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Theoretical Modelling of Modulational Instability of a Lower Hybrid Wave in a Complex Plasma

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Abstract—The modulational instability of a lower hybrid wave is investigated in a dusty plasma slab by developing a non-local theory of this four wave parametric interaction process. The immersed dust grains modify the dispersion relation and growth rate expression of low frequency unstable mode. A numerical analysis shows that the frequencies and growth rate of unstable mode is higher in dusty plasma than in that without dust grains. The growth rate of the unstable mode is proportional to pump amplitude and has strong dependence on pump frequency.

1. INTRODUCTION

There has been extensive research in the field of parametric instabilities associated with electrostatic [1–5] and electromagnetic waves [6, 7] of large amplitude. This study and research is significant because of its profound applicability to space plasma, rf heating of fusion devices [8], and laboratory experiments [9–12]. Parametric instabilities also play a crucial role in laser interaction with plasma, e.g., laser driven fusion [13]. Four wave interaction processes such as modulational instability (MI) [14] also belong to this category.

Considerable emphasis in last three decades has been given for theoretical and experimental investigation of electrostatic waves in dusty plasma [15–37]. Chow and Rosenberg [17, 18] developed a model for electrostatic ion cyclotron (EIC) instability using Vlasov theory with dust grains immersed in plasma which was later found consistent with experimental findings of Barkan et al. [19]. Barkan et al. showed that growth rate of EIC waves increased with parameter δ (where δ is the ratio b/w ion and electron density). Sharma and Gahlot [36, 37] showed that drift wave instability is reduced in cylindrical dusty plasma by using lower hybrid (LH) pump wave with and without incorporating the effect of collisionality.

The immersed dust particles both in unmagnetized [38–40] and magnetized plasmas [41] influence parametric process involving three waves. Modulational instability (MI) of Langmuir and ion acoustic waves have also been analysed with keen interest [42, 43]. Liu and Tripathi [9] considered the MI of lower hybrid (LH) wave in infinite plasma. Konar et al. [44] have studied the MI of a LH wave in a plasma slab in absence of dust particles. This manuscript examines the MI of LH waves in presence of dust particles in a slab of plasma.

The process is explained as: A low frequency plasma mode (ω_l, k_l) combines with LH pump wave (ω_0, k_0) to give LH sidebands $(\omega_{1,2} = \omega_l \mp \omega_0, k_{1,2} = k_l \mp k_0)$ of high frequency. The sidebands thus produced interact with pump providing pondermotive force at (ω_l, k_l) that drives original plasma mode (ω_l, k_l). Section 2 illustrates the instability analysis using fluid treatment. Results and discussion are given in Section 3 while conclusion is mentioned in Section 4.

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2. INSTABILITY ANALYSIS

Consider a plasma slab filled with homogeneous dusty plasma that is infinite in Z-direction and bounded b/w $x = 0$ and $x = a_0$. It is immersed in a magnetic field $\vec{B}_s = B_s\hat{z}$. In equilibrium, the charge, densities, mass and temperature of electrons, ions and dust grains in the plasma slab are denoted by ($-e$, n_{e0}, m_e, T_e), (e, n_{i0}, m_i, T_i) and ($-Q_{d0}$, n_{d0}, m_d, T_d) respectively. MI involves four-wave interaction in which a large amplitude lower hybrid pump wave couples to a electrostatic perturbation (ω_l, k_l) and two lower hybrid sidebands $(\omega_{1,2}, k_{1,2})$ (cf. Fig. 1). We assume the potentials of the four waves of the form

$$\begin{aligned}\phi_0 &= \phi_0(x)\exp\left[-i\left(\omega_0 t - k_{0z}z\right)\right]\\ \phi_1 &= \phi_1(x)\exp\left[-i\left(\omega_1 t - k_{1z}z\right)\right]\\ \phi_2 &= \phi_2(x)\exp\left[-i\left(\omega_2 t - k_{2z}z\right)\right]\\ \phi &= \phi(x)\exp\left[-i\left(\omega_l t - k_{lz}z\right)\right]\end{aligned}$$

The mode structure equation for the lower hybrid (LH) pump wave is given as

$$\frac{\partial^2\phi_0}{\partial x^2} + K_0^2\phi_0 = 0, \tag{1}$$

where $K_{0\perp}^2 = \frac{\omega_{LH}^2 m_i}{\omega_0^2 m_e}k_{0z}^2 - k_{0z}^2$, $\omega_{LH} = \frac{\omega_{pi}}{\sqrt{1+\frac{\omega_{pe}^2}{\omega_{ce}^2}}}$, $\omega_{pe}(=\sqrt{\frac{4\pi n_{e0}e^2}{m_e}})$, $\omega_{pi}(=\sqrt{\frac{4\pi n_{i0}e^2}{m_i}})$, and $\omega_{ce}(=\frac{eB_s}{m_e c})$ are the lower hybrid, electron plasma, ion plasma and electron cyclotron frequency, respectively.

The equation of motion for plasma electrons in the case of high frequency LH waves is given by

$$m_e\frac{d\vec{v}}{dt} = -e\vec{E} - \frac{e}{c}\left(\vec{v}\times\vec{B}_s\right) \tag{2}$$

On linearization, Eq. (2) gives perturbed velocity as

$$V_j = \frac{-e\nabla\phi_j\times\vec{\omega}_{ce}}{m_e\omega_{ce}^2}, \tag{3}$$

$V_{jz} = \frac{-ek_{jz}\phi_j}{m_e\omega_j}$, where $j = 0,\ 1,\ 2$.

As $\omega_l \ll \omega_{ce}$, ponderomotive force (F_{pz}) exerted by LH pump wave and the sidebands ($\phi_{1,2}$) on the electrons is given by

$$F_{pz} = -iek_z\phi_p \tag{4}$$

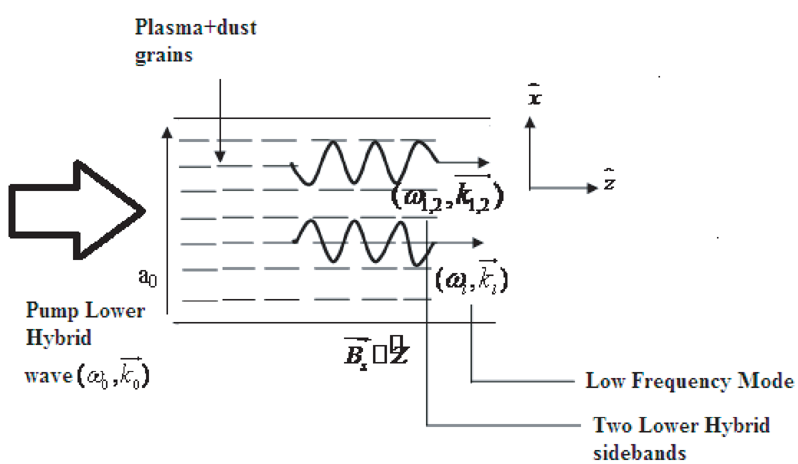

Figure 1. Schematic diagram of four wave parametric interaction in a plasma slab with negatively charged dust grains.

where

$$\phi_p = \frac{-ek_{0z}^2}{2m_e\omega_0^2}\left[\phi_0\phi_1 + \phi_0^*\phi_2\right]. \tag{5}$$

Electron response to ϕ_p and self-consistent potential ϕ turns out to be

$$n_{e1} = \frac{-n_{e0}ek_l^2\left(\phi + \phi_p\right)}{m_e\omega_l^2}, \tag{6}$$

where n_{e1} is the perturbed density of electrons.

The response of plasma species ion and dust at (ω_l, k_l) is obtained using eq. of motion and eq. of continuity as

$$n_{i1} = \frac{n_{i0}ek_l^2\phi}{m_i\omega_l^2}, \tag{7}$$

$$n_{d1} = -\frac{n_{d0}Q_{d0}k_l^2\phi}{m_d\omega_l^2}. \tag{8}$$

where n_{i1} and n_{d1} are the perturbed ion and dust density respectively.

We obtain dust charge fluctuation by following Jana et al. [16] & Varma et al. [45], i.e.,

$$Q_{d1} = \frac{|I_{e0}|}{i(\omega_l + i\eta_{dp})}\left(\frac{n_{i1}}{n_{i0}} - \frac{n_{e1}}{n_{e0}}\right), \tag{9}$$

where η_{dp} is the dust charging rate expressed as $\eta_{dp} = 0.79a(\frac{\omega_{pi}}{\lambda_{Di}})(\frac{1}{\delta_d})(\frac{m_i}{m_e}\frac{T_i}{T_e})^{\frac{1}{2}} \sim 10^{-2}\omega_{pe}(\frac{a}{\lambda_{De}})\frac{1}{\delta_d}$. λ_{Di}, λ_{De} and 'a' are ion Debye length, electron Debye length and dust grain size, respectively.

In Eq. (9), we have assumed that the dust charging time (η_{dp}^{-1}) is nearly equal to wave period (ω_l^{-1}).

Substituting the perturbed densities of electron and ion from Eqs. (6) and (7) in Eq. (9), we get

$$Q_{d1} = \frac{|I_{e0}|\,ek_l^2}{i(\omega_l + i\eta_{dp})\omega_l^2}\left[\frac{\phi}{m_i} + \frac{(\phi + \phi_p)}{m_e}\right]. \tag{10}$$

The quasineutrality condition satisfied at equilibrium is

$$-en_{i0} + en_{e0} + Q_{d0}n_{d0} = 0 \tag{11}$$

$\frac{n_{i0}}{n_{e0}} = 1 + \frac{n_{d0}}{n_{e0}}\frac{Q_{d0}}{e}$ or $\frac{n_{d0}}{n_{e0}} = (\delta_d - 1)\frac{e}{Q_{d0}}$, where $\delta_d = n_{i0}/n_{e0}$.

Substituting the perturbed quantities in the Poisson's equation, $\nabla^2\phi = 4\pi[n_{e1}e - n_{i1}e + n_{d0}Q_{d1} + Q_{d0}n_{d1}]$, we obtain

$$\begin{aligned}\nabla^2\phi \;=\; & \frac{-4\pi n_{e0}e^2k_l^2\left(\phi + \phi_p\right)}{m_e\omega_l^2} - \frac{4\pi n_{i0}e^2k_l^2\phi}{m_i\omega_l^2} \\ & + \frac{4\pi n_{d0}\,|I_{e0}|\,ek_l^2}{i\left(\omega_l + i\eta_{dp}\right)\omega_l^2}\left[\frac{\phi}{m_i} + \frac{(\phi + \phi_p)}{m_e}\right] - \frac{4\pi n_{d0}Q_{d0}^2k_l^2\phi}{m_d\omega_l^2}\end{aligned}$$

Substituting $\nabla^2 = -k_l^2$ for infinite geometry, we get

$$\phi = \frac{-\chi_{ed}\left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right]\phi_p}{\varepsilon_d}, \tag{12}$$

where

$$\varepsilon_d = 1 + \chi_{ed}\left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] + \chi_{id}\left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] + \chi_d,$$

$\chi_{ed} = -\frac{\omega_{pe}^2}{\omega_l^2}$, $\chi_{id} = -\frac{\omega_{pi}^2}{\omega_l^2}$, $\chi_d = -\frac{\omega_{pd}^2}{\omega_l^2}$, $\omega_{pd}(= \sqrt{\frac{4\pi n_{d0}Q_{d0}^2}{m_d}})$ and $\beta_{dp} = \frac{|I_{e0}|n_{d0}}{en_{e0}}$ is the coupling parameter expressed as $\beta_{dp} = 0.397(1 - \frac{1}{\delta_d})(\frac{a}{v_{te}})\omega_{pi}^2(\frac{m_i}{m_e})$. χ_{ed}, χ_{id}, χ_d are electron, ion and dust susceptibility, respectively, while ω_{pd} is the dust plasma frequency.

Nonlinear lower and upper sideband electron density perturbation is given by

$$n_1^{nl} = \frac{\nabla \cdot (n_{e1} v_0^*)}{2i\omega_1} = -\frac{e k_{0z}^2 \phi_0^* n_{e1}}{2 m_e \omega_0^2} \tag{13}$$

and

$$n_2^{nl} = \frac{\nabla \cdot (n_{e1} v_0)}{2i\omega_2} = -\frac{e k_{0z}^2 \phi_0 n_{e1}}{2 m_e \omega_0^2}, \tag{14}$$

where $\omega_1 \approx -\omega_0$ and $\omega_2 \approx \omega_0$.

Substituting Eqs. (13) and (14) in the Poisson's equation, the following nonlinear mode-structure equations for lower and upper sidebands are obtained:

$$\frac{\partial^2 \phi_1}{\partial x^2} + K_{1d}^2 \phi_1 = \frac{e^2 k_{0z}^4 k_l^2 \phi_0^* \chi_{ed}}{4 m_e^2 \omega_0^4 \varepsilon_d M} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] \left\{1 + \chi_{id} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] + \chi_d\right\} [\phi_0 \phi_1 + \phi_0^* \phi_2] \tag{15}$$

and

$$\frac{\partial^2 \phi_2}{\partial x^2} + K_{2d}^2 \phi_2 = \frac{e^2 k_{0z}^4 k_l^2 \phi_0 \chi_{ed}}{4 m_e^2 \omega_0^4 \varepsilon_d M} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] \left\{1 + \chi_{id} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] + \chi_d\right\} [\phi_0 \phi_1 + \phi_0^* \phi_2], \tag{16}$$

where

$$K_{1d}^2 = \frac{\frac{\omega_{pi}^2}{\omega_1^2} \frac{m_i}{m_e} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] k_{1z}^2 - k_{1z}^2}{1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] - \frac{\omega_{pi}^2}{\omega_1^2} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] - \frac{\omega_{pd}^2}{\omega_1^2}}, \tag{17}$$

$$K_{2d}^2 = \frac{\frac{\omega_{pi}^2}{\omega_2^2} \frac{m_i}{m_e} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] k_{2z}^2 - k_{2z}^2}{1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] - \frac{\omega_{pi}^2}{\omega_2^2} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] - \frac{\omega_{pd}^2}{\omega_2^2}} \tag{18}$$

and

$$M = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] - \frac{\omega_{pi}^2}{\omega_0^2} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] - \frac{\omega_{pd}^2}{\omega_0^2}.$$

If the R.H.S of Eqs. (15) and (16) are zero, then these equations represent the linear response at $(\omega_{1,2}, k_{1,2})$, and solutions are represented by ϕ_{1n_1} and ϕ_{1n_2}, respectively.

Expanding the solutions of Eqs. (15) and (16), i.e., ϕ_1 and ϕ_2 in terms of a complete set of orthonormal functions ϕ_{1n_1} and ϕ_{1n_2}, we get

$$\phi_1 = \sum_{n_1} A_{n_1}^{(1)} \phi_{1n_1} \tag{19}$$

and

$$\phi_2 = \sum_{n_2} A_{n2}^{(2)} \phi_{2n_2}. \tag{20}$$

When no pump wave is present, Eq. (15) becomes

$$\frac{\partial^2 \phi_1}{\partial x^2} + K_{1dn_1}^2 \phi_1 = 0 \tag{21}$$

Now subtracting Eq. (21) from Eq. (15), we get

$$\left[K_{1d}^2 - K_{1dn_1}^2\right] \phi_1 = \frac{e^2 k_{0z}^4 k_l^2 \phi_0^* \chi_{ed}}{4 m_e^2 \omega_0^4 \varepsilon_d M} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] \left\{1 + \chi_{id} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] + \chi_d\right\} [\phi_0 \phi_1 + \phi_0^* \phi_2]$$

Substituting the values of ϕ_1 and ϕ_2 from Eqs. (19) and (20), we get

$$\begin{aligned}\left[K_{1d}^2 - K_{1dn_1}^2\right] \sum_{n_1} A_{n_1}^{(1)} \phi_{1n_1} &= \frac{e^2 k_{0z}^4 k_l^2 \phi_0^* \chi_e}{4m_e^2 \omega_0^4 \varepsilon_d M} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] \left\{1 + \chi_{id} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] + \chi_d\right\} \\ &\times \left[\phi_0 \phi_0^* \sum_{n_1} A_{n_1}^{(1)} \phi_{1n_1} + \phi_0^* \phi_0^* \sum_{n_2} A_{n_2}^{(2)} \phi_{2n_2}\right]\end{aligned}$$

Above equation, when multiplied by $\phi_{1m_1}^*$ and integrated over 'x', gives

$$\int \left[K_{1d}^2 - K_{1dn_1}^2\right] \sum_{n_1} A_{n_1}^{(1)} \phi_{1n_1} \phi_{1m_1}^* dx = \int \eta_1 \phi_{1m_1}^* \left[\phi_0 \phi_0^* \sum_{n_1} A_{n_1}^{(1)} \phi_{1n_1} + \phi_0^* \phi_0^* \sum_{n_2} A_{n_2}^{(2)} \phi_{2n_2}\right] dx, \quad (22)$$

where

$$\eta_1 = \frac{e^2 k_{0z}^4 k_l^2 \chi_{ed}}{4m_e^2 \omega_0^4 \varepsilon_d M} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right] \left\{1 + \chi_{id} \left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] + \chi_d\right\}$$

Taking only one value $n_1 = m_1$, we obtain

$$\left[K_{1d}^2 - K_{1dn_1}^2 - \eta_1 \int \phi_0 \phi_0^* \phi_{1n_1}^* \phi_{1n_1} dx\right] A_{n_1}^{(1)} = \eta_1 \sum_{n_2} A_{n_2}^{(2)} \int \phi_0^* \phi_0^* \phi_{2n_2} \phi_{1n_1}^* dx. \quad (23)$$

Similarly we can write for upper sideband

$$\left[K_{2d}^2 - K_{2dn_2}^2 - \eta_1 \int \phi_0^* \phi_0 \phi_{2n_2} \phi_{2n_2}^* dx\right] A_{n_2}^{(2)} = \eta_1 \sum_{n_1} A_{n_1}^{(1)} \int \phi_0 \phi_0 \phi_{1n_1} \phi_{2n_2}^* dx. \quad (24)$$

Multiplying Eqs. (23) and (24) and taking $n_1 = n_2 = n$, i.e., the mode number for lower and upperside bands to be the same, nonlinear dispersion relation for four coupled waves becomes

$$\begin{aligned}&\left[K_{1d}^2 - K_{1dn}^2 - \eta_1 \int |\phi_0|^2 |\phi_{1n}|^2 dx\right] A_{n_1}^{(1)} \left[K_{2d}^2 - K_{2dn}^2 - \eta_1 \int |\phi_0|^2 |\phi_{2n}|^2 dx\right] A_{n_2}^{(2)} \\ &= \eta_1^2 A_{n_2}^{(2)} A_{n_1}^{(1)} \int \phi_0^* \phi_0^* \phi_{2n} \phi_{1n}^* dx \int \phi_0 \phi_0 \phi_{1n} \phi_{2n}^* dx\end{aligned}$$

or

$$\left[K_{1d}^2 - K_{1dn}^2 - \delta_1\right] \left[K_{2d}^2 - K_{2dn}^2 - \delta_2\right] = \mu, \quad (25)$$

where $\delta_1 = \eta_1 \int |\phi_0|^2 |\phi_{1n}|^2 dx$, $\delta_2 = \eta_1 \int |\phi_0|^2 |\phi_{2n}|^2 dx$ and $\mu = \eta_1^2 \int \phi_0^* \phi_0^* \phi_{2n} \phi_{1n}^* dx \int \phi_0 \phi_0 \phi_{1n} \phi_{2n}^* dx$.

As we know for modulational instability (MI) $k_{lz} \ll k_{0z}$, $\omega_l \ll \omega_0$, we can expand K_{1d}^2, K_{2d}^2 using Taylor's series for a function of two variables as

$$K_{1d}^2 = K_{1d}^2(-\omega_0, -k_0) + \omega_l \left.\frac{\partial K_{1d}^2}{\partial \omega_1}\right|_{-\omega_0} + k_{lz} \left.\frac{\partial K_{1d}^2}{\partial k_{1z}}\right|_{-k_{0z}} + \frac{\omega_l^2}{2} \left.\frac{\partial^2 K_{1d}^2}{\partial \omega_1^2}\right|_{-\omega_0} + \frac{k_{lz}^2}{2} \left.\frac{\partial^2 K_{1d}^2}{\partial k_{1z}^2}\right|_{-k_{0z}}, \quad (26)$$

$$K_{2d}^2 = K_{2d}^2(\omega_0, k_0) + \omega_l \left.\frac{\partial K_{2d}^2}{\partial \omega_2}\right|_{\omega_0} + k_{lz} \left.\frac{\partial K_{2d}^2}{\partial k_{2z}}\right|_{k_{0z}} + \frac{\omega_l^2}{2} \left.\frac{\partial^2 K_{2d}^2}{\partial \omega_2^2}\right|_{\omega_0} + \frac{k_{lz}^2}{2} \left.\frac{\partial^2 K_{2d}^2}{\partial k_{2z}^2}\right|_{k_{0z}}. \quad (27)$$

Let $\omega_l = \omega_r + i\gamma$ where ω_r represents the real part of unstable mode frequency and γ its growth rate. Now using the condition for modulational instability (MI), i.e., $\frac{\omega_r}{k_{lz}} \approx \frac{\partial \omega_0}{\partial k_{0z}}$, we obtain

$$\omega_r = \frac{\omega_0}{k_{0z}} \left\{1 - \frac{\left[1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1 + \frac{i\beta_{dp}}{\omega_l + i\eta_{dp}}\right)\right]\omega_0^2}{\omega_{pi}^2\left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})}\right]\frac{m_i}{m_e} + \omega_{pi}^2\left[1 + \frac{i\beta_{dp}}{(\omega_l + i\eta_{dp})\,\delta_d}\right] + \omega_{pd}^2}\right\} k_{lz} \quad (28)$$

and

$$\gamma = \frac{\sqrt{\mu - \delta_1\delta_2 + B_1(\delta_1 + \delta_2 - B_1)}}{A_1}, \quad (29)$$

where

$$A_1 = \frac{2\omega_{pi}^2 m_i}{\omega_0^3 m_e M}\left[1+\frac{i\beta_{dp}}{(\omega_l+i\eta_{dp})}\right]k_{0z}^2,$$

$$B_1 = \frac{3\omega_r^2\omega_{pi}^2 m_i}{\omega_0^4 m_e M}\left[1+\frac{i\beta_{dp}}{(\omega_l+i\eta_{dp})}\right]k_{0z}^2+\frac{k_{lz}^2}{M}\left\{\frac{\omega_{pi}^2 m_i}{\omega_0^2 m_e}\left[1+\frac{i\beta_{dp}}{(\omega_l+i\eta_{dp})}\right]-1\right\}.$$

The dispersion relation of Konar et al. [44] (cf. pages 3799 and 3800) when no dust grain is present is recovered by putting $\delta_d = 1$ and $\beta_{dp} = 0$.

3. RESULTS AND DISCUSSION

We solve Eqs. (28) and (29) numerically to obtain real frequency (ω_r) and growth rate (γ) of the unstable mode using following parameters: $n_{i0} = 5.0\times10^{10}\,\mathrm{cm}^{-3}$, $n_{d0} = 2.0\times10^4\,\mathrm{cm}^{-3}$, $T_e = T_i = 0.2\,\mathrm{eV}$, $m_i/m_e \approx 7.16\times10^4$ (Potassium), $a = 10^{-4}\,\mathrm{cm}$, $\omega_0 = 7.0\times10^9\,\mathrm{rad/sec.}$, $k_{0z} = 3.25\,\mathrm{cm}^{-1}$ and $k_{lz} = 0.035\,\mathrm{cm}^{-1}$. We vary δ_d from 1.0 to 5.0.

Figure 2 shows the variation of ω_r (rad./sec.) of the unstable mode with $\delta_d(= n_{io}/n_{eo})$ for magnetic field values $B_s = 2\,\mathrm{KG}$ and $B_s = 3\,\mathrm{KG}$. It can be seen from Fig. 2 that ω_r increases with δ_d and gets saturated for higher values of δ_d.

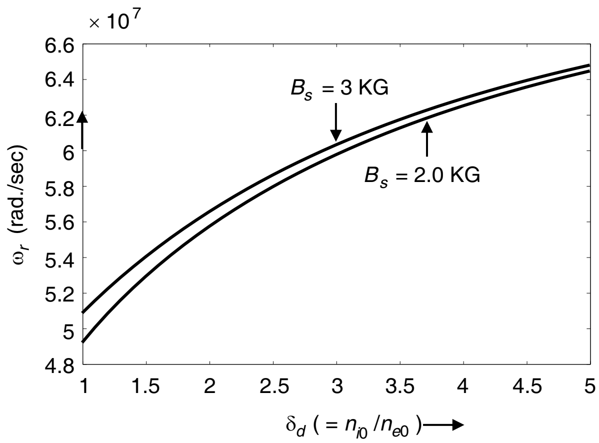

Figure 2. Dispersion curves of the unstable mode as a function of the density ratio of negatively charged dust grains to electrons $\delta_d(= n_{i0}/n_{e0})$ for different values of magnetic field B_s (in KG). The parameters are given in the text.

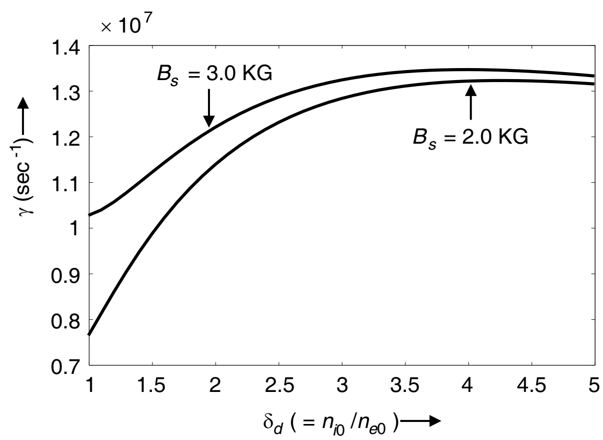

Figure 3. Growth rate γ (sec^{-1}) of the unstable mode as a function of δ_d for the same parameters as in Fig. 2 and for different values of magnetic field B_s (in KG).

Figure 3 depicts the variation of γ (sec.$^{-1}$) as a function of δ_d for the pump amplitude $\phi_0 = 0.023$ esu. Fig. 3 shows that γ increases by a factor ~ 1.73 (for $B_s = 2\,\mathrm{KG}$) and by a factor ~ 1.3 (for $B_s = 3.0\,\mathrm{KG}$) as δ_d is varied from one to four. The growth rate results thus obtained are consistent with the experimental finding of Barkan et al. [19] where growth rate is almost doubled under similar circumstances. Fig. 3 shows that γ increases initially with increase in δ_d but starts decreasing for higher values of δ_d. Thus the contribution of Landau damping becomes more significant at higher values of δ_d and magnetic field (B_s). In Eq. (29), $\mu \approx \delta_1\delta_2$, and since B_1 is positive, the growth is only possible when $\delta_1 + \delta_2 > B_1$, and this condition is satisfied when $\omega_r^2 > \omega_{pi}^2 I$, where $I = 1 + \frac{i\beta_{dp}}{(\omega_l+i\eta_{dp})\delta_d}$. The growth rate is found proportional to pump amplitude as $\delta_1 \approx \delta_2$ and $B_1 < 2\delta_1$. Thus a lower hybrid pump can be more modulationally unstable in the presence of dust grains to low frequency quasimode for reasonable pump power.

Figures 4 and 5 depict the variation of ω_r and growth rate (γ) with pump frequency (ω_0) for different values of δ_d. ω_r increases by 2.28% and γ by 31% corresponding to $\delta_d = 1.0$ (absence of dust

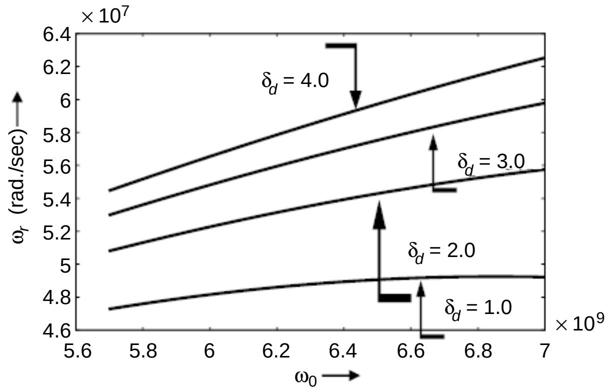

Figure 4. Real frequency (ω_r) of the unstable mode as a function of pump frequency (ω_0) for the same parameters as in Fig. 2 and for different values of δ_d.

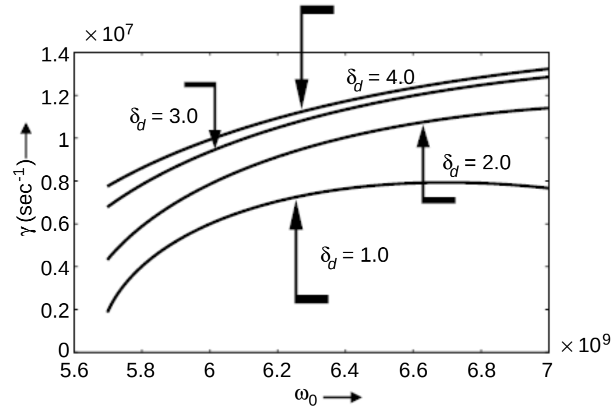

Figure 5. Growth rate γ ($\sec^{-1}$) of the unstable mode as a function of pump frequency $\delta_d = 3.0$ for the same parameters as in Fig. 2 and for different values of δ_d.

grains) while they increase by 10.8% and 33.6%, respectively, corresponding to $\delta_d = 4.0$ when ω_0 varies from 6.0×10^9 to 7.0×10^9 rad./sec. Thus the impact of pump frequency (ω_0) on ω_r and γ is enhanced in the presence of dust grains.

4. CONCLUSION

We developed a nonlocal theory of four wave parametric interaction to study the modulational instability (MI) of lower hybrid (LH) wave in a dusty plasma slab. Both wave frequency (ω_r) and growth rate (γ) of low frequency mode increase with increase in δ_d and are strongly dependent on pump frequency (ω_0) and magnetic field (B_s). The ion mass also effects ω_r and γ while landau damping has significant effect at larger values of δ_d. The growth rate of the unstable mode is proportional to pump amplitude, and it is observed that instability is possible only if unstable mode frequency $\omega_r^2 > \omega_{pi}^2 I$.

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