Highly Selective Lossy Dual-Band Bandstop Filter Synthesis and Design Based on Predistortion Hybrid Dual-Band Elliptic Reflection Function

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Abstract—This paper demonstrates a new class of highly selective bandstop filter based on cascading two identical lossy hybrid dual-band bandstop filters of low resonator Q factor. Each filter is synthesized based on multi-stage predistortion reflection mode technique. To demonstrate the approach, 4th order hybrid dual-band elliptic filter network which is a product of elliptic lowpass and highpass network functions has been predistorted and synthesized with low calculated Q factor. The lossy dual-band bandstop filters are fabricated and realized on microstrip planar structure. Both theoretical and experimental results clearly show good agreements with two stopband rejections up to 35 dB for one stage and 50 dB for two stages with passband loss of more than 10 dB.

1. INTRODUCTION

Among the challenges of the recent advancement in wireless communication systems, multi-channel interferences and spurious signals are the major concerns due to the dense utilization of frequency spectrum and the nonlinearity of wireless devices [1]. Consequently, it is highly desirable to have a compact single bandstop filter structure which produces high multi-band rejections in order to effectively suppress any unwanted spurious and inter-channel interference signals. Several narrow dual-band and triple-band bandstop filters have been reported recently [2–6]. As usual in filter design, losses are not included in synthesis by assuming a filter network has high Q resonator factor or is lossless. However, losses are inevitable in filter prototype realization. For instance, a bandstop filter design suffers from limited stopband rejection and poor roll-off due to the loss in its resonator. Hence, losses must be taken into account in filter synthesis to address the limitation. As reported in [7–9] the predistortion technique has been introduced to compensate this loss and improve poor stopband rejection with finite unloaded Q factor. The predistortion technique can restore a sharp response and stopband attenuation. Fig. 1 describes the response of predistortion reflection mode bandstop filter compared to the conventional bandstop filter which losses cause the rounding of passbands and reduce the stopband attenuations. However, it suffers from poor return loss. To overcome this poor return loss, a predistortion reflection-mode technique has been further introduced with the use of a circulator [10, 11]. At resonant frequency, the transmission characteristic from port 1 to port 3 of the circulator is that of a bandstop resonator with infinite unloaded Q factor. Therefore, a lossy high rejection bandstop filter can be realized to suppress any interference signal where loss can be tolerated.

This paper presents a new cascaded lossy dual-band bandstop filter (DBBSF) prototype using hybrid dual-band elliptic filtering functions based on predistortion reflection-mode technique. This is to achieve even higher stopband rejection as an extension work of authors' recent work reported in [12]. The hybrid dual-band filtering function is formed by the product of an elliptic lowpass and an elliptic
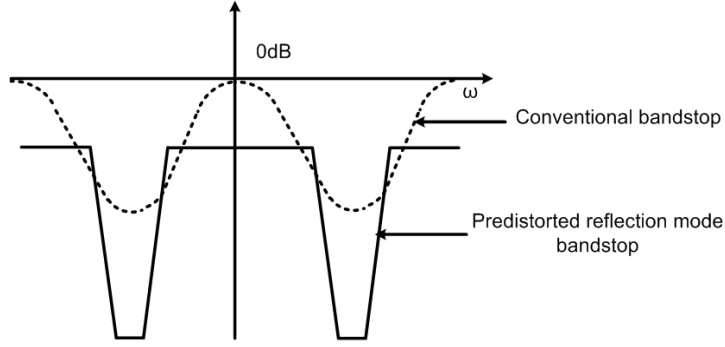
Figure 1. Conventional and predistortion reflection BSF's responses.

highpass lossless filtering functions. The realization of the lossy one port network is based on the multi-stage predistortion reflection mode technique is discussed. A high rejection DBBSF is achieved by cascading two predistortion reflection-mode networks in higher stages. Finally, DBBSF for one and two stages are designed and fabricated on printed circuit board to prove the concepts.

2. THEORY AND SYNTHESIS

2.1. Lossless Hybrid Dual-Band Elliptic Lowpass Network Function

A new class of hybrid filter network function is a dual-band network function formed by taking the product of two appropriate lowpass and highpass transmission functions. Let $F_L(\omega)$ and $F_H(\omega)$ be elliptic filtering functions of a l order lowpass and an h order highpass networks respectively. The amplitude-squared of nth $(l + h)$ order hybrid dual-band transfer function is defined as

$$|S_{21}(j\omega)|^2_{HB} = \frac{1}{1 + F_L^2(\omega)} \times \frac{1}{1 + F_H^2(k\omega)}. \tag{1}$$

The elliptic lowpass network function has a normalized cutoff frequency at $\pm 1\,\mathrm{rad/s}$, and the elliptic highpass network function has a normalized cutoff frequency scaled horizontally. To form the hybrid dual-band response, the highpass network cutoff frequency has to be in the interval of $[-1, 1]$ or $[-\frac{1}{k}, \frac{1}{k}]$. Fig. 2 depicts how the hybrid filtering response can be generated based on the product of lowpass and highpass filtering functions. From Fig. 2 it can be seen that the location of highpass cutoff frequency has an effect on the dual-band inner stopband. For $0 < k < 1$, the highpass cutoff frequency is greater than one; hence, Eq. (1) is reaching zero for all real frequency. For $k > 1$, the highpass response shrinks horizontally resulting in a narrow dual-band response. If the value of k increases, the inner stopband becomes narrower. Therefore, k primarily determines the desired bandwidth of the inner stopband of the hybrid dual-band response. It provides a full control over the inner band of the hybrid function. The example of a 4th order hybrid elliptic transmission functions based on different k values is illustrated in Fig. 3. Having formed the hybrid dual-band transmission function magnitude squared, the nth order reflection function magnitude squared is obtained by applying the conservation of energy for a lossless
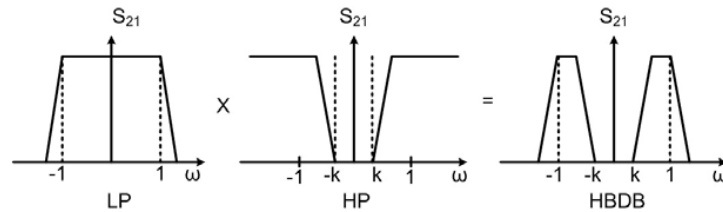
Figure 2. Hybrid dual-band transmission response.
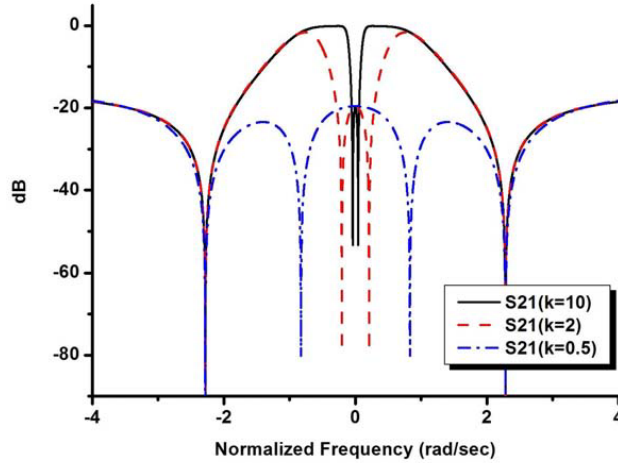
Figure 3. Transmission responses of a hybrid dual-band elliptic filter with different values of k.

passive network

$$|S_{11}(p)|^2_{HB} = 1 - |S_{21}(p)|^2_{HB}. \tag{2}$$

The reflection function coefficient is derived by solving and factorizing poles and zeros of Eq. (2) and given by

$$S_{11}(p) = \frac{N(p)}{D(p)} \tag{3}$$

where $N(p)$ and $D(p)$ are polynomials of degree n in a complex frequency variable p. $N(p)$ can be formed by factoring any negative zeros of $|S_{11}(p)|^2_{HB}$ while $D(p)$ is a Hurwitz polynomial formed by choosing the left half-plane poles of $|S_{11}(p)|^2_{HB}$. Thus, the reflection function of a dual-band hybrid filter type can be formed.

2.2. High Rejection Dual-Band Bandstop Filter Network Prototype Synthesis

The single and multi-stage predistortion reflection mode syntheses were presented in [11]. A reflection characteristic of a one port dissipated network is converted into a transmission characteristic of a two port network by the use of a circulator. With this approach a lossy single band bandstop filter having a transmission characteristic close to the lossless approximation is achieved. In order to obtain the DBBSF characteristic, this method is applied to the nth order hybrid reflection function in Eq. (3). The hybrid function is first predistorted by the loss factor α_1 and then multiplied by an arbitrary constant K. In predistortion, all zeros and poles of the network function are shifted to the right. For a system to be in the stable condition, all its poles must lie in the left half plane. Thus $0 < \alpha_1 < \alpha_{\max}$, where $\alpha_{\max}$ is the smallest real part of the function poles to be distorted. At the same time the maximum value of K is evaluated for any specific values of α_1 in such a way that the resultant hybrid network is passive. It is achieved by choosing K so that the magnitude squared of $[K|S_{11}(p - \alpha_1)|]^2$ is maximum 1 at real frequencies where $p = \pm j\omega_0$.

$$K^2 S_{11}^2(j\omega_0 - \alpha_1) = 1. \tag{4}$$

Taking derivative of both sides of the equation with respect to ω_0 gives

$$\frac{K^2 dS_{11}^2(j\omega_0 - \alpha_1)}{d\omega_0} = 0 \tag{5}$$

Having found the values of K and α_1 from Eqs. (4) and (5), the predistorted input impedance of the hybrid network is obtained

$$Z(p) = \frac{1 + KS_{11}(p - \alpha_1)}{1 - KS_{11}(p - \alpha_1)} = \frac{Z_n(p)}{Z_d(p)}. \tag{6}$$

The driving-point input impedance $Z(p)$ may be synthesized as a cascade of subnetworks if it is a rational positive real function. It is said to be a positive real function of the complex variable p if for all positive real p, real of $Z(p) \geq 0$ and $Z(p)$ is analytic in the strict right half-plane [13–15]. Now consider the even part of $Z(p)$

$$Z_e(p) = \frac{Z(p) + Z(-p)}{2}. \tag{7}$$

The realization of the subnetworks is based on types of zeros of $Z_e(p)$. If the zeros are purely imaginary, a Brune section can be realized and extracted. If the zeros are real and complex, C and D sections are realized and extracted respectively. After extracting the subnetwork, the remaining impedance is also the positive real function of degrees less than that of the overall impedance. It is a function of the extracted subnetwork transfer matrix and defined as

$$Z_1(p) = \frac{BZ_d(p) - DZ_n(p)}{CZ_n(p) - AZ_d(p)} \tag{8}$$

where A, B, C and D are the elements of the transfer matrix. From the remaining impedance, reflection stage function can be formed. A second stage predistortion is applied once again to the stage reflection function with the uniform loss and a second stage loss factor of α_2. Again, the subnetwork extraction is based on the type of zeros of the even part of the remaining impedance. For a single zero at ∞, an inline resistor and a lossy RC ladder network (shunt RC) can be extracted. This process is applied until the degree of the remaining impedance is zero. Eventually, losses are added back correspondingly to each stage in order to achieve a transmission response close to the lossless approximation characteristic with high rejection and constant passband loss of $20\log(K)$. To realize the lossy DBBSF network, the one-port predistorted reflection network is matched to one port of a circulator before performing bandpass frequency transformation. A reflection characteristic of the one-port dissipated network is converted into a transmission characteristic of a two-port network at port 3 of the circulator as shown in Fig. 4. With this approach a lossy one stage dual-band bandstop filter network having a transmission characteristic close to the lossless approximation can be achieved.
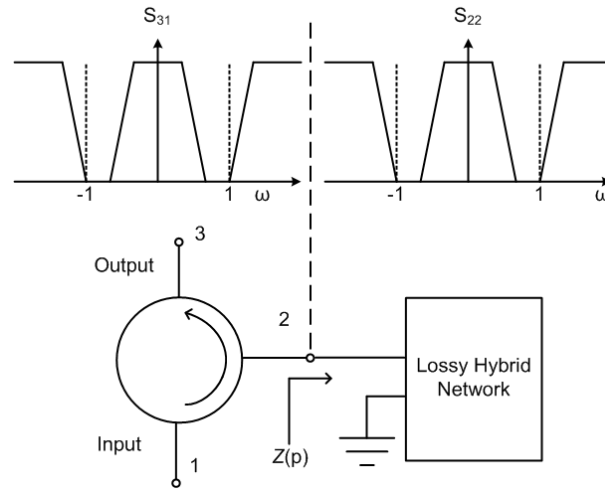
Figure 4. Lossy hybrid dual-band bandstop filter transmission characteristic response.

By cascading each synthesized one stage bandstop network prototype in such a way that the output of the first network is fed into the input of the second network, the DBBSF can achieve even higher stopband rejections with the increase of passband loss. Fig. 5 shows the two stages hybrid lossy DBBSF realized by cascading two identical hybrid lossy DBBSFs in series. At its resonant frequency, the bandstop transmission characteristic from port 1 to port 3 of the first network is fed into port 1 of the second network circulator. Let S_{31} and $S_{3'1'}$ be the transmission signal from port 1 to port 3 of each one port network respectively. The two stages hybrid lossy dual-band bandstop filter network can
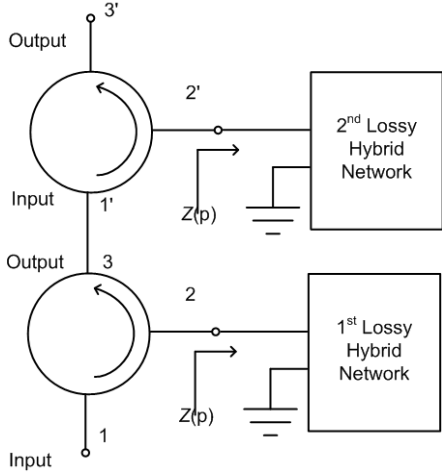
Figure 5. Two stages hybrid lossy DBBSF network.

Figure 6. Signal flow diagram representation for a cascaded network.

be represented by a decomposition of signal flow diagram as depicted in Fig. 6. From the signal flow diagram in Fig. 6, the output transmission signal for the overall network is [16]

$$|S_{3'1}(p)|^2 = |S_{31}(p)|^2 \times |S_{3'1'}(p)|^2. \tag{9}$$

Since the two networks are identical, (9) can be written as

$$|S_{3'1}(p)|^2 = |S_{31}(p)|^2 \times |S_{31}(p)|^2 \tag{10}$$

and the total transmission signal in dB is

$$|S_{3'1}(p)|_{dB} = |S_{31}(p)|_{dB} + |S_{31}(p)|_{dB}. \tag{11}$$

This expression shows that the overall network prototype transmission signal is 2 times greater than the transmission signal produced by the single stage network.

$$|S_{3'1}(p)|_{dB} = 2|S_{31}(p)|_{dB}. \tag{12}$$

Therefore, by having multiple n individual filter prototypes cascading in series, the selectivity of the filter can be improved by n times compared to that of a single stage network. This higher stages lossy bandstop filter could still be selective for any interference suppression. However, it may cause a system to suffer from high passband loss. To overcome this loss performance, it may be required to deploy low noise amplifier(s) at any stage of the system wherever necessary or to synthesize the network in higher Q factor where the technology is appropriate.

3. MICROSTRIP PROTOTYPE REALIZATIONS

Consider the 4th order hybrid dual-band network function generated from a 2nd order elliptic lowpass network of 20 dB attenuation in stopband and a 2nd order elliptic highpass network of 20 dB rejection in stopband. The highpass cutoff frequency is horizontally shrunk by the factor $k = 10$. The lossless 4th order hybrid dual-band transmission function magnitude squared is expressed as

$$|S_{21}(j\omega)|^2_{HB} = \frac{\left(\begin{array}{c} 0.012400299\omega^8 - 0.12920375\omega^6 \\ +0.33677820\omega^4 - 0.0011544993\omega^2 \end{array} \right)}{\left(\begin{array}{c} 0.41678167\omega^8 - 0.29866317\omega^6 \\ +0.38028854\omega^4 - 0.0043407501\omega^2 \\ +0.00009042093 \end{array} \right)}, \tag{13}$$

and the reflection function, $S_{11}(p)$ is derived as

$$S_{11}(p)_{HB} = \frac{\left(\begin{array}{c} p^4 + 0.5950771625p^3 + 0.3865876215p^2 \\ +0.06015627855p + 0.01487127191 \end{array}\right)}{\left(\begin{array}{c} p^4 + 1.234035734p^3 + 1.119719010p^2 \\ +0.1502340914p + 0.01472923457 \end{array}\right)}. \tag{14}$$

The hybrid reflection function is predistorted by a loss factor $\alpha_1 = 0.053125$ in the conditions of the resonant quality factor $Q = 200$ and the bandwith $\Delta f = 0.09411$. With this loss the passive one port network has a constant passband loss of $K = 10\,\text{dB}$ at $\omega_0 = 0.098571704872$ (rad/s). By applying the multi-stage predistortion technique, the first cross-coupled Brune section elements can be found

$$\begin{cases} K_1 = -1.207951 \\ K_2 = 0.202190 \\ K_3 = 0.084303 \end{cases}. \tag{15}$$

After extracting the first cross-coupled Brune section, the remaining stage reflection function is

$$S'_{11}(p) = \frac{\left(\begin{array}{c} -0.3162638193p^2 + 0.1468208732p \\ +0.1935004790 \end{array}\right)}{\left(\begin{array}{c} 2.3162638193p^2 + 2.3768686597p \\ +2.1622604411 \end{array}\right)}. \tag{16}$$

This subnetwork reflection function is of degree 2 and is predistorted by the second stage predistortion loss factor, $\alpha_2 = 0.396523$. The normalized element values for the second Brune section are

$$\begin{cases} K'_1 = -2.237100 \\ K'_2 = 0.596790 \\ K'_3 = 0.754446 \end{cases}. \tag{17}$$

After extracting the second Brune section, the remaining impedance is of degree 0; hence the remaining load admittance is 0.151805. Having synthesized the network, the losses must be added to the network prototype by $p \to p + \alpha_1$ for the first stage and $p \to p + \alpha_1 + \alpha_2$ for the second stage. The conventional bandpass frequency transformation is then applied to the lossy one-port network in order to obtain the DBBSF network characteristic. Fig. 7 shows the synthesized 4th order hybrid dual-band lowpass elliptic filter network with two pairs of calculated resonator Q factor of 200 and 23.6 for the first stage and second stage respectively. Since the synthesized network has the low finite low resonator Q factor of 200, it is suitable to fabricate and realize the prototype on planar structure such as microstrip. The first stage DBBSF was designed and fabricated on RT/Roger Duroid 5880 microstrip substrate with a thickness of 787 µm, relative dieletric constant $\epsilon_r = 2.2$, and loss tangent $\tan \sigma = 0.0009$. The surface mount circulator of part number UIYSC25A825T885, from UIY Technology has been used in constructing the prototype. The operational frequency of the circulator is from 825 MHz to 885 MHz with a passband loss of 0.35 dB and isolation of 20 dB giving the reflection signal at port 1 close to 0
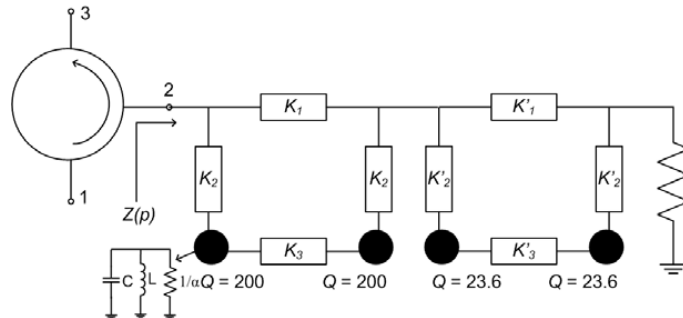
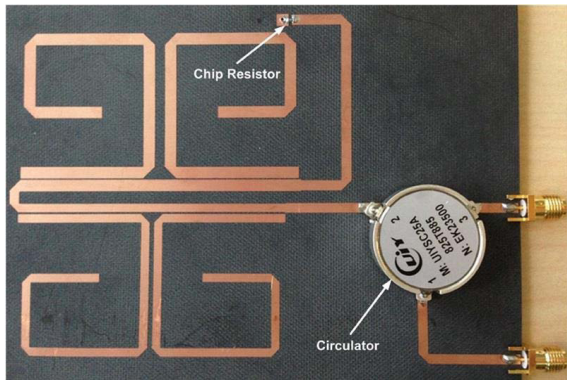**Figure 7.** Lossy reflection dual-band bandstop filter.
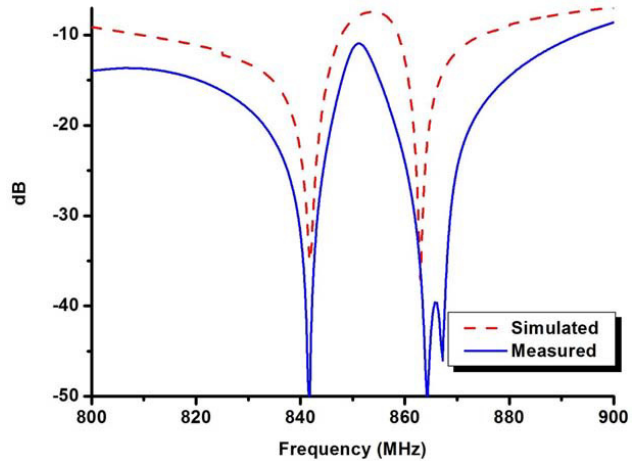
Figure 8. Dual-band bandstop filter prototype.

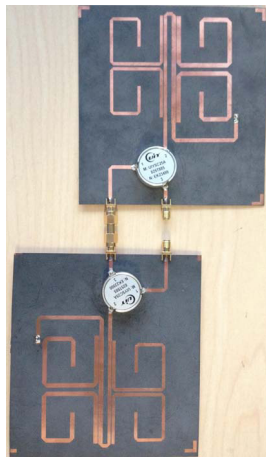**Figure 9.** Simulated and measured transmission responses for one stage.

Figure 10. 2 stages of dual-band bandstop filter prototype.
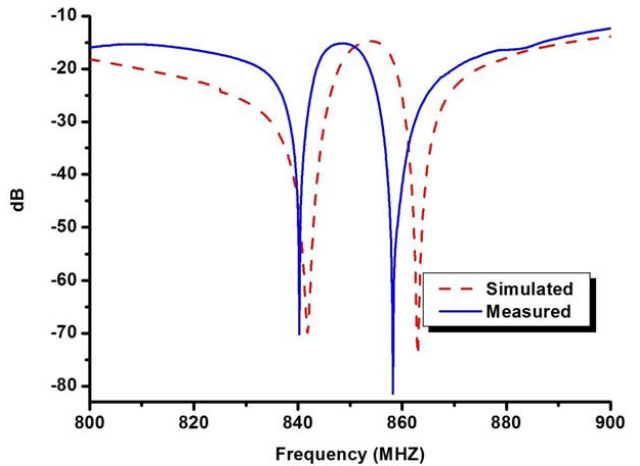

Figure 11. Simulated and measured transmission responses for two stages.

in magnitude. The photograph of DBBSF microstrip prototype is shown in Fig. 8. Fig. 9 compares the measured performance using ZVL network analyser from R&S and the simulated performance by Advanced Design System (ADS) from Keysight Technologies. The results show that the prototype can produce two high rejection bands up to 35 dB at 842 MHz and 867 MHz which are close to the simulated response. The measured and simulated passband losses are 14 dB and 10 dB respectively. The higher measured passband loss is due to the loss performance of the circulator and the low Q microstrip.

Two fabricated network prototypes of the synthesized 4th order DBBSF are cascaded in two stages as shown in Fig. 10. The output from the first prototype is fed into the input of the second prototype. Fig. 11 shows the simulated transmission response using ADS software and measured transmission response using ZVL network analyzer. Based on this configuration, it can be seen that the prototype produces two high rejection bands up to 60 dB at 840 MHz and 857 MHz with bandwidths of 1.2% and 2% respectively. However, the two stopband frequencies are shifted slightly lower with the passband loss of 15 dB. The discrepancy of the rejection bands may be due to the tolerant in fabrication and higher stage connections of the two prototypes. It can be noticed that the attenuation and the passband loss for the two-stage prototype are ~ 1.5 times higher than those of the single stage prototype (see Fig. 9).

4. CONCLUSION

In this article, a new class of filter type known as hybrid dual-band bandstop filter (DBBSF) is synthesized and realized based on predistortion reflection mode network. The hybrid dual-band lowpass network function is formed by the product of two elliptic lowpass and highpass network functions. With this new hybrid filter network function, the inner passband of the realized DBBSF can be fine-tuned by adjusting the highpass cutoff frequency scaling factor, k. The proposed filter prototype can produce two high rejection bands with finite low Q resonator factors. This leads to the achievable miniaturization and cost reduction compared to the low loss and high Q resonator of the same rejection levels. Moreover, it is beneficial to have multiple rejection bands in one single structure to address interference challenges of the unprecedented wireless communication systems. The microstrip prototypes of the 4th order hybrid dual-band bandstop filter were designed and fabricated to prove the the synthesis approach. The measured result shows that two stages lossy DBBSF can achieve higher stopband rejections about 1.5 times greater than that of a single stage prototype. However, it has higher loss performance in passband which can be further improved by synthesizing and realizing the prototype using higher Q resonator.

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