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Formation of Radiation Fields of Linear Vibrator Arrays by Using Impedance Synthesis

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Abstract—A new method of impedance synthesis of antenna array radiation fields based on a single methodological conception is presented. At first, an approximate solution for the current in the thin vibrator with variable impedance was obtained using the partial averaging operation of the integral-differential equation. The variable impedance of the vibrator was taken into account in the form of an integral coefficient averaged along the vibrator length. The approach turns out to be common for radiators with impedance coatings of different configurations and/or different distributions of lumped impedances. It is established that the shape of the vibrator radiation pattern (RP) does not depend on the form of the impedance distribution function, and it is determined only by the averaged value of the impedance distribution along the vibrator axis. The solution shows that the impedance coating of a symmetrical thin vibrator excited at the center by the voltage δ-generator affects the shape of the radiation pattern in the wave zone, and the effect is directly proportional to the small natural parameter of the problem. The synthesis problem of the radiator impedances for the spatial scanning of the RP was solved for the linear vibrator array. The analytical solution of the problem was obtained for the equidistant array of symmetric vibrators with equal excitation currents. The possibility of changing the RP shape over a wide range by varying the intrinsic complex impedances of the vibrators is demonstrated for an equidistant linear array consisting of 5 half-wave vibrators located at a distance of one eighth wavelength from each other in the free space.

1. INTRODUCTION

The problem of antenna array (AA) synthesis consists in finding a form, dimensions and amplitude-phase distribution (APD) of currents in the array elements by using a RP in the wave zone [1]. Since the shape of the antenna array is supposed to be known in most cases, the problem solution is reduced to determining numbers of the radiators, distances between them and complex current amplitudes in them. At present, the problems of current APD synthesis is fairly well understood theoretically, but unfortunately, it cannot help practical implementation of a particular antenna array design. Alternative problems of an AA constructive synthesis, which were apparently first considered in [2], are still poorly covered in the literature, since they are much more complex than the problems in a conventional formulation. Taking into account constraints arising in practice, these problems are usually inverse boundary value problems, which can often be reduced to nonlinear and multiextremal problems. Therefore, the problem can be solved by sophisticated numerical methods, whose results cannot be directly applied as a basis for array control algorithms required, e.g., for spatial scanning of the RP in the wave zone. Therefore, the solutions of the constructive synthesis problems in a spatial frequency representation can be obtained by using analytical or numerical analytic methods. In the

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latter case, the current practice is to obtain satisfactory results by using analytical or numerical and analytical methods.

On the other hand, the problems of constructive synthesis are actual for issues of electromagnetic compatibility of electronic equipment. In that case, field profiles in the wave and near-field radiation zones are studied (see, for example, [3–6]). Complex structures containing several integrated circuits can often be modelled based on the use of dipole arrays. The electromagnetic radiation in the near zone can be defined by using direct numerical methods developed for frequency-time domain. However, extrapolation of these methods to simulation of fields in the wave zone may encounter considerable difficulties [4].

In this article, a new approach to scanning of the RP of a linear vibrator arrays is presented based on impedance synthesis of radiators. This conception has been aroused from the analysis of impedance vibrator arrays [7, 8], and it can be considered as an improved method for synthesis of antenna arrays with reactive loads [9, 10].

2. ANALYTICAL CURRENT PRESENTATION ON A VIBRATOR WITH VARIABLE IMPEDANCE

Consider a problem of electromagnetic wave excitation by a thin impedance vibrator in infinite medium with material parameters $(\varepsilon_1;\mu_1)$. The vibrator intrinsic impedance is supposed to be an arbitrary complex-valued function of the vibrator length. The initial integral equation of the problem satisfying a boundary condition on the vibrator surface S can be written as [4]

$$\frac{1}{i\omega\varepsilon_1}(\text{graddiv}+k_1^2)\int\limits_S \hat{G}(\vec{r},\vec{r}\,')\vec{J}(\vec{r}\,')\mathrm{d}\vec{r}\,' = -\vec{E}_0(\vec{r}) + z_i(\vec{r})\vec{J}(\vec{r}), \tag{1}$$

where $z_i(\vec{r})$ is the intrinsic linear impedance ([Ohm/m]), $\vec{E}_0(\vec{r})$ the field of extraneous sources, $\hat{G}(\vec{r},\vec{r}\,')$ the tensor Green's function of a free space for electric vector potential $k_1 = k\sqrt{\varepsilon_1\mu_1}, k = 2\pi/\lambda$, and λ the wavelength in a free space. The equation was derived under conditions that all quantities depend on time t as $e^{i\omega t}$.

In the thin wire approximations, Equation (1) can be reduced to an integral equation with a quasi-one-dimensional kernel [8]

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}s^2}+k_1^2\right)\int\limits_{-L}^{L} J(s')\frac{e^{-ik_1R(s,s')}}{R(s,s')}\mathrm{d}s' = -i\omega\varepsilon_1 E_{0s}(s) + i\omega\varepsilon_1 z_i\,(s)\,J(s), \tag{2}$$

where L is the vibrator half-length, $R(s,s') = \sqrt{(s-s')^2+r^2}$, r the vibrator radius, and $E_{0s}(s)$ a projection of extraneous source fields at the vibrator axis. We will solve Equation (2) by a small parameter method [11]. Let us isolate a logarithmic singularity of the Equation (2) kernel with the help of an artificial technique [8, 11]

$$\int\limits_{-L}^{L} J(s')\frac{e^{-ik_1R(s,s')}}{R(s,s')}\mathrm{d}s' = \Omega(s)J(s) + \int\limits_{-L}^{L} \frac{J(s')e^{-ik_1R(s,s')}-J(s)}{R(s,s')}\,\mathrm{d}s'. \tag{3}$$

Here

$$\Omega(s) = \int\limits_{-L}^{L} \frac{\mathrm{d}s'}{\sqrt{(s-s')^2+r^2}} = \Omega + \gamma(s), \tag{4}$$

where $\gamma(s) = \ln\frac{[(L+s)+\sqrt{(L+s)^2+r^2}][(L-s)+\sqrt{(L-s)^2+r^2}]}{4L^2}$ is a function which is equal to zero at the vibrator center and reaches maximal values at the vibrator end. The current at the vibrator ends is zero since the boundary conditions $J(\pm L) = 0$ are fulfilled, and $\Omega = 2\ln\frac{2L}{r}$ is the large parameter. Then, taking into account Equation (3), Equation (2) can be converted to the integral-differential equation for the vibrator electric current

$$\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + k_1^2 J(s) = \alpha \left\{ i\omega\varepsilon_1 E_{0s}(s) + F[s, J(s)] - i\omega\varepsilon_1 z_i(s) J(s) \right\}, \tag{5}$$

where $\alpha = \frac{1}{2\ln[r/(2L)]}$ is the natural small parameter of the problem ($|\alpha| \ll 1$). A functional

$$\begin{aligned} F[s, J(s)] \;=\; & -\frac{\mathrm{d}J(s')}{\mathrm{d}s'}\frac{e^{-ik_1 R(s,s')}}{R(s,s')}\bigg|_{-L}^{L} + \left[\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + k_1^2 J(s)\right]\gamma(s) \\ & + \int_{-L}^{L} \frac{\left[\frac{\mathrm{d}^2 J(s')}{\mathrm{d}s'^2} + k_1^2 J(s')\right] e^{-ik_1 R(s,s')} - \left[\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + k_1^2 J(s)\right]}{R(s,s')}\mathrm{d}s' \end{aligned} \tag{6}$$

defines the vibrator self-field in the spatial domain. Let us introduce a denomination

$$\tilde{k}^2(s) = k_1^2\left[1 + i\alpha\omega\varepsilon_1 z_i(s)/k_1^2\right] = k_1^2\left[1 + i2\alpha \bar{Z}_S(s)/(\mu_1 kr)\right], \tag{7}$$

where

$$\bar{Z}_S(s) = 2\pi r z_i(s)/Z_0 \tag{8}$$

is distribution of surface impedance, normalized to the characteristic impedance of the medium $Z_0 = \sqrt{\mu_1/\varepsilon_1}$ [Ohm]. Then, Equation (5) can be written as

$$\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + \tilde{k}^2(s) J(s) = \alpha \left\{ i\omega\varepsilon_1 E_{0s}(s) + F[s, J(s)] \right\}. \tag{9}$$

The integral-differential Equation (9) has a variable parameter $\tilde{k}(s)$; therefore, it cannot be solved by methods proposed for vibrators with a constant impedance [8, 11]. However, an approximate analytical solution of Equation (9) can be obtained, since parameter $\tilde{k}(s)$ in expression (7) has two terms, and the second term is proportional to the small parameter α. Therefore, it would be appropriate to approximately represent the vibrator impedance $\bar{Z}_s(s)$ by its mean value along the vibrator length. Equation (9) can be written as

$$\frac{\mathrm{d}^2 J(s)}{\mathrm{d}s^2} + \tilde{k}_m^2 J(s) = \alpha \left\{ i\omega\varepsilon_1 E_{0s}(s) + F[s, J(s)] \right\}. \tag{10}$$

where $\tilde{k}_m^2 = k_1^2(1 + \frac{i2\alpha}{\mu_1 kr} \cdot \frac{1}{2L}\int_{-L}^{L} \bar{Z}_S(s)\mathrm{d}s)$ is the mean value of coefficient $\tilde{k}(s)$. The solution of Equation (10) can be called the first approximation to the solution of Equation (9) [12, 13].

The differential Equation (10) can be solved by the series expansion in the small parameter α

$$\begin{aligned} \frac{\mathrm{d}^2 J_1(s)}{\mathrm{d}s^2} + \tilde{k}_m^2 J_1(s) &= i\omega\varepsilon_1 E_{0s}(s), \\ \frac{\mathrm{d}^2 J_2(s)}{\mathrm{d}s^2} + \tilde{k}_m^2 J_2(s) &= F[s, J_1(s)], \\ &\dots \\ \frac{\mathrm{d}^2 J_n(s)}{\mathrm{d}s^2} + \tilde{k}_m^2 J_n(s) &= F[s, J_{n-1}(s)]. \end{aligned} \tag{11}$$

The solution of differential equations at each step should be determined by taking into account the boundary conditions for the currents $J_1(\pm L) = J_2(\pm L) = \ldots = J_n(\pm L) = 0$. The general solution for the current can be obtained as an expansion in powers of the small parameter α

$$J(s) = \alpha J_1(s) + \alpha^2 J_2(s) + \ldots \tag{12}$$

If the system in Eq. (11) is formally supplemented by homogeneous equation for the zero-approximation of the current J_0, it will have a solution $J_0(s) = C_1 \cos \tilde{k}_m s + C_2 \sin \tilde{k}_m s$, which does not depend upon the exciting field $E_{0s}(s)$.

If any losses are present in the medium or vibrator, the trigonometric functions in the solution become complex and, hence, cannot be equal to zero for any arguments. Then to satisfy the boundary conditions $J_0(\pm L) = 0$, the constants C_1 and C_2 should be equal to zeroes. Therefore, identities $J_0 \equiv 0$ and $F[s, J_0(s)] \equiv 0$ hold for any vibrator length, and the first approximation of the vibrator current becomes equal to

$$J(s) \approx \alpha J_1(s) = -\alpha \frac{i\omega\varepsilon_1/\tilde{k}}{\sin 2\tilde{k}_m L} \times \begin{cases} \sin \tilde{k}_m (L - s) \int\limits_{-L}^{s} E_{0s}(s') \sin \tilde{k}_m (L + s')\, \mathrm{d}s' \\ + \sin \tilde{k}_m (L + s) \int\limits_{s}^{L} E_{0s}(s') \sin \tilde{k}_m (L - s')\, \mathrm{d}s', \end{cases} \tag{13}$$

and one can see that it does not depend upon the vibrator eigenfield in Eq. (6).

If the vibrator is excited in the middle by a δ-generator with the voltage amplitude V_0, i.e., $E_{0s}(s') = V_0 \delta(s - s')$, Formula (13) can be reduced to

$$J(s) \approx \alpha J_1(s) = -\frac{i\omega\varepsilon_1 \alpha V_0}{2\tilde{k}_m \cos \tilde{k}_m L} \sin \tilde{k}_m (L - |s|) = J_0 \sin \tilde{k}_m (L - |s|) = J_0 f(s). \tag{14}$$

As can be seen, the approximate solution for the current on the vibrator with the variable impedance in Eq. (14) automatically ensures that the vibrator current continuity is preserved even for piecewise-constant impedance distributions. A characteristic property of the solution consists in that the variable impedance is taken into account as averaged integral coefficient $\frac{1}{2L} \int\limits_{-L}^{L} \bar{Z}_S(s)\mathrm{d}s$. Unlike the current distribution functions known from [14, 15], function $f(s)$ in Equation (14) contains the information about the impedance distribution along vibrator $\bar{Z}_S(s)$. Moreover, this approach is valid for vibrators with one or several local inclusions of lumped impedance loads.

3. RP OF A SYMMETRICAL VIBRATOR WITH VARIABLE IMPEDANCE

When the vibrator current and RP are known, the vibrator radiation field, i.e., the dependence of the radiated field intensity upon the direction can be obtained by summation of fields produced by all vibrator sections. The field in the wave zone at a distance R ($R \gg \lambda_1$, λ_1 is the wavelength in the medium) induced by the current element $J(s')ds'$ can be calculated by neglecting terms, which decrease with distance faster than $1/R$. Under this condition, the field induced in the wave zone by a symmetrical vibrator whereby the current flows in Eq. (14) can be represented as

$$E = E_\theta = \frac{60\pi i}{\lambda_1} \int\limits_{-L}^{L} \sin \theta_S \frac{e^{-ik_1 R_S}}{R_S} J(s')\mathrm{d}s'. \tag{15}$$

The formula is written in a spherical coordinate system whose axis coincides with the vibrator axis (Fig. 1).

Since distance R_S can be expanded in powers of the variable s' so that $R_S = R - s' \cos\theta + \frac{(s')^2 \sin^2\theta}{R} - \ldots$, we can assume that $R_S \approx R$ and $R_S \approx R - s' \cos\theta$ in the denominator and exponent of Formula (15), respectively. If the distance to the observation point is large, we can assume that $\sin\theta_S = \sin\theta$. Taking into account the above approximation, we can rewrite Eq. (15) as

$$E_\theta = \frac{60\pi i}{\lambda_1} \cdot \frac{e^{-ik_1 R}}{R} \sin\theta \int\limits_{-L}^{L} J(s') e^{ik_1 s' \cos\theta} ds'. \tag{16}$$

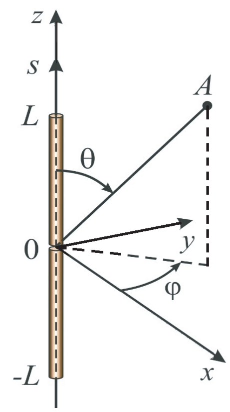

Figure 1. The problem geometry.

Analysis of expression (16) shows that small deviations of the current distribution $J(s')$ from the exact shape are averaged during the integration and do not significantly affect the spatial shape of the RP. Therefore, we can use the approximate current distribution in Eq. (14). After substituted Eq. (14) into Eq. (16), we obtain the following formula

$$E_\theta = \frac{120\pi i}{\lambda_1} \cdot \frac{e^{-ik_1 R}}{R} J_0 \frac{\cos(k_1 L\cos\theta) - \cos\left(\tilde{k}_m L\right)}{\tilde{k}_m^2 - (k_1\cos\theta)^2} \tilde{k}_m \sin\theta. \tag{17}$$

Before proceeding to the synthesis problem, we perform some identical transformations in Eq. (17). Let us introduce the notation $\bar{\beta} = \frac{1}{2L\mu_1 kr} \int\limits_{-L}^{L} \bar{Z}_S(s)ds$. Then, taking into account the relations in Eq. (7), the expression $\tilde{k}_m^2 - (k_1\cos\theta)^2$ in the denominator of Eq. (17) can be written as $k_1^2(1+2i\alpha\bar{\beta}) - k_1^2\cos^2\theta = k_1^2(\sin^2\theta + 2i\alpha\bar{\beta})$. Since the parameter α is small, we can write $\tilde{k}_m^2 = k_1^2(1+2i\alpha\bar{\beta}) \approx k_1^2(1+2i\alpha\bar{\beta}-\alpha^2\bar{\beta}^2) = k_1^2(1+i\alpha\bar{\beta})^2$ and, hence, $\tilde{k}_m = k_1(1+i\alpha\bar{\beta})$. Thus, the multiplier $\frac{\cos(k_1 L\cos\theta)-\cos(\tilde{k}_m L)}{\tilde{k}_m^2-(k_1\cos\theta)^2}\tilde{k}_m\sin\theta$ in Eq. (17) can be represented as

$$\frac{\cos(k_1 L\cos\theta) - \cos\left(\tilde{k}_m L\right)}{\tilde{k}_m^2 - (k_1\cos\theta)^2}\tilde{k}_m\sin\theta = \frac{\left(1+i\alpha\bar{\beta}\right)\sin\theta}{k_1\left(\sin^2\theta + 2i\alpha\bar{\beta}\right)}\left[\cos(k_1 L\cos\theta) - \cos(k_1 L + i\alpha\beta k_1 L)\right]. \tag{18}$$

Taking into account the series expansions of trigonometric functions $\sin x = x - x^3/3! + x^5/5! \mp \ldots$ and $\cos x = 1 - x^2/2! + x^4/4! \mp \ldots$, we can write expression (18) with an accuracy up to the parameter α as

$$\begin{aligned}
&\frac{\left(1+i\alpha\bar{\beta}\right)\sin\theta}{k_1\left(\sin^2\theta+2i\alpha\bar{\beta}\right)}\left[\cos(k_1 L\cos\theta) - \cos(k_1 L) + i\alpha\bar{\beta}k_1 L\sin(k_1 L)\right] \\
&= \frac{\left[Fc(\theta) + i\alpha\bar{\beta}\left(Fc(\theta) + k_1 L\sin(k_1 L)\right)\right]\sin\theta}{k_1\left(\sin^2\theta + 2i\alpha\bar{\beta}\right)} \\
&= \frac{1}{k_1\sin^3\theta}\left[Fc(\theta) + i\alpha\bar{\beta}\left(Fc(\theta) + k_1 L\sin(k_1 L)\right)\right]\left(\sin^2\theta - 2i\alpha\bar{\beta}\right) \\
&= \frac{1}{k_1\sin\theta}\left[Fc(\theta) - i\alpha\bar{\beta}\left(Fc(\theta)\frac{1+\cos^2\theta}{\sin^2\theta} - k_1 L\sin(k_1 L)\right)\right],
\end{aligned} \tag{19}$$

where $Fc(\theta) = \cos(k_1 L\cos\theta) - \cos(k_1 L)$. The final expression for the electric field E_θ can be presented as

$$E_\theta = \frac{120\pi}{\lambda_1} \cdot \frac{e^{-ik_1R}}{R} \cdot \frac{J_0}{k_1 \sin\theta} \left[iFc(\theta) + \alpha\bar{\beta} \left(Fc(\theta) \frac{1+\cos^2\theta}{\sin^2\theta} - k_1 L \sin(k_1 L) \right) \right]$$
$$= \frac{e^{-ik_1R}}{R} \cdot \frac{60 J_0}{\sqrt{\varepsilon_1 \mu_1} \sin\theta} \left[iFc(\theta) + \alpha\bar{\beta} \left(Fc(\theta) \frac{1+\cos^2\theta}{\sin^2\theta} - k_1 L \sin(k_1 L) \right) \right]. \quad (20)$$

The first term in square brackets of expression (20) determines the RP of a perfectly conducting vibrator, and the second term defines contribution of the impedance vibrator coating to the radiation field. If $\bar{Z}_S(s) = 0$, expression (20) is reduced to the well-known formula for the RP of a symmetric perfectly conducting vibrator in the wave zone (see, for example, [8]). This fact confirms a correctness of the approximation.

Formula (20) directly allows us to make two important conclusions. First, the impedance coating of a thin symmetrical vibrator excited in the center by a voltage generator influences the RP in the wave zone. The effect is directly proportional to the product of the parameter $\alpha = \frac{1}{2\ln[r/(2L)]}$ and the mean value of the variable impedance $\frac{1}{2L}\int_{-L}^{L} \bar{Z}_S(s)ds$, and it is inversely proportional to the coefficient $(\mu_1 kr)$. Consequently, the shape of the vibrator RD cannot be varied in wide limits by applying the impedance coating to the vibrator. Second, the form of the RP does not depend upon the function $\overline{Z_S}(s)$ if the mean value of the impedance is $\frac{1}{2L}\int_{-L}^{L} \bar{Z}_S(s)ds$.

4. PROBLEM OF IMPEDANCE SYNTHESIS OF A LINEAR VIBRATORY ARRAY

Let us consider a linear array of N symmetrical impedance vibrators (Fig. 2). The array spacing is d, and all vibrators are of length $2L$ and characterized by the variable impedances $\bar{Z}_{Sn}(s)$. If the vibrators are resonantly tuned by selecting intrinsic resistors of the δ-generators, mutual influences between them can be neglected, and the total field radiated by the array becomes equal to the sum of the field radiated by all vibrators, taking into account the phases of waves arriving to an observation point A. The difference of propagation paths between neighboring vibrators and the observation point in the wave zone is equal to $\mathrm{d}\sin\theta\cos\varphi$.

Let us introduce a notation $u = kd\sin\theta\cos\varphi$. Then, taking into account Eq. (20), we can write the

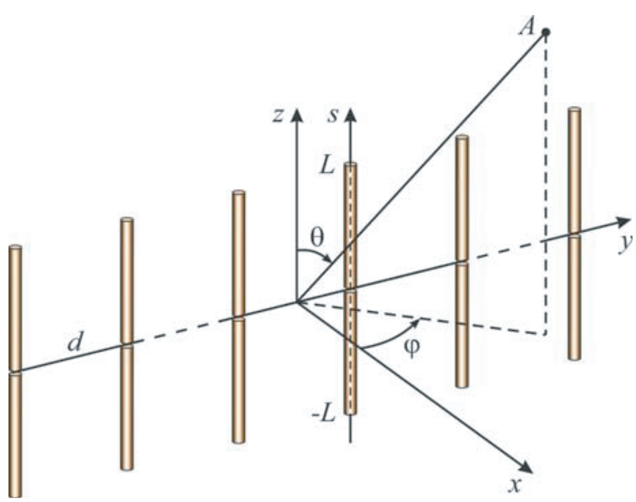

Figure 2. A linear vibrator array.

expression for the resulting field in the free space as [7, 10]

$$E_\theta = 60\frac{e^{-ikR}}{R}e^{-i(N-1)u/2}\left\{\begin{array}{l} \frac{J_1}{\sin\theta}\left[iFc\left(\theta\right)+\alpha\bar{\beta}_1\left(Fc\left(\theta\right)\frac{1+\cos^2\theta}{\sin^2\theta}-kL\sin\left(kL\right)\right)\right] \\ +\frac{J_2e^{iu}}{\sin\theta}\left[iFc\left(\theta\right)+\alpha\bar{\beta}_2\left(Fc\left(\theta\right)\frac{1+\cos^2\theta}{\sin^2\theta}-kL\sin\left(kL\right)\right)\right] \\ +\frac{J_3e^{i2u}}{\sin\theta}\left[iFc\left(\theta\right)+\alpha\bar{\beta}_3\left(Fc\left(\theta\right)\frac{1+\cos^2\theta}{\sin^2\theta}-kL\sin\left(kL\right)\right)\right]\ldots \\ +\frac{J_Ne^{i(N-1)u}}{\sin\theta}\left[iFc\left(\theta\right)+\alpha\bar{\beta}_N\left(Fc\left(\theta\right)\frac{1+\cos^2\theta}{\sin^2\theta}-kL\sin\left(kL\right)\right)\right] \end{array}\right\}, \tag{21}$$

where $\bar{\beta}_n = \frac{1}{2Lkr}\int\limits_{-L}^{L}\bar{Z}_{Sn}(s)\mathrm{d}s$ and $\{J_n\}$ is the current of the n-th vibrator ($n = 1, 2, \ldots, N$).

If the vibrators are perfectly conducting and the vibrator currents are equal, i.e., $J_1 = J_2 = \ldots = J_N = J_0$, expression (21) can be reduced to

$$E_\theta = 60iJ_0\frac{e^{-ikR}}{R}\frac{Fc\left(\theta\right)}{\sin\theta}e^{-i(N-1)u/2}\left\{1+e^{iu}+e^{i2u}+\ldots+e^{i(N-1)u}\right\}. \tag{22}$$

According to Formula (22), the maximum of the array RP is reached in direction ($\theta = \frac{\pi}{2}; \varphi = \frac{\pi}{2}$) when $u = 0$. On the other hand, it is known [7, 10] that if the phase shift between neighboring vibrators is equal to $-\Delta u$, the maximum of the array RP is shifted in the direction ($\theta_{\max}; \varphi_{\max}$) determined from the relation $\sin\theta_{\max}\cos\varphi_{\max} = \Delta u/kd$ [3, 6]. In this case, we can write

$$E_\theta = 60iJ_0\frac{e^{-ikR}}{R}\frac{Fc\left(\theta\right)}{\sin\theta}e^{-i(N-1)(u-\Delta u)/2}\left\{1+e^{i(u-\Delta u)}+e^{i2(u-\Delta u)}+\ldots+e^{i(N-1)(u-\Delta u)}\right\}. \tag{23}$$

Since amplitudes of the vibrator currents are arbitrary, we rename $J_0e^{i(N-1)\Delta u/2}$ by J_0, perform several identical transformations in Eq. (21), and obtain

$$E_\theta = 60iJ_0\frac{e^{-ikR}}{R}\frac{Fc\left(\theta\right)}{\sin\theta}e^{-i(N-1)(u-\Delta u)/2}\left\{\begin{array}{l} \left[1-i\alpha\bar{\beta}_1\left(\frac{1+\cos^2\theta}{\sin^2\theta}-\frac{kL\sin\left(kL\right)}{Fc\left(\theta\right)}\right)\right] \\ +e^{iu}\left[1-i\alpha\bar{\beta}_2\left(\frac{1+\cos^2\theta}{\sin^2\theta}-\frac{kL\sin\left(kL\right)}{Fc\left(\theta\right)}\right)\right] \\ +e^{i2u}\left[1-i\alpha\bar{\beta}_3\left(\frac{1+\cos^2\theta}{\sin^2\theta}-\frac{kL\sin\left(kL\right)}{Fc\left(\theta\right)}\right)\right]\ldots \\ +e^{i(N-1)u}\left[1-i\alpha\bar{\beta}_N\left(\frac{1+\cos^2\theta}{\sin^2\theta}-\frac{kL\sin\left(kL\right)}{Fc\left(\theta\right)}\right)\right] \end{array}\right\}. \tag{24}$$

Expressions (23) and (24) become identical if

$$e^{-i(n-1)kd\sin\theta\cos\varphi} = \left[1-i\alpha\bar{\beta}_n\left(\frac{1+\cos^2\theta}{\sin^2\theta}-\frac{kL\sin\left(kL\right)}{Fc\left(\theta\right)}\right)\right]\Bigg|_{\theta=\theta_{\max};\varphi=\varphi_{\max}}, \tag{25}$$

for every $n = 1, 2, \ldots, N$. The relations in Eq. (25) for the predefined angles ($\theta_{\max}; \varphi_{\max}$) can be used to uniquely find a solution vector for unknowns $\{\bar{\beta}_n\}$. Thus, we can affirm that the maximum of the antenna RP in the wave zone can be redirected to predefined position by synthesizing certain impedances on the array vibrators.

5. NUMERICAL RESULTS

Let us consider the vibrator array consisting of five ($N = 5$) symmetrical half-wave ($2L = \lambda/2$; $kL = \pi$) impedance vibrators with equal radii ($r = L/25 = \lambda/100$). The problem can be formulated as searching such impedances $\bar{Z}_{Sn}(s)$ that the maximum of the RP in the main plane $\theta_{\max} = \pi/2$ will be in the direction of the predefined angle $\varphi_{\max}$. In this case, it is not difficult to verify that the relations in Eq. (25) can be simplified and reduced to

$$e^{-i(n-1)kd\cos\varphi_{\max}} = 1 - i\alpha\bar{\beta}_n. \tag{26}$$

Without losing the approach commonality, we assume that the complex vibrator impedances $\bar{Z}_{Sn}(s) = \bar{R}_{Sn} + i\bar{X}_{Sn}$ do not depend upon the variable s. Then, taking into account that $\bar{\beta}_n = \frac{\bar{R}_{Sn}+i\bar{X}_{Sn}}{kr}$, the relations in Eq. (26) can be represented in the following form

$$\cos\left(kd\left(n-1\right)\cos\varphi_{\max}\right) - i\sin\left(kd\left(n-1\right)\cos\varphi_{\max}\right) = 1 - i\alpha\frac{\bar{R}_{Sn}+i\bar{X}_{Sn}}{kr}. \tag{27}$$

Equating the real and imaginary components of the left and right sides of Equation (27), we finally obtain the formulas ready for the numeric computing

$$\begin{aligned} \bar{R}_{Sn} &= 2kr\ln[r/(2L)]\sin\left[kd\left(n-1\right)\cos\varphi_{\max}\right], \\ \bar{X}_{Sn} &= -2kr\ln[r/(2L)]\left\{1-\cos\left[kd\left(n-1\right)\cos\varphi_{\max}\right]\right\}. \end{aligned} \tag{28}$$

Formulas (25) and (28) are valid for any number of vibrators in the array and for arbitrary distances between the vibrators. However, in general, we cannot guarantee that the impedances computed in such a way and defined here as effective physical quantities have the positive real parts, $\bar{R}_{Sn} \geq 0$. Namely, this condition, put forward from the energy considerations, determines the possibility of practical realization of the impedance $\bar{Z}_{Sn} = \bar{R}_{Sn} + i\bar{X}_{Sn}$ as the vibrator intrinsic impedance. From physical considerations,

Table 1. The estimated values of the impedances $\bar{Z}_{Sn}$ for various angles of radiation maxima.

n	$\varphi_{\max} = 100°$		$\varphi_{\max} = 110°$		$\varphi_{\max} = 120°$		$\varphi_{\max} = 130°$	
	$\bar{R}_{Sn}$	$\bar{X}_{Sn}$	$\bar{R}_{Sn}$	$\bar{X}_{Sn}$	$\bar{R}_{Sn}$	$\bar{X}_{Sn}$	$\bar{R}_{Sn}$	$\bar{X}_{Sn}$
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0081	0.089	0.031	0.172	0.066	0.246	0.107	0.306
3	0.032	0.175	0.121	0.323	0.246	0.426	0.382	0.479
4	0.072	0.255	0.258	0.432	0.492	0.492	0.705	0.443
5	0.124	0.327	0.424	0.487	0.737	0.426	0.934	0.213

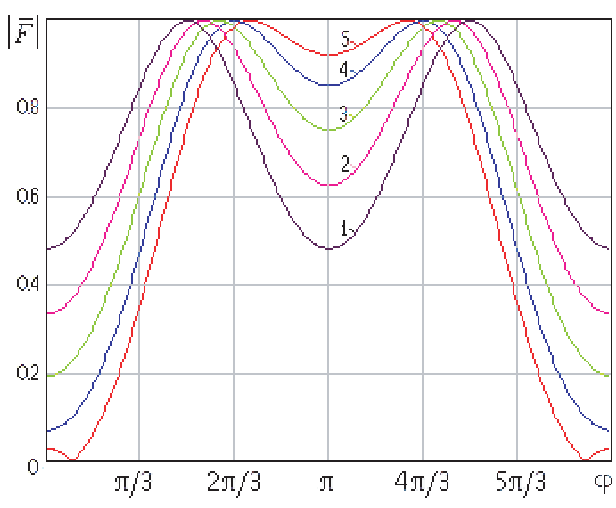

Figure 3. The normalized RP of the linear antenna array: **1** — $\varphi_{\max} = 90°$; **2** — $\varphi_{\max} = 100°$; **3** — $\varphi_{\max} = 110°$; **4** — $\varphi_{\max} = 120°$; **5** — $\varphi_{\max} = 130°$.

we can assume that the conditions $\bar{R}_{Sn} \geq 0$ can be satisfied for arrays with a single directional lobe. The study of more exact requirements for the array dimensions is beyond the scope of this article. As known, the antenna array can have a single label RP oriented in the direction $\varphi_{\max}$ in the range of real angles under condition $\frac{d}{\lambda} < \frac{1}{1+|\cos\varphi_{\max}|}$ [10, 16]. Therefore, we select the distance between the vibrators equal to $\lambda/8$, $(k\mathrm{d} = \pi/4)$. Following from formula (28), if the direction of the RP main lobe is defined by the angle $\varphi_{\max} = 90°$, the impedances $\bar{Z}_{sn}$ should be equal to zeros. Estimated values of the impedance $\bar{Z}_{Sn}$ for $\varphi_{\max}$ equal to 100°, 110°, 120° and 130° are shown in Table 1.

Table 1 shows that to direct the main lobe maximum at the angle other than 90°, the modulus of the impedances $|\bar{Z}_{Sn}|$ should be increased. Note, the impedance $\bar{Z}_{Sn}$ of the vibrator with the number $n = 1$ must be equal to zero for all angles $\varphi_{\max}$. The normalized RPs, $|\bar{F}|$, obtained using Eq. (24) as function of the angle $\varphi \in [0, 2\pi]$ are plotted for the angle $\varphi_{\max}$ given in Table 1. As can be seen from Fig. 3, the results of calculations confirm the possibility of varying the shape of the RP by changing the intrinsic impedances of the vibrators. The radiation pattern can be varied over a wide range: from a quasi-homogeneous to table-like form, directed along the longitudinal axis of the array.

6. CONCLUSION

A new method of impedance synthesis of antenna array radiation patterns based on a single methodological conception is presented. At first, an approximate solution for the current in the thin vibrator with variable impedance was obtained using the partial averaging operation of the integral-differential equation. The variable impedance of the vibrator was taken into account in the form of an integral coefficient averaged along the vibrator length. The approach turns out to be common for radiators with impedance coatings of different configurations and/or different distributions of lumped impedances. It was established that the shape of the vibrator RP does not depend on the form of the impedance distribution function, and it is determined only by the averaged value of the impedance distribution along the vibrator axis. The solution shows that the impedance coating of a symmetrical thin vibrator excited at the center by the voltage δ-generator affects the shape of the radiation pattern in the wave zone, and the effect is directly proportional to the small natural parameter of the problem. The synthesis problem of the radiator impedances for the spatial scanning of the RP was solved for the linear vibrator array. The analytical solution of the problem was obtained for the equidistant array of symmetric vibrators with equal excitation currents. The possibility of changing the RP shape over a wide range by varying the intrinsic complex impedances of the vibrators was demonstrated.

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