Progress In Electromagnetics Research M, Vol. 59, 103–109, 2017

The Weakened Weibel Electromagnetic Instability of Ultra-Intense MeV Electron Beams in Multi-Layer Solid Structure

Leng Liao and Ruiqiang Zhao*

Abstract—The Weibel instability of intense and collimated MeV fast electron beams in multi-layer structure is investigated. It is found that the electromagnetic instability of fast electron beams can be significantly suppressed by this structure. A strong magnetic field will be created at the interfaces between materials with different resistivities as these fast electrons are injected into this structure. It obstructs the transverse movement of the fast electrons and confines them to propagate along the interfaces. In consequence, the positive feedback loop between magnetic field perturbation and electrons density perturbation is broken, and the Weibel instability is thus weakened. Furthermore, the calculated results for Au/Si multi-layer structure by a hybrid Particle in Cell code have proven this weakening effect on the Weibel instability of intense fast electron beams. Because of the high energy-density delivered by the MeV electrons, these results indicate applications in high-energy physics, such as radiography, fast electron beam focusing, and perhaps fast ignition.

1. INTRODUCTION

When ultraintense lasers ($> 10^{18}\,\mathrm{W/cm^2}$) interact with solid targets, the relativistic fast electrons (MeV energy level) can be massively generated [1]. These laser-driven fast electron beams constitute mega-ampere currents and have many potential applications, such as astrophysical plasmas [2], radiography [3], and fusion physics [4–6], but the transportation of a very high current electron beam through solid density plasma is a critical problem for these applications, which all require a long transport distance. However, when these fast electrons are generated in a solid target, the return "cold" electron currents are concurrently produced in the background [7] for the requirement of charge neutrality in plasma. The equilibrium between fast electron beam and the return currents is unstable for the Weibel instabilities, which will produce strong electromagnetic perturbation and filaments of fast electron beam. It makes the energy transport effectiveness very poor and the intense fast electrons cannot transport beyond the filament length [8–13]. For any useful applications, it is therefore of great interest to find methods to weaken Weibel instabilities and increase the transport distance of intense fast electron beams in overdense plasma. Recently, many efforts in this direction have been reported [14–17]. Especially, in the work of [18], Mishra et al. present an approach to achieve suppression/complete stabilization of the transverse electromagnetic beam Weibel instability by periodically modifying the electron density of background with an equilibrium density ripple, shorter than the skin depth. Recent experiments have shown that the intense laser-driven MeV electron beam can effectively transport over millimeters in carbon nanotube array, 100 times longer than the typical filament length in solid target [19].

In the present work, we propose a nanometer multi-layer solid structure constituting two materials with different Z (here, Z indicates the atomic number of atoms constituting this material). This is a solid density structure, but the Weibel instability of fast electron beams can be effectively suppressed in this

---

Received 22 May 2017, Accepted 30 July 2017, Scheduled 6 August 2017

* Corresponding author: Ruiqiang Zhao (rqzhao@cqjtu.edu.cn).
The authors are with the School of Materials Science and Engineering, Chongqing Jiaotong University, Chongqing 400074, China.

structure. Because of the resistive mismatch at the interface between adjacent layers, a resistive magnetic field is generated on the fast electron beam arriving. This field push fast electrons to high-resistivity layers. By this resistive magnetic field, the fast electrons are mainly confined in the high-resistivity layers, and the transverse movement of fast electrons is suppressed. Without the transverse supplies of fast electrons, the growing of filaments in general Weibel instability is consequently suppressed, and the transverse movement of fast electron beams is also suppressed.

2. THEORY

The generation of magnetic fields during the propagation of fast electrons through a solid target can be described by combining a simple Ohm's law $E = -\eta j_{f^-}$ with Faraday's law to yield

$$\frac{\partial \vec{B}}{\partial t} = \eta \nabla \times \vec{j}_f + (\nabla \eta) \times \vec{j}_f \tag{1}$$

where η is the resistivity, and j_f is the fast electron current density. The first term on the right-hand side generates a magnetic field that pushes fast electrons towards regions of higher fast electron current density. The first term of Eq. (1) is the origin of electromagnetic instability. Giving a ripple of fast electron current density, normal to the streaming direction (i.e., normal to the direction of j_f), the generated magnetic field by the first term would push more fast electrons into the high density region from the low density region (the higher the density of fast electrons is, the higher the current density is: j_f). More and more fast electrons are squeezed together, which further strengthens the density ripple. Consequently, the generated field gets stronger further according to Eq. (1). It is a positive feedback: the magnetic field gets stronger and stronger, and the density ripple magnitude gets larger and larger. As a result, the Weibel instability is created.

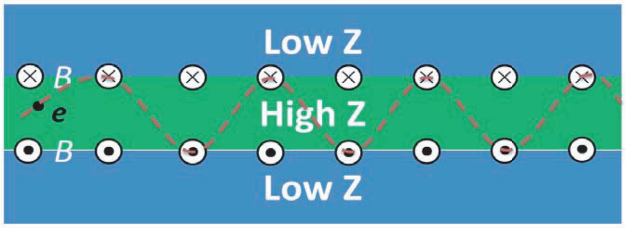

Figure 1. (Color Online) The plot of magnetic field at the interfaces between layers consisting of materials with different Z.

For the second term of Eq. (1), the generated magnetic field pushes fast electrons towards regions of higher resistivity. It implies that a target with a sharp resistivity boundary can build up strong magnetic fields, which push the fast electrons into the high resistivity regions. Here, we use this resistive magnetic field to design a target structure for fast electron transportation. The target consists of many alternately aligning high Z and low Z layers as depicted in Fig. 1, where Z indicates the average atomic number of atoms constituting this material. The layer thickness is chosen as tens of nanometers shorter than the skin depth. The higher Z material also has higher resistivity according to the Spitzer resistivity model [20]:

$$\eta = 10^{-4} Z \ln \Lambda / T_c^{1.5} \tag{2}$$

where T_c is the temperature of the cold electrons (in eV), and $\ln \Lambda$ is the Coulomb logarithm. There is accordingly an abrupt change of resistivity at the interfaces between adjacent layers with different Z. According to Eq. (1), the gradient of resistivity at boundaries is therefore very large, and the generated field by the second terms of Eq. (1) is dominated. During the propagation of intense fast electron flux along y direction, the fast electron beams are bended by the magnetic field into the high Z layers. In the transverse direction (x direction), these high Z layers act as magnetic traps, and the fast electrons are confined in these regions to propagate forward. Without enough transverse kinetic energy or force on fast electrons, these electrons cannot escape from these high Z layers.

Consider that there exist electron density ripples in the transverse direction. Because of the second term of Eq. (1), the transverse movement of these fast electrons is impeded by the magnetic field near interfaces, and the fast electrons are difficult to move across one layer to another. If the ripple size is larger than the aligning period of layers, the growing rate of perturbation will be suppressed, because the fast electron transportation from the low-density regions to high density regions is impeded. On the other hand, for the ripples whose transverse size smaller than the layer width, if the layer period is engineered shorter than the skin depth, it is a well-known fact that the transverse electromagnetic perturbations are also weakened [18]. Consequently, we can get results: for a multi-layer structure, the magnetic instability (with large k, i.e., small size) can be suppressed by decreasing the layer width, while the magnetic instability across layers (with small k, i.e., large size) can be suppressed by increasing the resistive magnetic field at the interfaces between adjacent layers with different Z.

3. SIMULATION AND DISCUSSION

The above theory discussion just gives a way to lower growth rate of Weibel instability by a multi-layer structure, but the weakening efficiency needs a further quantitative investigation, and the numerical calculation is required. Without loss of generality, we choose Au as the high Z material and Si as the low Z material in simulation. For comparison's purpose, the growing of instability in bulk Au and Si targets and Au/Si multi-array target are all numerically modeled by a hybrid algorithm which means a code using a particle-in-cell algorithm to describe the kinetic fast electrons, where the background electrons are treated as a Spitzer resistive conductor [20]. Because of the intense background return current induced by the fast electrons, Ohm heating in a short time ionizes the backgrounds into plasma. The ionization process is thus not taken into consideration, and we assume that the background is instantaneously ionized and heated to 100 eV on fast electrons arriving. Hence, we use the Spitzer resistivity model of hot plasma for background. The Ohm heating of the background plasma gives:

$$\frac{\partial T_c}{\partial t} = \frac{\eta j_c^2}{C} \tag{3}$$

where C is the heat capacity of cold electron specified by the ideal gas heat capacity: $C = 1.5n_c$ where n_c and j_c are cold electron density and current. Here, j_c is given by generalized Ohm law ignoring heat pressure and inertia term:

$$\vec{j}_c = \left(E - \frac{\vec{j}_c \times \vec{B}}{en}\right)/\eta \tag{4}$$

The simulations are carried out in a domain of 6.0 µm by 6.0 µm (x by y), represented by a 4096 by 4096 grid. The target is modeled by a group of alternatively aligning layers (the widths of high Z and low Z layers are both 50 nm, i.e., $ck/\omega_{pc} \approx 5 > 1$ for one layers in transverse direction) as shown in Fig. 2. The intense fast electron flow is uniformly injected from bottom of the box. The initial fast electron energetic distribution is set Maxwellian of temperature: $kT = 2.0\,\text{MeV}$, and the initial velocity is along the y axis. The cold electrons in background are assumed to be preheated to 100 eV.

By this model, we first simulate the evolution of the fast electron distributions in bulk Au and Si targets, which are contained in Figs. 3(a), (b) and 4(a), (b). Filamentations of fast electrons are quickly created after the intense fast electrons are injected in the bulk targets. Figs. 3(c), (d) and 4(c), (d) show the evolution of self-generated magnetic field in these two targets. The mechanism of this beam Weibel instability is well known. While the high density fast electron beams propagate in the uniform plasma, a return cold electron beam with equal current is created, and the equilibrium with two counter beams is formed. A density ripple in fast electron beam, normal to the streaming direction, leads to a magnetic field, which repulses the return current out of the high density region and reinforces magnetic field again, by which, at the same time, more and more fast electrons are gathered in high density region, thus providing a positive feedback responsible for the current sheet separation depicted in Figs. 3(a), (b) and 4(a), (b) and very strong self-generated magnetic field as shown in Figs. 3(c), (d) and 4(c), (d). It is clear that the transverse movement of electrons driven by magnetic field plays an essential role. If the transverse movement of electrons is obstructed, Weibel instability might be weakened.

For this reason, we design a target for fast electron transportation as shown in Fig. 2, which is a nanometer scale multi-layer structure consisting of Au and Si. The fast electrons propagate along the

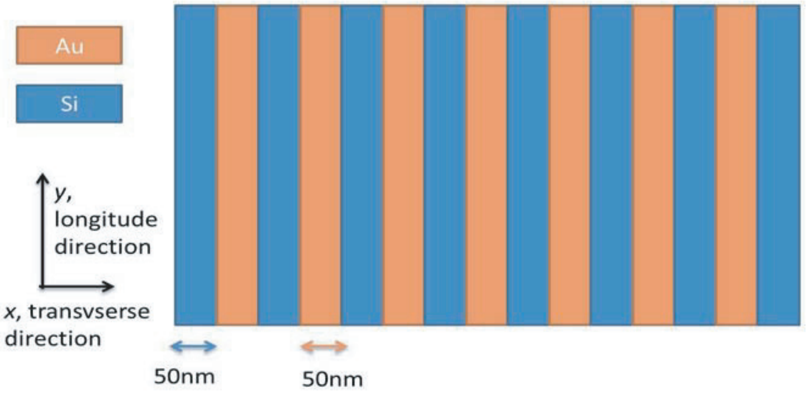

Figure 2. (Color Online) The plot of periodic multi-layer structure consisting with Au (high Z) and Si (low Z) material and the fast electrons are injected from the bottom. The yellow present Au layers and the blue present Si layers. The width of layers is 50 nm.

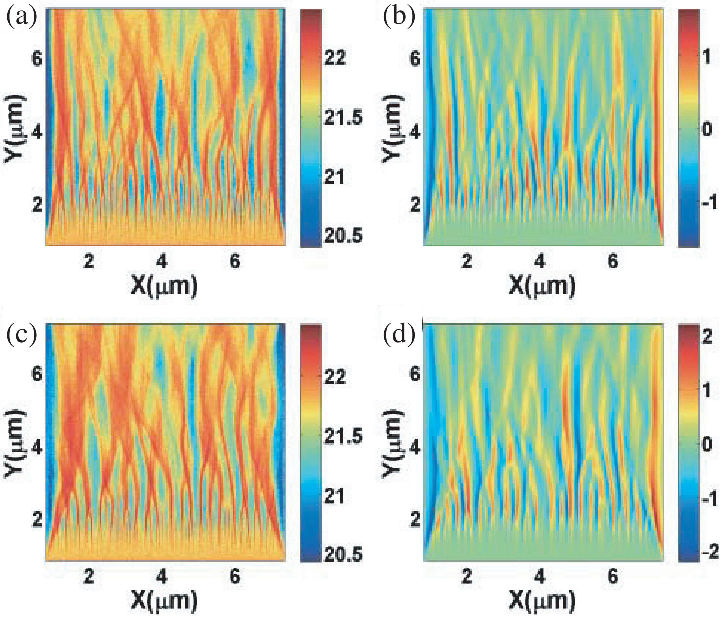

Figure 3. (Color online) The log 10 plot of fast electron distribution in bulk Si target in (a) at 50 fs and in (b) at 100 fs. The (c) and (d) present the corresponding magnetic field distribution as (c) 50 fs and (d) 100 fs.

direction of layer aligning (y axis, i.e., the longitude direction). Because of the second term of Eq. (1), self-generated magnetic fields are created at the interfaces of adjacent Au and Si layers as shown in Figs. 5(c), (d) for the large mismatch of resistivity, which push the fast electrons into the Au layers for the higher resistivity. These magnetic fields act as a magnetic "trap" to trap fast electrons in Au layers, by which the transverse movement of fast electrons is significantly suppressed. If there exist fast electron density ripples, the generated magnetic field by density ripple must conquer resistive magnetic field at interface to make fast electrons move in transverse direction. It weakens the positive feedback between magnetic field perturbation and electrons density perturbation. Consequently, the magnetic field cannot grow strong enough to separate current sheets as in bulk background. The fast electron distribution in a multi-layer structure is shown in Figs. 5(a), (b), in which fast electrons are effectively constrained in the Au layers, and there are no significant fast electron filaments as in bulk Au and Si targets.

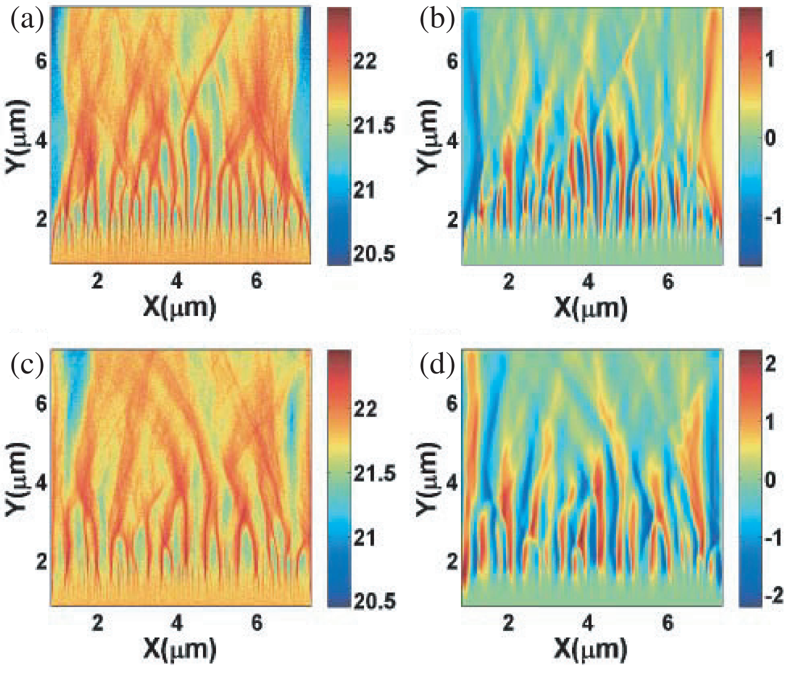

Figure 4. (Color online) The log 10 plot of fast electron distribution in bulk Au target in (a) at 50 fs and in (b) at 100 fs. The (c) and (d) present the corresponding magnetic field distribution as (c) 50 fs and (d) 100 fs.

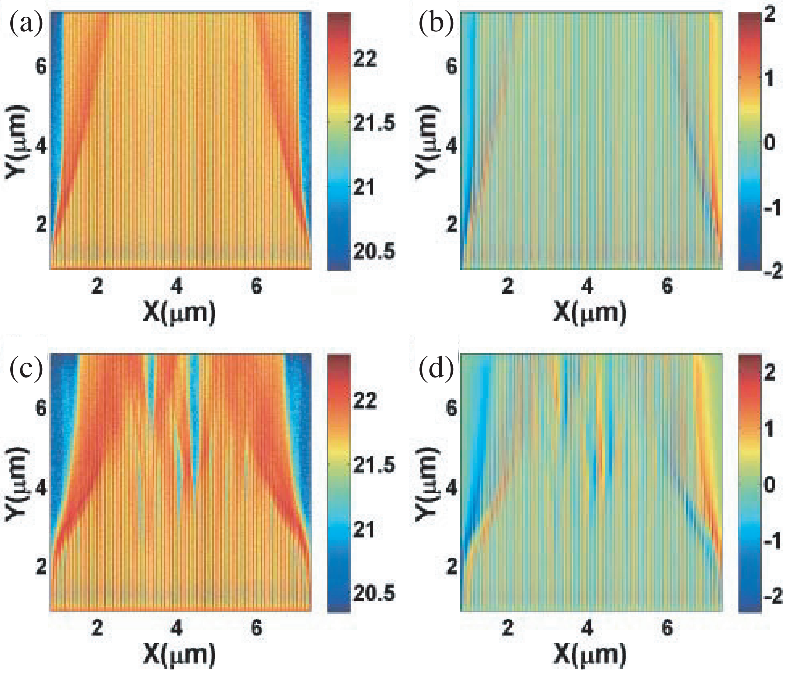

Figure 5. (Color online) The log 10 plot of fast electron distribution in Au/Si multi-layer target in (a) at 50 fs and in (b) at 100 fs. The (c) and (d) present the corresponding magnetic field distribution as (c) 50 fs and (d) 100 fs.

4. CONCLUSION

In summary, we propose a scheme by a multi-layer structure to prevent/weaken Weibel instability of fast electron beams. The self-generated magnetic field created at interfaces between successive layers can effectively trap the fast electrons in the layers consisting of high Z material. It prevents the transverse movement of fast electrons and breaks/weakens the positive feedback loop between magnetic field perturbation and electrons density perturbation. Furthermore, the calculated results by hybrid Particle in Cell code has proven this weakening effect for Weibel instability. Because of the high energy-density delivered by the MeV electrons, these results indicate applications in high-energy physics, such as radiography, fast electron beam focusing, and perhaps fast ignition.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (Grant No. 11404045), the Social Livelihood Science and Technology Innovation Special of Chongqing (Grant No. cstc2015shmszx30012), the Science and Technology Planning Project of Yunnan Province of China (Grant No. 2017IB025), and the State Key Laboratory Base of the Mountain Bridge and Tunnel Engineering (Chongqing Jiaotong University:CQSLBF-Y14-1).

Original Contribution

A scheme by multi-layer structure to weaken the Weibel electromagnetic instability of fast electron beams is proposed. The calculated results by hybrid Particle in Cell code has proven this weakening effect for Weibel instability induced by the multi-layer structure.

REFERENCES

1. Perry, M. D. and G. Mourou, "Terawatt to petawatt subpicosecond lasers," *Science*, Vol. 264, 917–924, 1994.
2. Gibbon, P., *Short Pulse Laser Interactions with Matter: An Introduction*, College Press, London, 2005.
3. Park, H. S., D. M. Chambers, H. K. Chung, R. J. Clarke, et al., "High-energy K alpha radiography using high-intensity, short-pulse lasers," *Phys. Plasmas*, Vol. 13, 056309, 2006.
4. Kodama, R., P. A. Norreys, K. Mima, A. E. Dangor, et al., "Fast heating of ultrahigh-density plasma as a step towards laser fusion ignition," *Nature*, Vol. 412, 798–802, 2001.
5. Atzeni, S. and J. Meyer-tar-Vehn, *Inertial Fusion-beam Plasma Interaction, Hydrodynamic, Dense Plasma Physics*, Clarendon, Oxford, 2003.
6. Tabak, M., J. Hammer, M. E. Glinsky, et al., "Ignition and high-gain with ultrapowerful lasers," *Phys. Plasmas*, Vol. 1, 1626–1634, 1994.
7. Weibel, E. S., "Spontaneously growing transverse waves in a plasma due to an anisotropic velocity distribution," *Phys. Rev. Lett.*, Vol. 2, 83–84, 1959.
8. Green, J. S., V. M. Ovchinnikov, R. G. Evans, K. U. Akli, et al., "Effect of laser intensity on fast-electron-beam divergence in solid-density plasmas," *Phys. Rev. Lett.*, Vol. 100, 015003, 2008.
9. Kodama, R., Y. Sentoku, Z. L. Chen, et al., "Plasma devices to guide and collimate a high density of MeV electrons," *Nature*, Vol. 432, 1005–1008, 2004.
10. Lancaster, K. L., J. S. Green, D. S. Hey, et al., "Measurements of energy transport patterns in solid density laser plasma interactions at intensities of $5 \times 10^{20}\,\mathrm{W\,cm}^{(-2)}$," *Phys. Rev. Lett.*, Vol. 98, 125002, 2007.
11. Santos, J. J., F. Amiranoff, S. D. Baton, et al., "Fast electron transport in ultraintense laser pulse interaction with solid targets by rear-side self-radiation diagnostics," *Phys. Rev. Lett.*, Vol. 89, 207–213, 2002.
12. Sentoku, Y., K. Mima, S. Kojima, et al., "Magnetic instability by the relativistic laser pulses in overdense plasmas," *Phys. Plasmas*, Vol. 7, 689–695, 2000.

13. Stephens, R. B., R. A. Snavely, Y. Aglitskiy, et al., "K-alpha fluorescence measurement of relativistic electron transport in the context of fast ignition," *Phys. Rev. E*, Vol. 69, 039901, 2004.
14. Robinson, A. P. L., M. Sherlock, and P. A. Norreys, "Artificial collimation of fast-electron beams with two laser pulses," *Phys. Rev. Lett.*, Vol. 100, 025002, 2008.
15. Bell, A. R. and R. J. Kingham, "Resistive collimation of electron beams in laser-produced plasmas," *Phys. Rev. Lett.*, Vol. 91, 035003, 2003.
16. McKenna, P., A. P. L. Robinson, D. Neely, et al., "Effect of lattice structure on energetic electron transport in solids irradiated by ultraintense laser pulses," *Phys. Rev. Lett.*, Vol. 106, 185004, 2011.
17. Ramakrishna, B., S. Kar, A. P. L. Robinson, et al., "Laser-driven fast electron collimation in targets with resistivity boundary," *Phys. Rev. Lett.*, Vol. 105, 135001, 2010.
18. Mishra, S. K., P. Kaw, A. Das, et al., "Stabilization of beam-weibel instability by equilibrium density ripples," *Phys. Plasmas*, Vol. 21, 012108, 2014.
19. Chatterjee, G., P. K. Singh, S. Ahmed, et al., "Macroscopic transport of mega-ampere electron currents in aligned carbon-nanotube arrays," *Phys. Rev. Lett.*, Vol. 108, 235005, 2012.
20. Spitzer, L. and R. Harm, "Transport phenomena in a completely ionized gas," *Phys. Rev.*, Vol. 89, 977–981, 1953.