Plane Wave Diffraction by a Finite Parallel-Plate Waveguide with Sinusoidal Wall Corrugation

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Abstract—The diffraction by a finite parallel-plate waveguide with sinusoidal wall corrugation is analyzed for the E-polarized plane wave incidence using the Wiener-Hopf technique combined with the perturbation method. Assuming that the corrugation amplitude of the waveguide walls is small compared with the wavelength and expanding the boundary condition on the waveguide surface into the Taylor series, the problem is reduced to the diffraction by a flat, finite parallel-plate waveguide with a certain mixed boundary condition. Introducing the Fourier transform for the unknown scattered field and applying an approximate boundary condition together with a perturbation series expansion for the scattered field, the problem is formulated in terms of the zero-order and the first-order Wiener-Hopf equations. The Wiener-Hopf equations are solved via the factorization and decomposition procedure leading to the exact and asymptotic solutions. Taking the inverse Fourier transform and applying the saddle point method, a scattered far field expression is derived explicitly. Scattering characteristics of the waveguide are discussed in detail via numerical examples of the radar cross section (RCS).

1. INTRODUCTION

In microwave and optical engineering, there are many devices with periodic structures such as resonators, filters, reflector antennas, and couplers composed of gratings. Therefore the analysis of the scattering and diffraction by periodic structures is an important subject in electromagnetic theory and optics. Various analytical and numerical methods have been developed thus far and diffraction phenomena have been investigated for a number of periodic structures [1]. It is well known that the Riemann-Hilbert problem technique [2–4], the analytical regularization methods [4–7], the Yasuura method [8–10], the integral and differential method [11], the point matching method [12], and the Fourier series expansion method [13, 14] are efficient for the analysis of diffraction problems involving periodic structures. The Wiener-Hopf technique [15–18] is known as a powerful approach for analyzing electromagnetic wave problems associated with canonical geometries rigorously, and can be applied efficiently to the problems of diffraction by specific periodic structures such as gratings. There are significant contributions to the analysis of the diffraction by gratings and other related structures based on the Wiener-Hopf technique [19–25]. In the previous papers [26–29], we have analyzed the diffraction problems involving transmission-type gratings with the aid of the Wiener-Hopf technique, where rigorous solutions valid over a broad frequency range have been obtained.

It is to be noted that the analysis in most of the above-mentioned papers are restricted to periodic structures of infinite extent and plane boundaries. Therefore, it is important to investigate scattering problems involving periodic structures without these restrictions. As an example of infinite periodic structures with non-plane boundaries, Das Gupta [30] analyzed the plane wave diffraction by a half-plane with sinusoidal corrugation by means of the Wiener-Hopf technique together with the perturbation method. The method developed in [30] has been generalized thereafter by Chakrabarti and Dowerah [31]

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for the Wiener-Hopf analysis of the H-polarized plane wave diffraction by two parallel sinusoidal half-planes. In [32, 33], we have analyzed the problem considered by Chakrabarti and Dowerah via a different Wiener-Hopf approach for both E and H polarizations, and derived various new expressions of the scattered field. We have also considered a finite sinusoidal grating as another important generalization to Das Gupta [30] and analyzed the plane wave diffraction for both E and H polarizations via a hybrid Wiener-Hopf and perturbation approach [34–36].

The aim of this paper is to provide further generalization to our previous analysis carried out for the diffraction problems involving the semi-infinite parallel-plate waveguide with sinusoidal corrugation [32, 33] and the finite sinusoidal grating [34–36]. We shall analyze in this paper the plane wave diffraction by a finite parallel-plate waveguide with sinusoidal wall corrugation for the E-polarized plane wave incidence. The method is based on the use of the Wiener-Hopf technique with the perturbation method.

Assuming that the corrugation amplitude of the waveguide walls is small compared with the wavelength, the original problem is replaced by the problem of diffraction by a flat, finite parallel-plate waveguide with an impedance-type, boundary condition. Introducing the Fourier transform for the unknown scattered field and applying boundary conditions in the transform domain, the problem is formulated in terms of the simultaneous Wiener-Hopf equations satisfied by unknown spectral functions. By using a perturbation series expansion for the scattered field, these Wiener-Hopf equations are separated into the zero-order and first-order Wiener-Hopf equations, which are then solved exactly via the factorization and decomposition procedure. However, the solution is formal since infinite series with unknown coefficients as well as branch-cut integrals with unknown integrands are involved. In order to obtain explicit approximate solutions of the Wiener-Hopf equations, we shall apply the method based on a rigorous asymptotics established recently by Kobayashi [37]. For the infinite series with unknown coefficients, we shall derive highly accurate, approximate expressions by taking into account the edge condition explicitly. For the branch-cut integrals with unknown integrands, we assume that the waveguide length is large compared with the incident wavelength and derive high-frequency asymptotic expressions of the branch-cut integrals. Based on these results, approximate solutions of the Wiener-Hopf equations, efficient for numerical computation, are explicitly derived. Taking the Fourier inverse of the solution in the transform domain and applying the saddle point method, a scattered far field in the real space is derived. Representative numerical examples of the radar cross section (RCS) are shown for various physical parameters, and the effect of sinusoidal corrugation of the waveguide walls is investigated in detail.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

2. FORMULATION OF THE PROBLEM

We consider the diffraction of an E-polarized plane wave by a finite parallel-plate waveguide with sinusoidal wall corrugation as shown in Figure 1, where the surface of the two planes is assumed to be finitely thin, perfectly electric conducting, and uniform in the y-direction, being defined by

$$x = \pm b + h \sin mz, \quad |z| \le a, \tag{1}$$

where $2h$ is the corrugation amplitude and $m\,(>0)$ is the periodicity (surface roughness) parameter. Taking into account the geometry of the waveguide together with the fact that the electric field is parallel to the y-axis, this scattering problem is reduced to a two-dimensional problem.

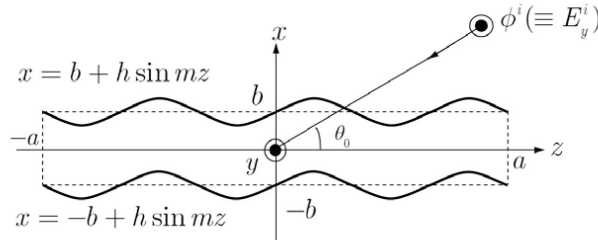Let us define the total electric field $\phi^t(x, z)\,[\equiv E_y^t(x, z)]$ by

$$\phi^t(x, z) = \phi^i(x, z) + \phi(x, z), \tag{2}$$

where $\phi^i(x, z)$ is the incident field of E polarization given by

$$\phi^i(x, z) = e^{-ik(x\sin\theta_0 + z\cos\theta_0)}, \quad 0 < \theta_0 < \pi/2 \tag{3}$$

with $k[\equiv \omega(\varepsilon_0\mu_0)^{1/2}]$ being the free-space wavenumber. The scattered field $\phi(x, z)$ satisfies the two-dimensional Helmholtz equation

$$\left(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2\right)\phi(x, z) = 0. \tag{4}$$

Figure 1. Geometry of the problem.

Nonzero components of the scattered electromagnetic fields are derived from the following relation:

$$(E_y, H_x, H_z) = \left(\phi, \frac{i}{\omega\mu_0} \frac{\partial\phi}{\partial z}, \frac{1}{i\omega\mu_0} \frac{\partial\phi}{\partial x} \right). \tag{5}$$

The total electric field ϕ^t satisfies the perfect conductor condition

$$\phi^t(\pm b + h\sin mz, z) = 0, \quad |z| < a \tag{6}$$

on the waveguide walls. We assume that the corrugation amplitude $2h$ is small compared with the wavelength and expand Eq. (6) in terms of the Taylor series. Then by ignoring the $O(h^2)$ terms from the Taylor expansion, we obtain that

$$\phi^t(\pm b, z) + h\sin mz \frac{\partial\phi^t(\pm b, z)}{\partial z} + O(h^2) = 0, \quad |z| < a. \tag{7}$$

Equation (7) is the approximate boundary condition used throughout the remaining part of this paper. We note that, by letting $h \to 0$ in Eq. (7), the problem reduces to the diffraction problem involving a flat, finite parallel-plate waveguide.

For convenience of analysis, we assume that the medium is slightly lossy as in $k = k_1 + ik_2$ with $0 < k_2 \ll k_1$. The solution for real k is obtained by letting $k_2 \to +0$ at the end of analysis. In view of the radiation condition, it follows that the scattered field $\phi(x, z)$ behaves like the diffracted field for fixed x as $|z| \to \infty$. Hence we can show that

$$\phi(x, z) \sim CH_0^{(1)}(k\rho) \sim C'\rho^{-1/2}e^{ik_1\rho}e^{-k_2\rho} = O\left(e^{-k_2|z|}\right) = O\left(e^{-k_2|z|\cos\theta_0}\right) \tag{8}$$

as $|z| \to \infty$, where $\rho = (x^2 + z^2)^{1/2}$, and C and C' are constants. In Eq. (8), $H_0^{(1)}(\cdot)$ is the Hankel function of the first kind.

We introduce the Fourier transform of the scattered field $\phi(x, z)$ as

$$\Phi(x, \alpha) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \phi(x, z)e^{i\alpha z} dz, \tag{9}$$

where $\alpha = \text{Re}\alpha + i\text{Im}\alpha(\equiv \sigma + i\tau)$. In view of Eq. (8), it follows that $\Phi(x, \alpha)$ is regular in the strip $|\tau| < k_2 \cos\theta_0$ of the complex α-plane. We also introduce the Fourier integrals as in

$$\Phi_{\pm}(x, \alpha) = \pm(2\pi)^{-1/2} \int_{\pm a}^{\pm\infty} \phi(x, z)e^{i\alpha(z\mp a)} dz, \tag{10}$$

$$\Phi_1(x, \alpha) = (2\pi)^{-1/2} \int_{-a}^{a} \phi(x, z)e^{i\alpha z} dz, \tag{11}$$

Then it is seen that $\Phi_{\pm}^{(n)}(x, \alpha)$ are regular in $\tau \gtrless \mp k_2 \cos\theta_0$ whereas $\Phi_1(x, \alpha)$ is an entire function. It follows from Eqs. (9)–(11) that

$$\Phi(x, \alpha) = e^{-i\alpha a}\Phi_-(x, \alpha) + \Phi_1(x, \alpha) + e^{i\alpha a}\Phi_+(x, \alpha). \tag{12}$$

Taking the Fourier transform of Eq. (4) and making use of Eq. (8), we derive that

$$\left[d^2/dx^2 - \gamma^2(\alpha)\right] \Phi(x, \alpha) = 0, \tag{13}$$

where $\gamma(\alpha) = (\alpha^2 - k^2)^{1/2}$. Since $\gamma(\alpha)$ is a double-valued function of α, we choose its proper branch so that $\gamma(\alpha)$ reduces to $-ik$ when $\alpha = 0$. The solution of Eq. (13) is expressed as

$$
\begin{aligned}
\Phi(x,\alpha) &= A(\alpha)e^{-\gamma(\alpha)x}, \quad x > b, \\
&= B(\alpha)e^{-\gamma(\alpha)x} + C(\alpha)e^{\gamma(\alpha)x}, \quad |x| < b, \\
&= D(\alpha)e^{\gamma(\alpha)x}, \quad x < -b,
\end{aligned}
\tag{14}
$$

where $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, and $D(\alpha)$ are unknown functions. For convenience of analysis, we introduce the Fourier integrals as in

$$
P_\pm(\alpha) = \pm(2\pi)^{-1/2} \int_{\pm a}^{\pm\infty} \left[\phi(b+0,z) + h\sin mz \frac{\partial\phi(b+0,z)}{\partial x} \right] e^{i\alpha(z\mp a)} dz,
\tag{15}
$$

$$
Q_\pm(\alpha) = \pm(2\pi)^{-1/2} \int_{\pm a}^{\pm\infty} \left[\phi(-b-0,z) + h\sin mz \frac{\partial\phi(-b-0,z)}{\partial x} \right] e^{i\alpha(z\mp a)} dz,
\tag{16}
$$

$$
\left. \begin{aligned} M_1(\alpha) \\ N_1(\alpha) \end{aligned} \right\} = (2\pi)^{-1/2} \int_{-a}^{a} \left[\frac{\partial\phi(\pm b+0,z)}{\partial x} - \frac{\partial\phi(\pm b-0,z)}{\partial x} \right] e^{i\alpha z} dz,
\tag{17}
$$

$$
F_{1,2}(\alpha) = (2\pi)^{-1/2} \int_{-a}^{a} \left[\phi(\pm b,z) + h\sin mz \frac{\partial\phi(\pm b,z)}{\partial x} \right] e^{i\alpha z} dz.
\tag{18}
$$

Taking into account the approximate boundary condition on the waveguide surface as given by Eq. (7) and carrying out some manipulations, we find from Eqs. (10), (11), and (12) that

$$
P_+(\alpha) + P_-(\alpha) + F_1(\alpha) = \Phi(b+0,\alpha) + \frac{h}{2i} \left[\Phi'(b+0,\alpha+m) - \Phi'(b+0,\alpha-m) \right],
\tag{19}
$$

$$
Q_+(\alpha) + Q_-(\alpha) + F_2(\alpha) = \Phi(-b-0,\alpha) + \frac{h}{2i} \left[\Phi'(-b-0,\alpha+m) - \Phi'(-b-0,\alpha-m) \right],
\tag{20}
$$

where the prime denotes differentiation with respect to x. Substituting the scattered field expression in Eq. (14) into Eqs. (19), (20), and (17), it follows that

$$
\begin{aligned}
P_+(\alpha) + P_-(\alpha) + F_1(\alpha) = A(\alpha)e^{-\gamma(\alpha)b} &+ \frac{ih}{2}\left[\gamma(\alpha+m)A(\alpha+m)e^{-\gamma(\alpha+m)b}\right. \\
&\left. -\gamma(\alpha-m)A(\alpha-m)e^{-\gamma(\alpha-m)b}\right],
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
Q_+(\alpha) + Q_-(\alpha) + F_2(\alpha) = D(\alpha)e^{-\gamma(\alpha)b} &- \frac{ih}{2}\left[\gamma(\alpha+m)D(\alpha+m)e^{-\gamma(\alpha+m)b}\right. \\
&\left. -\gamma(\alpha-m)D(\alpha-m)e^{-\gamma(\alpha-m)b}\right],
\end{aligned}
\tag{22}
$$

$$
M_1(\alpha) = -\gamma(\alpha)\left[A(\alpha)e^{-\gamma(\alpha)x} - B(\alpha)e^{-\gamma(\alpha)x} + C(\alpha)e^{\gamma(\alpha)x} \right],
\tag{23}
$$

$$
N_1(\alpha) = -\gamma(\alpha)\left[B(\alpha)e^{\gamma(\alpha)x} - C(\alpha)e^{-\gamma(\alpha)x} + D(\alpha)e^{-\gamma(\alpha)x} \right],
\tag{24}
$$

$$
\Phi''(b+0,\alpha) - \Phi''(b-0,\alpha) = \gamma^2(\alpha)\left\{ [A(\alpha)-B(\alpha)]e^{-r(\alpha)b} - C(\alpha)e^{r(\alpha)b} \right\},
\tag{25}
$$

$$
\Phi(-b+0,\alpha) - \Phi(-b-0,\alpha) = B(\alpha)e^{r(\alpha)b} + [C(\alpha)-D(\alpha)]e^{-r(\alpha)b},
\tag{26}
$$

where the prime denotes differentiation with respect to x. Making use of the continuity of tangential electric fields across $x = \pm b$ and Eq. (2), we deduce the following relations:

$$
\Phi(-b+0,\alpha) - \Phi(-b-0,\alpha) = (2\pi)^{-1/2}\frac{ih}{2}\left[N_1(\alpha+m) - N_1(\alpha-m) \right],
\tag{27}
$$

$$
\Phi''(b+0,\alpha) - \Phi''(b-0,\alpha) = (2\pi)^{-1/2}\frac{ih}{2}\left[M_1(\alpha+m) - M_1(\alpha-m) \right].
\tag{28}
$$

Substituting Eqs. (25) and (26) into Eqs. (28) and (27) respectively, we can derive equations which relate $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, and $D(\alpha)$ with $M_1(\alpha)$ and $N_1(\alpha)$. Solving these equations for $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, and $D(\alpha)$, we find that

$$A(\alpha) = -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{N_1(\alpha)}{\gamma(\alpha)} - (2\pi)^{-1/2}\frac{ih}{2}\left[N_1(\alpha+m) - N_1(\alpha-m)\right]\right\}$$
$$-\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{M_1(\alpha)}{\gamma(\alpha)} - (2\pi)^{-1/2}\frac{ih}{2}\left[M_1(\alpha+m) - M_1(\alpha-m)\right]\right\}, \tag{29}$$

$$B(\alpha) = -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{N_1(\alpha)}{\gamma(\alpha)} - (2\pi)^{-1/2}\frac{ih}{2}\left[N_1(\alpha+m) - N_1(\alpha-m)\right]\right\}, \tag{30}$$

$$C(\alpha) = -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{M_1(\alpha)}{\gamma(\alpha)} + (2\pi)^{-1/2}\frac{ih}{2}\left[M_1(\alpha+m) - M_1(\alpha-m)\right]\right\}, \tag{31}$$

$$D(\alpha) = -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{M_1(\alpha)}{\gamma(\alpha)} + (2\pi)^{-1/2}\frac{ih}{2}\left[M_1(\alpha+m) - M_1(\alpha-m)\right]\right\}$$
$$-\frac{e^{\gamma(\alpha)b}}{2}\left\{\frac{N_1(\alpha)}{\gamma(\alpha)} + (2\pi)^{-1/2}\frac{ih}{2}\left[N_1(\alpha+m) - N_1(\alpha-m)\right]\right\}. \tag{32}$$

Substituting Eqs. (29) and (32) into Eqs. (21) and (22), respectively and using the boundary conditions, we arrive at

$$S(\alpha) + G_1(\alpha) = -K(\alpha)U_1(\alpha) + \frac{ih}{4}\left\{\left[e^{-2\gamma(\alpha+m)b} - (2\pi)^{-1/2}e^{-2\gamma(\alpha)b} + (2\pi)^{-1/2} - 1\right]V_-(\alpha+m)\right.$$
$$\left. + \left[(2\pi)^{-1/2}e^{-2\gamma(\alpha)b} - e^{-2\gamma(\alpha-m)b} - (2\pi)^{-1/2} + 1\right]V_-(\alpha-m)\right\}, \tag{33}$$

$$D(\alpha) + G_2(\alpha) = -L(\alpha)V_1(\alpha) + \frac{ih}{4}\left\{\left[(2\pi)^{-1/2}e^{-2\gamma(\alpha)b} - e^{-2\gamma(\alpha+m)b} + (2\pi)^{-1/2} - 1\right]U_-(\alpha+m)\right.$$
$$\left. + \left[e^{-2\gamma(\alpha-m)b} - (2\pi)^{-1/2}e^{-2\gamma(\alpha)b} - (2\pi)^{-1/2} + 1\right]U_-(\alpha-m)\right\}, \tag{34}$$

where

$$S(\alpha) = [P_+(\alpha) + P_-(\alpha)] + [Q_+(\alpha) + Q_-(\alpha)], \tag{35}$$
$$D(\alpha) = [P_+(\alpha) + P_-(\alpha)] - [Q_+(\alpha) + Q_-(\alpha)], \tag{36}$$
$$U_1(\alpha) = M_1(\alpha) + N_1(\alpha), \quad V_1(\alpha) = M_1(\alpha) - N_1(\alpha), \tag{37}$$

$$G_{1,2}(\alpha) = F_1(\alpha) \pm F_2(\alpha) = e^{-ikb\sin\theta_0}\left\{\frac{e^{-i\alpha a}A_0 - e^{i\alpha a}B_0}{\alpha - k\cos\theta_0} + \frac{ikh\sin\theta_0}{2}\sum_{n=1}^{2}(-1)^{n+1}\frac{e^{-i\alpha a}A_n - e^{i\alpha a}B_n}{\alpha - k\cos\theta_n}\right\}$$
$$\pm e^{ikb\sin\theta_0}\left\{\frac{e^{-i\alpha a}A_0 - e^{i\alpha a}B_0}{\alpha - k\cos\theta_0} + \frac{ikh\sin\theta_0}{2}\sum_{n=1}^{2}(-1)^{n+1}\frac{e^{-i\alpha a}A_n - e^{i\alpha a}B_n}{\alpha - k\cos\theta_n}\right\}, \tag{38}$$

$$A_0 = -(2\pi)^{-1/2}ie^{ika\cos\theta_0}, \quad B_0 = -(2\pi)^{-1/2}ie^{-ika\cos\theta_0}, \tag{39}$$
$$A_n = (2\pi)^{-1/2}e^{ika\cos\theta_n}, \quad B_n = (2\pi)^{-1/2}e^{-ika\cos\theta_n} \tag{40}$$
$$\cos\theta_{1,2} = \cos\theta_0 \mp m/k, \tag{41}$$
$$K(\alpha) = \frac{e^{-\gamma(\alpha)b}\cosh\gamma(\alpha)b}{\gamma(\alpha)}, \quad L(\alpha) = \frac{e^{-\gamma(\alpha)b}\sinh\gamma(\alpha)b}{\gamma(\alpha)}. \tag{42}$$

Equations (33) and (34) are the simultaneous Wiener-Hopf equations satisfied by $S(\alpha)$, $D(\alpha)$, $U_1(\alpha)$, and $V_1(\alpha)$, which hold for any α in the strip $|\tau| < k_2\cos\theta_0$. In the above, $K(\alpha)$ and $L(\alpha)$ defined by Eq. (42) are kernel functions.

3. ZERO- AND FIRST-ORDER WIENER-HOPF EQUATIONS

In order to solve the Wiener-Hopf Equations (33) and (34), we express the unknown functions $S(\alpha)$, $D(\alpha)$, $U_1(\alpha)$, and $V_1(\alpha)$ in terms of perturbation series expansions in h omitting $O(h^2)$ as

$$S(\alpha) = S^0(\alpha) + hS^1(\alpha) + O(h^2), \tag{43}$$

$$D(\alpha) = D^0(\alpha) + hD^1(\alpha) + O(h^2), \tag{44}$$

$$U_1(\alpha) = U_1^{(0)}(\alpha) + hU_1^{(1)}(\alpha) + O(h^2), \tag{45}$$

$$V_1(\alpha) = V_1^{(0)}(\alpha) + hV_1^{(1)}(\alpha) + O(h^2). \tag{46}$$

We can also express the known functions $G_1(\alpha)$ and $G_2(\alpha)$ defined by Eq. (38) in the form of a perturbation series in h as in

$$G_1(\alpha) = G_1^0(\alpha) + hG_1^1(\alpha) + O(h^2), \tag{47}$$

$$G_2(\alpha) = G_2^0(\alpha) + hG_2^1(\alpha) + O(h^2). \tag{48}$$

In view of Eqs. (35) and (36), $S^0(\alpha)$ and $S^1(\alpha)$ in Eq. (43) and $D^0(\alpha)$ and $D^1(\alpha)$ in Eq. (44) can be expressed as follows:

$$S^0(\alpha) = e^{i\alpha a}S_+^0(\alpha) + e^{-i\alpha a}S_-^0(\alpha) = e^{i\alpha a}\left[P_+^0(\alpha) + Q_+^0(\alpha)\right] + e^{-i\alpha a}\left[P_-^0(\alpha) + Q_-^0(\alpha)\right], \tag{49}$$

$$S^1(\alpha) = e^{i\alpha a}S_+^1(\alpha) + e^{-i\alpha a}S_-^1(\alpha) = e^{i\alpha a}\left[P_+^1(\alpha) + Q_+^1(\alpha)\right] + e^{-i\alpha a}\left[P_-^1(\alpha) + Q_-^1(\alpha)\right], \tag{50}$$

$$D^0(\alpha) = e^{i\alpha a}D_+^0(\alpha) + e^{-i\alpha a}D_-^0(\alpha) = e^{i\alpha a}\left[P_+^0(\alpha) - Q_+^0(\alpha)\right] + e^{-i\alpha a}\left[P_-^0(\alpha) - Q_-^0(\alpha)\right], \tag{51}$$

$$D^1(\alpha) = e^{i\alpha a}D_+^1(\alpha) + e^{-i\alpha a}D_-^1(\alpha) = e^{i\alpha a}\left[P_+^1(\alpha) - Q_+^1(\alpha)\right] + e^{-i\alpha a}\left[P_-^1(\alpha) - Q_-^1(\alpha)\right]. \tag{52}$$

We substitute Eqs. (43)–(48) into Eqs. (33) and (34), and make use of Eqs. (49)–(52) in the resultant equations. After ignoring the $O(h^2)$ terms, the original Wiener-Hopf equations can be separated into the $O(1)$ equations

$$K(\alpha)U_1^{(0)}(\alpha) + e^{i\alpha a}\tilde{S}_{(+)}^0(\alpha) + e^{-i\alpha a}\tilde{S}_-^0(\alpha) = 0, \tag{53}$$

$$L(\alpha)V_1^{(0)}(\alpha) + e^{i\alpha a}\tilde{D}_{(+)}^0(\alpha) + e^{-i\alpha a}\tilde{D}_-^0(\alpha) = 0 \tag{54}$$

and the $O(h)$ equations

$$K(\alpha)U_1^{(1)}(\alpha) + e^{i\alpha a}\tilde{S}_{(+)}^1(\alpha) + e^{-i\alpha a}\tilde{S}_-^1(\alpha) - \frac{i}{4}\left\{\left[e^{-2\gamma(\alpha+m)b} - (2\pi)^{-1/2}e^{-2\gamma(\alpha)b} + (2\pi)^{-1/2} - 1\right]\right.$$

$$V_1^{(0)}(\alpha+m) + \left[(2\pi)^{-1/2}e^{-2\gamma(\alpha)b} - e^{-2\gamma(\alpha-m)b} - (2\pi)^{-1/2} + 1\right]V_1^{(0)}(\alpha-m)\right\} = 0, \tag{55}$$

$$L(\alpha)V_1^{(1)}(\alpha) + e^{i\alpha a}\tilde{D}_{(+)}^1(\alpha) + e^{-i\alpha a}\tilde{D}_-^1(\alpha) - \frac{i}{4}\left\{\left[(2\pi)^{-1/2}e^{-2\gamma(\alpha)b} - e^{-2\gamma(\alpha+m)b} + (2\pi)^{-1/2} - 1\right]\right.$$

$$U_1^{(0)}(\alpha+m) + \left[e^{-2\gamma(\alpha-m)b} - (2\pi)^{-1/2}e^{-2\gamma(\alpha)b} - (2\pi)^{-1/2} + 1\right]U_1^{(0)}(\alpha-m)\right\} = 0 \tag{56}$$

for $|\tau| < k_2\cos\theta_0$, where

$$\tilde{S}_{(+)}^0(\alpha) = P_+^0(\alpha) + Q_+^0(\alpha) - 2B_0\frac{\cos(kb\sin\theta_0)}{\alpha - k\cos\theta_0}, \tag{57}$$

$$\tilde{S}_-^0(\alpha) = P_-^0(\alpha) + Q_-^0(\alpha) + 2A_0\frac{\cos(kb\sin\theta_0)}{\alpha - k\cos\theta_0}, \tag{58}$$

$$\tilde{D}_{(+)}^0(\alpha) = P_+^0(\alpha) - Q_+^0(\alpha) + 2iB_0\frac{\sin(kb\sin\theta_0)}{\alpha - k\cos\theta_0}, \tag{59}$$

$$\tilde{D}_-^0(\alpha) = P_-^0(\alpha) - Q_-^0(\alpha) - 2iA_0\frac{\sin(kb\sin\theta_0)}{\alpha - k\cos\theta_0}, \tag{60}$$

$$\tilde{S}_{(+)}^1(\alpha) = P_+^1(\alpha) + Q_+^1(\alpha) + \sum_{n=1}^{2}(-1)^n\frac{B_n(C_1 + C_2)}{\alpha - k\cos\theta_n}, \tag{61}$$

$$\tilde{S}_-^1(\alpha) = P_-^1(\alpha) + Q_-^1(\alpha) - \sum_{n=1}^{2} (-1)^n \frac{A_n(C_1 + C_2)}{\alpha - k\cos\theta_n}, \tag{62}$$

$$\tilde{D}_{(+)}^1(\alpha) = P_+^1(\alpha) - Q_+^1(\alpha) + \sum_{n=1}^{2} (-1)^n \frac{B_n(C_1 - C_2)}{\alpha - k\cos\theta_n}, \tag{63}$$

$$\tilde{D}_-^1(\alpha) = P_-^1(\alpha) - Q_-^1(\alpha) - \sum_{n=1}^{2} (-1)^n \frac{A_n(C_1 - C_2)}{\alpha - k\cos\theta_n}, \tag{64}$$

$$C_n = \frac{ik\sin\theta_0}{2} e^{\pm ikb\sin\theta_0} \tag{65}$$

for $n = 1, 2$.

Equations (53), (54) and (55), (56) are the zero- and first-order Wiener-Hopf equations, respectively. The zero-order problem corresponds to the diffraction by a flat, finite parallel-plate waveguide, whereas the first-order problem is important since it contains the effect due to the sinusoidal corrugation.

4. EXACT AND ASYMPTOTIC SOLUTIONS

The kernel function $K(\alpha)$ and $L(\alpha)$ defined by Eq. (42) are factorized as in [15–18]

$$K(\alpha) = K_+(\alpha)K_-(\alpha) = K_+(\alpha)K_+(-\alpha), \tag{66}$$
$$L(\alpha) = L_+(\alpha)L_-(\alpha) = L_+(\alpha)L_+(-\alpha), \tag{67}$$

where

$$\begin{aligned}
K_+(\alpha) &= (\cos kb)^{1/2} e^{i\pi/4}(k+\alpha)^{-1/2} \exp\left[\frac{i\gamma(\alpha)b}{\pi}\ln\frac{\alpha - \gamma(\alpha)}{k}\right] \\
&\cdot \exp\left[\frac{i\alpha b}{\pi}\left(1 - C + \ln\frac{\pi}{2kb} + i\frac{\pi}{2}\right)\right] \prod_{n=1,\,\text{odd}}^{\infty}\left(1 + \frac{\alpha}{i\gamma_n}\right)e^{2i\alpha b/n\pi},
\end{aligned} \tag{68}$$

$$\begin{aligned}
L_+(\alpha) &= \left(\frac{\sin kb}{k}\right)^{1/2} \exp\left[\frac{i\gamma(\alpha)b}{\pi}\ln\frac{\alpha - \gamma(\alpha)}{k}\right] \\
&\cdot \exp\left[\frac{i\alpha b}{\pi}\left(1 - C + \ln\frac{2\pi}{kb} + i\frac{\pi}{2}\right)\right] \prod_{n=2,\,\text{even}}^{\infty}\left(1 + \frac{\alpha}{i\gamma_n}\right)e^{2i\alpha b/n\pi}
\end{aligned} \tag{69}$$

with $C(= 0.57721566\ldots)$ being Euler's constant and

$$\gamma_n = \left[(n\pi/2b)^2 - k^2\right]^{1/2}. \tag{70}$$

It seems from Eqs. (66) and (67) that $K_\pm(\alpha)$ and $L_\pm(\alpha)$ are regular and nonzero in $\tau \gtrless \mp k_2$. We can also verify that

$$K_\pm(\alpha) \sim (\mp 2i\alpha)^{-1/2}, \quad L_\pm(\alpha) \sim (\mp 2i\alpha)^{-1/2} \tag{71}$$

as $\alpha \to \infty$ with $\tau \gtrless \mp k_2$. We shall now solve the zero-order Wiener-Hopf Equations (53), (54) and the first-order Wiener-Hopf Equations (55), (56) to derive the exact and asymptotic solutions.

4.1. Solution of the Zero-Order Wiener-Hopf Equations (53) and (54)

The zero-order equations (53) and (54) are the simultaneous Wiener-Hopf equations arising in the diffraction problem for a flat, finite parallel-plate waveguide. Multiplying both sides of Eqs. (53) and (54) by $e^{\pm i\alpha a}/K_\mp(\alpha)$ and $e^{\pm i\alpha a}/L_\mp(\alpha)$, respectively and applying the decomposition procedure, we arrive

at the exact solution with the result that

$$\tilde{S}^0_{(+)}(\alpha) = -K_+(\alpha)\left\{\frac{2B_0\cos(kb\sin\theta_0)}{K_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{1}{2}\left[\sigma^s_{u0}(\alpha) - \sigma^d_{u0}(\alpha)\right]\right.$$

$$\left. -\frac{1}{2}\left[u^s_0(\alpha) - u^d_0(\alpha)\right]\right\} = 0, \tag{72}$$

$$\tilde{S}^0_-(\alpha) = -K_-(\alpha)\left\{-\frac{2A_0\cos(kb\sin\theta_0)}{K_-(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{1}{2}\left[\sigma^s_{u0}(-\alpha) - \sigma^d_{u0}(-\alpha)\right]\right.$$

$$\left. -\frac{1}{2}\left[u^s_0(-\alpha) - u^d_0(-\alpha)\right]\right\} = 0, \tag{73}$$

$$\tilde{D}^0_{(+)}(\alpha) = -L_+(\alpha)\left\{-\frac{2iA_0\sin(kb\sin\theta_0)}{L_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{1}{2}\left[\sigma^s_{v0}(\alpha) - \sigma^d_{v0}(\alpha)\right]\right.$$

$$\left. -\frac{1}{2}\left[v^s_0(\alpha) - v^d_0(\alpha)\right]\right\} = 0, \tag{74}$$

$$\tilde{D}^0_-(\alpha) = -L_-(\alpha)\left\{\frac{2iA_0\sin(kb\sin\theta_0)}{L_-(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{1}{2}\left[\sigma^s_{v0}(-\alpha) + \sigma^d_{v0}(-\alpha)\right]\right.$$

$$\left. -\frac{1}{2}\left[v^s_0(-\alpha) + v^d_0(-\alpha)\right]\right\} = 0, \tag{75}$$

where

$$u^{s,d}_0(\alpha) = \frac{1}{\pi i}\int_k^{k+i\infty}\frac{e^{2i\beta a}\gamma(\beta)K_+(\beta)\tilde{S}^{s0,d0}_{(+)}(\beta)}{\beta + \alpha}d\beta, \tag{76}$$

$$v^{s,d}_0(\alpha) = \frac{1}{\pi i}\int_k^{k+i\infty}\frac{e^{2i\beta a}\gamma(\beta)L_+(\beta)\tilde{D}^{s0,d0}_{(+)}(\beta)}{\beta + \alpha}d\beta, \tag{77}$$

$$\sigma^{s,d}_u(\alpha) = \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\left(\frac{n\pi}{2b}\right)^2\frac{K_+(i\gamma_n)e^{-2a\gamma_n}\tilde{S}^{s0,d0}_{(+)}(i\gamma_n)}{ib\gamma_n(\alpha + i\gamma_n)}, \tag{78}$$

$$\sigma^{s,d}_v(\alpha) = \sum_{\substack{n=2\\ \text{even}}}^{\infty}\left(\frac{n\pi}{2b}\right)^2\frac{L_+(i\gamma_n)e^{-2a\gamma_n}\tilde{D}^{s0,d0}_{(+)}(i\gamma_n)}{ib\gamma_n(\alpha + i\gamma_n)} \tag{79}$$

Introducing the functions

$$\tilde{S}^{s0,d0}_{(+)}(\alpha) = \tilde{S}^0_{(+)}(\alpha) \pm \tilde{S}^0_-(-\alpha), \quad \tilde{D}^{s0,d0}_{(+)}(\alpha) = \tilde{D}^0_{(+)}(\alpha) \pm \tilde{D}^0_-(-\alpha), \tag{80}$$

Equations (72)–(75) can be rearranged as

$$\frac{\tilde{S}^{s0,d0}_{(+)}(\alpha)}{b} = b^{-1/2}K_+(\alpha)\left[\pm b^{-1/2}u^{s,d}_0(\alpha)\mp\sum_{n=2}^{\infty}\frac{a_n p_n s^{so,d0}_n}{b\,(\alpha + i\gamma_{2n-3})} \mp \frac{A^u_0}{b(\alpha + k\cos\theta_0)} - \frac{B^u_0}{b(\alpha - k\cos\theta_0)}\right], \tag{81}$$

$$\frac{\tilde{D}^{s0,d0}_{(+)}(\alpha)}{b} = b^{-1/2}L_+(\alpha)\left[\pm b^{-1/2}v^{s,d}_0(\alpha)\mp\sum_{n=2}^{\infty}\frac{\tilde{a}_n \tilde{p}_n d^{so,d0}_n}{b\,(\alpha + i\gamma_{2n-2})} \pm \frac{A^v_0}{b(\alpha + k\cos\theta_0)} + \frac{B^v_0}{b(\alpha - k\cos\theta_0)}\right], \tag{82}$$

where

$$a^0_1 = kb, \quad \tilde{a}^0_1 = kb, \tag{83}$$

$$a^0_n = \frac{(2n-3)^2\pi^2 e^{-2a\gamma_{2n-3}}}{4bi\gamma_{2n-3}}, \quad \tilde{a}^0_n = \frac{(2n-2)^2\pi^2 e^{-2a\gamma_{2n-2}}}{4bi\gamma_{2n-2}}, \quad n \geq 2, \tag{84}$$

$$p^0_1 = b^{-1/2}K_+(k), \quad \tilde{p}^0_1 = b^{-1/2}L_+(k), \tag{85}$$

$$p^0_n = b^{-1/2}K_+(i\gamma_{2n-3}), \quad \tilde{p}^0_n = b^{-1/2}L_+(i\gamma_{2n-2}), \quad n \geq 2, \tag{86}$$

$$s^{s0,d0} = b^{-1}\tilde{S}^{s0,d0}_{(+)}(k), \quad d^{s0,d0} = b^{-1}\tilde{D}^{s0,d0}_{(+)}(k), \tag{87}$$

$$s^{s0,d0}_n = b^{-1}\tilde{S}^{s0,d0}_{(+)}(i\gamma_{2n-3}), \quad d^{s0,d0}_n = b^{-1}\tilde{D}^{s0,d0}_{(+)}(i\gamma_{2n-2}), \quad n \geq 2. \tag{88}$$

Equations (81) and (82) are formal since they contain the branch-cut integrals with unknown integrands and the infinite series with unknown coefficients. Therefore it is necessary to develop approximation procedures for obtaining explicit approximate solutions for numerical computation.

Regarding the branch-cut integrals $u_0^{s,d}(\alpha)$ and $v_0^{s,d}(\alpha)$ with the unknown integrands $\tilde{S}^{s0,d0}_{(+)}(\beta)$ and $\tilde{D}^{s0,d0}_{(+)}(\beta)$, we apply the rigorous asymptotic method developed by Kobayashi [37]. Omitting the details, we obtain a high-frequency expressions of $u_0^{s,d}(\alpha)$ and $v_0^{s,d}(\alpha)$ for large $k|a|$ as in

$$u_0^{s,d}(\alpha) \sim kK_+(k)\tilde{S}^{s0,d0}_{(+)}(k)\xi(\alpha), \quad v_0^{s,d}(\alpha) \sim kL_+(k)\tilde{D}^{s0,d0}_{(+)}(k)\xi(\alpha), \tag{89}$$

where

$$\xi(\alpha) = -\frac{2a^{1/2}e^{2ika}}{\pi}\Gamma_1\left[1/2, -2i(\alpha+k)a\right]. \tag{90}$$

In Eq. (90), $\Gamma_1(\cdot,\cdot)$ is the generalized gamma function introduced by Kobayashi [38] and is defined by

$$\Gamma_m(u,v) = \int_0^\infty \frac{t^{u-1}e^{-t}}{(t+v)^m}dt \tag{91}$$

for $\mathrm{Re}\,u > 0$, $|v| > 0$, $|\arg v| < \pi$, and positive integer m.

We next evaluate the infinite series $\sigma_u^{s,d}(\alpha)$ and $\sigma_v^{s,d}(\alpha)$ with the unknown coefficients $s_n^{s0,d0}$ and $d_n^{s0,d0}$. Taking into account the edge condition, we can show that $\tilde{S}^{s0,d0}_{(+)}(\alpha)$ and $\tilde{D}^{s0,d0}_{(+)}(\alpha)$ have the asymptotic behavior

$$\tilde{S}^{s0,d0}_{(+)}(\alpha), \; \tilde{D}^{s0,d0}_{(+)}(\alpha) = O(\alpha^{-3/2}), \quad \alpha \to \infty. \tag{92}$$

Therefore, the infinite series contained in Eqs. (81) and (82) are approximated with the choice of a large positive integer N by

$$\sum_{n=2}^\infty \frac{a_n p_n s_n^s}{b\left(\alpha + i\gamma_{2n-3}\right)} \approx \sum_{n=2}^N \frac{a_n p_n s_n^s}{b\left(\alpha + i\gamma_{2n-3}\right)} + K_u^{(1)}S_{uN}^{(1)}(\alpha), \tag{93}$$

$$\sum_{n=2}^\infty \frac{\tilde{a}_n \tilde{p}_n d_n^s}{b\left(\alpha + i\gamma_{2n-2}\right)} \approx \sum_{n=2}^N \frac{\tilde{a}_n \tilde{p}_n d_n^s}{b\left(\alpha + i\gamma_{2n-2}\right)} + K_v^{(1)}S_{vN}^{(1)}(\alpha), \tag{94}$$

where $K_u^{(1)}$ and $K_v^{(1)}$ are unknown constants independent of n, and

$$S_{uN}^{(1)}(\alpha) = \sum_{n=N+1}^\infty \frac{a_n\left(b\gamma_{2n-3}\right)^{-2}}{b\left(\alpha + i\gamma_{2n-3}\right)}, \tag{95}$$

$$S_{vN}^{(1)}(\alpha) = \sum_{n=N+1}^\infty \frac{a_n\left(b\gamma_{2n-2}\right)^{-2}}{b\left(\alpha + i\gamma_{2n-2}\right)}. \tag{96}$$

By substituting Eqs. (89), (93), and (94) into Eqs. (81) and (82), we arrive at the explicit approximate solutions of the Wiener-Hopf Equations (53) and (54) with the result that

$$\frac{\tilde{S}^{s0,d0}_{(+)}(\alpha)}{b} \approx b^{-1/2}K_+(\alpha)\left[\pm a_1^0 p_1^0 s^{s0,d0}\xi(\alpha) \mp \sum_{n=2}^N \frac{a_n p_n s_n^{so,d0}}{b\left(\alpha + i\gamma_{2n-3}\right)}\right.$$
$$\left. \mp K_u^{(1)}S_{uN}^{(1)}(\alpha) \mp \frac{A_0^u}{b\left(\alpha + k\cos\theta_0\right)} - \frac{B_0^u}{b\left(\alpha - k\cos\theta_0\right)}\right], \tag{97}$$

$$\frac{\tilde{D}^{s0,d0}_{(+)}(\alpha)}{b} \approx b^{-1/2}L_+(\alpha)\left[\pm \tilde{a}_1^0 \tilde{p}_1^0 d^{s0,d0}\xi(\alpha) \mp \sum_{n=2}^N \frac{\tilde{a}_n \tilde{p}_n d_n^{so,d0}}{b\left(\alpha + i\gamma_{2n-2}\right)}\right.$$
$$\left. \mp K_v^{(1)}S_{vN}^{(1)}(\alpha) \pm \frac{A_0^v}{b\left(\alpha + k\cos\theta_0\right)} + \frac{B_0^v}{b\left(\alpha - k\cos\theta_0\right)}\right]. \tag{98}$$

4.2. Solution of the First-Order Wiener-Hopf Equations (55) and (56)

Multiplying both sides of Eq. (55) by $e^{\pm i\alpha a}/K_{\mp}(\alpha)$, Eq. (56) by $e^{\pm i\alpha a}/L_{\mp}(\alpha)$ respectively, and applying the decomposition procedure, we arrive at the exact solution with the result that

$$\frac{\tilde{S}^{s1,d1}_{(+)}(\alpha)}{K_+(\alpha)} + \sum_{n=1}^{2}(-1)^{n+1}\left[\frac{B_n(C_1+C_2)}{K_+(k\cos\theta_n)(\alpha-k\cos\theta_n)} \pm \frac{e^{2ika\cos\theta_n}B_n(C_1+C_2)}{K_-(k\cos\theta_n)(\alpha+k\cos\theta_n)}\right.$$

$$\left. +\sigma^{s,d}_{u1}(\alpha)-u^{s,d}_1(\alpha)\right] - \frac{T^{s1,d1}(\alpha)}{K_+(\alpha)} = 0, \tag{99}$$

$$\frac{\tilde{D}^{s1,d1}_{(+)}(\alpha)}{L_+(\alpha)} + \sum_{n=1}^{2}(-1)^{n+1}\left[\frac{B_n(C_1-C_2)}{L_+(k\cos\theta_n)(\alpha-k\cos\theta_n)} \pm \frac{e^{2ika\cos\theta_n}B_n(C_1-C_2)}{L_-(k\cos\theta_n)(\alpha+k\cos\theta_n)}\right.$$

$$\left. +\sigma^{s,d}_{v1}(\alpha)-v^{s,d}_1(\alpha)\right] - \frac{Q^{s1,d1}(\alpha)}{L_+(\alpha)} = 0, \tag{100}$$

where

$$T^{s1,d1}(\alpha) = (ie^{-i\alpha a}/4)\left[e^{-2\gamma(\alpha-m)b}+C_3(\alpha)\right]\left\{\left[V^{(0)}_1(\alpha-m)\mp V^{(0)}_1(-\alpha-m)\right]\right.$$

$$\left. +\left[e^{-2\gamma(\alpha+m)b}+C_3(\alpha)\right]\left[V^{(0)}_1(\alpha+m)\mp V^{(0)}_1(-\alpha-m)\right]\right\}, \tag{101}$$

$$Q^{s1,d1}(\alpha) = (ie^{-i\alpha a}/4)\left\{\left[e^{-2\gamma(\alpha-m)b}+C_4(\alpha)\right]\left[U^{(0)}_1(\alpha-m)\mp U^{(0)}_1(-\alpha+m)\right]\right.$$

$$\left. -\left[e^{-2\gamma(\alpha+m)b}+C_4(\alpha)\right]\left[U^{(0)}_1(\alpha+m)\mp U^{(0)}_1(-\alpha-m)\right]\right\}, \tag{102}$$

$$C_{3,4}(\alpha) = -(2\pi)^{-1/2}\left[e^{-2\gamma(\alpha)b}\mp 1\right]\mp 1, \tag{103}$$

$$u^{s,d}_1(\alpha) = \frac{1}{\pi i}\int_k^{k+i\infty}\frac{e^{2iua}\gamma(u)K_+(u)\tilde{S}^{s1,d1}_{(+)}(u)}{u+\alpha}du, \tag{104}$$

$$v^{s,d}_1(\alpha) = \frac{1}{\pi i}\int_k^{k+i\infty}\frac{e^{2iua}\gamma(u)L_+(u)\tilde{D}^{s1,d1}_{(+)}(u)}{u+\alpha}du, \tag{105}$$

$$\sigma^{s,d}_{u1}(\alpha) = \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\left(\frac{n\pi}{2b}\right)^2\frac{K_+(i\gamma_n)e^{-2a\gamma_n}\tilde{S}^{s1,d1}_{(+)}(i\gamma_n)}{ib\gamma_n(\alpha+i\gamma_n)}, \tag{106}$$

$$\sigma^{s,d}_{v1}(\alpha) = \sum_{\substack{n=2\\ \text{even}}}^{\infty}\left(\frac{n\pi}{2b}\right)^2\frac{L_+(i\gamma_n)e^{-2a\gamma_n}\tilde{D}^{s1,d1}_{(+)}(i\gamma_n)}{ib\gamma_n(\alpha+i\gamma_n)} \tag{107}$$

with

$$\tilde{S}^{s1,d1}_{(+)}(\alpha) = \tilde{S}^1_{(+)}(\alpha) \pm \tilde{S}^1_-(-\alpha), \quad \tilde{D}^{s1,d1}_{(+)}(\alpha) = \tilde{D}^1_{(+)}(\alpha) \pm \tilde{D}^1_-(-\alpha). \tag{108}$$

In order to derive approximate solutions for numerical computation, it is required to evaluate the unknown functions $u^{s,d}_1(\alpha)$ and $v^{s,d}_1(\alpha)$ defined by Eqs. (104) and (105) as well as $\sigma^{s,d}_{u1}(\alpha)$ and $\sigma^{s,d}_{v1}(\alpha)$ defined by Eqs. (106) and (107). To this end, we may apply a procedure similar to that employed for the zero-order Wiener-Hopf equations. Omitting the whole details, we arrive at the approximate solutions as in

$$\tilde{S}^{s1\cdot d1}_{(+)}(\alpha) \approx \frac{K_+(\alpha)}{b^{1/2}}\left\{\pm a^1_1 p^1_1 s^{s1,d1}\xi(\alpha) \mp \sum_{n=2}^{N-1}\frac{a^1_n p^1_n s^{s1,d1}_n}{b(\alpha+i\gamma_{2n-3})} \pm K^{(2)}_u S^{(2)}_{uN}(\alpha)\right.$$

$$+b^{1/2}\sum_{n=1}^{2}(-1)^n\left[\frac{B_n(C_1+C_2)}{K_+(k\cos\theta_n)(\alpha-k\cos\theta_n)}\pm\frac{e^{2ika\cos\theta_n}B_n(C_1+C_2)}{K_-(k\cos\theta_n)(\alpha+k\cos\theta_n)}\right]\Bigg\}+T^{s1,d1}(\alpha),$$

(109)

$$\tilde{D}_{(+)}^{s1\cdot d1}(\alpha)\approx\frac{L_+(\alpha)}{b^{1/2}}\Bigg\{\pm\tilde{a}_1^1\tilde{p}_1^1 d^{s1,d1}\xi(\alpha)\mp\sum_{n=2}^{N-1}\frac{\tilde{a}_n^1\tilde{p}_n^1 d_n^{s1,d1}}{b(\alpha+i\gamma_{2n-2})}\mp K_v^{(2)}S_{vN}^{(2)}(\alpha)$$

$$+b^{1/2}\sum_{n=1}^{2}(-1)^n\left[\frac{B_n(C_1-C_2)}{L_+(k\cos\theta_n)(\alpha-k\cos\theta_n)}\pm\frac{e^{2ika\cos\theta_n}B_n(C_1-C_2)}{L_-(k\cos\theta_n)(\alpha+k\cos\theta_n)}\right]\Bigg\}+Q^{s1,d1}(\alpha),$$

(110)

where

$$a_1^1 = kb, \quad \tilde{a}_1^1 = kb,$$

(111)

$$a_n^1 = \frac{(2n-3)^2\pi^2 e^{-2a\gamma_{2n-3}}}{4bi\gamma_{2n-3}}, \quad \tilde{a}_n^1 = \frac{(2n-2)^2\pi^2 e^{-2a\gamma_{2n-2}}}{4bi\gamma_{2n-2}}, \quad n\geq 2,$$

(112)

$$p_1^1 = b^{-1/2}K_+(k), \quad \tilde{p}_1^1 = b^{-1/2}L_+(k),$$

(113)

$$p_n^1 = b^{-1/2}K_+(i\gamma_{2n-3}), \quad \tilde{p}_n^1 = b^{-1/2}L_+(i\gamma_{2n-2}), \quad n\geq 2,$$

(114)

$$s^{s1,d1} = \tilde{S}_{(+)}^{s1,d1}(k), \quad d^{s1,d1} = \tilde{D}_{(+)}^{s1,d1}(k),$$

(115)

$$s_n^{s1,d1} = \tilde{S}_{(+)}^{s1,d1}(i\gamma_{2n-3}), \quad d_n^{s1,d1} = \tilde{D}_{(+)}^{s1,d1}(i\gamma_{2n-2}), \quad n\geq 2.$$

(116)

$$S_{uN}^{(2)}(\alpha) = \sum_{n=N+1}^{\infty}\frac{a_n(b\gamma_{2n-3})^{-2}}{b(\alpha+i\gamma_{2n-3})},$$

(117)

$$S_{vN}^{(2)}(\alpha) = \sum_{n=N+1}^{\infty}\frac{a_n(b\gamma_{2n-2})^{-2}}{b(\alpha+i\gamma_{2n-2})},$$

(118)

$$u_{s,d}^1(\alpha) \sim kK_+(k)\tilde{S}_{(+)}^{s1,d1}(k)\xi(\alpha),$$

(119)

$$v_{s,d}^1(\alpha) \sim kL_+(k)\tilde{D}_{(+)}^{s1,d1}(k)\xi(\alpha)$$

(120)

as $ka\to\infty$.

4.3. Determination of the Unknown Coefficients

Equations (97), (98), (109), and (110) are approximate solutions to the simultaneous Wiener-Hopf Equations (53)–(56), and hold uniformly in θ_0 for large N and $|k|a$, where the unknowns $s_n^{s1,d1}$, $s_n^{s0,d0}$, $d_n^{s0,d0}$, and $d_n^{s1,d1}$ for $n=1,2,3,\ldots,N$ as well as $K_u^{(1)}$, $K_v^{(1)}$, $K_u^{(2)}$, and $K_v^{(2)}$ are contained.

In order to determine these unknowns, we set $\alpha=k, i\gamma_{2m-3}$ for $m=2,3,4,\ldots,N+1$ in Eqs. (97) and (109). We also set $\alpha=k, i\gamma_{2m-2}$ for $m=2,3,4,\ldots,N+1$ in Eqs. (98) and (110). These procedures lead to the two sets of $(N+1)\times(N+1)$ equations, where $s_{N+1}^{s1,d1}$ and $d_{N+1}^{s1,d1}$ are involved. Since N is a large positive number, we can employ Eq. (92) to replace $s_{N+1}^{s1,d1}$ and $d_{N+1}^{s1,d1}$ by their asymptotic expressions containing $K_u^{(1)}$, $K_v^{(1)}$, $K_u^{(2)}$, and $K_v^{(2)}$. Thus the two sets of $(N+1)\times(N+1)$ matrix equations with the $N+1$ unknowns are derived, which can be solved numerically with high accuracy. Since the approximation procedures developed in this section are based on a rigorous asymptotics with the aid of the edge condition, the above approximate solutions are valid over a wide frequency range as long as the waveguide length $2a$ is not too small compared with wavelength. The solution in the complex domain is complete and we are now in a position to derive explicitly a scattered field in the real space by taking the inverse Fourier transform.

5. SCATTERED FAR FIELD

In this section, we shall derive an asymptotic expression of the field outside the waveguide. The region outside the waveguide actually includes $|z| > a$ with $|x| < b$, but contributions from this region are negligibly small at large distance from the origin. Therefore, only the scattered far field for $|x| > b$ will be discussed in the following. The scattered field $\phi(x, z)$ in the real space can be derived by taking the inverse Fourier transform of Eq. (9) according to the formula

$$\phi(x, z) = (2\pi)^{-1/2} \int_{-\infty+ic}^{\infty+ic} \Phi(x, \alpha)e^{-i\alpha z}d\alpha, \tag{121}$$

where c is a constant satisfying $|c| < k_2 \cos\theta_0$.

Substituting Eq. (37) into Eqs. (45) and (46), we obtain that

$$\left.\begin{aligned} M_1(\alpha) \\ N_1(\alpha) \end{aligned}\right\} = \frac{1}{2}\left\{\left[U_1^{(0)}(\alpha) \pm V_1^{(0)}(\alpha)\right] + h\left[U_1^{(1)}(\alpha) \pm V_1^{(1)}(\alpha)\right]\right\}. \tag{122}$$

Furthermore substituting Eq. (122) into Eqs. (29) and (32) and taking into account Eqs. (14) and (121), we arrive at an integral representation for the scattered field with the result that

$$\phi(x, z) = \phi^{(0)}(x, z) + h\phi^{(1)}(x, z), \tag{123}$$

where

$$\begin{aligned} \phi^{(0)}(\pm b, z) &= -(2\pi)^{-1/2} \int_{-\infty+ic}^{\infty+ic} \frac{1}{2\gamma(\alpha)}\left[U_1^{(0)}(\alpha)\cosh\gamma(\alpha)b \right.\\ &\quad \left. \pm V_1^{(0)}\sinh\gamma(\alpha)b\right]e^{\mp\gamma(\alpha)x-i\alpha z}d\alpha, \quad x \gtrless b, \end{aligned} \tag{124}$$

$$\begin{aligned} \phi^{(1)}(\pm b, z) &= (2\pi)^{-1/2} \int_{-\infty+ic}^{\infty+ic} \left(-\frac{h}{2\gamma(\alpha)}\right)\left[U_1^{(1)}(\alpha)\cosh\gamma(\alpha)b \pm V_1^{(1)}(\alpha)\sinh\gamma(\alpha)b\right] \\ &\quad \pm(2\pi)^{-1/2}\frac{ih}{4}\left\{\left[U_1^{(0)}(\alpha+m) - U_1^{(0)}(\alpha-m)\right]\cosh\gamma(\alpha)b \right.\\ &\quad \left. \pm\left[V_1^{(0)}(\alpha+m) - V_1^{(0)}(\alpha-m)\right]\sinh\gamma(\alpha)b\right\}e^{\mp\gamma(\alpha)x-i\alpha z}d\alpha, \quad x \gtrless \pm b. \end{aligned} \tag{125}$$

As seen above, Eqs. (124) and (125) are the zero- and first-order scattered fields, respectively. The zero-order scattered field corresponds to the problem of diffraction by a flat, finite parallel-plate waveguide. In the following, we shall derive asymptotic expressions of the zero- and first-order scattered far fields explicitly.

Since the integrand of Eq. (124) has branch points at $\alpha = \pm k$, evaluation in closed form is in general difficult. However, we may apply the saddle point method to derive an asymptotic expression at large distances from the origin. Introduce the cylindrical coordinates (ρ, θ) centered at the origin as follows:

$$x = \rho\sin\theta, \quad z = \rho\cos\theta, \quad \text{for} \quad -\pi < \theta < \pi. \tag{126}$$

By means of the saddle point method, we can derive a far field asymptotic expression of the scattered field with the result that

$$\begin{aligned} \phi^{(0)}(\rho, \theta) &\sim -\frac{1}{2\gamma(-k\cos\theta)}\left[U_1^{(0)}(-k\cos\theta)\cosh\gamma(-k\cos\theta)b \right.\\ &\quad \left. \pm V_1^{(0)}(-k\cos\theta)\sinh\gamma(-k\cos\theta)b\right]k\sin|\theta|\frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}} \end{aligned} \tag{127}$$

for $x \gtrless \pm b$ as $k\rho \to \infty$.

Next we shall derive an asymptotic expression of the first-order scattered field $\phi^{(1)}(x, z)$. In view of Eq. (125), the first-order scattered field can be written as

$$\phi^{(1)}(x, z) = \phi_u^{(1)}(x, z) + \phi_v^{(1)}(x, z), \tag{128}$$

where

$$\phi_u^{(1)}(x,z) = -(2\pi)^{-1/2} \int_{-\infty+ic}^{\infty+ic} \frac{1}{2\gamma(\alpha)} h \left[U_1^{(1)}(\alpha) \cosh\gamma(\alpha)b \right.$$

$$\left. \pm V_1^{(1)}(\alpha) \sinh\gamma(\alpha)b \right] e^{\mp\gamma(\alpha)x - i\alpha z} d\alpha, \quad x \gtrless \pm b, \tag{129}$$

$$\phi_v^{(1)}(x,z) = \pm(2\pi)^{-1/2} \cdot \int_{-\infty+ic}^{\infty+ic} \frac{ih}{4} \left\{ \left[U_1^{(0)}(\alpha+m) - U_1^{(0)}(\alpha-m) \right] \cosh\gamma(\alpha)b \right.$$

$$\left. \pm \left[V_1^{(0)}(\alpha+m) - V_1^{(0)}(\alpha-m) \right] \sinh\gamma(\alpha)b \right\} e^{\mp\gamma(\alpha)x - i\alpha z} d\alpha, \quad x \gtrless \pm b. \tag{130}$$

Applying the saddle point method with the aid of the cylindrical coordinate defined by Eq. (126), it is found that $\phi_u^{(1)}(x,z)$ has the asymptotic expression

$$\phi_u^{(1)}(\rho,\theta) \sim -\frac{1}{2\gamma(-k\cos\theta)} \left[U_1^{(1)}(-k\cos\theta) \cosh\gamma(-k\cos\theta)b \right.$$

$$\left. \pm V_1^{(1)}(-k\cos\theta) \sinh\gamma(-k\cos\theta)b \right] k \sin|\theta| \frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}} \tag{131}$$

for $x \gtrless \pm b$ as $k\rho \to \infty$.

An asymptotic evaluation of $\phi_v^{(1)}(x,z)$ defined by Eq. (130) is in general difficult, since the integrand has branch points at $\alpha = \pm k + m, \pm k - m$ as well as $\alpha = \pm k$. For simplicity, we assume $|m/k| \ll 1$, which implies that the period of corrugation is large compared with the wavelength. Then in the process of asymptotic evaluation, we can ignore contributions from branch-cut integrals occurring due to the branch points at $\alpha = \pm k + m, \pm k - m$. Therefore the simple saddle point method may be employed to obtain a far field expression with the result that

$$\phi_v^{(1)}(\rho,\theta) \sim \mp\frac{i}{4} \left\{ \left[U_1^0(-k\cos\theta^{(1)}) - U_1^0(-k\cos\theta^{(2)}) \right] \cosh\gamma(-k\cos\theta)b \right.$$

$$\left. \pm \left[V_1^0(-k\cos\theta^{(1)}) - V_1^0(-k\cos\theta^{(2)}) \right] \sinh\gamma(-k\cos\theta)b \right\} k \sin|\theta| \frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}} \tag{132}$$

for $x \gtrless \pm b$ as $k\rho \to \infty$. where

$$\theta^{(1)} = \cos^{-1}(\cos\theta - m/k), \quad \theta^{(2)} = \cos^{-1}(\cos\theta + m/k). \tag{133}$$

As seen above, substitution of Eqs. (131) and (132) into Eq. (128) yields an explicit far field expression of the first-order scattered far field, and holds for arbitrary incidence and observation angles.

6. NUMERICAL RESULTS AND DISCUSSION

In this section, we shall present numerical examples of the RCS and discuss far field scattering characteristics of the waveguide in detail. Since this is a two-dimensional problem, the RCS per unit length is given by

$$\rho = \lim_{\rho\to\infty} \left(2\pi\rho|\phi|^2/|\phi^i|^2 \right), \tag{134}$$

where

$$\phi(\rho,\theta) = \phi^{(0)}(\rho,\theta) + h\phi^{(1)}(\rho,\theta). \tag{135}$$

In computing Eq. (135), we have used the asymptotic expressions given by Eqs. (127), (128), (131) and (132). As has been mentioned in Section 2, it is essential to reduce the original problem to the diffraction by a flat, finite parallel-plate waveguide with a Leontovich-type boundary condition as given by Eq. (7) under the small-depth approximation. By careful numerical experimentation, we have verified that, if the corrugation depth $2h$ satisfies $2h \leq 0.1\lambda$ with λ being the free-space wavelength, then Eq. (7) can be employed to simulate a perfectly conducting sinusoidal surface with sufficient accuracy. On the

other hand, in order to validate the far field asymptotic expression of $\phi_v^{(1)}(x, z)$ given by Eq. (132), the ratio m/k has been taken as $m/k \leq 0.2$ in numerical computations. Under this condition, contributions from branch-cut integrals due to the branch points at $\alpha = \pm k + m$ and $\pm k - m$ arising in the process of asymptotic evaluation of Eq. (130) are small compared with the saddle point contribution and hence, Eq. (132) can be used with reasonable accuracy.
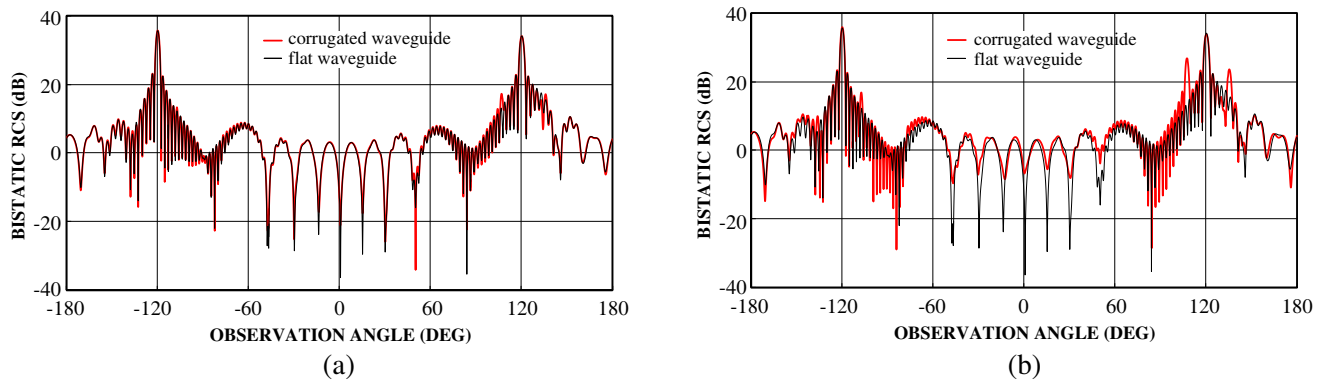
Figures 2–5 show numerical examples of the bistatic RCS σ/λ as a function of observation angle θ
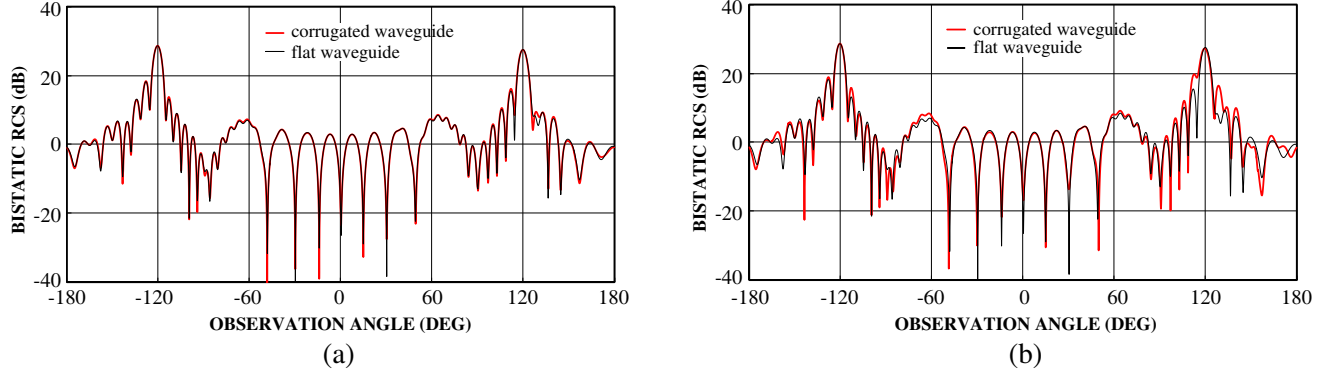
Figure 2. Bistatic RCS σ/λ [dB] of a flat waveguide and a corrugated waveguide for $\theta_0 = 60°$, $N = 1$, $2a = 10\lambda$, $m/k = 0.1$, $2b = 4\lambda$. (a) $2h = 0.02\lambda$. (b) $2h = 0.1\lambda$.
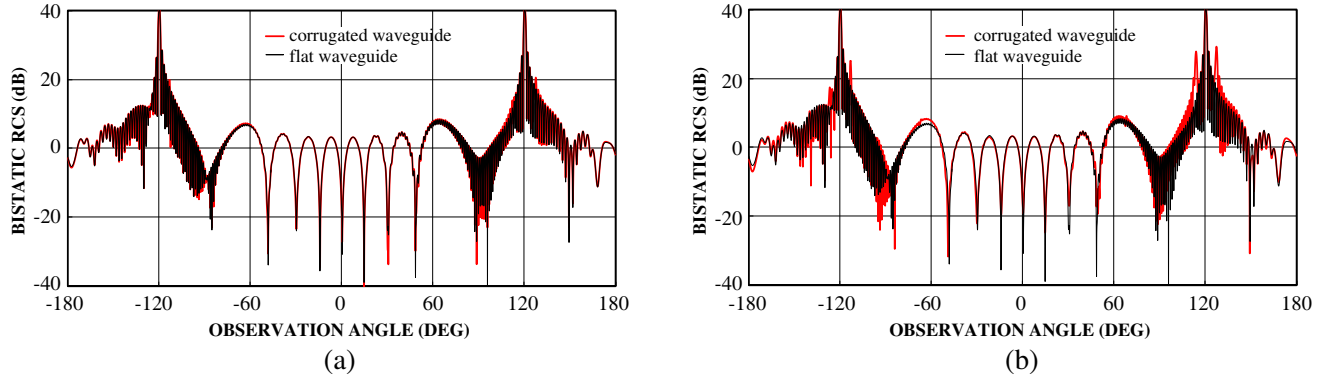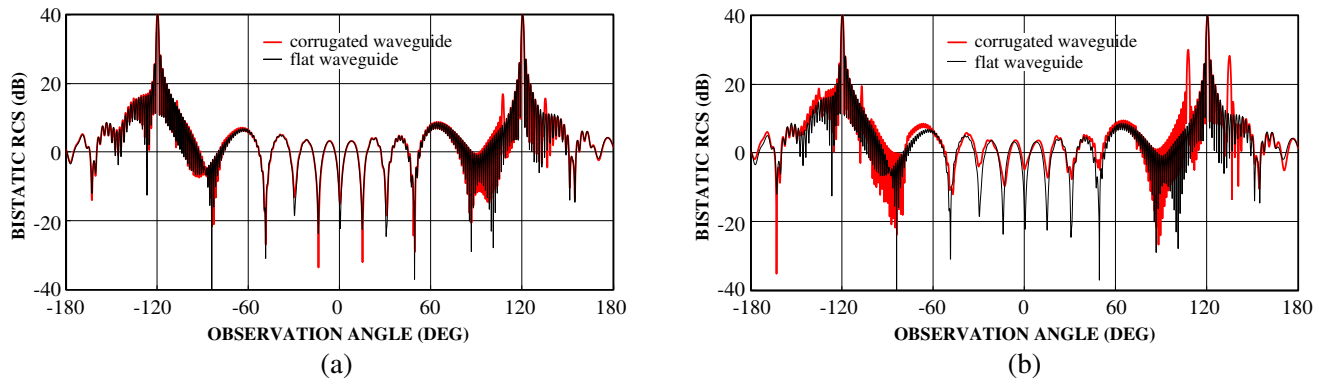

Figure 3. Bistatic RCS σ/λ [dB] of a flat waveguide and a corrugated waveguide for $\theta_0 = 60°$, $N = 5$, $2a = 50\lambda$, $m/k = 0.1$, $2b = 4\lambda$. (a) $2h = 0.02\lambda$. (b) $2h = 0.1\lambda$.


Figure 4. Bistatic RCS σ/λ [dB] of a flat waveguide and a corrugated waveguide for $\theta_0 = 60°$, $N = 5$, $2a = 25\lambda$, $m/k = 0.2$, $2b = 4\lambda$. (a) $2h = 0.02\lambda$. (b) $2h = 0.1\lambda$.

for various values of N, $2a$, m/k, and $2h$, where the incidence angle θ_0 and the waveguide width $2b$ are fixed as $60°$ and 4λ, respectively. All these results provide comparisons on the scattering characteristics between the flat waveguide (black lines) and the corrugated waveguide (red lines). In the figures, the parameter m/k is important in numerical computation and is physically the periodicity (surface roughness) parameter. In addition, $N(\equiv (2a/\lambda)(m/k))$ implies the number of periods of the corrugation of the waveguide walls. The periodicity parameter m/k is chosen as 0.1 and 0.2 in Figures 2 and 3 and Figures 4 and 5, respectively. We have chosen the waveguide length and the corrugation depth as $2a = 10\lambda, 25\lambda, 45\lambda, 50\lambda$ and $2h = 0.02\lambda, 0.1\lambda$, respectively.

It is seen from all the figures that the bistatic RCS has maximum peaks at $\theta = -120°$ and $120°$, which correspond to the incident and reflected shadow boundaries, respectively. Comparing the results for the corrugated waveguide with those for the flat waveguide, we observe that the effect of the sinusoidal corrugation of the waveguide walls is noticeable in the reflection region $90° < \theta < 180°$, and the bistatic RCS has sharp peaks at two particular observation angles around the specularly-reflected direction at $\theta = \pi - \theta_0 (= 120°)$. Consideration on the infinite periodic structure may offer physical understanding of the scattering mechanism at these particular observation angles. Referring to (41), it is seen that $\pi - \theta_1$ and $\pi - \theta_2$ are, respectively, propagation directions of the (-1) and $(+1)$ order diffracted waves involved in the Floquet mode arising in periodic structures of infinite extent [17]. The angles $\pi - \theta_1, \pi - \theta_2$ are, respectively, $113.6°, 126.9°$ in Figures 2 and 3, and $107.5°, 134.4°$ in Figures 4 and 5, where somewhat large reflection is expected. In fact, we see that the observation angles associated with the two peaks around $\pi - \theta_0$ are precisely coincident with the directions at $\pi - \theta_1$ and $\pi - \theta_2$. On the other hand, the peaks along the specular reflection $\pi - \theta_0$ is also expected from the grating theory since they exactly correspond to the propagation direction of the zero-order Floquet mode. Therefore it is confirmed that

Figure 5. Bistatic RCS σ/λ [dB] of a flat waveguide and a corrugated waveguide for $\theta_0 = 60°$, $N = 9$, $2a = 45\lambda$, $m/k = 0.2$, $2b = 4\lambda$. (a) $2h = 0.02\lambda$. (b) $2h = 0.1\lambda$.
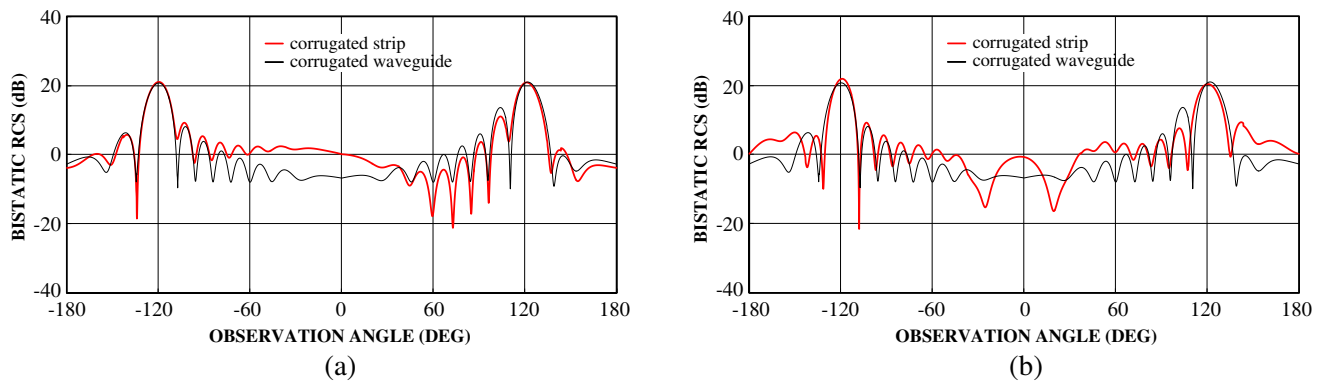
Figure 6. Bistatic RCS σ/λ [dB] of a corrugated strip and a corrugated waveguide for $\theta_0 = 60°$, $N = 1$, $2a = 5\lambda$, $m/k = 0.2$, $2h = 0.1\lambda$. (a) $2b = 0.4\lambda$. (b) $2b = 1.2\lambda$.
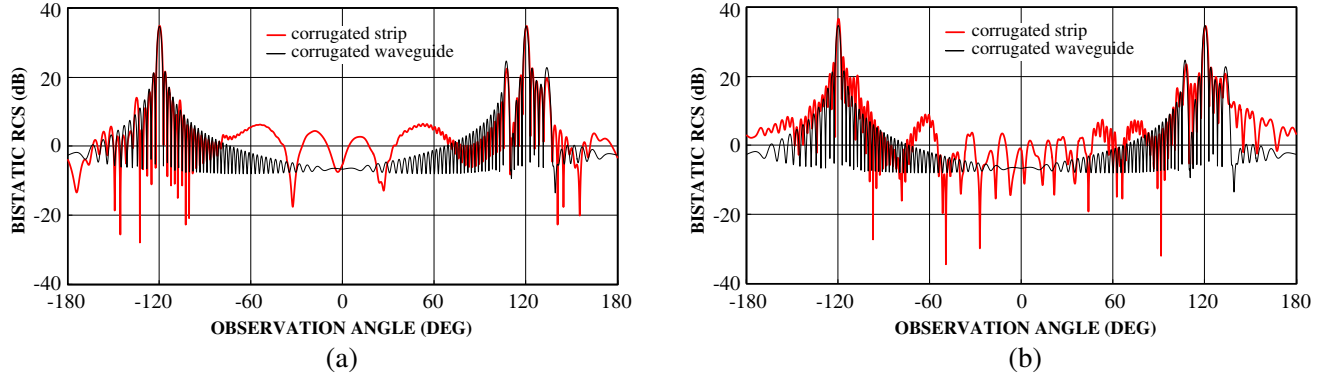
Figure 7. Bistatic RCS σ/λ [dB] of a corrugated strip and a corrugated waveguide for $\theta_0 = 60°$, $N = 5$, $2a = 25\lambda$, $m/k = 0.2$, $2h = 0.1\lambda$. (a) $2b = 2\lambda$. (b) $2b = 6\lambda$.

the three peaks at $\pi - \theta_0, \pi - \theta_1$, and $\pi - \theta_2$ in the results for the sinusoidal wall corrugation are due to the periodicity of the sinusoidal surface of the waveguide.

It can also be observed by comparing Figures 2(a), 3(a), 4(a), and 5(a) with Figures 2(b), 3(b), 4(b), and 5(b) that the peaks occurring at the $\pi - \theta_1$ and $\pi - \theta_2$ directions become sharper with an increase of $2h$. On comparing Figure 2 for $N = 1$ with Figure 3 for $N = 5$, we see the peaks along $\pi - \theta_1$ and $\pi - \theta_2$ more clearly for larger N. This is because, if N increases, then the surface of the waveguide walls approaches a periodic structure and hence, waves along the propagation directions of the particular Floquet modes are strongly excited.

We shall now investigate the scattering characteristics depending on the number of strips, namely a single corrugated strip and two corrugated strips (corrugated parallel-plate waveguide). Figures 6 and 7 show comparisons between these two structures. Scattering characteristics in the neighborhood of the main-lobe directions at $\theta = \pm 120°$ show close features for both cases as can be expected. The differences between a single strip and two strips occur noticeably for $|\theta| < 60°$ and $150° < \theta < 180°$. This is because, in this region, the effect due to the radiation from waveguide modes becomes stronger.

7. CONCLUDING REMARKS

In this paper, we have analyzed the diffraction by a finite parallel-plate waveguide with sinusoidal wall corrugation using the Wiener-Hopf technique combined with the perturbation method. Assuming that the corrugation amplitude is small compared with the wavelength and expanding the scattered field in the form of a perturbation series, the problem has been reduced to the diffraction by a flat, finite parallel wall waveguide with a certain mixed boundary condition. Using this approximate boundary condition, the problem has been formulated in terms of the simultaneous Wiener-Hopf equations. The Wiener-Hopf equations have been solved via the factorization and decomposition procedure leading to the exact and approximate solutions. Taking the inverse Fourier transform and applying the saddle point method, an asymptotic expression of the scattered far field has been derived.

Based on the results, we have carried out numerical computation of the RCS for various physical parameters and investigated the effect of sinusoidal corrugation of the waveguide walls in detail. As a result, it has been confirmed that, in the reflection region, scattered waves are strongly excited along the specific directions corresponding to the three dominant Floquet modes. We have compared scattering characteristics in detail between the flat waveguide and the corrugated waveguide and those between the single strip and the corrugated waveguide (i.e., two corrugated strips). Our solution is valid for the case where the corrugation depth and the corrugation period are small and large compared with the wavelength, respectively. Hence, the method of solution developed in this paper may become less accurate for deep or dense corrugation. In this paper, our idea for approximating the boundary condition on the sinusoidal surface was to use only the zero- and first-order terms in the Taylor expansion around the average surface. Hence, by keeping up to higher order terms in the Taylor series, it can be possible to extend the range of applicability of the method. This may serve as a future topic of research.

REFERENCES

1. Byelobrov, V. O., T. L. Zinenko, K. Kobayashi, and A. I. Nosich, "Periodicity matters: Grating or lattice resonances in the scattering by sparse arrays of subwavelength strips and wires," *IEEE Antennas Propagat. Mag.*, Vol. 57, No. 6, 34–45, 2015.

2. Shestopalov, V. P., *The Riemann-Hilbert Method in the Theory of Diffraction and Propagation of Electromagnetic Waves*, Kharkov University Press, Kharkov, 1971 (in Russian).

3. Nosich, A. I., "Green's function-dual series approach in wave scattering by combined resonant scatterers," *Analytical and Numerical Methods in Electromagnetic Wave Theory*, Chap. 9, M. Hashimoto, M. Idemen, and O. A. Tretyakov (eds.), Science House, Tokyo, 1993.

4. Shestopalov, V. P., L. N. Litvinenko, S. A. Masalov, and V. G. Sologub, *Diffraction of Waves by Gratings*, Kharkov University Press, Kharkov, 1973 (in Russian).

5. Shestopalov, V. P., A. A. Kirilenko, and S. A. Masalov, *Convolution-type Matrix Equations in the Theory of Diffraction*, Naukova Dumka Publishing, Kiev, 1984 (in Russian).

6. Nosich, A. I., "The method of analytical regularization in wave-scattering and eigenvalue problems: Foundations and review of solutions," *IEEE Antennas Propagat. Mag.*, Vol. 41, No. 3, 34–49, 1999.

7. Nosich, A. I., "Method of analytical regularization in computational photonics," *Radio Sci.*, Vol. 51, 1421–1430, 2016.

8. Ikuno, H. and K. Yasuura, "Improved point-matching method with application to scattering from a periodic surface," *IEEE Trans. Antennas Propagat.*, Vol. 21, No. 5, 657–662, 1973.

9. Okuno, Y., "The mode-matching method," *Analysis Methods for Electromagnetic Wave Problems*, Chap. 4, E. Yamashita (ed.), Artech House, Boston, 1990.

10. Okuno, Y., "An introduction to the Yasuura method," *Analytical and Numerical Methods in Electromagnetic Wave Theory*, Chap. 11, M. Hashimoto, M. Idemen, and O. A. Tretyakov (eds.), Science House, Tokyo, 1993.

11. Petit, R. (ed.), *Electromagnetic Theory of Gratings*, Springer-Verlag, Berlin, 1980.

12. Hinata, T. and T. Hosono, "On the scattering of electromagnetic wave by plane grating placed in homogeneous medium — Mathematical foundation of point-matching method and numerical analysis," *Trans. IECE Japan*, Vol. J59-B, No. 12, 571–578, 1976 (in Japanese).

13. Yamasaki, T., K. Isono, and T. Hinata, "Analysis of electromagnetic fields in inhomogeneous media by Fourier series expansion methods — The case of a dielectric constant mixed a positive and negative regions," *IEICE Trans. Electron.*, Vol. E88-C, No. 12, 2216–2222, 2005.

14. Ozaki, R., T. Yamasaki, and T. Hinata, "Scattering of electromagnetic waves by multilayered inhomogeneous columnar dielectric gratings," *IEICE Trans. Electron.*, Vol. E90-C, No. 2, 295–303, 2007.

15. Noble, B., *Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations*, Pergamon, London, 1958.

16. Weinstein, L. A., *The Theory of Diffraction and the Factorization Method*, The Golem Press, Boulder, 1969.

17. Mittra, R. and S.-W. Lee, *Analytical Techniques in the Theory of Guided Waves*, Macmillan, New York, 1971.

18. Kobayashi, K., "Wiener-Hopf and modified residue calculus techniques," *Analysis Methods for Electromagnetic Wave Problems*, Chap. 8, E. Yamashita (ed.), Artech House, Boston, 1990.

19. Baldwin, G. L. and A. E. Heins, "On the diffraction of a plane wave by an infinite plane grating," *Math. Scand.*, Vol. 2, 103–118, 1954.

20. Hills, N. L. and S. N. Karp, "Semi-infinite diffraction gratings — I," *Comm. Pure Appl. Math.*, Vol. 18, No. 1–2, 203–233, 1965.

21. Lüneburg, E. and K. Westpfahl, "Diffraction of plane waves by an infinite strip grating," *Ann. Phys.*, Vol. 27, No. 3, 257–288, 1971.

22. Lüneburg, E., "Diffraction by an infinite set of parallel half-planes and by an infinite strip grating," *Analytical and Numerical Methods in Electromagnetic Wave Theory*, Chap. 7, M. Hashimoto,

M. Idemen, and O. A. Tretyakov (eds.), Science House, Tokyo, 1993.

23. Serbest, A. H., A. Kara, and E. Lüneburg, "Scattering of plane waves at the junction of two corrugated half-planes," *Electromagnetics*, Vol. 25, No. 1, 21–38, 2005.

24. Idemen, M. and A. Alkumru, "Diffraction of two-dimensional high-frequency electromagnetic waves by a locally perturbed two-part impedance plane," *Wave Motion*, Vol. 42, 53–73, 2005.

25. Ayub, M., M. Ramzan, and A. B. Mann, "Acoustic diffraction by an oscillating strip," *Applied Mathematics and Computation*, Vol. 214, 201–209, 2009.

26. Kobayashi, K., "Diffraction of a plane wave by the parallel plate grating with dielectric loading," *Trans. IECE Japan*, Vol. J64-B, No. 10, 1091–1098, 1981 (in Japanese).

27. Kobayashi, K., "Diffraction of a plane electromagnetic wave by a parallel plate grating with dielectric loading: The case of transverse magnetic incidence," *Can. J. Phys.*, Vol. 63, No. 4, 453–465, 1985.

28. Kobayashi, K. and T. Inoue, "Diffraction of a plane wave by an inclined parallel plate grating," *IEEE Trans. Antennas Propagat.*, Vol. 36, No. 10, 1424–1434, 1988.

29. Kobayashi, K. and K. Miura, "Diffraction of a plane wave by a thick strip grating," *IEEE Trans. Antennas Propagat.*, Vol. 37, No. 4, 459–470, 1989.

30. Das Gupta, S. P., "Diffraction by a corrugated half-plane," *Proc. Vib. Prob.*, Vol. 3, No. 11, 413–424, 1970.

31. Chakrabarti, A. and S. Dowerah, "Traveling waves in a parallel plate waveguide with periodic wall perturbations," *Can. J. Phys.*, Vol. 62, No. 3, 271–284, 1984.

32. Zheng, J. P. and K. Kobayashi, "Diffraction by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation: Combined perturbation and Wiener-Hopf analysis," *Progress In Electromagnetics Research B*, Vol. 13, 75–110, 2009.

33. Zheng, J. P. and K. Kobayashi, "Combined Wiener-Hopf and perturbation analysis of the *H*-polarized plane wave diffraction by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation," *Progress In Electromagnetics Research B*, Vol. 13, 203–236, 2009.

34. Kobayashi, K. and T. Eizawa, "Plane wave diffraction by a finite sinusoidal grating," *IEICE Trans.*, Vol. E74, No. 9, 2815–2826, 1991.

35. Eizawa, T. and K. Kobayashi, "Wiener-Hopf analysis of the *H*-polarized plane wave diffraction by a finite sinusoidal grating," *Progress In Electromagnetics Research*, Vol. 149, 1–13, 2014.

36. Kobayashi, K., "Some diffraction problems involving modified Wiener-Hopf geometries," *Analytical and Numerical Methods in Electromagnetic Wave Theory*, Chap. 4, M. Hashimoto, M. Idemen, and O. A. Tretyakov (eds.), Science House, Tokyo, 1993.

37. Kobayashi, K., "Solutions of wave scattering problems for a class of the modified Wiener-Hopf geometries," *IEEJ Transactions on Fundamentals and Materials*, Vol. 133, No. 5, 233–241, 2013.

38. Kobayashi, K., "On generalized gamma functions occurring in diffraction theory," *J. Phys. Soc. Japan*, Vol. 60, 1501–1512, 1991.