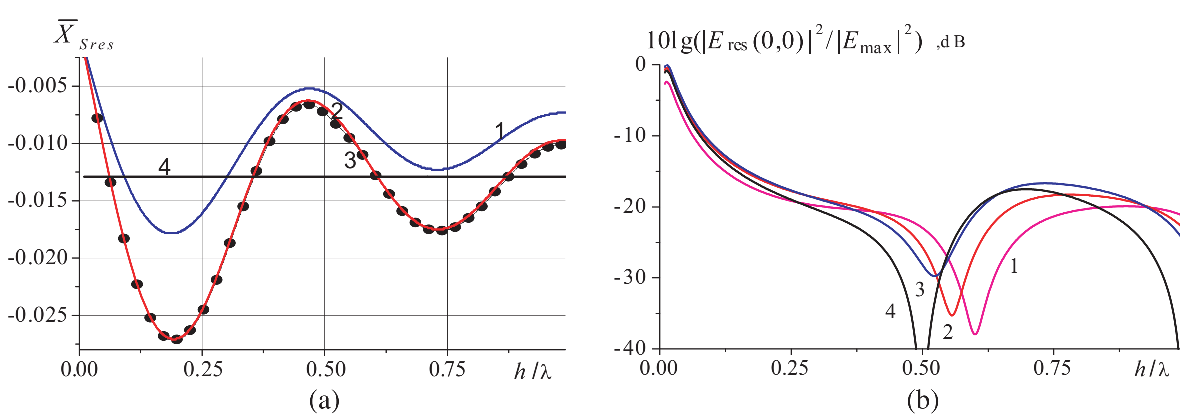

Figure 4. Resonant impedance $\bar{X}_{Sres}^{Hs}$ of the vibrator vs the ratio h/λ: (a) 1 — for the infinite screen (approximate formula); 2 — for the infinite screen and 3 — for the square screen $L/\lambda = 0.8 \ldots 1.2$ (calculation from the field maximum); 4 — for the vibrator in the free space; (b) square modulus of resonant field for the screen $L/\lambda = 0.6$, 0.9, 1.2 and for infinite plane (curves 1, 2, 3 and 4 accordingly) vs h/λ.

the surface impedance $\bar{X}_{Sres}^{Hs}$ as functions of distances h between the single half-wave vibrator with ratio $l/r = 75$ and the screen with various dimensions. There are presented plots for the vibrator over an infinite screen, obtained by using the approximate formula derived from condition $X_{in} = 0$ [5, 6] (curve 1); for the vibrator over an infinite screen, obtained from calculating the field maximum (curve 2); for the vibrator over the square screen with dimensions $L/\lambda = 0.8,\ 0.9,\ 1,\ 1.2$ ($\bar{X}_{Sres}^{Hs}$-values coincide with curve 3). For comparison the resonant impedance of the vibrator in the free space $\bar{X}_{Sres}^{Fs} = -0.013$ (Fig. 4(a), curve 4) is presented here. As can be seen, curves 2 and 3 coincide within graphical accuracy, i.e., the screen dimensions of the wavelength order practically do not affect the values of the vibrator resonant impedance, but the curve 1 differs substantially

In Fig. 4(b), the dependences of the normalized square of the resonant field modulus in decibels, viz. $10\lg|E_{res}(0,0)|^2/|E|^2_{\max}$, on the distance h/λ in the cases of the square screen with $L/\lambda = 0.6,\ 0.9,\ 1.2$ and for infinite plane (curves 1, 2, 3 and 4 accordingly) are shown. The $|E_{res}(0,0)|^2$-values for each distance h are reached at the resonant impedances $\bar{X}_{Sres}^{Hs}$, shown in Fig. 4(a) (curves 2 and 3).

Let us pass to synthesis of the circularly polarized radiation by the crossed vibrators exited in-phase and placed over the screen. We have developed the fast active algorithms and created the software to define the crossed vibrators impedances $\bar{X}_{S_1}^{cp}$ and $\bar{X}_{S_2}^{cp}$, at which a circularly polarized wave with the ellipticity $|\sigma(0,0)| \approx 1$ forms in the normal direction to the screen. Figs. 5(a), (b) show the appropriate impedances $\bar{X}_{S_1}^{cp}$ and $\bar{X}_{S_2}^{cp}$ for the crossed half-wave vibrators excited in-phase, which ensure $|\sigma(0,0)| \approx 1$ in the case of the screen with $L = 1.2\lambda$ versus the ratio h/λ. In Fig. 5(c), the dependences of $10\lg|E_{cp}(0,0)|^2/|E|^2_{\max}$ are shown on h/λ in the cases of exciting the screen with $L = 1.2\lambda$ by the

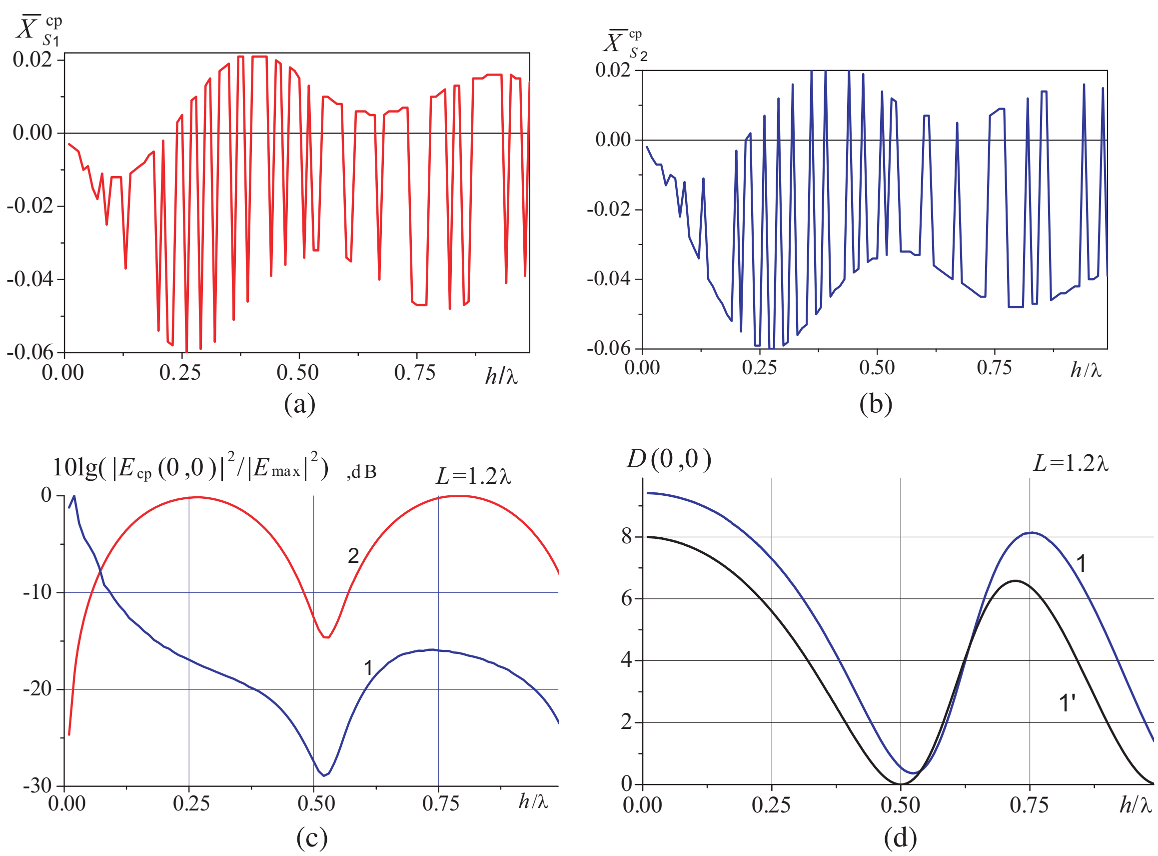

Figure 5. Vibrator impedances (a) $\bar{X}_{S_1}^{cp}$ and (b) $\bar{X}_{S_2}^{cp}$, at which the ellipticity $|\sigma(0,0)| \approx 1$, vs h/λ; (c) square modulus of circularly polarized field in cases of exciting the screen $L = 1.2\lambda$ by the crossed vibrators with the impedances $\bar{X}_{S_{1,2}}^{cp}$ (curve 1) and by the turnstile antenna (curve 2) vs h/λ; (d) directivity factor $D(0,0)$ vs h/λ: screen $L = 1.2\lambda$ (curve 1), infinite screen (curve 2).

crossed vibrators with the impedances $\bar{X}^{cp}_{S_{1,2}}$ (curve 1) and using the turnstile antenna ($\bar{Z}_{S_{1,2}} = 0$, curve 2).

From comparing curve 1 (Fig. 5(c)) with curve 3 (Fig. 4(b)) we can state the equality of the normalized square modulus of circularly polarized field $|\dot{E}_{cp}(0,0)|^2$ and resonant field $|\dot{E}_{res}(0,0)|^2$ in the direction normal to the screen $L = 1.2\lambda$, excited by two crossed orthogonal vibrators and by the single vibrator respectively. From this fact follows the equality of the directivity factor $D(0,0)$ for the both radiating systems (Fig. 5(d)). In [10], it was shown that the maximum directivity factor $D(0,0)$ was formed for the impedance vibrator, as well as for the perfectly conducting vibrator, removed from the screen on $h = 0.25\lambda$, in the case of the optimal screen dimensions $L = W = (1.15 \div 1.2)\lambda$, $D_{\max}(0,0) = 7.29$.

Figure 6(a) shows equal-value contour of the ellipticity in the normal direction to the screen $\sigma(0,0)$ calculated by the formula (18) and plotted in coordinates $\bar{X}_{S_1}$ and $\bar{X}_{S_2}$ in the case of the screen with $L = 1.2\lambda$ and $h = 0.25\lambda$. From the data array of $\sigma(0,0)$ we find $\sigma_{cp}(0,0)$ and the corresponding $\bar{X}^{cp}_{S_1}$ and $\bar{X}^{cp}_{S_2}$. In given case the circularly polarized radiation along the screen normal is formed when $\sigma_{cp} = -0.99$ ($\bar{X}^{cp}_{S_1} = 0.005$, $\bar{X}^{cp}_{S_2} = -0.059$) and $\sigma_{cp} = 0.99$ ($\bar{X}^{cp}_{S_1} = -0.059$, $\bar{X}^{cp}_{S_2} = 0.005$). Fig. 6(b) presents equal-value contour of the corresponding square of the field modulus in decibels plotted in coordinates $\bar{X}_{S_1}$ and $\bar{X}_{S_2}$. The field maximum, $|E_{\max}(0,0)|^2$, is achieved if the resonant values of impedances $\bar{X}^{Hs}_{Sres_1}$ and $\bar{X}^{Hs}_{Sres_2}$ are equal to -0.025, which coincide with the resonant impedance of the single vibrator with the screen ($L = 1.2\lambda$ and $h = 0.25\lambda$ (Fig. 4(a), curve 3)).

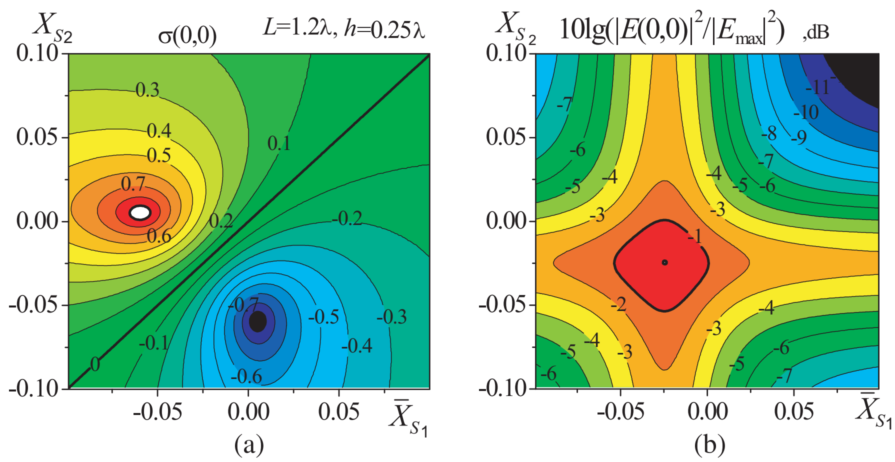

Figure 6. Equal-value contours (a) of the ellipticity and (b) of the corresponding square of the field modulus in the direction normal to the screen $L = 1.2\lambda$ at $h = 0.25\lambda$ plotted in coordinates $\bar{X}_{S_1}$ and $\bar{X}_{S_2}$.

To define the conditions for the formation of the circular polarized radiation with the maximal achievable directivity in the normal direction to the screen, we develop an algorithm and create the respective software for computing the directivity factor $D(0,0)$ and the radiation resistance $R_{\Sigma cp}$ after the formulas

$$D(0,0) = 120\frac{|E(0,0)|^2}{R_\Sigma}, \quad R_\Sigma = 30 I_\Sigma / (\left|I_0^{Hs}\right|^2 \pi). \tag{21}$$

Here $I_\Sigma = \int\limits_0^{2\pi} d\varphi \int\limits_0^{\pi} E^2(\theta',\varphi')\sin\theta' d\theta'$, $|E|^2(\theta',\varphi') = |E_\theta|^2(\theta',\varphi') + |E_\varphi|^2(\theta',\varphi')$.

Let us analyze the parameters of the optimal radiating system: the impedances $\bar{X}^{cp}_{S_1}$ and $\bar{X}^{cp}_{S_2}$ of the orthogonal vibrators, the screen dimension L_{opt}/λ, and the distance between the vibrator and screen h_{opt}/λ. Figs. 7(a), (b) show the equal-value contours of the impedances $\bar{X}^{cp}_{S_1}$ and $\bar{X}^{cp}_{S_2}$ of the crossed vibrators at which $|\sigma(0,0)| \approx 1$, plotted in coordinates L/λ and h/λ. When the impedances $\bar{X}^{cp}_{S_{1,2}}(L,h)$

and the currents $\dot{I}^{Hs}_{01,2}(h)$ were determined, the equal-value contours of the square module of the field in the normal direction to the screen (Fig. 7(c)), of the directivity factor $D(0,0)$ (Fig. 7(d)) and of the radiation resistance $R_{\Sigma cp}$ (Fig. 7(e)) were plotted in the coordinates L/λ and h/λ.

The maximal values of $D_{\max}(0,0)$ and $R_{\Sigma\max}$ are determined by analyzing the data array of Figs. 7(d), (e). These values and corresponding optimal values L_{opt}/λ and h_{opt}/λ are given in Table 2.

The extreme values of $|\sigma_{cp}|$, the corresponding impedances $\bar{X}^{cp}_{S_1}$ and $\bar{X}^{cp}_{S_2}$, electrical lengths and current amplitudes of the vibrators **1** and **2** for the optimal circular polarized radiating system ($L_{opt} = 1.2\lambda$ and $h_{opt} = 0.25\lambda$) are given in Table 3.

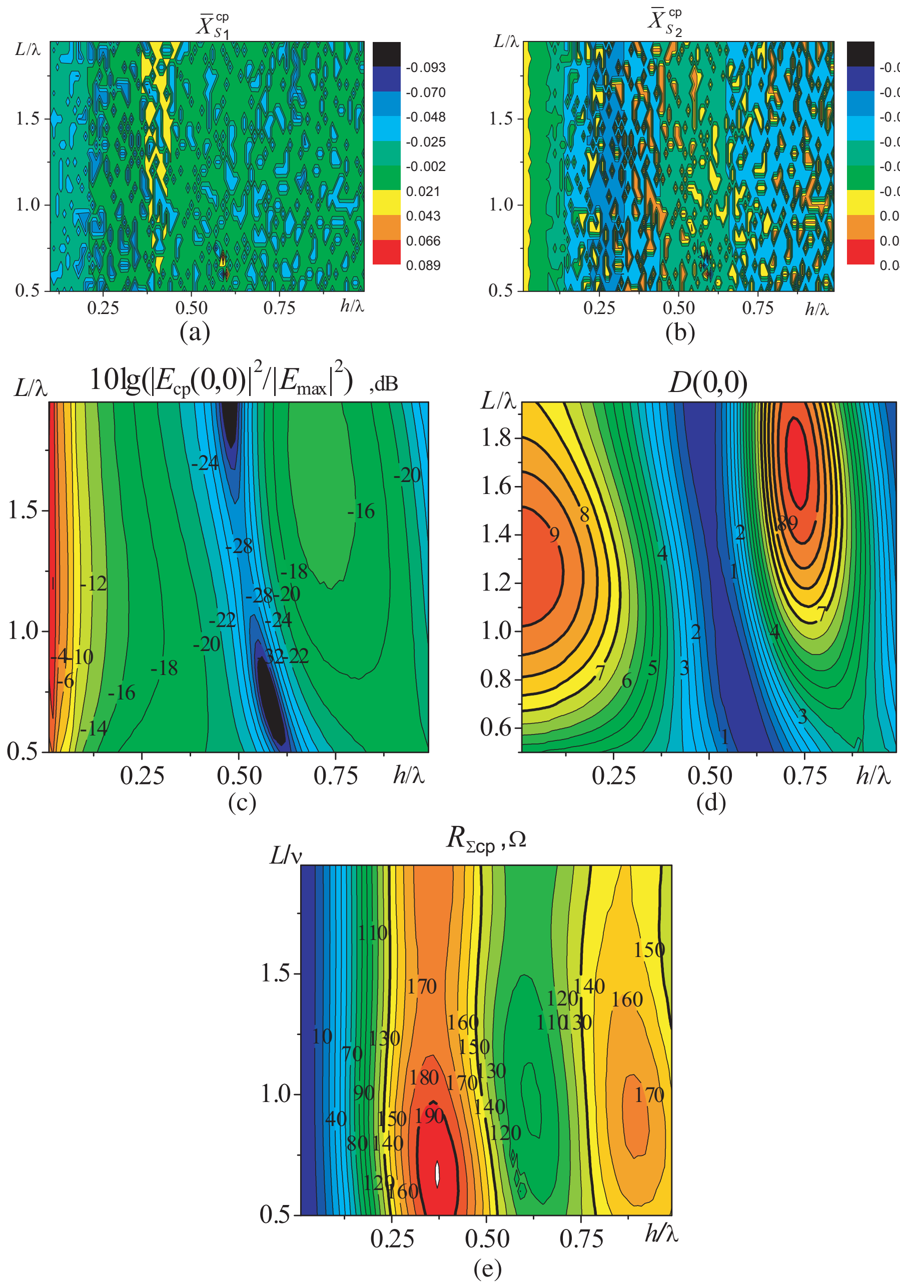

Figure 7. Equal-value contours of the parameters for the crossed vibrators in the coordinates L/λ andh/λ: (a) impedance $\bar{X}^{cp}_{S_1}$; (b) impedance $\bar{X}^{cp}_{S_2}$; (c) square module of field in dB; (d) directivity; (e) radiation resistance.

Table 2. The maximal values of $D_{\max}(0,0)$, $R_{\Sigma\max}$ and corresponding values of L_{opt}/λ and h_{opt}/λ.

$D(0,0)$	$R_{\Sigma cp}$, Ohm	L_{opt}/λ	h_{opt}/λ
$(5.1 \div 8.1)$	$(202 \div 120)$	$(0.85 \div 1.25)$	$(0.34 \div 0.2)$
$(8 \div 9.6)$	$(127 \div 140)$	$(1.25 \div 1.75)$	$0.71 \div 0.75)$
$(4.1 \div 5.7)$	$(208 \div 180)$	$(0.7 \div 1.25)$	$(0.37 \div 0.32)$
7.3	152	1.2	0.25

Table 3. The extreme parameters for optimal radiating system.

σ_{cp}	$\bar{X}_{S_1}^{cp}$	$\bar{X}_{S_2}^{cp}$	$\tilde{k}_1' l$	$\tilde{k}_2' l$	$\dot{I}_{cp1}^{Hs}$	$\dot{I}_{cp2}^{Hs}$
-0.99	0.005	-0.059	0.512π	0.359π	$(-16.45; -16.23)$	$(20.35; -20.79)$
0.99	-0.059	0.005	0.359π	0.512π	$(20.35; -20.79)$	$(-16.45; -16.23)$

5. DIRECTIONAL AND POLARIZATION RADIATION CHARACTERISTICS OF THE OPTIMAL RADIATING SYSTEM

Let us now consider the directional and polarization radiation characteristics of the optimal circular polarized radiating system ($\bar{X}_{S_1}^{cp} = 0.005$, $\bar{X}_{S_2}^{cp} = -0.059$, $L_{opt} = 1.2\lambda$ and $h_{opt} = 0.25\lambda$). To understand the mechanism of the total field formation in the main observation planes $\varphi' = 0°$ and $\varphi' = 90°$, we consider the RPs of the field components $F_{\theta,\varphi}(\theta,\varphi) = 20\lg|\dot{E}_{\theta,\varphi}(\theta,\varphi)|/|\dot{E}|_{\max}$ separately for each vibrator (Figs. 8(a), (b)). The cone angles of the edge wave shadows related to the edges $n = 1, 2$ ($\beta_{1,2}'$) and $n = 3, 4$ ($\beta_{3,4}'$) (Fig. 1) of the rectangular screen are defined by the formulas [1, 7]

$$\beta_{1,2}' = \operatorname{arctg}\left[2\sqrt{h^2 + (L/2)^2}/W\right], \quad \beta_{3,4}' = arctg\left[2\sqrt{h^2 + (W/2)^2}/L\right]. \tag{22}$$

The cone angles $\beta_{1,2}'$ and $\beta_{3,4}'$ of the edge wave shadow for all edges calculated by formula (22) are equal to 47.3°. In the observation plane $\varphi' = 0°$, the light region formed by the edges **1** and **2** is restricted by angular sectors $\theta' \in (0°, 42.7°)$ and $\theta' \in (137.3°, 180°)$, while that from edges 3 and 4 is restricted by $\theta' \in (0°, 180°)$. In the observation plane $\varphi' = 90°$, the light region created by the edges 1, 2 and 3, 4 is restricted by angular sectors $\theta' \in (0°, 180°)$ and $\theta' \in (0°, 42.7°)$ and $\theta' \in (137.3°, 180°)$ respectively.

As can be seen, due to the diffraction at screen edges 3 and 4, vibrator **1** induces $E_{\theta'1}$ component of the main and cross polarizations in the planes $\varphi' = 0$ (Fig. 8(a), curve 1) and $\varphi' = 90°$ (Fig. 8(a), curve 2), respectively. Analogously, the vibrator **2** induces $E_{\theta'2}$ component of the cross and main polarizations in the planes $\varphi' = 0$ (Fig. 8(b), curve 1) and $\varphi' = 90°$ (Fig. 8(b), curve 2). Figs. 8(c)–8(d) show the genesis of the RP formation of the $E_{\theta'}$ component of the total field of crossed vibrators (curve 1 is the vibrator **1** RP; curve 2 is of the vibrator **2** RP, curve 3 is the RP of the total field).

The RPs, $F_{\theta,\varphi}(\theta,\varphi)$, of the orthogonal field components in the observation planes $\varphi' = 0°$, 45° and 90° are shown in Figs. 8(e)–8(f). If the screen is square, the RPs of E_θ components created by vibrators **1** and **2** in the observation planes $\varphi' = 0°$ and $\varphi' = 90$ (Fig. 8(e), curve 1) are identical. The $E_{\varphi'}$ component (Fig. 8(f), curve 1) is formed by vibrator **1** in the plane $\varphi' = 90°$ and by vibrator **2** in the plane $\varphi' = 0°$. The RPs in the plane $\varphi' = 45°$ are shown as curve 2. The power RP, $G = 10\lg(|\dot{E}|^2/|\dot{E}|^2_{\max})$, is shown in Fig. 8(g) in the observation planes $\varphi' = 0°$ and 90° by curve 1 (RPs coincide) and in the plane $\varphi' = 45°$ by curve 2.

The ellipticities in the observation planes $\varphi' = 0°$, $\varphi' = 90°$ and $\varphi' = 45°$ are shown in Fig. 8(h) (curve 1 and curve 2). The sign of the ellipticity changes when the screen plane ($\theta' = 90°$) is crossed. If the vibrator impedances are $\bar{X}_{S_1} = 0.005$ and $\bar{X}_{S_2} = -0.059$, the elepticity is less than zero in the front half-space (left polarization), and it is greater than zero (right polarization) in the shadow half-space. If the vibrator impedances are $\bar{X}_{S_1} = -0.059$ and $\bar{X}_{S_2} = 0.005$, the elepticity sign is changed.

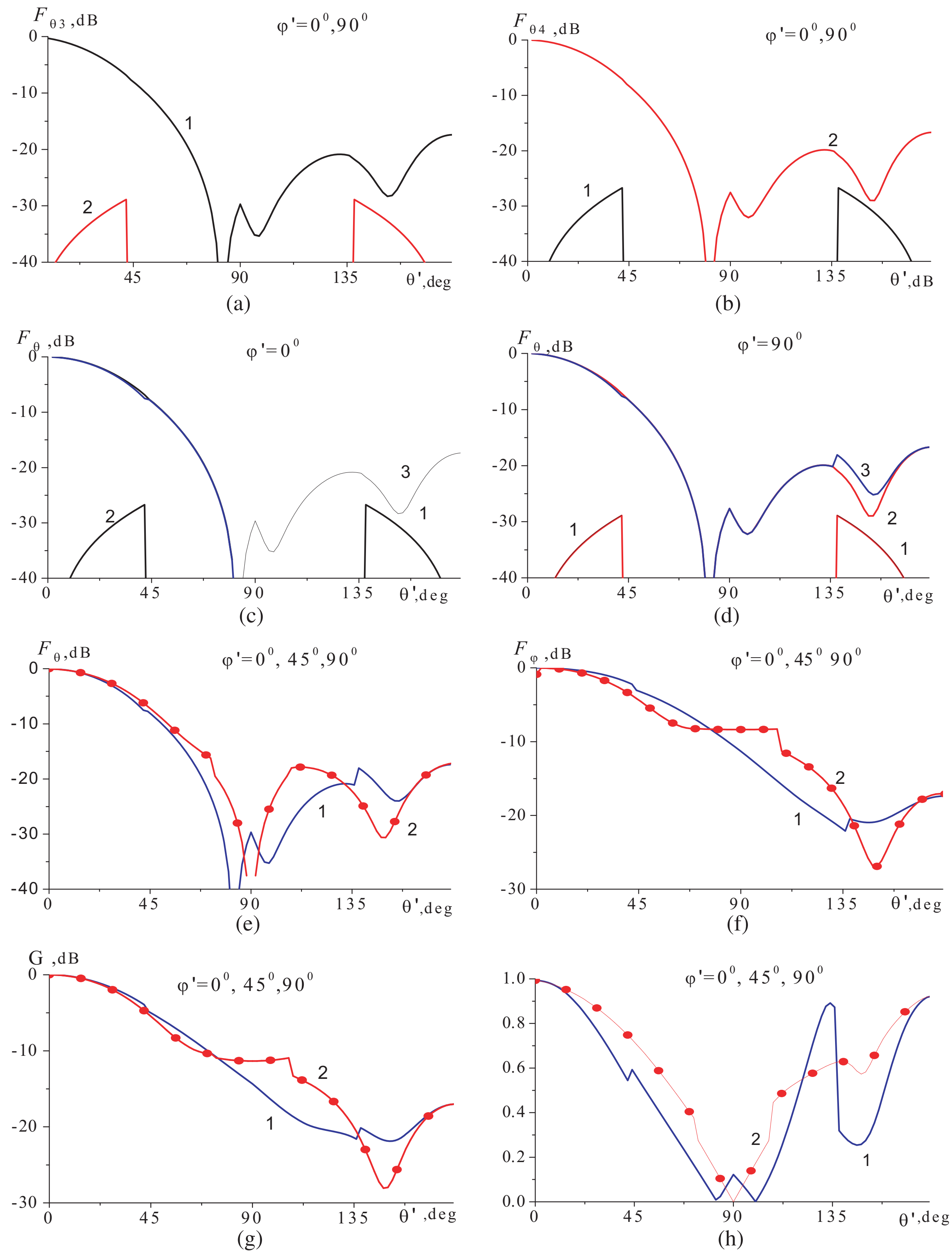

Figure 8. RPs of the fields of vibrators **1**, **2** and total field in the observation plane $\varphi' = 0°$, $45°$, $90°$ ($\bar{X}_{S_1} = 0.005$, $\bar{X}_{S_2} = -0.059$, $h = 0.25\lambda$ and $L = W = 1.2\lambda$).

The spatial RPs of the field components, power, and polarization are shown in Figs. 9(a)–9(d). The ellipticity σ and orientation angle of the polarization ellipse β are calculated by formulas (14) and (18). If the vibrator impedances are $\bar{X}_{S_1} = -0.059$ and $\bar{X}_{S_2} = 0.005$, all RPs are shifted by 90° relative to the observation angle φ'; the direction of the polarization is also changed.

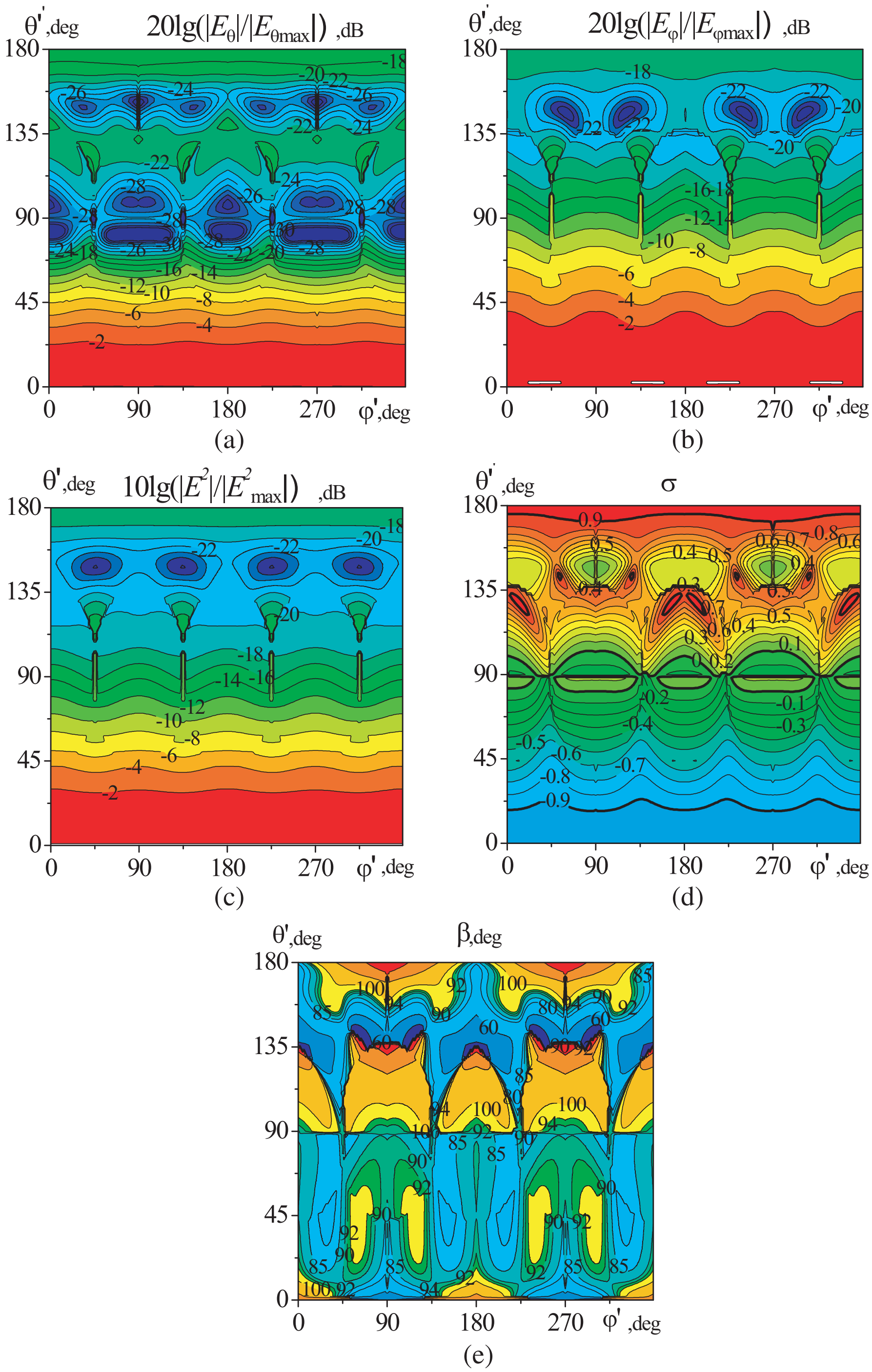

Figure 9. The spatial RPs of the field components, power, and polarization in the coordinates θ', φ' at $\bar{X}_{S_1} = 0.005$, $\bar{X}_{S_2} = -0.059$, $h = 0.25\lambda$, and $L = W = 1.2\lambda$.

6. CONCLUSION

In the framework of the UGTD, an asymptotic solution of a 3D vector diffraction problem for the two crossed impedance vibrators excited in-phase and placed over a rectangular perfectly conducting screen of finite dimensions is obtained by using the asymptotic expressions for the current of the impedance horizontal vibrator over the perfectly conducting plane. An algorithm and respective software for determining the far zone field components are developed. It is shown that a phase shift between the currents of the vibrators close to 90° can be obtained by the choice of the impedances $\bar{X}_{S_{1,2}} = \bar{X}^{cp}_{S_{1,2}}$ of the vibrators with the equal geometric lengths. The screen dimensions of the wavelength order do not affect resonant impedance $\bar{X}^{Hs}_{Sres_{1,2}}$ of a thin vibrator, but substantially influence on the $\bar{X}^{cp}_{S_{1,2}}$-values.

The formation of the orthogonal field components in the main observation planes is analyzed in detail. It is shown that each vibrator field in the normal direction of the screen is determined only by the component of the basic polarization, while the fields diffracted on the square screens have equal amplitudes. Therefore, the radiation field in the normal direction to square screens is circularly polarized for all distances between the vibrators and the screen.

It is also shown that the maximum radiation resistance and directivity in the direction of the normal to the screen for the circularly polarized radiating system can be achieved by selecting the optimal screen dimensions and the distance between the vibrator and screen.

REFERENCES

1. Yeliseyeva, N. P. and N. N. Gorobets, *Diffraction of Radiation of the Wire Antenna on the Rectangular and Corner Screens*, V. N. Karazin Kharkiv National University, Kharkiv, 2009 (in Russian).
2. Nesterenko, M. V., V. A. Katrich, and V. M. Dakhov, "Formation of the radiation field with the set spatial-polarization characteristics by the crossed impedance vibrators system," *Radiophysics and Quantum Electronics*, Vol. 53, No. 5–6, 371–378, 2010.
3. Kouyoumjian, R. G. and P. H. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proceedings of the IEEE*, Vol. 62, 1448–1461, 1974.
4. Pathak, P. H., "High frequency techniques for antenna analysis," *Proceedings of the IEEE*, Vol. 80, No. 1, 44–65, 1992.
5. Nesterenko, M. V., V. A. Katrich, Y. M. Penkin, V. M. Dakhov, and S. L. Berdnik, *Thin Impedance Vibrators. Theory and Applications*, Springer Science+Business Media, New York, 2011.
6. Yeliseyeva, N. P., S. L. Berdnik, V. A. Katrich, and M. V. Nesterenko, "Electrodynamic characteristics of horizontal impedance vibrator located over a finite-dimensional perfectly conducting screen," *Progress In Electromagnetics Research B*. Vol. 63, 275–288, 2015.
7. Gorobetz, N. N. and N. P. Yeliseyeva, "Radiation characteristics of an electric dipole of arbitrary orientation placed above a plane screen," *Telecommunications and Radio Engineering*, Vol. 60, No. 3&4, 30–47, 2003.
8. Born, M. and E. Wolf, *Principles of Optics*, Pergamon Press, 1975.
9. Yeliseyeva, N. P., S. L. Berdnik, V. A. Katrich, and M. V. Nesterenko, "Formation of circularly polarized wave by the oblique impedance wire dipole located over rectangular screen," *Radiofizika and Radioastronomia*, Vol. 21, No. 3, 216–230, 2016 (in Russian).
10. Yeliseyeva, N. P., S. L. Berdnik, V. A. Katrich, and M. V. Nesterenko, "Directional and polarization radiation characteristics of a horizontal impedance vibrator located above a rectangular screen," *Journal of Communications Technology and Electronics*, Vol. 61, No. 2, 99–111, 2016.