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Side Lobe Level Reduction of Any Type of Linear Equally Spaced Array Using the Method of Convolution

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Abstract—In most applications of antenna arrays, side lobe levels (SLLs) are commonly unwanted. Especially, the first side lobe level which determines maximum SLL is the main source of electromagnetic interference (EMI), and hence, it should be lowered. A procedure of finding the optimum side lobe-minimizing weights for an arbitrary linear equally spaced array is derived, which holds for any scan direction, beam width, and type of antenna element used. In this science article, the use of convolution procedure and the time scaling property reduces the side lobe level for any type of linear equally spaced array. Results show that by this procedure, the side lobe level is reduced about two times or even more.

1. INTRODUCTION

Array antenna design and application has become very popular in several fields of engineering in recent decades [1–4]. An antenna array (often called a phased array) is a set of N spatially separated antennas. The number of antennas in an array can be as small as two or as large as several thousands. Radiating elements might be dipoles, open-ended waveguides, slotted waveguides, microstrip antennas, helices, spirals, etc. A phased array antenna offers the possibility to steer the beam by means of electronic control. Array antennas provide the designer additional degree of freedom relating to a single antenna due to existence of a number of radiating elements. By properly adjusting the relative phase or amplitude of the array elements, radiation pattern of the array is adjusted in a desired direction, or the main beam is suppressed along undesired directions. The antenna array can also be used to increase the overall gain, provide diversity reception, cancel out interference from a particular set of directions, determine the direction of arrival of the incoming signals, maximize the signal to interference plus noise ratio (SINR), etc.

In most applications of antenna arrays, side lobe levels (SLLs) are commonly unwanted. Especially, the first SLL which determines maximum SLL is the main source of electromagnetic interference (EMI), and hence, it should be reduced without disturbing the width of main beam of the pattern. SLL reduction is a crucial topic in some applications such as radar systems, which plays a significant role in anti-jamming methods. A procedure of finding the optimum side lobe-minimizing weights for an arbitrary linear equally spaced array is derived, which holds for any scan direction, beam width, and type of antenna element used. In this article, by use of convolution procedure and the time scaling property, reduce SLL for any type of linear equally spaced array. We know that time compression of a signal results in its spectral expansion and that time expansion of the signal results in its spectral compression. Therefore, the resulting array has low SLL but wide half power beam width. Results show that by this procedure, the SLL is reduced about two times or even more.

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2. THEORY

Techniques for designing antenna array can be divided into two main objects: 1) finding the excitations and 2) finding the positions of antenna elements that obtain a set of trade-off solutions between mean beam width and SLL. In this paper, we will concentrate on uniform spacing for any type of linear antenna array. Figure 1 exhibits a linear equally spaced array in the z axes. Let us assume that the number of elements forming the array is N. The nth component has weight a_n. The z-directed elements are spaced d apart. The output of a linear phase array can be written as

$$AF = \sum_{n=0}^{N-1} a_n e^{jn\psi} = \sum_{n=0}^{N-1} a_n e^{j(nkd\cos(\theta)+\beta)} \tag{1}$$

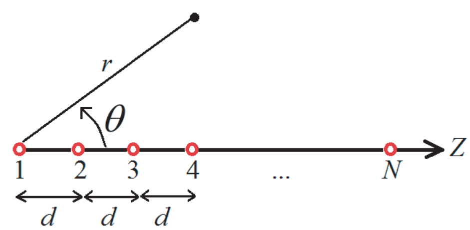

Figure 1. Geometry of linear equally spaced array antenna.

In the above, k is the wave vector, which specifies the variation of the phase as a function of position, and ψ is the phase between the elements. Also, β represents the phase by which the current in each component leads the current of the preceding element. The phase β can be written as

$$\beta = -kd\cos(\theta_0) \tag{2}$$

where θ_0 is beam steering. The weight a_n may be uniform or may be in any form according to the designer's needs. This can include various weights such as the Gaussian, Kaiser-Bessel, Hamming, or Blackman weights.

From antenna theory, we know that the excitation distribution and far-field array factor are related by Fourier transforms. Array factor formula represents a finite Fourier series that relates the element excitation coefficients (a_n) of the array. For the normalized array factor, if element excitation coefficients of antenna array convolve by themselves, the corresponding normalized array factor is multiplied by itself. By this work, the proposed array can provide SLL values lower than those achievable by an arbitrary excited array. Moreover, the new array factor possesses a narrower main beam than the old array factor, indicating that smaller half-power beam widths are produced. It is also noted that the primary array factor and new array factor are periodic functions of ψ with period of 2π and have similar general far-field radiation pattern structures, and the maximum and minimum point positions will be unchanged. From mathematics theory, we know that if vectors x_n and x_m have n and m elements, respectively, convolution of two vectors has $n + m - 1$ members. According to mentioned rule, disadvantage of this method is increasing the number of array elements. Thus, for overcoming this problem and fixing the length of array, the new element excitation coefficients must be rescaled. Let us denote element excitation coefficients of new and primary arrays by a_n and a_m, respectively. The element excitation coefficients of new array can be obtained by convoluting a_m by itself as follow.

$$a_n = a_m * a_m \tag{3}$$

According to the rule mentioned above, vector a_n has $2N - 1$ elements, but we want to have an array with N elements. Therefore, we should rescale the element excitation coefficients a_n. Hence, we keep odd elements of vector a_n. This sampling method known as the time scaling property of Fourier transform. As a rule, the time scaling property of Fourier transform implies that time compression of a signal results in its spectral expansion and that time expansion of the signal results in its spectral compression. Therefore, the new array has low SLL but wide beam width.

For a specific normalized amplitude distribution plotted in Figure 2, the amplitude broad banding factor L_a and the place broad banding factor L_x can be defined. In the next section, these two coefficients will be considered to be the important parameters of aperture distribution of a linear equally spaced array. According to the mentioned definition, the amplitude broad banding factor L_a can be calculated by the difference between maximum and minimum excitation coefficient amplitudes. Computing procedure of the place broad banding factor L_x consists of two steps. First step includes the process of finding the middle point normalized amplitude distribution in the vertical direction. After the middle point is determined, the second step is to draw a horizontal line passing this point. Finally, the place broad banding factor L_x can be computed by difference points of intersection of horizontal line with normalized amplitude distribution. In the future, these two coefficients will be used to the analysis of directivity, half power beam width and SLL. Also, it will be shown that variation of directivity, half power beam width and SLL is related to variation of the amplitude broad banding factor and the place broad banding factor.

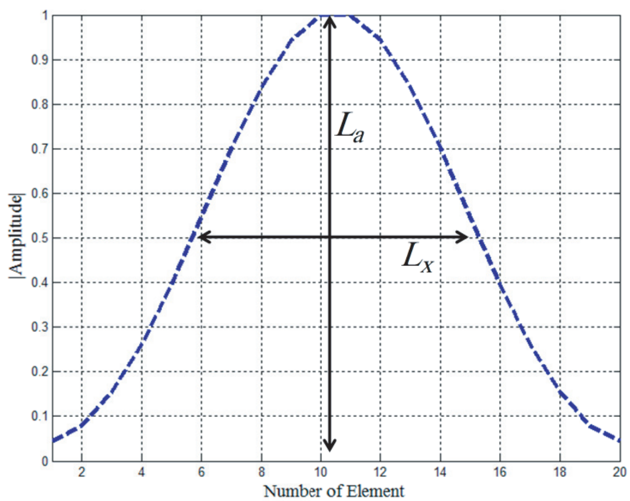

Figure 2. Amplitude distribution of specific normalized amplitude distribution.

It can be assumed that the aperture antennas are similar to array antennas. In aperture antennas, half power beadwidth and aperture dimension in one direction are approximately related to the following formula [5].

$$HP_{\text{aperture-antenna}} = \frac{K}{L/\lambda} \tag{4}$$

where L is length of the aperture, and K is a constant number. According to assumed rule, half power beamwidth of an array can be approximated by similar formula as follow.

$$HP_{\text{array}} = \frac{K}{L_x/\lambda} \tag{5}$$

In the above formula, L_x is the place broad banding factor. In fact, we know that the excitation distribution and far-field array factor are related by Fourier series. On the other hand, the time scaling property of Fourier series implies that time compression of a signal results in its spectral expansion and that time expansion of the signal results in its spectral compression. Hence, half power beamwidth and L_x factor related to each other inversely. By applying convolution method, L_x factor becomes great. Place broad banding factor compression of the aperture distribution results in its pattern expansion, and then half power beamwidth becomes wide. Let us denote place broad banding factor of new and primary array by L_{xn} and L_{xo}, respectively. Also, we depict HPBWs of new and primary arrays by HP_n and HP_o, respectively. From Eq. (5), HP_n and HP_o are related approximately to each other by the following formula.

$$\frac{HP_o}{HP_n} \cong \frac{L_{xn}}{L_{xo}} \tag{6}$$

It is clear from the property of convolution that SLLs of new (SLL_n) and primary (SLL_o) arrays are related to each other by the following formula.

$$\frac{SLL_n}{SLL_o}(dB) \simeq 2 \tag{7}$$

A linear equally spaced array with constant spacing d between the elements and the element excitation coefficients a_n oriented along the z axis have directivity as follows [6].

$$D = \frac{\left|\sum\limits_{n=0}^{N-1} a_n\right|^2}{\sum\limits_{m=0}^{N-1}\sum\limits_{p=0}^{N-1} a_m a_p^* e^{j(m-p)\beta} \mathrm{sinc}[(m-p)kd]} \tag{8}$$

From antenna theory, it can be remembered that for an array without grating lobe in far-field radiation pattern, HPBW and directivity are related to each other inversely. We show directivities of new and primary arrays by D_n and D_o, respectively. According to the mentioned note and from Eqs. (6) and (8), following formula presents relationship between D_n and D_o.

$$\frac{D_n}{D_o} = \frac{HP_o}{HP_n} \simeq \frac{L_{xn}}{L_{xo}} = \frac{\left|\sum\limits_{n=0}^{N-1} b_n\right|^2 \sum\limits_{m=0}^{N-1}\sum\limits_{p=0}^{N-1} a_m a_p^* e^{j(m-p)\beta} \mathrm{sinc}[(m-p)kd]}{\left|\sum\limits_{n=0}^{N-1} a_n\right|^2 \sum\limits_{m=0}^{N-1}\sum\limits_{p=0}^{N-1} b_m b_p^* e^{j(m-p)\beta} \mathrm{sinc}[(m-p)kd]} \tag{9}$$

where b_n and a_n are excitation coefficients of new and primary arrays. Note that $b_n = a_n * a_n$.

3. RESULTS AND DISCUSSION

To verify the accuracy of the proposed method, four examples are presented. In the first one, a specific uniform array with $N = 12$, $d = 0.5\lambda$, $\theta_0 = 60°$ is considered. Figure 3 represents the comparison between primary array factor (PAF) and new array factor (NAF). In the second example, a specific Tschebyscheff array with $d = 0.5\lambda$, $\theta_0 = 65°$, $N = 20$, $SLL = -23$ dB is considered. Radiation patterns of the two arrays can be seen in Figure 4. In the third example, a specific Taylor one-parameter array with parameters $N = 16$, $SLL = -15$ dB, $d = 0.75\lambda$, $\theta_0 = 80°$ is considered. Figure 5 presents the comparison between PAF and NAF. A specific array with parameters $N = 24$, $n_L = 8$, $n_R = 5$, $SLL_L = -20$ dB, $SLL_R = -40$ dB, $d = 0.5\lambda$, $\theta_0 = 60°$ is considered in the fourth example with Elliot method. Figure 6 displays the radiation pattern of the initial and modified arrays.

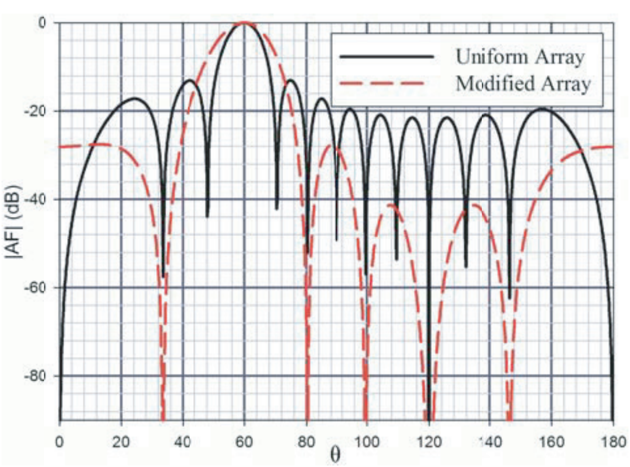

Figure 3. Radiation pattern of a uniform array.

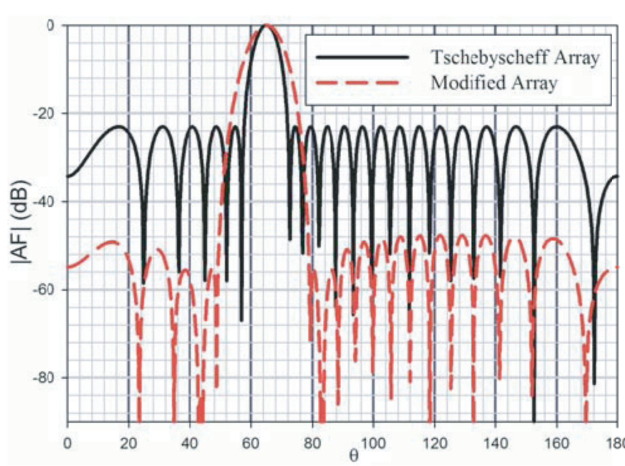

Figure 4. Array factor of Tschebyscheff array.

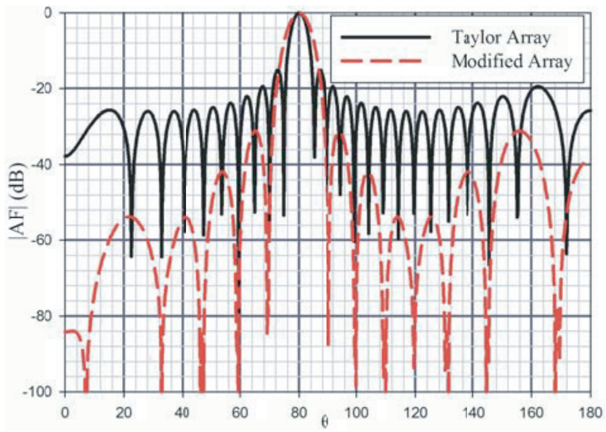

Figure 5. Radiation pattern of Taylor one parameters array.

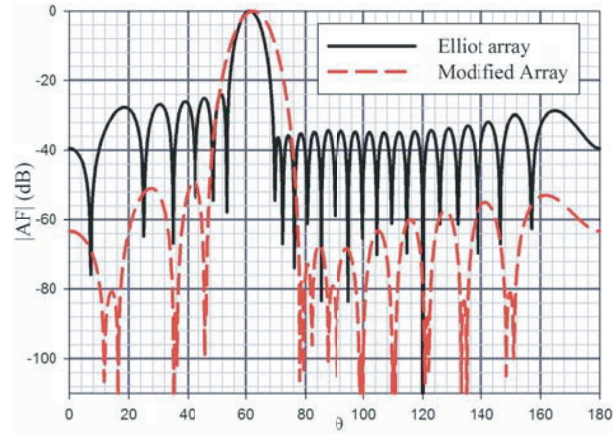

Figure 6. Radiation pattern of array synthesize with Elliot method.

As predicted, in all examples, SLL is reduced very well so that the SLL of NAF is almost twice of the PAF, and $HPBW$ of NAF is greater than PAF because by reducing SLL, half power beamwidth is increased. This can be considered as a penalty for the proposed method. Also, it can be seen that the general form of the radiation pattern of the arrays is maintained approximately. Therefore, results confirm the relationships of Eqs. (6), (7) and (9) with good accuracy. Due to rescaling, the element excitation coefficients, maximum and minimum point's positions in radiation pattern are changed. By changing the maximum and minimum point's positions of the radiation pattern, the zero's location of the array factors will also change. However, this notification does not have effect on the goal of this paper. Using the MATLAB software to program the process of the proposed method, we can get the SLL, $HPBW$, directivity, L_a and L_x of assumed examples. Table 1 and Table 2 show SLL, $HPBW$, directivity, L_x and L_a of example array, respectively. Indices n and o depict the new and primary array parameters. Results shown in theses tables confirm the relationships of Eqs. (6), (7) and (9) with good accuracy.

Table 1. SLL, $HPBW$ and directivity of assumed examples.

		SLL_n (dB)	SLL_o (dB)	HP_n (deg)	HP_o (deg)	D_n	D_o
Example I		−27	−13	14.42	10.1	9.5	12.51
Example II		−55	−23	8.92	6.4	14.2	19.7
Example III		−31	−15	6.6	4.6	10.83	14.97
Example IV	Left	−49	−24	8.64	6.12	15.8	23.22
	Right	−78	−36	8.3	6.12		

Figure 7 shows the new and primary amplitude distributions, for different arrays assumed. Since the number of elements and distances between elements for each array are specified, with a given amplitude distribution, the amplitude excitation coefficients can be obtained easily. It is observed that as the SLL increases the amplitude distribution from the center element(s) toward those at the edges is smoother and monotonically decreases for the whole array. As a result, after applying the proposed method, the amplitude broad banding factor for the modified array (L_{an}) becomes greater than the amplitude broad banding factor for primary array (L_{ao}). It means that $L_{an} > L_{ao}$.

This method can be applied several times to any linear equally spaced array with arbitrary steering angle, element spacing and number of elements to decrease SLL even more. Figure 8 shows variation of L_a versus iteration number for considered arrays. As can be seen, growing the number of iterations results in increasing the difference between maximum and minimum excitation coefficients. The arrays with large L_a are not very practical. Hence, it is recommended that the proposed method should not be used more than once.

Table 2. L_x and L_a of assumed examples.

	L_{xn}	L_{xo}	L_{an}	L_{ao}
Example I	6	11	0.91	0
Example II	4.8	5.5	0.96	0.59
Example III	4	5.35	0.96	0.25
Example IV	5.1	6.3	0.99	0.704

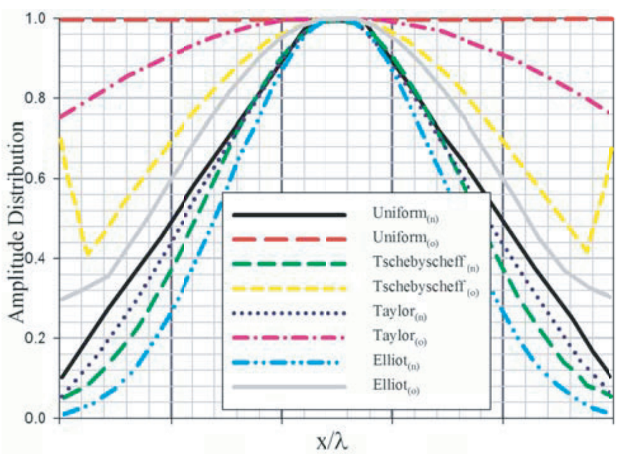

Figure 7. New and primary amplitude distribution, for different assumed array.

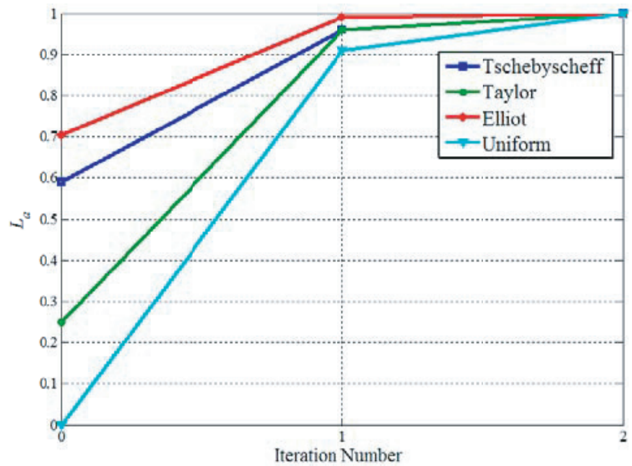

Figure 8. Variation of L_a versus iteration number for all considered arrays.

4. CONCLUSION

Since the first side lobe level which determines maximum *SLL* is the main source of electromagnetic interference, it should be lowered. A procedure of finding the optimum side lobe-minimizing weights for an arbitrary linear equally spaced array is derived, which holds for any scan direction, beamwidth, and type of antenna element used. In this paper, by use of convolution procedure and the time scaling property, the side lobe level for any type of linear equally spaced array is reduced. Results show that by this simple method, the side lobe level is reduced about two times or even more.

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