

# A New GE/PSO Antenna Arrays Synthesis Technique and Its Application to DoA Estimation

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**Abstract**—Direction of arrival estimation has a noteworthy significance in numerous applications, such as radar systems, smart antennas, sonar, mobile communications, and space communications. The algorithms used to estimate the direction of arrival are to some degree complex and time consuming. Also, the number of antenna elements is a discriminating parameter for assessing the performance of the DoA technique. For real time systems, quick and savvy techniques are required. Along these lines, decreasing the estimation time and also reducing the system cost while keeping a generally high precision are crucial issues. In this paper, a new technique for linear antenna arrays synthesis using optimized number of antenna elements and its application to direction of arrival estimation is introduced. The synthesized arrays exhibit approximately the same radiation pattern as the original arrays. The optimized antenna arrays are synthesized using reduced number of antenna elements. In this case, the number of antenna elements reduction will minimize the system cost and decrease the number of picked samples from the different signal sources. As the number of samples decreases, the dimensions of the steering matrix and data correlation matrix are reduced. In this context, the computational burden, estimation time, and system cost are optimized. The proposed technique can be applied to single or multi-snapshot DoA estimation techniques.

## 1. INTRODUCTION

Linear antenna arrays synthesis using reduced number of antenna elements has gained great research attention. Recently, the matrix pencil method (MPM) [1], forward-backward matrix pencil method (FBMPM) [2], and hybrid technique between the method of moments and the genetic algorithm (MoM/GA) [3] have been successfully applied to synthesizing linear antenna arrays. A new iterative technique for pencil beam pattern synthesis of linear and planar antenna arrays using minimized number of antenna elements is presented in [4]. The technique is based on a hybrid combination between the Nonuniform Fourier Transform (NUFFT) and a global optimization method. The NUFFT is utilized to determine excitation coefficients for a fixed positions nonuniform array. On the other hand, the simulated annealing (SA) is used alternatively to determine the optimal positions. The number of elements reduction is achieved by iteratively removing the elements that contribute the least to the array factor. Generally, the excitation coefficients and elements locations are iteratively recalculated until the SLL is unchanged, or the maximum iteration number is reached. A hybrid technique for nonuniform planar antenna arrays synthesis considering the coupling between the array elements is presented in [5]. The deterministic array factor is used for the preliminary evaluation of the array excitation coefficients and element positions for a desired radiation pattern. Afterwards, the obtained array structure is optimized using PSO in combination with the Multiport Network Model algorithm for fast modeling of spurious mutual coupling processes. In the same context, a modified differential

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evolution algorithm for radiation pattern synthesis of antenna arrays is presented in [6]. Two novel strategies for best of random mutation and randomized local search are introduced to enhance and accelerate the convergence of standard differential evolution algorithm (SDE). Although these arrays synthesis algorithms [4–6] provide high performance, they are time consuming.

In parallel, direction of arrival estimation (DoA) has importance in many applications such as radar systems, satellite communications, mobile communications, and smart antennas. Many research efforts attempted to estimate the DoAs of coherent and non-coherent sources either using single snapshot or multi-snapshots. Recently, a new high resolution DoA estimation technique “MV-SVD” for coherent signals is presented in [7]. It employs a hybrid combination between the virtual array extension, singular value decomposition (SVD), and modified MUSIC algorithms. It provides high resolution and stability at low SNR values, and increases the maximum number of detectable sources to  $M - 1$ , where  $M$  is the number of antenna array elements. A new modified MUSIC algorithm for coherent sources detection known as amendment MUSIC algorithm is presented in [8]. The amendment MUSIC spectrum is used instead of the original MUSIC spectrum to enhance the resolution of the DoA estimation process. The performance of the previously stated DoA estimation techniques can be optimized in terms of computational burden, estimation time, and system cost. In this paper, a new antenna array synthesis technique called GE/PSO is introduced. The GE/PSO technique is based on the Gaussian Elimination [9] and particle swarm optimization techniques [10]. This novel technique is applied to MV-SVD technique for the DoA estimation. It is worth mentioning that this technique can also be applied to other DoA techniques. The optimized arrays exhibit approximately the same radiation patterns as the original arrays using reduced number of antenna elements. The number of antenna elements reduction will minimize the system cost and decreases the number of picked samples from the different signal sources. As the number of samples decreases, the dimensions of the steering matrix and data correlation matrix are reduced. In this context, the computational burden, estimation time, and system cost are optimized.

## 2. PROPOSED GE/PSO ARRAY SYNTHESIS TECHNIQUE

In this section, a simple and accurate algorithm for linear antenna arrays synthesis using reduced number of equally spaced antenna elements is introduced. The proposed technique is simply a combination of the Gaussian Elimination method (GE) and Particle Swarm optimization (PSO). The desired array factor of a linear antenna array consisting of  $L$  isotropic elements located at  $z = ld$  along the  $z$ -axis as shown in Figure 1 is given by,

$$AF(\theta) = \sum_{l=0}^{L-1} a_l \exp(jkld \cos \Theta) \quad (1)$$

where  $\Theta$  denotes the azimuth angle,  $a_l$  the excitation coefficient of the  $l^{th}$  antenna element,  $k = 2\pi/\lambda$  the free space wave number [3],  $d$  the adjacent element spacing, and the position of the first element is at the origin. In order to synthesize the array by calculating the excitation coefficients, element spacing and number of elements, the following algorithm is introduced. Assume that the array factor of the synthesized antenna array consists of  $M$  isotropic elements located at  $z = md$  along the  $z$ -axis as shown in Figure 1.

$$AF_o(\theta) = \sum_{m=0}^{M-1} a_m \exp(jkmd_o \cos \Theta) \approx AF(\Theta) \quad (2)$$

where  $AF_o(\Theta)$  is the optimized array factor,  $a_m$  the excitation coefficient of the  $m^{th}$  antenna element,  $d_o$  the optimized adjacent element spacing, and the position of the first element is at the origin. Eq. (2) is expressed in matrix form as follows:

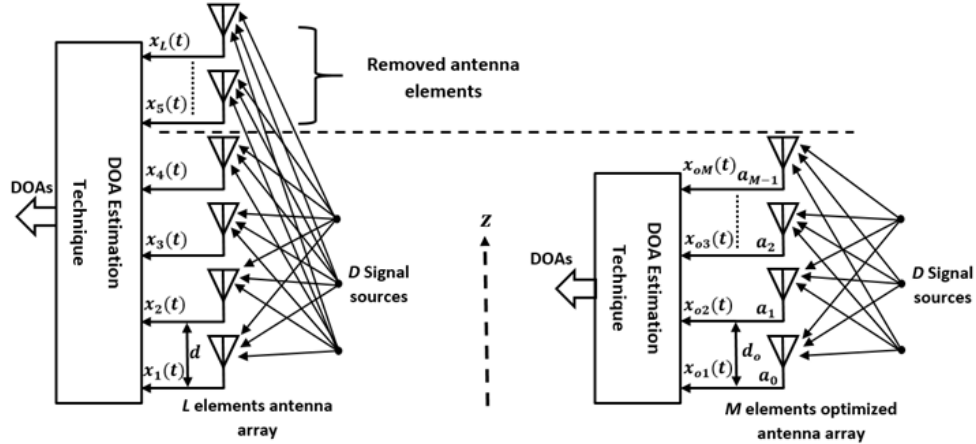
$$[X]_{M \times N} [A]_{M \times 1} = [Y]_{N \times 1} \quad (3)$$

where  $N$  is the number of samples used to represent the desired array factor. The elements of the matrix  $[X]_{M \times N}$  are given by,

$$X_{mn} = e^{jkmd_o \cos(\Theta_n)}, \quad m = 0, 1, 2, \dots, M-1 \quad \text{and} \quad n = 0, 1, 2, \dots, N \quad (4)$$

The elements of the vector  $[Y]_{N \times 1}$  are given by,

$$Y_n = AF(\Theta_n) \quad (5)$$



**Figure 1.** Block diagram of the ordinary L-elements and the optimized M-elements DoA estimation systems.

where  $\Theta_n$  are the angles at which the desired array factor is sampled.  $a_m$  are the elements of the vector  $[A]_{M \times 1}$ , where  $[A]_{M \times 1} = [a_0, a_1, a_2, \dots, a_{M-1}]^T$ . The optimized excitation coefficients  $a_m$  are determined by solving the linear system of Eq. (3) simply using GE method [9].

Because any antenna array is characterized by three main parameters, number of array elements ( $M$ ), element spacing ( $d_o$ ), and excitation coefficients ( $a_m$ ), the GE algorithm [9] is used to obtain the excitation coefficients while the PSO is utilized to get the optimum values of  $M$ , and  $d_o$ . PSO is one of the intelligent optimization algorithms. A swarm of particles (individuals) is randomly initialized in a search space, where each particle has a position. Each particle is searching for the optimum solution. Each particle moves hence it also has velocity. The initial particles' velocities are 1/4 of parameter space size or set to zero. The position of each particle is optimized through fitness function, then searching for the optima by updating its parameters. The cost function  $F(k)$  to be minimized is taken as the least mean square error between the optimized array factor and the desired array factor as follows

$$F(k) = \frac{1}{N} \sum_{n=1}^N [|AF(n) - AF_o(n)|]^2 \tag{6}$$

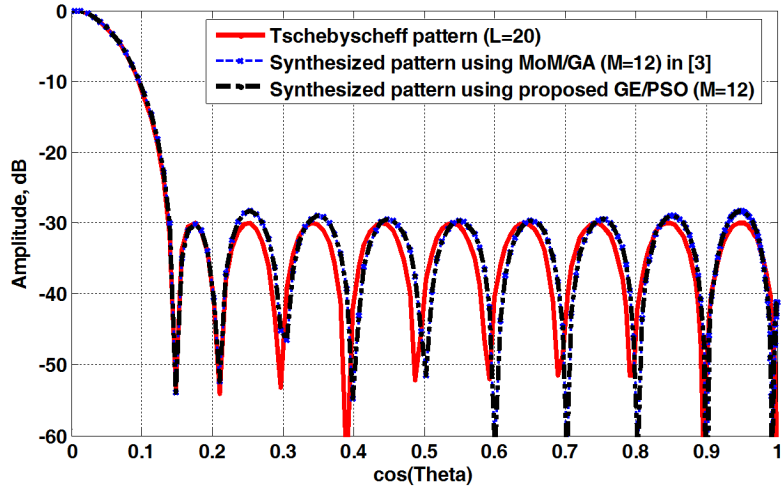
where  $N$  is the number of samples which are used to represent the desired array factor.

### 3. RESULTS AND DISCUSSIONS

In this section, three test cases of simulations are performed. In the first case, the proposed GE/PSO technique is carried out on a Chebyshev linear antenna array to verify its effectiveness over the MoM/GA technique [3]. The second case is intended to the application of the GE/PSO on a printed antenna array to study its effect on the coupling between the array elements. The third test case is dedicated to the application of the GE/PSO to enhance the performance of DoA estimation techniques.

#### 3.1. Case 1: Synthesis of Chebyshev Antenna Array

Consider Chebychev antenna array consisting of  $L=20$  array elements with side lobe level  $SLL=30$  dB and a half wavelength spacing between elements. This Chebyshev pattern is used as a desired pattern for both the MoM/GA and the proposed GE/PSO. After applying both techniques, the desired pattern is synthesized using only  $M=12$  uniformly spaced antenna elements. Figure 2 shows a very good agreement between the synthesized pattern and desired Chebyshev pattern. A comparison between the synthesized arrays using GE/PSO and MoM/GA and the ordinary 20-elements  $\lambda/2$  Chebyshev array is indicated in Table 1. The simulation results reveal that the two techniques have very close performances, but the complexity of the GE/PSO algorithm is much lower than the MoM/GA.



**Figure 2.** Synthesized radiation pattern using MoM/GA and the proposed GE/PSO compared to the ordinary 20-elements Chebyshev pattern.

**Table 1.** Comparison between the proposed GE/PSO, the MoM/GA, and the traditional 20-elements  $\lambda/2$  Chebyshev array.

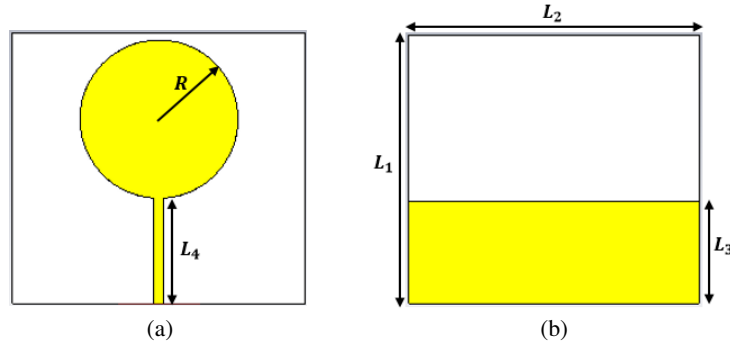
The analytical Chebyshev array		Synthesized array using MoM/GA in [3]		Synthesized array using GE/PSO	
$L = 20$		$M = 12$		$M = 12$	
$d/\lambda = 0.5$		$d/\lambda = 0.832$		$d_o/\lambda = 0.835$	
$a_1$	1	$a_1$	1.4371	$a_1$	1.4488
$a_2$	0.8771	$a_2$	1.9962	$a_2$	1.9695
$a_3$	1.2009	$a_3$	2.9227	$a_3$	2.8997
$a_4$	1.5497	$a_4$	3.8819	$a_4$	3.8766
$a_5$	1.9052	$a_5$	4.6715	$a_5$	4.6890
$a_6$	2.2465	$a_6$	5.1173	$a_6$	5.1506
$a_7$	2.5522				
$a_8$	2.8022				
$a_9$	2.9793				
$a_{10}$	3.0712				
HPBW = $6.1879^\circ$		HPBW = $6.1879^\circ$		HPBW = $6.1879^\circ$	

### 3.2. Case 2: Synthesis of a Printed Chebyshev Antenna Array

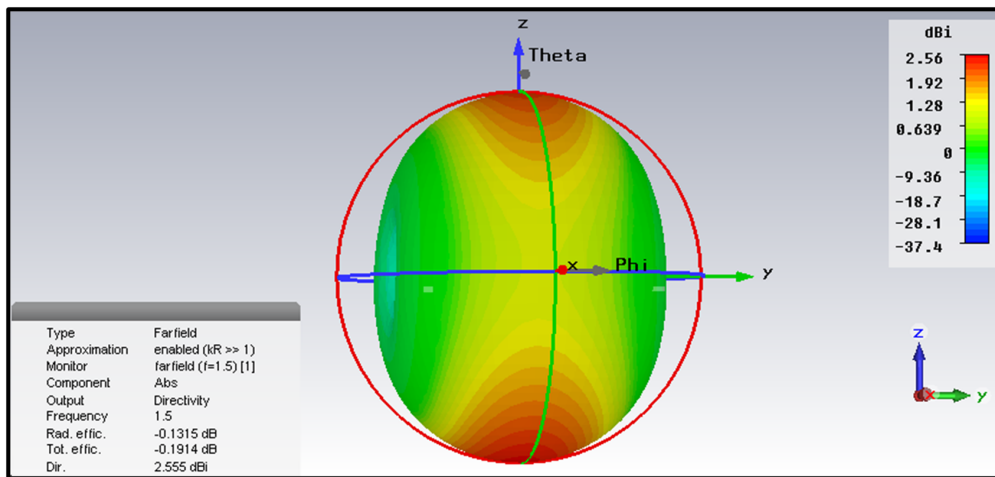
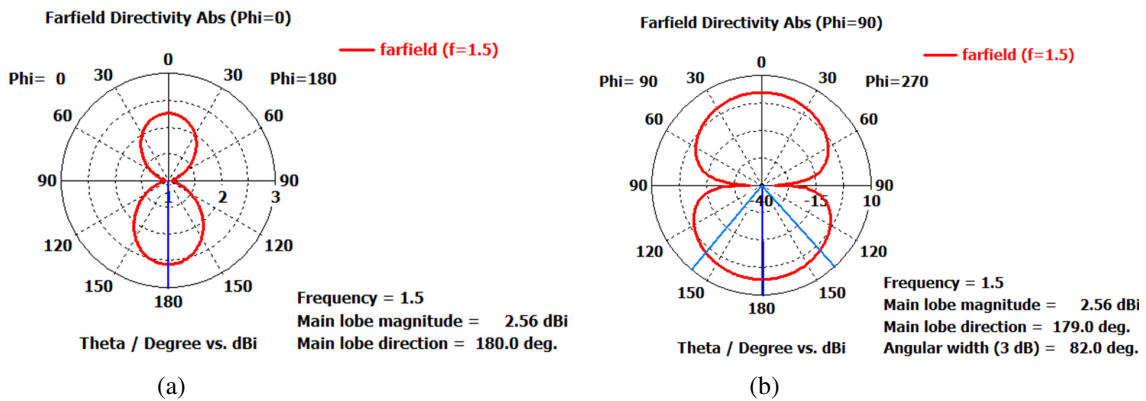
In this case, the GE/PSO technique is applied to the aforementioned Chebyshev array to study its effect on the coupling between the array elements. Consider a printed Chebyshev antenna array consisting of  $L = 20$  Ultra Wideband (UWB) circular patch antenna elements. Figure 3 shows the geometry

**Table 2.** Dimensions of the UWB antenna.

$L_1$	$L_2$	$L_3$	$L_4$	R
82 mm	88.8 mm	31 mm	31.8496 mm	24 mm

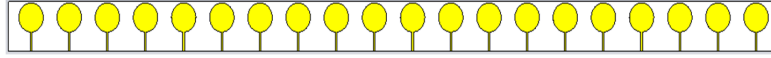


**Figure 3.** (a) Radiating patch. (b) Defected ground of the UWB antenna designed using CST microwave studio.

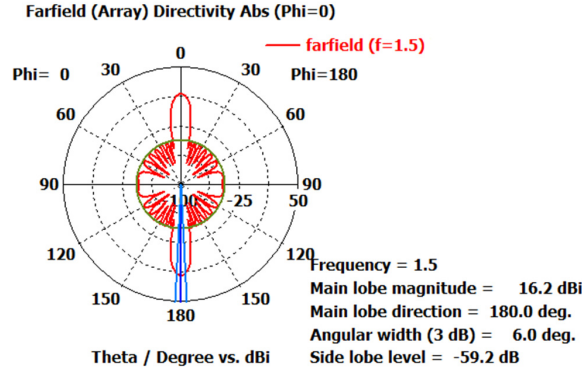


**Figure 4.** (a) polar plot in  $\varphi = 0^\circ$  plane, (b) polar plot in  $\varphi = 90^\circ$  plane, and (c) 3D plot of radiation pattern of the UWB antenna element.

and dimensions of the antenna with a circular aperture. The UWB antenna is designed using the CST-Microwave studio software where an FR4 (lossy) substrate with  $\epsilon_r = 4.3$  and  $h = 1.6$  mm is utilized. The antenna dimensions are listed in Table 2. The radiation pattern of the antenna element is relatively as the dipole antenna at  $f = 1.5$  GHz as shown in Figure 4. The designed antenna is then used to construct a Chebyshev array consisting of  $L = 20$  elements, with uniform element spacing  $d = \lambda/2 = 75$  mm at  $f = 1.5$  GHz as shown in Figure 5. Applying the excitation coefficients of the ordinary Chebyshev



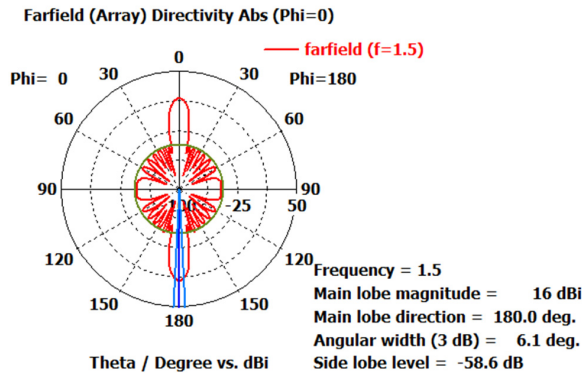
**Figure 5.** UWB Chebyshev array consisting of  $L = 20$  elements with uniform element spacing  $d = \lambda/2 = 75$  mm at  $f = 1.5$  GHz.



**Figure 6.** The radiation pattern of ordinary Chebyshev array consisting of  $L=20$  elements with uniform element spacing  $d = \lambda/2 = 75$  mm at  $f = 1.5$  GHz.



**Figure 7.** Synthesized Chebyshev array using  $M = 12$  elements with uniform element spacing  $d_o = 0.835 \lambda = 125.25$  mm at  $f = 1.5$  GHz.



**Figure 8.** The radiation pattern of the synthesized Chebyshev array using  $M = 12$  elements.

array listed in Table 1, the resulting radiation pattern will be as shown in Figure 6. The array provides  $SLL = -59.2$  dB,  $HPBW = 6^\circ$ , directivity  $D = 16.2$  dBi, and radiation efficiency  $-0.3421$  dB which corresponds to 92.42%. On the other hand, the synthesized Chebyshev array consisting of  $M = 12$  elements, with uniform element spacing  $d_o = 0.835\lambda = 125.25$  mm at  $f = 1.5$  GHz, is shown in Figure 7. Applying the excitation coefficients of the synthesized Chebyshev array listed in Table 1, the array radiation pattern is as shown in Figure 8. The array provides  $SLL = -58.6$  dB,  $HPBW = 6.1^\circ$ , directivity  $D = 16$  dBi, and radiation efficiency  $-0.1938$  dB which corresponds to 95.6%. Additionally, the coupling between the antenna elements  $S_{i,j}$  for ordinary and synthesized antenna arrays are plotted in Figure 9 and Figure 10, respectively. It is clear that the synthesized array provides less coupling than the ordinary array. For example, the coupling coefficient  $S_{11,1}$  for the ordinary array lies in the range  $-60.96$  dB  $< S_{11,1} < -43.7$  dB over the specified frequency range while the coupling coefficient

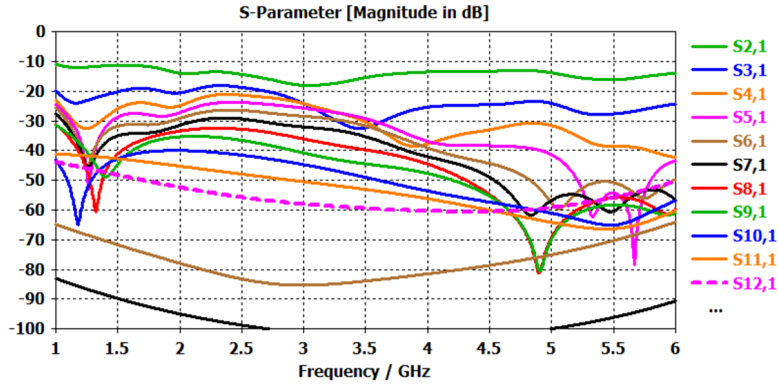


Figure 9. The scattering parameters  $S_{i,j}$  of the ordinary Chebyshev array.

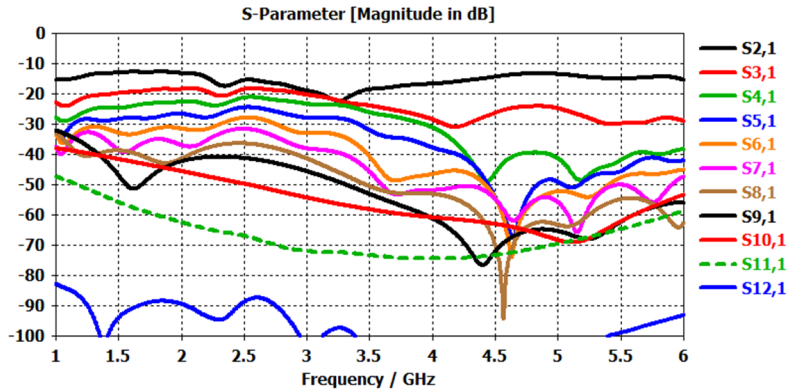


Figure 10. The scattering parameters  $S_{i,j}$  of the synthesized Chebyshev array.

$S_{11,1}$  for the synthesized array lies in the range  $-75 \text{ dB} < S_{11,1} < -47.344 \text{ dB}$  over the same frequency range. From these results, it is obvious that the proposed synthesis technique provides higher radiation efficiency and lower coupling between the antenna elements than the ordinary array.

### 3.3. Case 3: Application of the Proposed GE/PSO Array Synthesis Technique to DoA Estimation Techniques

Consider  $D$  narrowband signals incident on a uniform linear antenna array consisting of  $L$  uniformly spaced antenna elements with element spacing  $d = \lambda/2$ . Assume that the signals are received from the directions  $(\theta_1, \theta_2, \theta_3, \dots, \theta_D)$  where  $(D \leq L)$ . The output of each array element in the antenna array at time  $t$  is expressed in a matrix form as [7],

$$X(t) = AS(t) + n(t) \tag{7}$$

where

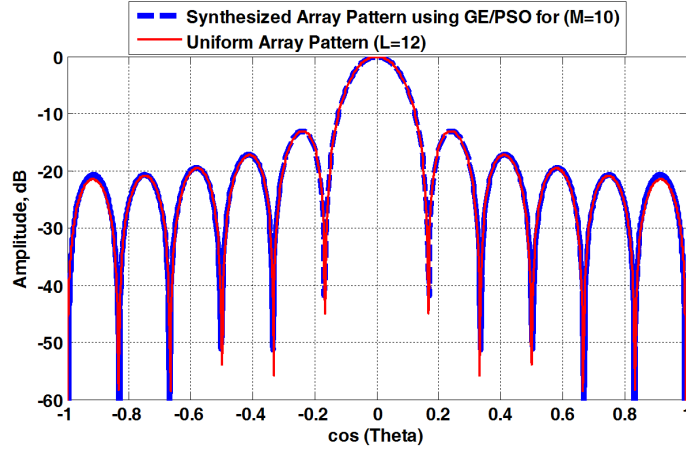
$$\begin{aligned} X(t) &= [x_1(t), x_2(t), \dots, x_L(t)]^T \\ S(t) &= [s_1(t), s_2(t), \dots, s_D(t)]^T \\ n(t) &= [n_1(t), n_2(t), \dots, n_L(t)]^T \\ A(\theta) &= [a(\theta_1), a(\theta_2), \dots, a(\theta_D)] \\ a(\theta_i) &= \left[ 1, e^{-j\frac{2\pi}{\lambda}d \sin \theta_i}, \dots, e^{-j(L-1)\frac{2\pi}{\lambda}d \sin \theta_i} \right]^T \end{aligned} \tag{8}$$

where  $x_l(t)$ ,  $(l = 1, 2, \dots, L)$  is the input of the  $l^{th}$  antenna element,  $s_d(t)$   $(d = 1, 2, \dots, D)$  the complex amplitude of the narrow band signals,  $A(\theta)$  the array manifold matrix, and  $a(\theta_i)$  the array steering

vector. The superscript  $T$  is vector or matrix transpose. By applying the proposed GE/PSO array synthesis technique, the antenna array is synthesized using only  $M$  elements. The set of Eqs. (7) and (8) can be rewritten using  $M < L$  as follows

$$\begin{aligned}
 X_{opt}(t) &= A_{opt}S(t) + n_{opt}(t) \\
 X_{opt}(t) &= [x_{o1}(t), x_{o2}(t), \dots, x_{oM}(t)]^T \\
 S(t) &= [s_1(t), s_2(t), \dots, s_D(t)]^T \\
 n_{opt}(t) &= [n_{o1}(t), n_{o2}(t), \dots, n_{oM}(t)]^T \\
 A_{opt}(\theta) &= [a_{opt}(\theta_1), a_{opt}(\theta_2), \dots, a_{opt}(\theta_D)] \\
 a_{opt}(\theta_i) &= \left[ a_0, a_1 e^{-j\frac{2\pi}{\lambda}d_o \sin \theta_i}, \dots, a_M e^{-j(M-1)\frac{2\pi}{\lambda}d_o \sin \theta_i} \right]^T
 \end{aligned} \tag{9}$$

These optimized parameters can be directly applied to any DoA estimation technique. In this case, the proposed GE/PSO technique is applied to the high resolution MV-SVD DoA estimation technique presented in [7]. Consider  $D=8$  coherent signals incident on  $L=12$  elements uniform linear array (ULA) with element spacing  $d=\lambda/2$  from the directions  $(-30^\circ, -20^\circ, -10^\circ, 0^\circ, 10^\circ, 20^\circ, 30^\circ$  and  $40^\circ)$  at  $SNR=-5$  dB. Applying the proposed GE/PSO technique, the ULA is synthesized using  $M=10$  antenna elements with uniform element spacing  $d_o=0.6007\lambda$ . Figure 11 shows a very close copy of the ULA pattern. The synthesized excitation coefficients using GE/PSO are listed in Table 3. These optimized parameters are directly applied to the set of equations, Eq. (7), Eq. (8), Eq. (9), and Eq. (10), respectively in [7] where the MV-SVD DoA estimation technique is presented. These equations are rewritten as follows.



**Figure 11.** Synthesized radiation pattern using 10 uniformly spaced elements compared to the 12-elements ULA pattern using the proposed GE/PSO.

**Table 3.** The synthesized excitation coefficients using the proposed GE/PSO.

$\mathbf{a}_1$	$\mathbf{a}_2$	$\mathbf{a}_3$	$\mathbf{a}_4$	$\mathbf{a}_5$	$\mathbf{a}_6$	$\mathbf{a}_7$	$\mathbf{a}_8$	$\mathbf{a}_9$	$\mathbf{a}_{10}$
1.1557	1.2145	1.1977	1.2014	1.2017	1.2017	1.2014	1.1977	1.2145	1.1557

Applying the virtual array extension on the optimized antenna array of  $M$  elements, the optimized received data matrix  $X_{opt}(t)$  is used to construct a  $(2M - 1) \times K$  dimensional matrix  $X_{1opt}$  instead of  $(2L - 1) \times K$  dimensional matrix,

$$X_{1opt} = \begin{bmatrix} X_{opt}(t)' \\ X_{opt}(t) \end{bmatrix} \tag{10}$$



where  $K$  is the number of received snapshots, and  $X_{opt}(t)'$  can be expressed as follows

$$X_{opt}(t)' = [x_{oM}(t)^*, x_{oM-1}(t)^*, x_{oM}(t)^*, \dots, x_{o2}(t)^*]^T. \quad (11)$$

The optimized virtual data covariance matrix  $R_{ox_1}$  is given by

$$R_{ox_1} = E [X_{1opt}X_{1opt}^H] = A_{o1}R_{ss}A_{o1}^H + \sigma_{on}^2 I_{2M-1} \quad (12)$$

where  $A_{o1}$  is the optimized array factor in matrix form,  $R_{ss} = E[SS^H]$  the source covariance matrix,  $E[n_{opt}(t)n_{opt}(t)^H] = \sigma_{on}^2 I$ ,  $\sigma_{on}^2$  the noise variance,  $I$  the identity matrix,  $H$  the conjugation transpose, and  $E[\cdot]$  the expectation.

The singular value decomposition is applied to the data covariance matrix  $R_{ox_1}$  to obtain the eigenvector  $e = [e_1, e_2, \dots, e_{2M-1}]$  which corresponds to the largest eigenvalues of the data covariance matrix  $R_{ox_1}$ . This eigenvector is used to construct the eigenvalues matrix  $Y_{opt}$  as follows

$$Y_{opt} = \begin{bmatrix} e_1 & e_2 & \dots & e_p \\ e_2 & e_3 & \dots & e_{p+1} \\ \dots & \dots & \dots & \dots \\ e_m & e_{m+1} & \dots & e_{2M-1} \end{bmatrix} \quad (13)$$

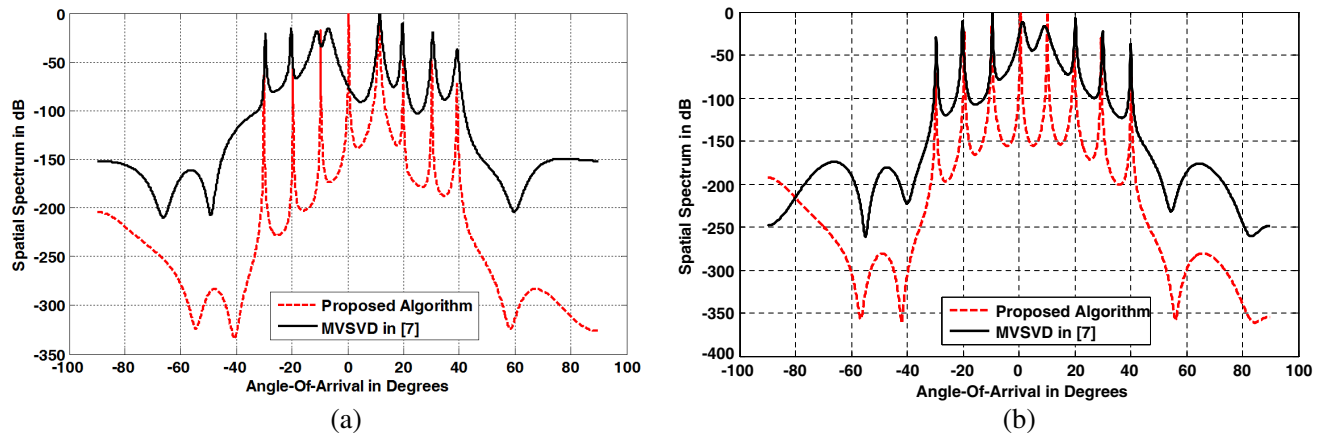
where  $m = M$ , and  $m + p - 1 = 2M - 1$ . In this case, the constructed matrix  $Y_{opt}$  is an  $M \times M$  dimensional matrix.  $Y_{opt}$  is used to construct the two matrices  $Y_0$  and  $Y_1$  applying Eqs. (14) and (15) as presented in [7].

$$Y_0 = Y_{opt}Y_{opt}^H \quad (14)$$

$$Y_1 = \frac{1}{2}(Y_0 + J_m Y_0^* J_m) \quad (15)$$

The directions of arrivals of the incident signals are estimated by locating the peaks of the modified MUSIC spectrum  $P_{MUSIC}(\theta)$  as presented in [7].

Figure 12 shows the spatial spectrum of the proposed optimized array based MV-SVD technique compared to the spatial spectrum of the ordinary array based MV-SVD technique. It is noted from Figures 12(a) and (b) that the ordinary array based MVSVD technique has different performances when running more than one time since it is based on a random process. Furthermore, all the sources are detected using the proposed technique while not all the sources are detected using the ordinary pattern based MVSVD algorithm since six of the eight sources are detected as in Figure 12(a). Also, one can notice that the new technique has a high selectivity (sharp spectrum peaks) compared to the ordinary array based MVSVD technique. So, we can say that the proposed technique provides more accurate results and higher stability. Also the execution time of the proposed technique is 0.0503 sec while that of the ordinary array based MVSVD is 0.0635 sec. It allows 20.7874% time saving.



**Figure 12.** (a) and (b): spatial spectrum of the proposed technique compared to the spatial spectrum of the MVSVD algorithm based on the ordinary array.

#### 4. CONCLUSION

In this paper, a new technique for linear antenna arrays synthesis using reduced number of antenna elements and its application to direction of arrival estimations techniques is developed. The optimized arrays exhibit approximately the same radiation patterns as the original arrays using reduced number of antenna elements. The simulation results reveal that the proposed technique reduces the coupling between the antenna array elements and increases the radiation efficiency. For DoA estimation, the number of antenna elements reduction will minimize the system cost and decreases the number of picked samples from the different signal sources. As the number of samples decreases, the dimensions of the steering matrix and the data correlation matrix are reduced. As a result, the computational burden, estimation time, and system cost are minimized. Furthermore, the proposed optimized array based DoA estimation technique provides more accurate results and higher stability than the ordinary array based DoA estimation techniques.

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