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Identification of Equivalent Circuit Based on Polygon Network for Nonreciprocal Lossy N-Port Device

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Abstract—In this paper, a technique to identify/synthesize an equivalent circuit of nonreciprocal lossy N-port device is presented. The technique joins the classical procedure discussed in the '60s to the polygon network recently proposed in the literature, which permits to draw an equivalent circuit for reciprocal lossless N-port device in a very simple way. The identification is applied to two microwave devices, a reciprocal lossy iris in WR90 waveguide and a 3-port nonreciprocal lossy circulator. The proposed equivalent circuit could give some information about the agreement of the manufactured device and its design, which usually is developed in the hypothesis of ideal lossless components.

1. INTRODUCTION

In the past, many researchers have studied the equivalent circuits of N-port microwave devices starting from the knowledge of the impedance matrix Z or scattering matrix S [1, 2]. Carlin proposed in '50s–'60s an efficient approach to synthesize equivalent circuits based on $2N$-ports transformer banks, which realize the desired circuit [3–6]. Cederbaum [7], Oono [8] and Youla et al. [9, 10] proposed some refinements to that approach to enhance the realization of the circuit. The main problem lies in the use of "complex transformers" [11] that cannot be realized in some scenarios. Moreover, the corresponding network can be very hard to manage if the device has many ports.

Recently, an equivalent circuit for lossless N-port S-matrix has been defined using a very simple and efficient synthesis technique [12], based on a polygon network, with susceptances placed at the sides and at the diagonals of N-port polygon, and on N transmission lines connecting the device ports to the polygon sides. For example, the equivalent circuit for a 4-port device is shown in Fig. 1(a). A square is drawn, and six susceptances are placed at the sides and at the diagonals (the reciprocal kernel in Fig. 1(a)). Finally four transmission lines connect the external input ports of the device to the sides of the polygon. Transformers are used to normalize the susceptances contained in the kernel. The electrical parameters are ten as the ten scattering coefficients, because the S-matrix is symmetric, and they can be easily evaluated as described in [12]. This equivalent circuit is able to also include the presence of evanescent modes [13, 14], and it has been applied to some microwave discontinuities, as inductive/capacitive irises, bends and T-junctions [15]. Moreover, similar equivalent circuits have been defined to reconstruct the experimental S-matrix of N-port device with measurements performed only with 2-port Vector Network Analyzer (VNA) [16].

For N-port reciprocal device, the equivalent circuit contains N input transmission lines and $N(N-1)/2$ susceptances, placed at the sides and at diagonals of N-side polygon. Globally, there are $N(N+1)/2$ electrical parameters, corresponding exactly to the $N(N+1)/2$ scattering parameters of the reciprocal N-port device. The easiness of drawing the equivalent circuit of N-port device should be appreciated.

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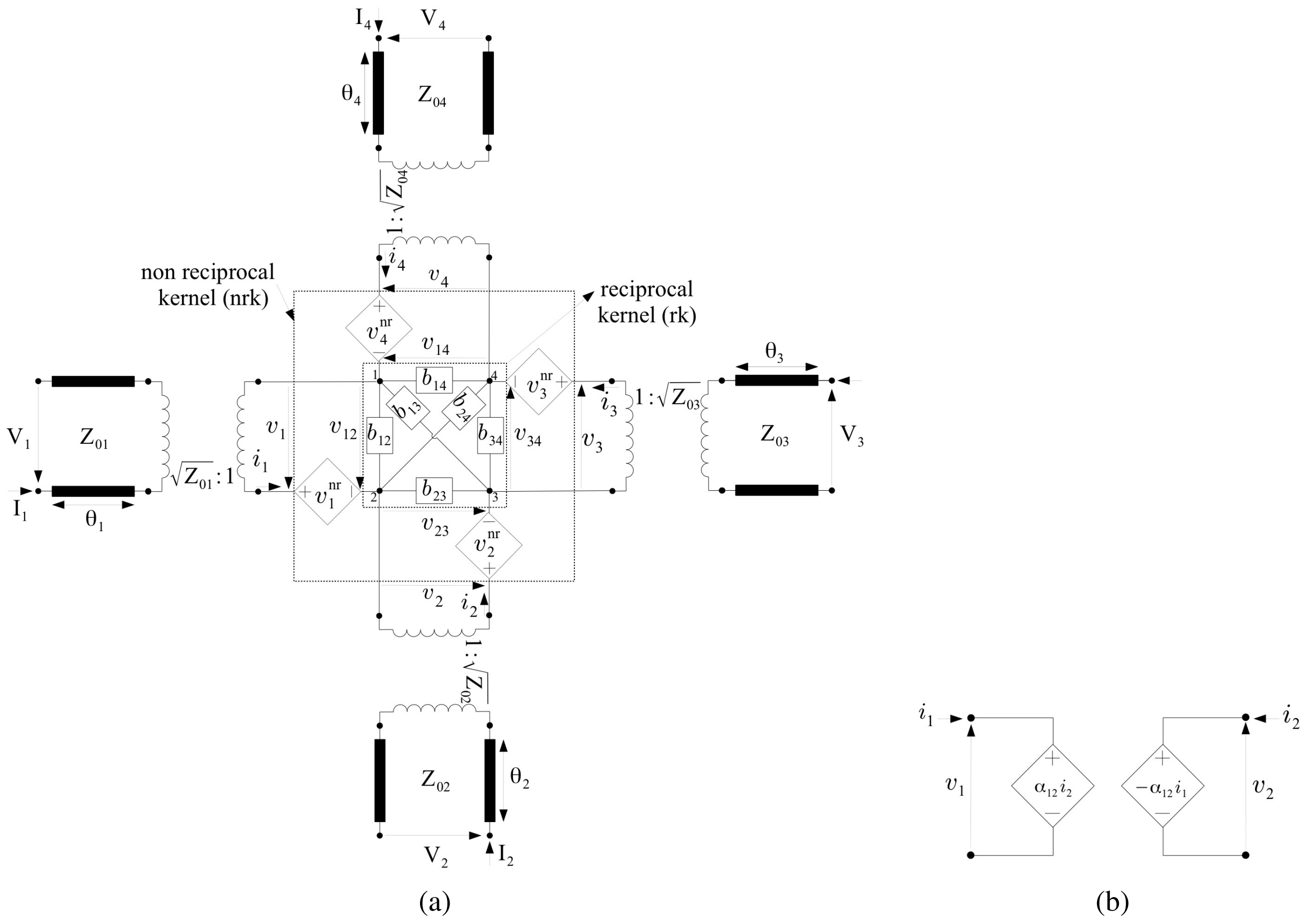


Figure 1. Equivalent circuits of (a) a reciprocal 4-port device and (b) a non reciprocal lossless gyrator.

The aim of this paper is to join these new equivalent circuits and the classical synthesis, replacing the transformer banks with the polygon network, that is easier to evaluate, to obtain a circuit able to represent the general case of nonreciprocal lossy N-port device. The equivalent circuit can be used in circuit synthesis or identification.

The circuit synthesis is the process that starts from the knowledge of a lossless S-matrix with the desired behavior and ends with the realization of a microwave device, usually assumed lossless in the design phase. In this case, the synthesized equivalent circuit gives the susceptance values that must be realized, for example, with irises, or cavities, or other microwave devices. The electrical lengths of the transmission lines in the circuit are the lengths of the waveguides that connect the N input ports to the circuit kernel, to obtain the desired phase relationships between ports. Nonreciprocity is obtained with gyrators connected to the circuit ports, realized with ferrite devices.

The circuit identification is the process that starts from the knowledge of measured lossy S-matrix and ends with the evaluation of an equivalent circuit that could contain information about its manufacturing. In fact, actual devices are realized with some mechanical tolerances which can cause unexpected behavior with respect to the desired response imposed in the design process, often developed in the hypothesis of lossless devices. The identified equivalent circuit can point out the effect of the losses and the discrepancies of some realized susceptances with the optimal value obtained in the design. Hence the identification can help the designer and manufacturer to improve the realized device, with a feedback process that can focus on the minimization of the losses and/or on the enhancement of the realization process.

2. THEORY

2.1. Non Reciprocal Lossless Device

The first step to extend this equivalent circuit to any kind of device is to consider the case of a nonreciprocal lossless device. The simplest way is to remember that nonreciprocity can be added with a gyrator. A gyrator is a nonreciprocal lossless device that has the following normalized impedance matrix

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \zeta^{\text{gyr}} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & \alpha_{12} \\ -\alpha_{12} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \Rightarrow \begin{cases} v_1 = \alpha_{12} i_2 \\ v_2 = -\alpha_{12} i_1 \end{cases} \tag{1}$$

Equation (1) represents two CCVSs (Current Controlled Voltage Source), at the two sides of the gyrator, which can be put in the form shown in Fig. 1(b). The amplitude α_{12} must be real to ensure that the gyrator is lossless.

Hence, to introduce nonreciprocity in the equivalent circuit shown in Fig. 1(a), the current in each port must control a voltage source at the other ports. This implies the presence of $N(N-1)/2 = 6$ gyrators which must be placed at the ports of the circuit reciprocal kernel. Each gyrator relates the voltage at each nonreciprocal kernel port to the current in the other ports, as shown in Fig. 2(a). With the gyrators, the number of circuit parameters is $N^2 = 16$, (4 transmission lines, 6 gyrators and 6 susceptances), just as the number of scattering parameters of a 4-port nonreciprocal device (equal to N^2, with $N = 4$). In this sense, the proposed circuit is a minimal realization.

The kernel circuit in Fig. 2(a) appears complex, but it can be drawn in more readable form, if the voltage at each port of the gyrators is included in a global CCVS at the same port. In doing so, four CCVSs are placed at the input ports of the circuit kernel. CCVS placed at port 1 depends on the current of ports 2, 3 and 4, similarly, for the other CCVSs. Hence, the nonreciprocal equivalent circuit can be drawn as shown in Fig. 2(b), with:

$$v_1^{\text{nr}} = \alpha_{12} i_2 + \alpha_{13} i_3 + \alpha_{14} i_4 \tag{2}$$

$$v_2^{\text{nr}} = -\alpha_{12} i_1 + \alpha_{23} i_3 + \alpha_{24} i_4 \tag{3}$$

$$v_3^{\text{nr}} = -\alpha_{13} i_1 - \alpha_{23} i_2 + \alpha_{34} i_4 \tag{4}$$

$$v_4^{\text{nr}} = -\alpha_{14} i_1 - \alpha_{24} i_2 - \alpha_{34} i_3 \tag{5}$$

or

$$v_k^{\text{nr}} = \sum_{j=k+1}^{N} \alpha_{kj} i_k - \sum_{j=1}^{k-1} \alpha_{jk} i_k \quad k = 1, \dots, N \tag{6}$$

with $N = 4$ for Fig. 2(b). The nonreciprocal kernel (nrk) shown in Fig. 2(b) is the combination of the CCVS's and the reciprocal kernel (rk), i.e., the polygon susceptance network. The normalized nonreciprocal Z-matrix of the nonreciprocal kernel, ζ^{nrk}, is obtained from the knowledge of ζ^{rk}, the reciprocal normalized Z-matrix of the reciprocal kernel, summing the effects of the gyrators:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \zeta^{\text{nrk}} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \left\{ \overbrace{\begin{bmatrix} 0 & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ -\alpha_{12} & 0 & \alpha_{23} & \alpha_{24} \\ -\alpha_{13} & -\alpha_{23} & 0 & \alpha_{34} \\ -\alpha_{14} & -\alpha_{24} & -\alpha_{34} & 0 \end{bmatrix}}^{\zeta^{\text{gyr}}} + j \overbrace{\begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{12} & x_{22} & x_{23} & x_{24} \\ x_{13} & x_{23} & x_{33} & x_{34} \\ x_{14} & x_{24} & x_{34} & x_{44} \end{bmatrix}}^{\zeta^{\text{rk}}} \right\} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \tag{7}$$

It should be noted that:

- the impedance matrix $\zeta^{\text{nrk}} = \zeta^{\text{gyr}} + \zeta^{\text{rk}}$ is lossless, because the reciprocal part ζ^{rk} is purely imaginary, and ζ^{gyr} is lossless.
- $\zeta_{ij}^{\text{nrk}} = -(\zeta_{ji}^{\text{nrk}})^*$, i.e., ζ^{nrk} is a "skew-hermitian" matrix because $\zeta^{\text{nrk}} = -(\zeta^{\text{nrk}})^\dagger$ (symbol $\dagger$ represents Transpose Conjugate matrix).

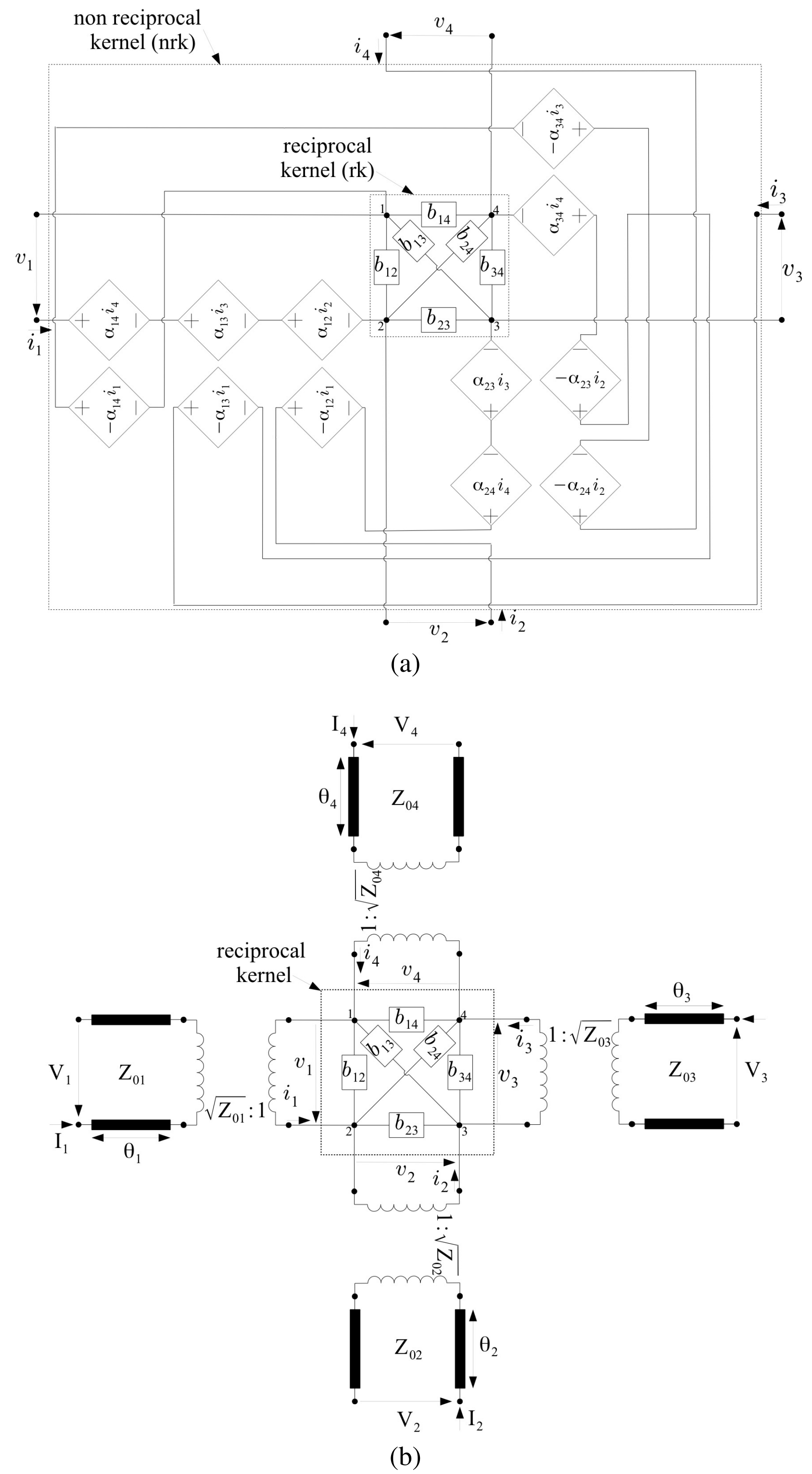


Figure 2. (a) The non reciprocal kernel of the equivalent circuit of a non reciprocal 4-port device, with six gyrators. (b) The equivalent circuit of a non reciprocal 4-port device, where the gyrators of Fig. 2(a) are replaced by four global CCVS's. CCVS amplitudes $v_1^{\mathrm{nr}}, v_2^{\mathrm{nr}}, v_3^{\mathrm{nr}}, v_4^{\mathrm{nr}}$ are as in Equations (2)–(5).

Having defined the equivalent circuits, the identification/synthesis procedure must be defined, starting from the knowledge of the overall nonreciprocal S-matrix. As discussed in the Introduction, the synthesis procedure described in [3–11] is very effective, but it makes use of transformer banks which are not simple to synthesize if the device has many ports. Moreover, it does not respect the geometry of the device because that network is a combination of reactive elements, transformers and gyrators, which does not take into account the physical realization of the device. In fact, any microwave device is realized with input lines which connect the inner kernel to the external ports, in order to delete the effects of the modes below cutoff excited by the kernel of the device. The synthesis procedure described in [3–11] includes input lines in the overall device, which is represented with reactive elements and transformers and putting the nonreciprocal gyrators exactly at the input ports.

On the contrary, the equivalent circuit shown in Fig. 2(b) takes into account the presence of the input lines and represents only the nonreciprocal kernel with gyrators and a polygon network as should be, for example, for a waveguide gyrators, where the nonreciprocity is due to the ferrite placed in the kernel of the actual device.

The synthesis procedure starts from the evaluation of the sum of the voltage drops at the ports of the nonreciprocal kernel. In fact, from the definition of ζ^{nrk}, Equation (7),

$$v_1+v_2+v_3+v_4=\sum_{i=1}^{4}\zeta_{i1}^{\mathrm{nrk}}i_1+\sum_{i=1}^{4}\zeta_{i2}^{\mathrm{nrk}}i_2+\sum_{i=1}^{4}\zeta_{i3}^{\mathrm{nrk}}i_3+\sum_{i=1}^{4}\zeta_{i4}^{\mathrm{nrk}}i_4 \tag{8}$$

and from the analysis of the circuit shown in Fig. 2(b) and from Equations (2)–(5)

$$\begin{aligned} v_1+v_2+v_3+v_4 &= v_{12}+v_1^{\mathrm{nr}}+v_{23}+v_2^{\mathrm{nr}}+v_{34}+v_3^{\mathrm{nr}}+v_{41}+v_4^{\mathrm{nr}} \\ &= (v_{12}+v_{23}+v_{34}+v_{41})+v_1^{\mathrm{nr}}+v_2^{\mathrm{nr}}+v_3^{\mathrm{nr}}+v_4^{\mathrm{nr}}=0+v_1^{\mathrm{nr}}+v_2^{\mathrm{nr}}+v_3^{\mathrm{nr}}+v_4^{\mathrm{nr}} \\ &= \sum_{i=1}^{4}\alpha_{i1}i_1+\sum_{i=1}^{4}\alpha_{i2}i_2+\sum_{i=1}^{4}\alpha_{i3}i_3+\sum_{i=1}^{4}\alpha_{i4}i_4 \end{aligned} \tag{9}$$

with $\alpha_{ii}=0$ and $\alpha_{ji}=-\alpha_{ij}$, if $j>i$. Equations (8) and (9) give

$$\sum_{i=1}^{4}\zeta_{i1}^{\mathrm{nrk}}i_1+\sum_{i=1}^{4}\zeta_{i2}^{\mathrm{nrk}}i_2+\sum_{i=1}^{4}\zeta_{i3}^{\mathrm{nrk}}i_3+\sum_{i=1}^{4}\zeta_{i4}^{\mathrm{nrk}}i_4=\sum_{i=1}^{4}\alpha_{i1}i_1+\sum_{i=1}^{4}\alpha_{i2}i_2+\sum_{i=1}^{4}\alpha_{i3}i_3+\sum_{i=1}^{4}\alpha_{i4}i_4 \tag{10}$$

If we suppose to excite the device at one port at time, we obtain four conditions on each column of ζ^{nrk}:

$$\frac{v_1+v_2+v_3+v_4}{i_1}=\sum_{i=1}^{4}\zeta_{i1}^{\mathrm{nrk}}=\sum_{i=1}^{4}\alpha_{i1}\in\Re \text{ if } i_2=i_3=i_4=0\ \forall i_1\Rightarrow \mathrm{Im}\left[\sum_{i=1}^{4}\zeta_{i1}^{\mathrm{nrk}}\right]=0 \tag{11}$$

$$\frac{v_1+v_2+v_3+v_4}{i_2}=\sum_{i=1}^{4}\zeta_{i2}^{\mathrm{nrk}}=\sum_{i=1}^{4}\alpha_{i2}\in\Re \text{ if } i_1=i_3=i_4=0\ \forall i_2\Rightarrow \mathrm{Im}\left[\sum_{i=1}^{4}\zeta_{i2}^{\mathrm{nrk}}\right]=0 \tag{12}$$

$$\frac{v_1+v_2+v_3+v_4}{i_3}=\sum_{i=1}^{4}\zeta_{i3}^{\mathrm{nrk}}=\sum_{i=1}^{4}\alpha_{i3}\in\Re \text{ if } i_1=i_2=i_4=0\ \forall i_3\Rightarrow \mathrm{Im}\left[\sum_{i=1}^{4}\zeta_{i3}^{\mathrm{nrk}}\right]=0 \tag{13}$$

$$\frac{v_1+v_2+v_3+v_4}{i_4}=\sum_{i=1}^{4}\zeta_{i4}^{\mathrm{nrk}}=\sum_{i=1}^{4}\alpha_{i4}\in\Re \text{ if } i_1=i_2=i_3=0\ \forall i_4\Rightarrow \mathrm{Im}\left[\sum_{i=1}^{4}\zeta_{i4}^{\mathrm{nrk}}\right]=0 \tag{14}$$

Equations (11)–(14) are the key to evaluate the electrical lengths $\theta_1,\theta_2,\theta_3,\theta_4$, starting from the knowledge of the S-matrix of the overall non reciprocal lossless device, named S^{nr}. In fact, the electrical lengths $\theta_1,\theta_2,\theta_3,\theta_4$ must have values such that the imaginary part of the normalized impedance matrix ζ^{nrk} must satisfy Equations (11)–(14).

Hence, if we connect numerically four lines with negative electrical lengths $-\theta_1,-\theta_2,-\theta_3,-\theta_4$ to the input ports of S^{nr}, we obtain a new S-matrix which coincides with S-matrix of the nonreciprocal kernel, S^{nrk}, if and only if the associated Z-matrix, ζ^{nrk}, satisfies Equations (11)–(14)

$$\mathrm{Im}\left[\sum_{i=1}^{N}\zeta_{ij}^{\mathrm{nrk}}\right]=0\quad j=1,\ldots,N\quad N=4 \tag{15}$$

being

$$\zeta^{\text{nrk}} = \left[I - S^{\text{nrk}}\right]^{-1}\left[S^{\text{nrk}} + I\right] \tag{16}$$

$$S^{\text{nrk}} = \text{diag}\left[e^{j\theta_1}, e^{j\theta_2}, e^{j\theta_3}, e^{j\theta_4}\right] \cdot S^{\text{nr}} \cdot \text{diag}\left[e^{j\theta_1}, e^{j\theta_2}, e^{j\theta_3}, e^{j\theta_4}\right] \tag{17}$$

I is the 4×4 Identity Matrix, and diag is a 4×4 Diagonal Matrix with entries contained in the brackets. Equations (15) represent a system of four equations in four unknowns, $\theta_1, \theta_2, \theta_3, \theta_4$, which can be solved numerically. Equations (15)–(17) replace the condition on the S-matrix (or Z-matrix) discussed in [12] for the reciprocal case.

Once the values of $\theta_1, \theta_2, \theta_3, \theta_4$ have been obtained, ζ^{nrk} can be extracted by Equations (16)–(17). From Equation (7), CCVS's values and the reciprocal Z-matrix of the polygon network in Fig. 2(b) are

$$\begin{bmatrix} 0 & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ -\alpha_{12} & 0 & \alpha_{23} & \alpha_{24} \\ -\alpha_{13} & -\alpha_{23} & 0 & \alpha_{34} \\ -\alpha_{14} & -\alpha_{24} & -\alpha_{34} & 0 \end{bmatrix} = \text{Re}\left[\zeta^{\text{nrk}}\right] \tag{18}$$

$$\zeta^{\text{rk}} = j\text{Im}\left[\zeta^{\text{nrk}}\right] \tag{19}$$

The susceptances contained in the polygon network can be evaluated solving a linear equation system, starting from the knowledge of ζ^{rk}, Equation (19), as described in [12]. The circuit parameters evaluated in the identification/synthesis procedure are related to the "kind" of the analyzed device. Hence, they can be constants or varied with the frequency, depending on the actual device.

Obviously, Equations (15)–(17) can be easily extended to the case of N-port device, replacing 4 with N. In this case, the equivalent circuit contains N^2 electrical parameters, just as the number of the scattering parameters of the $N \times N$ S-matrix. In fact, the total number of the circuit parameters are:

- N transmission lines of electrical lengths $\theta_k, k = 1, 2, \ldots, N$, connecting the input ports to the non reciprocal kernel.
- $N\frac{N-1}{2}$ gyrators placed at the ports. In fact, in the hypothesis that a gyrator links port i to port j:
  - $N-1$ gyrators are placed at the first port (to link the first port to the other $N-1$ ports).
  - $N-2$ gyrators are placed at the second port (to link the second port to the other $N-2$ ports, except the first port that is linked with the gyrator described in the previous item).
  - $N-3$ gyrators are placed at the third port (to link the third port to the other $N-3$ ports, except the first and the second ports that are linked with the gyrators described in the two previous items).
  - ....
  - 1 gyrator is placed at port $N-1$ (to link the port $N-1$ to the last port).
- N susceptances placed at the side of the polygon and $N\frac{N-3}{2}$ susceptances placed at the polygon diagonals (overall $N\frac{N-1}{2}$ susceptances).

The sum of the electrical parameters is: N (electrical lengths) + $N\frac{N-1}{2}$ (gyrators) $+N$ (susceptances at sides) $+N\frac{N-3}{2}$ (susceptances at diagonals) $= N^2$. Hence, the proposed equivalent circuit is a minimal representation for any N-port non reciprocal lossless device.

2.2. Nonreciprocal Lossy Device

The power loss in actual microwave device is due to the presence of waveguide discontinuities (inductive/capacitive irises, cavities, ...) in the device kernel where the field can be very strong. For example, in a capacitive diaphragm the electric field is perpendicular to the edges of the metallic partitions of the waveguide. On these edges, the electric field (sum of the incident and reflected fundamental mode and the excited evanescent modes) tends to be very strong, and it causes a "large" amount of power loss just on the diaphragm surface.

A second cause of loss is the attenuation that the fundamental mode suffers during propagation in the cavities, due to the conducting regular waveguide walls. The attenuation can be modeled with the classical approaches described in [1, 2]. This effect becomes substantial if the electromagnetic field propagates up and down many times, as it happens in a cavity, but it can be neglected if the electromagnetic field travels only one or two times in the waveguide, as in the input lines of the device. In fact, in these lines the electromagnetic field associated with the fundamental mode travels towards the kernel of the device is reflected and travels backwards the input port. Hence, for only two travels, the losses due to the conducting waveguide walls can be neglected, and the input lines can be considered lossless. Hence, the equivalent circuit should satisfy such a characteristic, and it should include the loss only in the kernel.

Starting from the circuit developed in Subsection 2.1, we can expect that the Z-matrix of the kernel device will be a complex quantity. A first hypothesis to define the lossy equivalent circuit can be to replace the susceptances of the polygon network with admittances, where the conductances should represent the effect of the power loss. This approach can be easily developed, but it causes a non-realizable equivalent circuit, because some conductances are positive while others are negative. The global amount of loss is exactly that imposed by the actual device, but this circuit implies the presence of active device that is not present in actual device. Hence, this simple approach is not correct.

The solution lies in following the approach developed in [3, 10] which suggests to represent the Z-matrix of the nonreciprocal lossy kernel, ζ^{lnrk}, which relates voltages v_{i} and currents i_{i}, $i = 1, 2, \ldots, N$, at the ports of the nonreciprocal lossy kernel, as the series of two Z-matrices:

$$\zeta^{\text{lnrk}} = \zeta^{\text{nrk}} + \zeta^{\text{loss}} \tag{20}$$

$$\zeta^{\text{nrk}} = \frac{1}{2}\left[\zeta^{\text{lnrk}} - \left(\zeta^{\text{lnrk}}\right)^{\dagger}\right] \tag{21}$$

$$\zeta^{\text{loss}} = \frac{1}{2}\left[\zeta^{\text{lnrk}} + \left(\zeta^{\text{lnrk}}\right)^{\dagger}\right] \tag{22}$$

The first, ζ^{nrk}, is the Z-matrix of the nonreciprocal lossless kernel relating voltages $v_{\text{i}}^{\text{nrk}}$ and currents i_{i}, and the second, ζ^{loss}, is the Z-matrix of the nonreciprocal lossy kernel of the device relating voltages $v_{\text{i}}^{\text{loss}}$ and currents i_{i}. Some properties of ζ^{nrk} and ζ^{loss} should be highlighted:

- from Equation (21), ζ^{nrk} is a lossless "skew-hermitian" matrix and satisfies

$$\zeta^{\text{nrk}} = -\left(\zeta^{\text{nrk}}\right)^{\dagger} \tag{23}$$

  with $\text{Re}[\zeta_{ii}^{\text{nrk}}] = 0$, i.e., the elements of the main diagonal are imaginary numbers. Moreover, $\zeta_{ji}^{\text{nrk}} = -\left(\zeta_{ij}^{\text{nrk}}\right)^{*}$, or $\zeta_{ji}^{\text{nrk}} = -\alpha_{ij} + jx_{ij}$ and $\zeta_{ij}^{\text{nrk}} = \alpha_{ij} + jx_{ij}$.

- from Equation (22), ζ^{loss} is a lossy "hermitian" matrix and satisfies

$$\zeta^{\text{loss}} = \left(\zeta^{\text{loss}}\right)^{\dagger} \tag{24}$$

  with $\zeta_{ji}^{\text{loss}} = (\zeta_{ij}^{\text{loss}})^{*}$ and $\text{Im}[\zeta^{\text{loss}} ii] = 0$, i.e., the elements of the main diagonal are real numbers.

Hence, ζ^{nrk} is the lossless skew-hermitian Z-matrix which has been discussed in Section 2.1, and it can be identified with the equivalent circuit of Fig. 2(a), with the input lines being lossless, as previously discussed. Joining this circuit and Equation (20), the 4-port equivalent circuit proposed in this approach to solve the case of nonreciprocal lossy device is shown Fig. 3, where ζ^{nrk} and ζ^{loss} represent the Z-matrices to be realized.

The four lines with electrical lengths $\theta_1, \theta_2, \theta_3, \theta_4$ are obtained applying conditions in Eqs. (15)–(17) to ζ^{nrk}, and ζ^{nrk} is represented with gyrators and the susceptances polygon network, as previously discussed. The second Z-matrix, ζ^{loss}, can be realized with an equivalent circuit with susceptances and resistance because it is a matrix of positive definite or semi-definite hermitian form [3, 8].

The synthesis of ζ^{loss} is little bit complex, and details are discussed in Appendix A. The following procedure should be applied to realize ζ^{loss}:

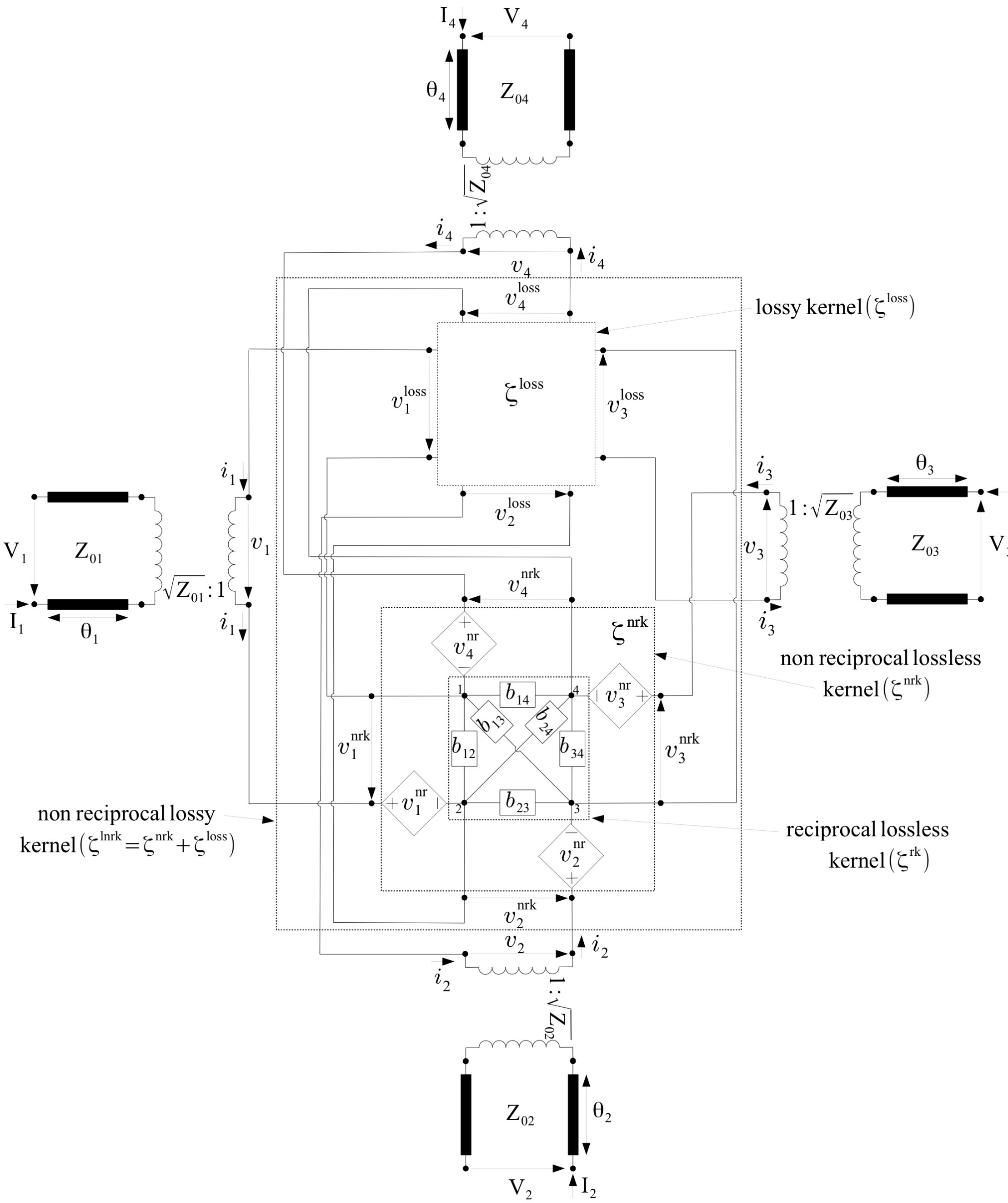


Figure 3. The equivalent circuit of 4-port non reciprocal lossy device.

- evaluation of the S-matrix $S^{\text{loss}\rightarrow\text{dia}}$ which transforms ζ^{loss} in a diagonal matrix, Equation (A11)

$$S^{\text{loss}\rightarrow\text{dia}} = \begin{bmatrix} 0_{4\times4} & u \\ u^{\dagger} & 0_{4\times4} \end{bmatrix} \tag{25}$$

  where u is the 4×4 matrix containing the eigenvectors of ζ^{loss}, written as columns.
- mathematically connect $2N = 8$ admittances $-jb_{\text{i}}^{\text{z}}$, $i = 1, 2, \ldots, 2N$ to the $2N$ ports of $S^{\text{loss}\rightarrow\text{dia}}$, obtaining a new S-matrix $S_{\text{z}}^{\text{loss}\rightarrow\text{dia}}$ which can be realized with a $2N$-port susceptances polygon network and $2N$ transmission lines. The choice of b_{i}^{z}, $i = 1, \ldots, 2N$ is arbitrary.

- once the polygon network has been evaluated, connect $2N$ admittances $jb_{\mathrm{i}}^{\mathrm{z}}$, $i = 1, \ldots, 2N$ to the corresponding $2N$ ports
- load the output $N = 4$ ports with N resistive loads $r_{\mathrm{i}}^{\mathrm{load}}$, equal to the i-th eigenvalue of ζ^{loss}, for $i = 1, \ldots, N$.

The resulting equivalent circuit of ζ^{loss} is shown in Fig. 4, where CCVSs are related to the gyrator amplitudes $\alpha_{kj}^{\mathrm{loss}}$, evaluated in the identification of ζ^{loss}, as discussed in Subsection 2.1:

$$v_k^{\mathrm{nrl}} = -\sum_{j=1}^{k-1} \alpha_{jk}^{\mathrm{loss}} i_k + \sum_{j=k+1}^{2N} \alpha_{kj}^{\mathrm{loss}} i_k \quad k = 1, 2, \ldots, 2N \ \ (N = 4) \tag{26}$$

This procedure ends the identification process.

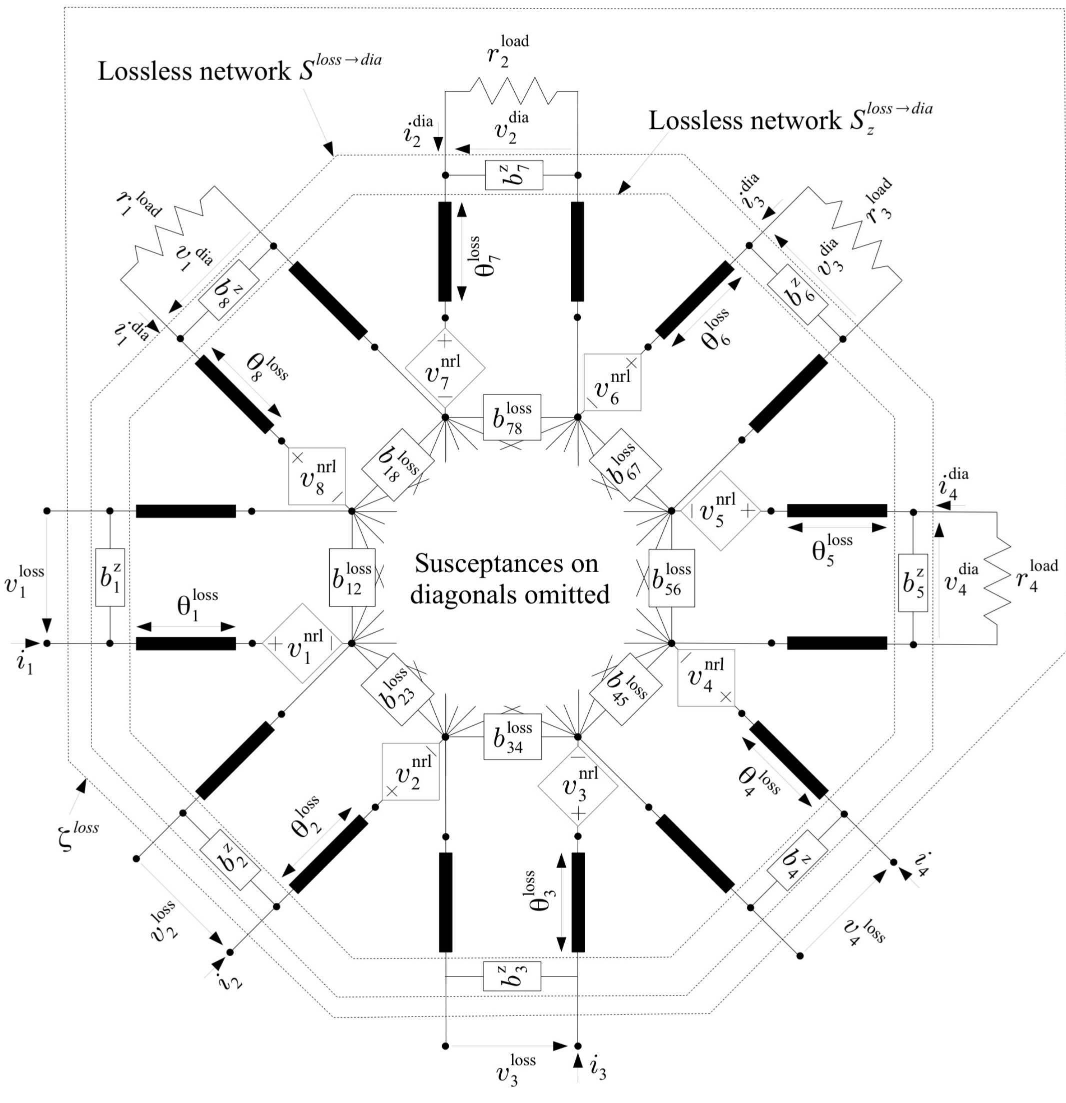
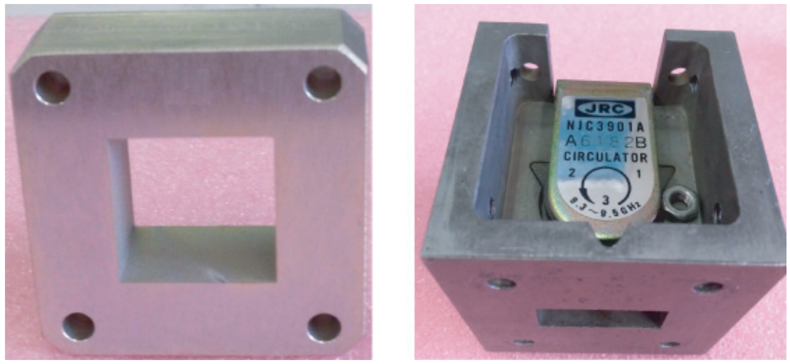
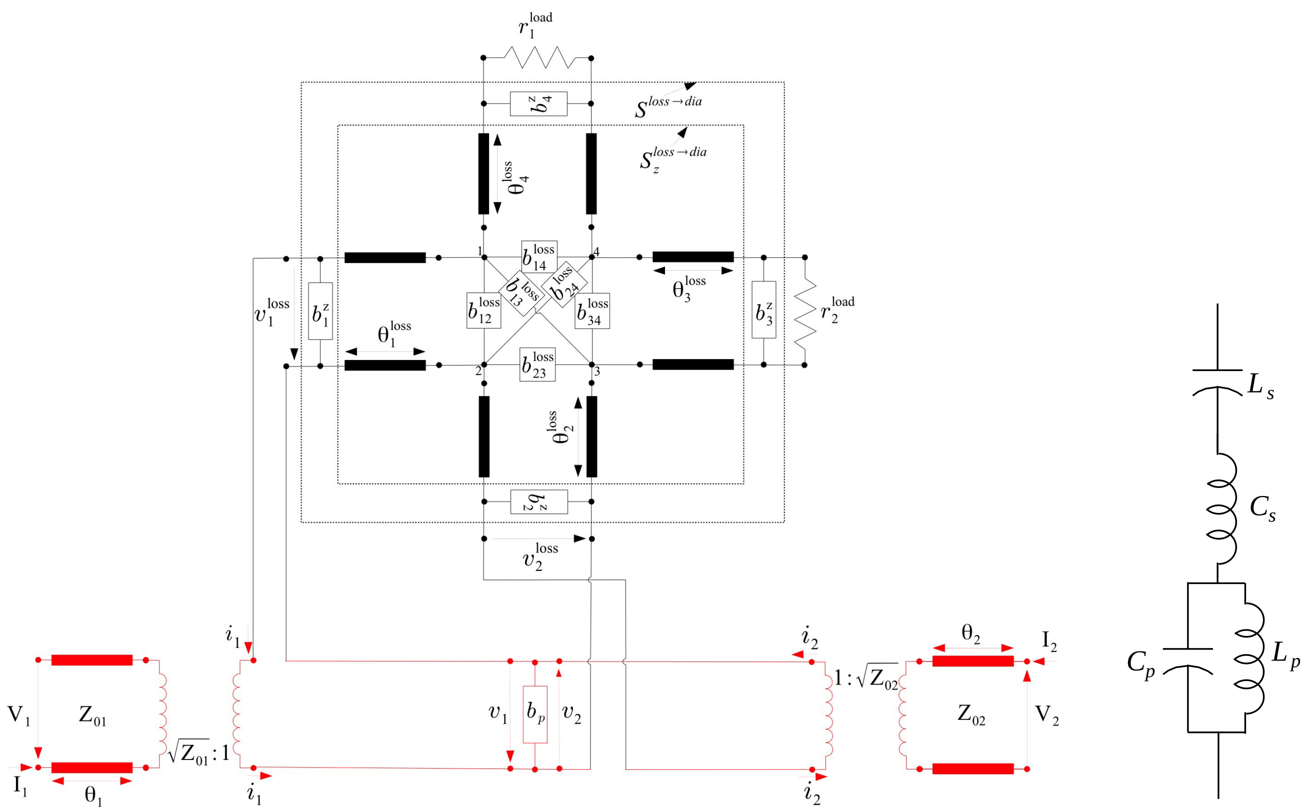


Figure 4. The equivalent circuit of ζ^{loss} for a 4-port device. Susceptances $b_{\mathrm{i}}^{\mathrm{z}}$ are placed at ports $1, 2, \ldots, 8$ to transform $S_{\mathrm{z}}^{\mathrm{loss}\rightarrow\mathrm{dia}}$ in $S^{\mathrm{loss}\rightarrow\mathrm{dia}}$. $r_1^{\mathrm{load}}, r_2^{\mathrm{load}}, r_3^{\mathrm{load}}, r_4^{\mathrm{load}}$ are equal to the eigenvalues of ζ^{loss} and are the resistive loads connected to the output ports of the diagonalized impedance matrix ζ^{dia}. The susceptances in the inner diagonals of the octagon are not drawn for simplicity.



(a) (b)

Figure 5. (a) Square reciprocal iris (side 19 mm, depth 15 mm) inserted between two rectangular WR90 waveguides. (b) 3-port circulator in rectangular WR90 waveguide.



(a) (b)

Figure 6. (a) The equivalent circuit of 2-port reciprocal lossy iris in WR90 waveguide. The lossless skew-hermitian part is drawn with red color and the lossy hermitian part with black color. (b) Realization of the susceptance b_p contained in the skew-hermitian lossless part of the equivalent circuit.

3. RESULTS

The first case is the identification of the S-matrix of a reciprocal lossy 2-port waveguide iris, as shown in Fig. 5(a). A square iris (side 19 mm, depth 15 mm) is placed between two rectangular WR90 waveguides ($a = 22.86$ mm, $b = 10.16$ mm). Agilent 8510C VNA has been used to measure its scattering parameters in the X band and, with these measurements, the equivalent circuit shown in Fig. 6(a) has been identified, and the corresponding electrical parameters are shown in Fig. 7. The normalized susceptances b_i^z which delete the eigenvalue $\lambda = 1$ of order 2 in $S^{\text{loss}\to\text{dia}}$ have been chosen to ensure that the diagonal elements of $S_z^{\text{loss}\to\text{dia}}$ have equal magnitude: $b_i^z = 0.535$, $i = 1, 2, 3, 4$. In doing so, some symmetries in the equivalent circuit of the lossy part can be obtained. In fact, it can be seen from Figs. 7(a)–7(b) that $\theta_1^{\text{loss}} = \theta_3^{\text{loss}}, \theta_2^{\text{loss}} = \theta_4^{\text{loss}}$ and $b_{12}^{\text{loss}} = b_{14}^{\text{loss}}, b_{23}^{\text{loss}} = b_{34}^{\text{loss}}$. Obviously, other choices for b_i^z are permitted.

The resistive loads shown in Fig. 7(c) are not equal because of the presence of measurement errors at the two ports of VNA and a non-perfect connection between the iris and the two waveguides.

In fact, a small misalignment causes a different effect on the losses at the two front surfaces of the iris, producing a small difference between the measured $\left|S_{11}^{\text{exp lossy}}\right|$ and $\left|S_{22}^{\text{exp lossy}}\right|$, as shown in Fig. 7(d) (black and blue dotted lines). This misalignment is confirmed also by the little spike appearing in $\left|S_{11}^{\text{exp lossy}}\right|$, $\left|S_{12}^{\text{exp lossy}}\right|$, $\left|S_{22}^{\text{exp lossy}}\right|$ at about 11.2 GHz. In fact, modes TE_{11} and TM_{11} of the square iris

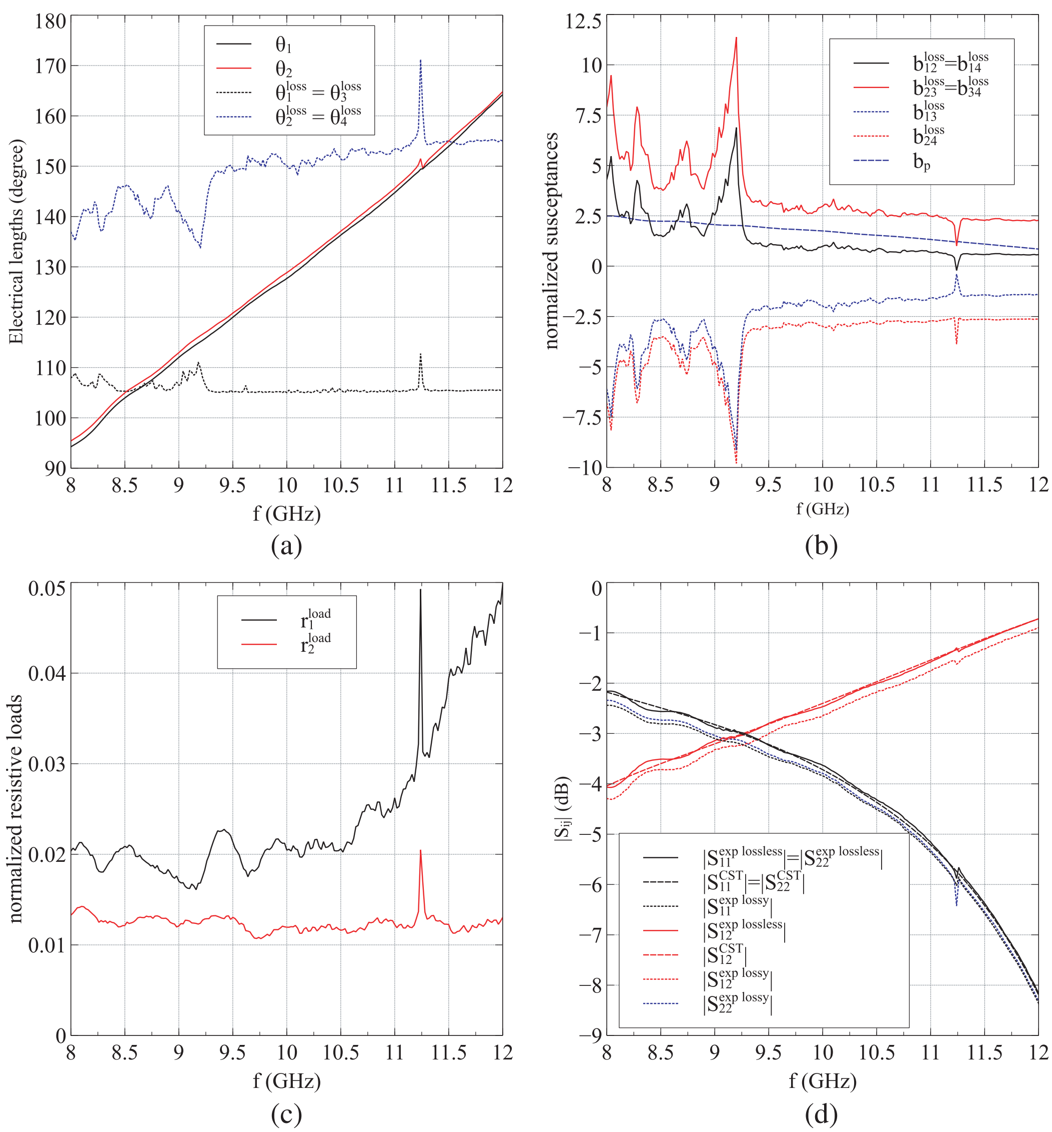


Figure 7. (a)–(c) Electrical lengths, susceptances and resistive loads of the equivalent circuit shown in Fig. 6(a), relative to the iris of Fig. 5(a). (d) Scattering coefficients of the "experimental lossless" iris (continuous lines), CST simulations (dashed lines) and the experimental results for the lossy actual iris (dotted lines).

have a frequency cutoff at about 11.16 GHz. If the alignment were perfect, these modes should not be excited by the discontinuity. Hence, the little spike at about 11.20 GHz means that these modes are excited by a little misalignment between the iris and the two waveguides. The effect of this spike is evident in the electric circuit parameters shown in Figs. 7(a)–7(d). Finally, the misalignment accounts for the little difference between θ_1 and θ_2 in Fig. 7(a), which should be equal for a centered iris.

The normalized susceptances b_p relative to the lossless skew-hermitian part can be realized with the circuit shown in Fig. 6(b), with $L_s = 5.63 \cdot 10^{-12}$, $C_s = 9.95 \cdot 10^{-9}$, $L_p = 1.49 \cdot 10^{-12}$, and $C_p = 1.05 \cdot 10^{-10}$. The normalized susceptances b_{ij}^{loss} and load resistances relative to the lossy hermitian part have a behavior quite difficult to be realized with the same precision used for b_p. Anyway, the effect of the lossy part is small compared to the lossless part of the circuit, because the iris has a low amount of loss. Hence, these electrical parameters can be approximated with a constant value obtained with an integral mean over the whole band: $r_1 \approx 0.029$, $r_2 \approx 0.012$, $b_{12}^{\mathrm{loss}} = b_{14}^{\mathrm{loss}} \approx 1.42$, $b_{23}^{\mathrm{loss}} = b_{34}^{\mathrm{loss}} \approx 3.63$, $b_{13}^{\mathrm{loss}} \approx -2.52$, $b_{24}^{\mathrm{loss}} \approx -3.54$. The S-matrix obtained with these inductances/capacitances/resistances is in a very good agreement with the experimental results shown in Fig. 7(d) (about ± 0.15 dB), and they are not shown for simplicity.

During measurements, the effect of the losses cannot be deleted, because they are inseparable from the actual device, but it is interesting to wonder what happens if the lossy part of the equivalent circuit is deleted, replacing r_1^{load} and r_2^{load} with short circuits. The readers may think that the "experimental results for lossless" iris are obtained. Hence, the scattering coefficients for the "experimental lossless" iris, obtained with that replacement, have been evaluated, and they are shown in Fig. 7(d) with continuous lines. The "experimental lossless" and the CST simulations (dashed lines) for the lossless iris are very close in Fig. 7(d). Hence, from the experimental results on lossy device, the "experimental lossless" results can be evaluated from the equivalent circuit and compared with numerical simulations in order to understand how close the simulations are to the "experimental lossless" device, which can never be measured because the losses are unavoidable. If they are very close, the differences between the

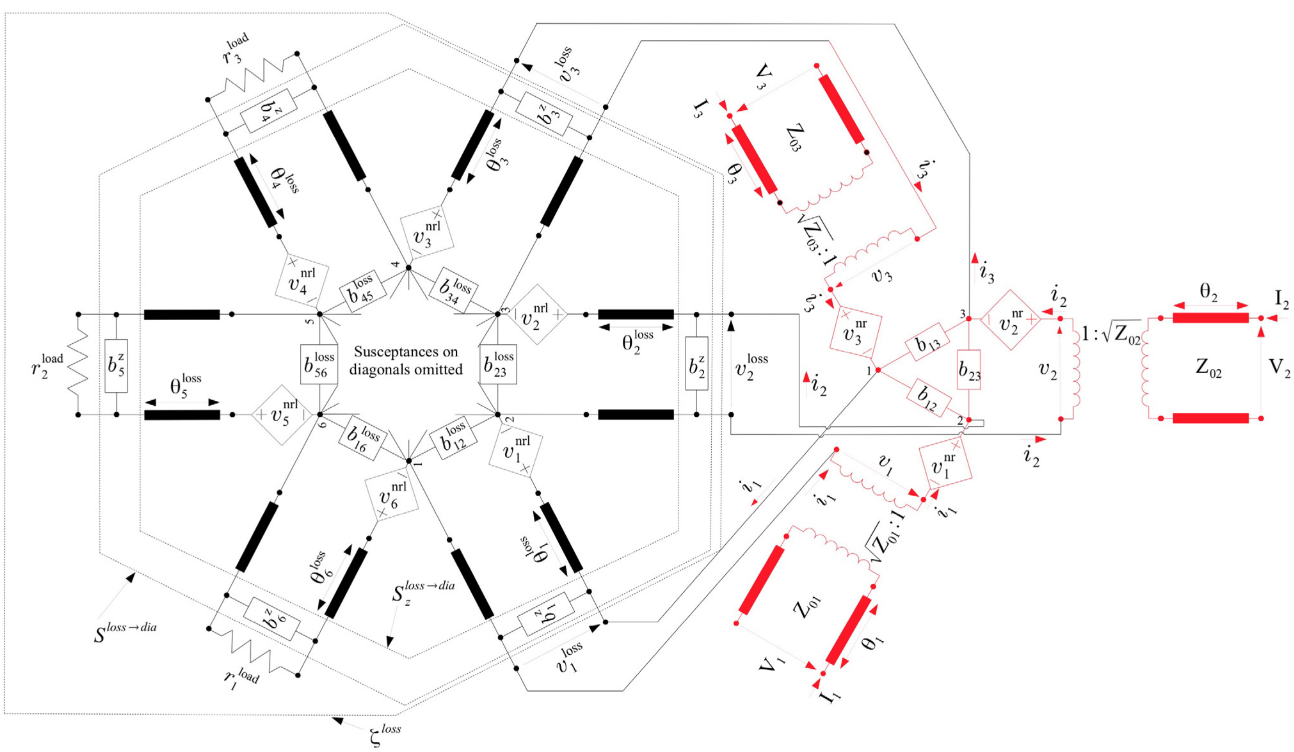
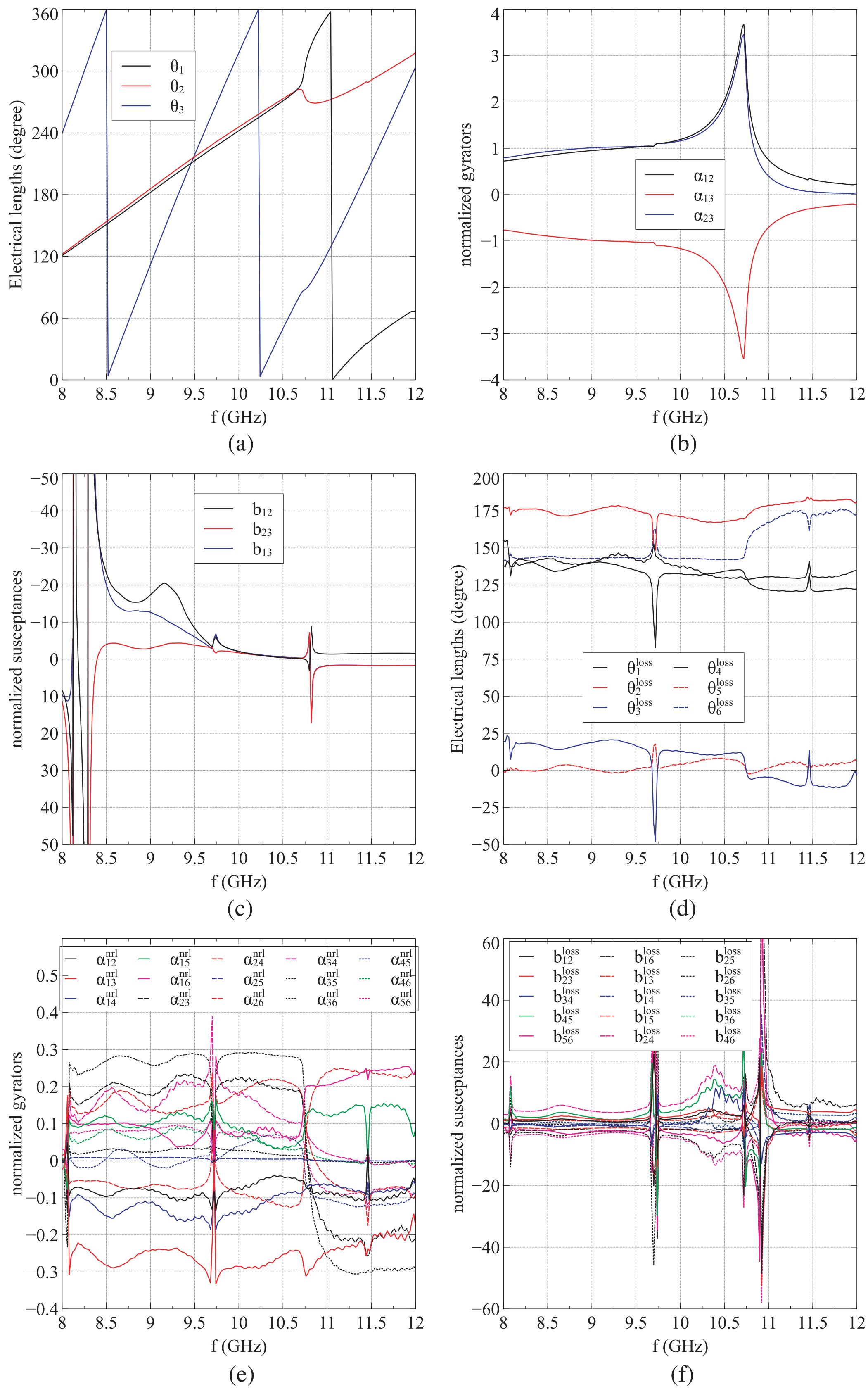


Figure 8. The equivalent circuit of 3-port non reciprocal lossy circulator in WR90 waveguide, shown in Fig. 5(b). The lossless skew-hermitian part is drawn with red color (right side) and the lossy hermitian part with black color (left side).


Electrical lengths (degree)
f (GHz)
θ_1
θ_2
θ_3
(a)
normalized gyrators
f (GHz)
α_{12}
α_{13}
α_{23}
(b)
normalized susceptances
f (GHz)
b_{12}
b_{23}
b_{13}
(c)
Electrical lengths (degree)
f (GHz)
θ_1^{loss} θ_4^{loss}
θ_2^{loss} θ_5^{loss}
θ_3^{loss} θ_6^{loss}
(d)
normalized gyrators
f (GHz)
α_{12}^{nrl} α_{15}^{nrl} α_{24}^{nrl} α_{34}^{nrl} α_{45}^{nrl}
α_{13}^{nrl} α_{16}^{nrl} α_{25}^{nrl} α_{35}^{nrl} α_{46}^{nrl}
α_{14}^{nrl} α_{23}^{nrl} α_{26}^{nrl} α_{36}^{nrl} α_{56}^{nrl}
(e)
normalized susceptances
f (GHz)
b_{12}^{loss} b_{16}^{loss} b_{25}^{loss}
b_{23}^{loss} b_{13}^{loss} b_{26}^{loss}
b_{34}^{loss} b_{14}^{loss} b_{35}^{loss}
b_{45}^{loss} b_{15}^{loss} b_{36}^{loss}
b_{56}^{loss} b_{24}^{loss} b_{46}^{loss}
(f)

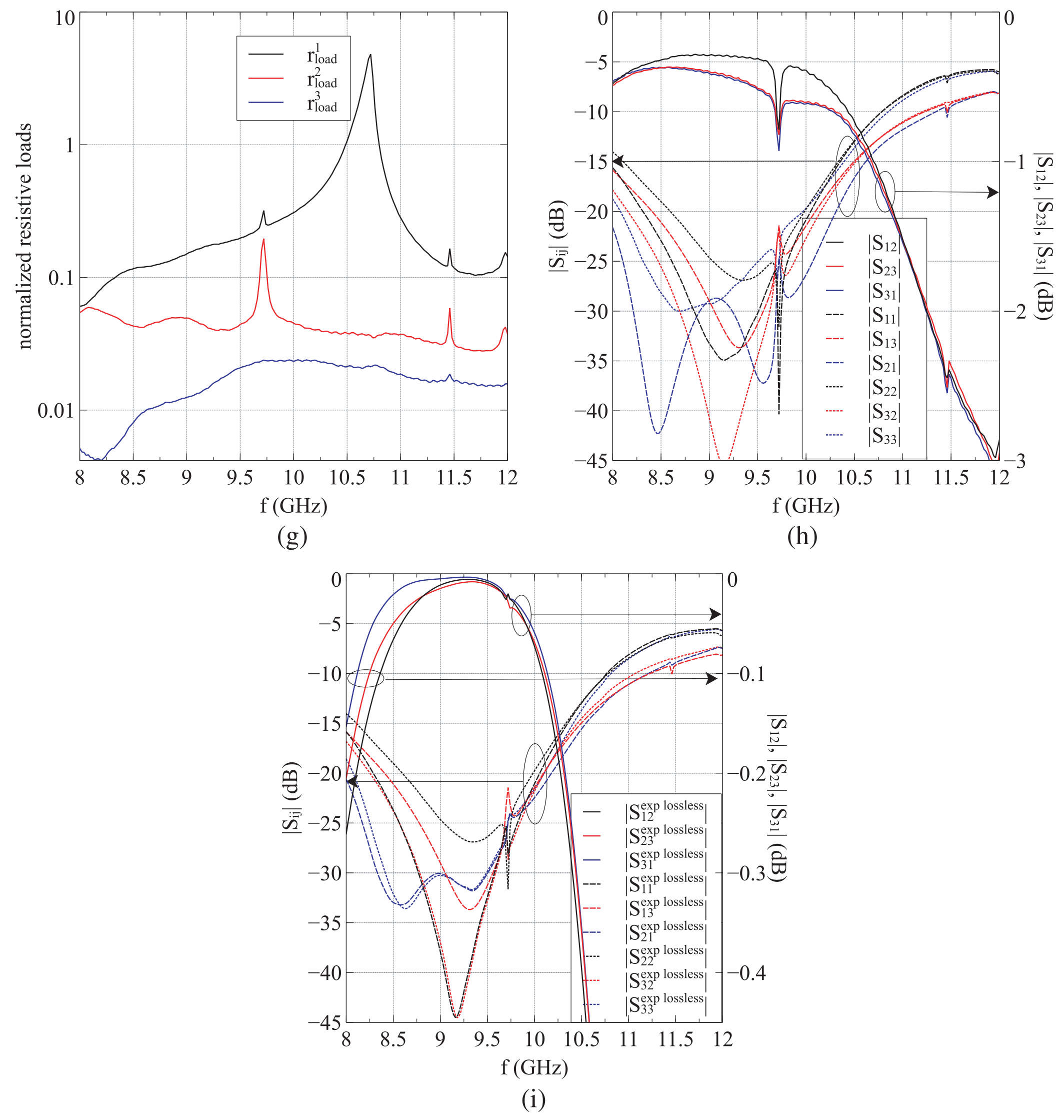


Figure 9. (a)–(f) Electrical lengths, gyrators and susceptances of the equivalent circuit shown in Fig. 8, relative to the circulator of Fig. 5(b). (g) Resistive loads of the equivalent circuit shown in Fig. 8, relative to the circulator of Fig. 5(b). (h) The experimental scattering coefficients of the circulator. (i) The scattering coefficients of the "experimental lossless" circulator, obtained from the circuit of Fig. 8 replacing the resistive loads with short circuits, to delete the effect of the losses.

experimental lossy results and the numerical simulations are simply due to the device loss. Otherwise, there can be some discrepancy between the realized irises and those obtained in the phase design. This can enhance the optimization of filters, diplexers and other devices.

The second analyzed device is the 3-port circulator in WR90 rectangular waveguide acting in the X-band, shown in Fig. 5(b). The experimental results for the 3-port circulator shown in Fig. 9(h) have been used to identify the circuit shown in Fig. 8: the nonreciprocal lossless skew-hermitian part is identified with a triangle of susceptances and three CCVSs at its ports (right side of the figure, red color); the nonreciprocal lossy hermitian part is identified with a polygon network with six sides and three ports $(4, 5, 6)$ closed on three resistive loads, equal to the eigenvalues of ζ^{loss} (left side of the figure, black color). The normalized susceptances b^{z}_{i} which delete the eigenvalue $\lambda = 1$ of order 3 in $S^{\text{loss}\to\text{dia}}$ have been chosen equal to $b^{\text{z}}_{\text{i}} = (-1)^{\text{i}} 0.5\text{i}$, $i = 1, \ldots, 6$. The electrical lengths, normalized susceptances, gyrator amplitudes and resistive loads are shown in Figs. 9(a)–9(g). It is interesting to evaluate the "experimental lossless" case, obtained replacing the three resistive loads with short circuits: these results are shown in Fig. 9(i). It should be noted that the "experimental lossless" circulator works well in the band 9.3–9.5 GHz, where $|S_{12}|, |S_{23}|, |S_{31}|$ are very similar, as declared by the manufacturer (see Fig. 5(b)). Hence, design is probably correct. On the other hand, comparing 9(h) and 9(i), it is evident that the losses act on $|S_{12}|$ in a different way from that on $|S_{23}|$ and $|S_{31}|$. This behavior can be of interest for the manufacturer, which can enhance the circulator design to make $|S_{12}| \approx |S_{23}| \approx |S_{31}|$ in the band of interest.

4. CONCLUSIONS

The equivalent circuit for nonreciprocal lossy N-port device has been obtained joining the classic synthesis technique and the recent polygon network, which permits to draw the equivalent circuit in a very simple way for any number of ports. The technique has been applied to the identification of the equivalent circuit of a reciprocal lossy iris and of a non reciprocal lossy 3-port circulator. The equivalent circuit can permit to delete the effect of the losses replacing the resistive loads, which represent the loss, with short circuits, to enhance the design of the device, which is usually done in the hypothesis of lossless components.

APPENDIX A.

The procedure to synthesize ζ^{loss} is little bit complex, and some preliminary remarks must be done.

Hermitian nonreciprocal Z-matrix ζ^{loss}, Equation (22), relates the voltages at the input, $v_{\text{i}}^{\text{loss}}$, $i = 1, 2, 3, 4$, to the current i_{i}, as shown in Fig. 3:

$$v^{\text{loss}} = \begin{bmatrix} v_1^{\text{loss}} \\ v_2^{\text{loss}} \\ v_3^{\text{loss}} \\ v_4^{\text{loss}} \end{bmatrix} = \zeta^{\text{loss}} i = \zeta^{\text{loss}} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \tag{A1}$$

The identification of ζ^{loss} is not simple as ζ^{nrk}, because ζ^{loss} satisfies Equation (24), which implies that the nonreciprocity afflicts the imaginary part of ζ^{loss}. Hence, non reciprocity of ζ^{loss} cannot be represented with real gyrators as done for ζ^{nrk}.

However, a solution can be found if a transformation on voltages $v_{\text{i}}^{\text{loss}}$ and currents i_{i} is applied, as suggested in [3, 8]. To do this, we must observe that ζ^{loss} is a Positive Definite (PD) matrix of an hermitian form, because it is related to the dissipated power in the device. In fact, if ζ^{loss} is excited by N currents, say the vector i_{exc} of dimensions $N \times 1$, the dissipated power in ζ^{loss}, P_{loss}, is

$$P_{\text{loss}} = (i_{\text{exc}})^{\dagger} \zeta^{\text{loss}} i_{\text{exc}} \tag{A2}$$

which is always a positive real quantity. This is just the definition of a Positive Definite hermitian matrix that has some properties [17]:

- the eigenvalues of a matrix of an hermitian PD form (ζ^{loss}) are always positive;
- if the eigenvectors of ζ^{loss} are written as the columns of a matrix u, then

$$u\, u^{\dagger} = u^{\dagger}\, u = I \Rightarrow u^{-1} = u^{\dagger} \tag{A3}$$

- a matrix of an hermitian PD form (ζ^{loss}) can be diagonalized with the help of its eigenvectors. In fact, if the eigenvectors are put in the columns of the matrix u, it is possible to define a diagonal matrix from ζ^{loss} such that

$$\zeta^{\text{loss}} = u\, \zeta^{\text{dia}}\, u^{\dagger} \quad \Rightarrow \quad \zeta^{\text{dia}} = u^{\dagger}\, \zeta^{\text{loss}}\, u \tag{A4}$$

where the elements of the diagonal matrix ζ^{dia} are just the real positive eigenvalues of ζ^{loss}.

From Equations (A1) and (A4):

$$v^{\mathrm{loss}} = \zeta^{\mathrm{loss}}\ i = u\ \zeta^{\mathrm{dia}}\ u^{\dagger}\ i \tag{A5}$$

or

$$u^{-1}\ v^{\mathrm{loss}} = u^{\dagger}\ v^{\mathrm{loss}} = \zeta^{\mathrm{dia}}\ u^{\dagger}\ i \tag{A6}$$

Setting

$$v^{\mathrm{dia}} = u^{\dagger}\ v^{\mathrm{loss}} \tag{A7}$$
$$i^{\mathrm{dia}} = u^{\dagger}\ i \tag{A8}$$

we obtain,

$$v^{\mathrm{dia}} = \zeta^{\mathrm{dia}}\ i^{\mathrm{dia}} = \mathrm{diag}\left[\lambda_1^{\zeta^{\mathrm{loss}}}, \lambda_2^{\zeta^{\mathrm{loss}}}, \lambda_3^{\zeta^{\mathrm{loss}}}, \lambda_4^{\zeta^{\mathrm{loss}}}\right] i^{\mathrm{dia}} = \mathrm{diag}\left[r_1^{\mathrm{load}}, r_2^{\mathrm{load}}, r_3^{\mathrm{load}}, r_4^{\mathrm{load}}\right] i^{\mathrm{dia}} \tag{A9}$$

where $\lambda_{\mathrm{i}}^{\zeta^{\mathrm{loss}}}$ is the i-th eigenvalue of ζ^{loss}. Equations (A7)–(A8) represent a linear transformation between (v^{loss}, i) and $(v^{\mathrm{dia}}, i^{\mathrm{dia}})$, which are related by Equation (A9), representing a diagonal matrix, ζ^{dia}, that connects the voltage at the i-th port to the current at the same port with a resistor equal to the i-th eigenvalue of ζ^{loss}, as shown in Fig. A1(a).

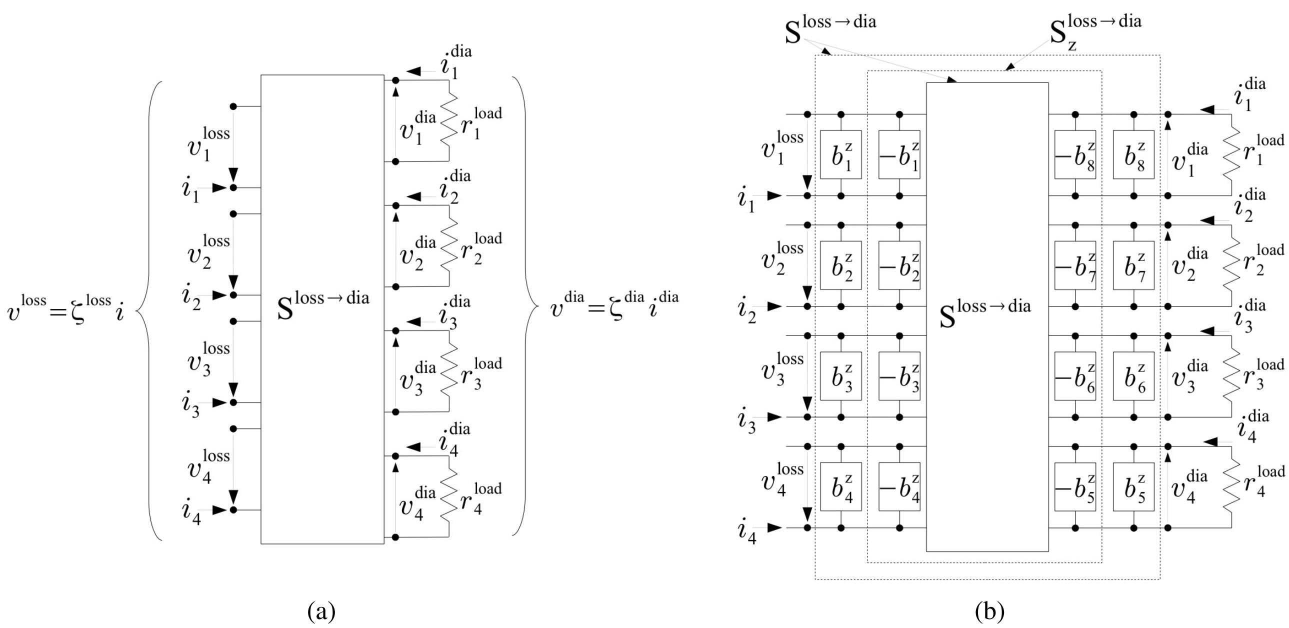


Figure A1. (a) Transformation of the hermitian matrix ζ^{loss} into the diagonal matrix ζ^{dia} connected to resistive loads. (b) Insertion of shunt susceptances at the ports of $S^{\mathrm{loss}\to\mathrm{dia}}$ to define the S-matrix $S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}}$ which can be realized with a polygon susceptances network.

The linear transformation in Equations (A7)–(A8) can be represented by the following matrix

$$\begin{bmatrix} v^{\mathrm{dia}} \\ i^{\mathrm{dia}} \end{bmatrix} = \begin{bmatrix} u^{\dagger}_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & u^{\dagger}_{N\times N} \end{bmatrix} \begin{bmatrix} v^{\mathrm{loss}} \\ i \end{bmatrix} = T^{\mathrm{loss}\to\mathrm{dia}} \begin{bmatrix} v^{\mathrm{loss}} \\ i \end{bmatrix} \tag{A10}$$

where $N = 4$ and $T^{\mathrm{loss}\to\mathrm{dia}}$ is a lossless $2N \times 2N$, (8×8), matrix representing the linear transformation to diagonalize ζ^{loss}. Hence, the realization of ζ^{loss} has been transformed in the realization of $T^{\mathrm{loss}\to\mathrm{dia}}$. This problem has been solved in [3, 8] making use of "complex transformers" which are quite difficult to realize [11], especially if the number of ports increases. The solution proposed in this approach exploits the polygon network approach previously defined.

First of all, the lossless S-matrix corresponding to $T^{\text{loss}\to\text{dia}}$ is:

$$S^{\text{loss}\to\text{dia}} = \begin{bmatrix} 0_{4\times4} & \left(u^{\dagger}\right)^{-1} \\ u^{\dagger} & 0_{4\times4} \end{bmatrix} = \begin{bmatrix} 0_{4\times4} & u \\ u^{\dagger} & 0_{4\times4} \end{bmatrix} \tag{A11}$$

where u is the 4×4 matrix containing the eigenvectors of ζ^{loss}, written as columns.

In order to obtain the lossless matrix $S^{\text{loss}\to\text{dia}}$, the eigenvectors of ζ^{loss} must be normalized to ensure that the sum of the squares of the magnitude components of each eigenvector is equal to 1: $\sum_{k=1}^{4}|u_{ki}|^2 = 1,\ i = 1,2,3,4$. In fact

$$\begin{aligned} S^{\text{loss}\to\text{dia}} \cdot \left(S^{\text{loss}\to\text{dia}}\right)^{\dagger} &= \begin{bmatrix} 0_{4\times4} & u \\ u^{\dagger} & 0_{4\times4} \end{bmatrix} \begin{bmatrix} 0_{4\times4} & u \\ u^{\dagger} & 0_{4\times4} \end{bmatrix}^{\dagger} = \begin{bmatrix} 0_{4\times4} & u \\ u^{\dagger} & 0_{4\times4} \end{bmatrix} \begin{bmatrix} 0_{4\times4} & u \\ u^{\dagger} & 0_{4\times4} \end{bmatrix} \\ &= \operatorname{diag}\left[\sum_{k=1}^{4}|u_{k1}|^2, \sum_{k=1}^{4}|u_{k2}|^2, \sum_{k=1}^{4}|u_{k3}|^2, \sum_{k=1}^{4}|u_{k4}|^2,\right. \\ &\qquad \left.\sum_{k=1}^{4}|u_{k1}|^2, \sum_{k=1}^{4}|u_{k2}|^2, \sum_{k=1}^{4}|u_{k3}|^2, \sum_{k=1}^{4}|u_{k4}|^2\right] = I \end{aligned} \tag{A12}$$

Recalling that for a block matrix

$$\det[M] = \det[M_{22}]\det\left[M_{11} - M_{12}M_{22}^{-1}M_{21}\right] \tag{A13}$$

where M_{ij} are the blocks of matrix M, the eigenvalues of $S^{\text{loss}\to\text{dia}}$ are obtained from Equations (A3) and (A11):

$$\begin{aligned} \det\left[S^{\text{loss}\to\text{dia}} - \lambda I\right] &= \det\begin{bmatrix} -\lambda I_{4\times4} & u \\ u^{\dagger} & -\lambda I_{4\times4} \end{bmatrix} = \det[-\lambda I_{4\times4}]\det\left[-\lambda I_{4\times4} - u\left(-\lambda I_{4\times4}\right)^{-1}u^{\dagger}\right] \\ &= \lambda^4\det\left[-\lambda I_{4\times4} + u\left(\frac{I_{4\times4}}{\lambda}\right)u^{\dagger}\right] = \lambda^4\det\left[\frac{\left(1-\lambda^2\right)I_{4\times4}}{\lambda}\right] = \\ &= \left(1-\lambda^2\right)^4 = 0 \end{aligned} \tag{A14}$$

Hence the eigenvalues of $S^{\text{loss}\to\text{dia}}$ are $\lambda = \pm1$ with order 4 and, by this reason, it is not possible to obtain the corresponding Z-matrix, $\zeta^{\text{loss}\to\text{dia}}$, from $S^{\text{loss}\to\text{dia}}$. In fact, the following relation holds

$$\zeta^{\text{loss}\to\text{dia}} = \left(I - S^{\text{loss}\to\text{dia}}\right)^{-1}\left(S^{\text{loss}\to\text{dia}} + I\right) \tag{A15}$$

and it is necessary that $I - S^{\text{loss}\to\text{dia}}$ is nonsingular, i.e., $\det(I - S^{\text{loss}\to\text{dia}}) \neq 0$. If $S^{\text{loss}\to\text{dia}}$ has eigenvalue $\lambda = 1$, from Equation (A14) it follows that $\det(I - S^{\text{loss}\to\text{dia}}) = 0$ and $\zeta^{\text{loss}\to\text{dia}}$ can not be defined. In this case, it is not possible to define the polygon network susceptances directly from $S^{\text{loss}\to\text{dia}}$, because such polygon network requires the definition of a Z-matrix.

Anyway, we can put at each port of $S^{\text{loss}\to\text{dia}}$ a shunt of two admittances, jb_{i}^{z} and $-jb_{\text{i}}^{\text{z}}$, without changing $S^{\text{loss}\to\text{dia}}$, as shown in Fig. A1(b). If we include the eight admittances $-jb_{\text{i}}^{\text{z}}$ in $S^{\text{loss}\to\text{dia}}$ we obtain a new S-matrix, $S_{\text{z}}^{\text{loss}\to\text{dia}}$, with all eigenvalues other than 1, as discussed in Appendix B. This ensures that the Z-matrix $\zeta_{z}^{\text{loss}\to\text{dia}}$ exists and that $S_{\text{z}}^{\text{loss}\to\text{dia}}$ can be identified with the susceptances polygon network and transmission lines. Actually only four admittances $-jb_{\text{i}}^{\text{z}}$ are sufficient to obtain the S-matrix $S_{\text{z}}^{\text{loss}\to\text{dia}}$ with all eigenvalues other than 1, but we prefer to include 8 admittances to impose some symmetry properties to $S_{\text{z}}^{\text{loss}\to\text{dia}}$ (for example, that all the diagonal elements of $S_{\text{z}}^{\text{loss}\to\text{dia}}$ have the same magnitude).

Hence, to realize ζ^{loss} the following procedure applies ($N = 4$ in Fig. A1):

- evaluation of the scattering matrix $S^{\text{loss}\to\text{dia}}$ which transforms ζ^{loss} in a diagonal matrix, based on the eigenvectors of ζ^{loss}, as shown in Equation (A11).
- mathematically connect $2N$ admittances $-jb_{\text{i}}^{\text{z}}$, $i = 1,2,\ldots,2N$ to the $2N$ ports of $S^{\text{loss}\to\text{dia}}$, obtaining the new S-matrix $S_{\text{z}}^{\text{loss}\to\text{dia}}$ which can be realized with a $2N$-port susceptances polygon network and $2N$ transmission lines. The choice of b_{i}^{z}, $i = 1,\ldots,2N$ is arbitrary.

- once the polygon network has been evaluated, connect $2N$ admittances $jb_{\mathrm{i}}^{\mathrm{z}}$, $i = 1, \ldots, 2N$ to the corresponding $2N$ ports.
- load the output N ports with N resistive loads $r_{\mathrm{i}}^{\mathrm{load}}$, equal to the i-th eigenvalue of ζ^{loss}, for $i = 1, \ldots, N$.

The resulting equivalent circuit of ζ^{loss} is shown in Fig. 4, where the CCVS's are related to the gyrator amplitudes $\alpha_{kj}^{\mathrm{loss}}$, evaluated in the identification of ζ^{loss}, as discussed in Subsection 2.1:

$$v_k^{\mathrm{nrl}} = -\sum_{j=1}^{k-1} \alpha_{jk}^{\mathrm{loss}} i_k + \sum_{j=k+1}^{2N} \alpha_{kj}^{\mathrm{loss}} i_k \quad k = 1, 2, \ldots, 2N \;\; (N = 4) \tag{A16}$$

APPENDIX B.

In this appendix, it is shown that the $2N$-port S-matrix $S^{\mathrm{loss}\to\mathrm{dia}}$, which does not posses the corresponding Z-matrix because $S^{\mathrm{loss}\to\mathrm{dia}} - I$ is singular, can be transformed in a new matrix $S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}}$, with $S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}} - I$ not singular, connecting N admittances jb_k, $k = 1, 2, \ldots, N$ to N ports of $S^{\mathrm{loss}\to\mathrm{dia}}$.

The $2N$-port S-matrix $S^{\mathrm{loss}\to\mathrm{dia}}$ has N eigenvalues equal to 1 and N eigenvalues equal to -1, as discussed in Appendix A, Equation (A14). The matrix can be divided in blocks as follows

$$\begin{aligned} S^{\mathrm{loss}\to\mathrm{dia}} &= \begin{bmatrix} 0_{N\times N} & u_{N\times N} \\ u^{\dagger}_{N\times N} & 0_{N\times N} \end{bmatrix} = \left[\begin{array}{cc:c} 0_{N\times N} & u_{N\times N-1} & u^{\mathrm{LC}}_{N\times 1} \\ u^{\dagger}_{N-1\times N} & 0_{N-1\times N-1} & 0_{N-1\times 1} \\ \hdashline u^{\dagger,\mathrm{LR}}_{1\times N} & 0_{1\times N-1} & 0_{1\times 1} \end{array}\right] = \\ &= \begin{bmatrix} S^{11}_{(2N-1)\times(2N-1)} & S^{12}_{(2N-1)\times 1} \\ S^{21}_{1\times(2N-1)} & 0_{1\times 1} \end{bmatrix} \end{aligned} \tag{B1}$$

where $u^{\dagger}_{N-1\times N}$ and $u_{N\times N-1}$ are blocks $u^{\dagger}_{N\times N}$ and $u_{N\times N}$ without the last row, $u^{\dagger,\mathrm{LR}}_{1\times N}$, and the last column, $u^{\mathrm{LC}}_{N\times 1}$, respectively. Matrix u satisfies the following conditions:

$$1) \qquad I = u\,u^{\dagger} = \begin{bmatrix} u_{N\times N-1} & u^{\mathrm{LC}}_{N\times 1} \end{bmatrix} \begin{bmatrix} u^{\dagger}_{N-1\times N} \\ u^{\dagger,\mathrm{LR}}_{1\times N} \end{bmatrix} = \left[u_{N\times N-1} u^{\dagger}_{N-1\times N} + u^{\mathrm{LC}}_{N\times 1} u^{\dagger,\mathrm{LR}}_{1\times N} \right] \tag{B2}$$

$$2) \qquad \mathrm{rank}\left[u_{N\times N-1} u^{\dagger}_{N-1\times N} \right] = N-1 \Rightarrow \det\left[u_{N\times N-1} u^{\dagger}_{N-1\times N} \right] = 0 \tag{B3}$$

$$3) \qquad \mathrm{rank}\left[u^{\mathrm{LC}}_{N\times 1} u^{\dagger,\mathrm{LR}}_{1\times N} \right] = 1 \Rightarrow \det\left[u^{\mathrm{LC}}_{N\times 1} u^{\dagger,\mathrm{LR}}_{1\times N} \right] = 0 \tag{B4}$$

Condition 2) is obtained recalling that for a matrix A, $\mathrm{rank}[AA^{\dagger}] = \mathrm{rank}[A]$, [18], and condition 3) holds because all columns of $u^{\mathrm{LC}}_{N\times 1} u^{\dagger,\mathrm{LR}}_{1\times N}$ are equal to column vector $u^{\mathrm{LC}}_{N\times 1}$ times u_{k1}, being k the column index, and its rank is equal to 1.

If the last port of $S^{\mathrm{loss}\to\mathrm{dia}}$ is connected to a shunt admittance jb (Fig. A1(b)), with S-matrix S_b equal to

$$S_b = \begin{bmatrix} S_b^{11} & S_b^{12} \\ S_b^{12} & S_b^{11} \end{bmatrix} = \begin{bmatrix} -\dfrac{jb}{2+jb} & \dfrac{2}{2+jb} \\ \dfrac{2}{2+jb} & -\dfrac{jb}{2+jb} \end{bmatrix} \tag{B5}$$

with

$$S_b^{11} S_b^{11} - S_b^{12} S_b^{12} = -S_b^{11} - S_b^{12} \tag{B6}$$

$$S_b^{12} - S_b^{11} = 1 \tag{B7}$$

the new overall S-matrix, $S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}}$, is:

$$S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}} = \begin{bmatrix} S^{11} + S_b^{11} S^{12} S^{21} & S_b^{12} S^{12} \\ S_b^{12} S^{21} & S_b^{11} \end{bmatrix} \tag{B8}$$

The eigenvalues of $S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}}$ are obtained from (A13):

$$\begin{aligned}\det\left[S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}}-\lambda I\right] &= \det\left[S_b^{11}-\lambda\right]\det\left[S^{11}+S_b^{11}S^{12}S^{21}-\lambda I_{2N-1\times 2N-1}-S^{12}S_b^{12}\frac{1}{S_b^{11}-\lambda}S^{21}S_b^{12}\right]\\ &= \frac{\det\left[\left(S_b^{11}-\lambda\right)\left(S^{11}+S_b^{11}S^{12}S^{21}-\lambda I_{2N-1\times 2N-1}\right)-S^{12}S_b^{12}S^{21}S_b^{12}\right]}{\left(S_b^{11}-\lambda\right)^{2N-2}}\\ &= \frac{\det\left[\left(S_b^{11}-\lambda\right)\left(S^{11}-\lambda I_{2N-1\times 2N-1}\right)+S^{12}S^{21}\left(S_b^{11}S_b^{11}-S_b^{11}\lambda-S_b^{12}S_b^{12}\right)\right]}{\left(S_b^{11}-\lambda\right)^{2N-2}}\end{aligned}\tag{B9}$$

From Equation (B1), $S^{12}S^{21}$ is the following $(2N-1)$ square matrix

$$S^{12}S^{21}=\begin{bmatrix} u_{N\times 1}^{\mathrm{LC}}u_{1\times N}^{\dagger,\mathrm{LR}} & 0_{N\times N-1}\\ 0_{N-1\times N} & 0_{N-1\times N-1}\end{bmatrix}\tag{B10}$$

and, from Equations (B1), (B6) and (B7), the matrix $A=(S_b^{11}-\lambda)(S^{11}-\lambda I_{2N-1\times 2N-1})+S^{12}S^{21}(S_b^{11}S_b^{11}-S_b^{11}\lambda-S_b^{12}S_b^{12})$ in Equation (B9) can be written as:

$$A=\begin{bmatrix} u_{N\times 1}^{\mathrm{LC}}u_{1\times N}^{\dagger,\mathrm{LR}}\left(-S_b^{11}-S_b^{12}-S_b^{11}\lambda\right)-\left(S_b^{11}-\lambda\right)\lambda I_{N\times N} & \left(S_b^{11}-\lambda\right)u_{N\times N-1}\\ \left(S_b^{11}-\lambda\right)u_{N-1\times N}^{\dagger} & -\left(S_b^{11}-\lambda\right)\lambda I_{N-1\times N-1}\end{bmatrix}\tag{B11}$$

Hence, from Equation (A13), (B2), (B6), (B7),

$$\begin{aligned}\det\left[S^{\mathrm{loss}\to\mathrm{dia}}-\lambda I\right] &= \frac{\det\left[-\left(S_b^{11}-\lambda\right)\lambda I_{N-1\times N-1}\right]}{\left(S_b^{11}-\lambda\right)^{2N-2}}\det\left[u_{N\times 1}^{\mathrm{LC}}u_{1\times N}^{\dagger,\mathrm{LR}}\left(-S_b^{11}-S_b^{12}-S_b^{11}\lambda\right)\right.\\ &\quad\left.-\left(S_b^{11}-\lambda\right)\lambda I_{N\times N}+\left(S_b^{11}-\lambda\right)^2u_{N\times N-1}\frac{I_{N-1\times N-1}}{\left(S_b^{11}-\lambda\right)\lambda}u_{N-1\times N}^{\dagger}\right]\\ &= \frac{\left(S_b^{11}-\lambda\right)^{N-1}(-\lambda)^{N-1}}{\left(S_b^{11}-\lambda\right)^{2N-2}}\det\left[\left(S_b^{11}+S_b^{12}+S_b^{11}\lambda\right)\left(u_{N\times N-1}u_{N-1\times N}^{\dagger}-I_{N\times N}\right)\right.\\ &\quad\left.-\left(S_b^{11}-\lambda\right)\lambda I_{N\times N}+u_{N\times N-1}u_{N-1\times N}^{\dagger}\frac{\left(S_b^{11}-\lambda\right)}{\lambda}\right]\\ &= \frac{\det\left[I_{N\times N}\lambda\left(\lambda^2-2S_b^{11}\lambda-S_b^{11}-S_b^{12}\right)+u_{N\times N-1}u_{N-1\times N}^{\dagger}S_b^{11}\left(\lambda^2+2\lambda+1\right)\right]}{\lambda\left(\lambda-S_b^{11}\right)^{N-1}}\\ &= \frac{\det\left[I_{N\times N}\lambda(\lambda+1)\left(\lambda-1-2S_b^{11}\right)+u_{N\times N-1}u_{N-1\times N}^{\dagger}S_b^{11}(\lambda+1)^2\right]}{\lambda\left(\lambda-S_b^{11}\right)^{N-1}}\\ &= \frac{(\lambda+1)^N\det\left[I_{N\times N}\lambda\left(\lambda-1-2S_b^{11}\right)+u_{N\times N-1}u_{N-1\times N}^{\dagger}S_b^{11}(\lambda+1)\right]}{\lambda\left(\lambda-S_b^{11}\right)^{N-1}}\end{aligned}\tag{B12}$$

Equation (B12) has:

- N roots $\lambda=-1$, due to factor $(\lambda+1)^N$, as the matrix $S^{\mathrm{loss}\to\mathrm{dia}}$;
- $N-1$ roots $\lambda=1$. In fact, for $\lambda=1$ and from Equations (B2) and (B3), Equation (B12) becomes

$$\det\left[2S_b^{11}\left(-I_{N\times N}+u_{N\times N-1}u_{N-1\times N}^{\dagger}\right)\right]=\det\left[-2S_b^{11}u_{N\times 1}^{\mathrm{LC}}u_{1\times N}^{\dagger,\mathrm{LR}}\right]=0\tag{B13}$$

  and $\lambda=1$ is a root with order $N-1$ because the $N\times N$ matrix $u_{N\times 1}^{\mathrm{LC}}u_{1\times N}^{\dagger,\mathrm{LR}}$ has rank equal to 1;.

- one root $\lambda = 1 + 2S_b^{11}$. In fact, for $\lambda = 1 + 2S_b^{11}$ and from (B4), Equation (B12) becomes

$$\det\left[u_{N\times N-1}u^{\dagger}_{N-1\times N}S_b^{11}\left(1+S_b^{11}\right)\right]=0 \tag{B14}$$

  and $\lambda = 1 + 2S_b^{11}$ is a simple root because the $N \times N$ matrix $u_{N\times N-1}u^{\dagger}_{N-1\times N}$ has rank equal to $N-1$.

Hence, the shunt admittance jb connected to port $2N$ has changed the order of the root $\lambda = 1$ to $N-1$. If other $N-1$ shunt susceptances are connected to other ports, we can delete all the roots $\lambda = 1$ from $S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}} - I$, obtaining $\det[S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}} - I] \neq 0$. In doing so, $S_{\mathrm{z}}^{\mathrm{loss}\to\mathrm{dia}}$ possesses a Z-matrix and it can be represented with the polygon susceptance network.

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