Investigation on an Effective Magnetic Permeability of the Rod-Shaped Ferrites

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Abstract—In this paper, we provide full-wave numerical results concerning the effective magnetic permeability of the rod-shaped ferrites which is useful in the calculation of the inductance or the efficiency of ferrite rod antennas. This study covers the circular cross-section ferrites with a centered winding fitting the diameter of the rod. A careful attention is taken to the simulation model, and obtained results are in better agreement with the measured coils than existing solution published earlier.

1. INTRODUCTION

The effective magnetic permeability of a rod (μ_{rod}) is known to be lower than the magnetic permeability of the material due to the magnetic field leakage caused by its opened shape. In 1969, Snelling [1] published a theoretical variation of μ_{rod} as a function of the length/diameter ratio (l/d ratio) of the rod for different relative material permeabilities, denoted by μ. Considering that μ is much greater than 1, the proposed model can be described by Equation (1), where D is a demagnetizing factor.

$$\mu_{rod} \approx \frac{\mu}{1 + \mu D} \tag{1}$$

Based on the demagnetization factor which exists for ellipsoidal and cylindrical rods, an analytical result shown in Fig. 1 has been obtained and still used by ferrite manufacturers [2, 3]. Around the same period, Khomich [4] published an equation which describes these curves based on measurement results:

$$\mu_{rod} = \frac{\mu}{1 + 0.84 \left(\dfrac{d}{l}\right)^{1.7} (\mu - 1)} \tag{2}$$

Another variation of the equation has been proposed in [5]:

$$\mu_{rod} = \frac{\mu}{1 + (\mu + 1)(d/l)^2 \Big(\ln(l/d) \left\{0.5 + 0.7 \times \left[1 - \exp(-0.001 \times \mu)\right]\right\} \Big)} \tag{3}$$

In all cases, an important condition considering the wire winding should be respected, i.e., the winding should cover the whole rod and be very close and tight. However, this condition is difficult to achieve in the practice and can lead to important errors. Since all empirical equations have been based on some finite amount of measured samples which are not clearly described, we want to provide in this paper a solution based on a simulated model without any extrapolation of obtained data.

Using an efficient FEM algorithm from HFSS, we could solve this problem with a high precision on real windings having natural small air gaps due to coating and/or manual winding.
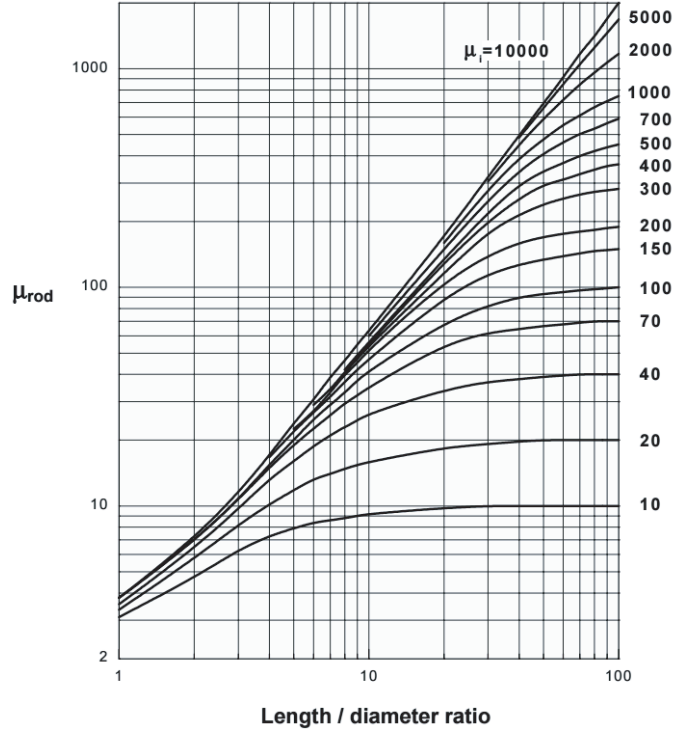
Figure 1. Analytical result of the effective permeability μ_{rod} for different l/d ratios and μ.

This paper is composed of two parts. The first deals with the description of the simulation model. The second is the presentation of results considering the μ_{rod} as a function of the l/d ratio, where we compare measured use cases to simulation results as well as results provided by existing methods or equations.

2. SIMULATION MODEL

The simulation has been performed with HFSS FEM solver. The winding is made with a 1 mm diameter copper wire coiled around a 10 mm diameter ferromagnetic rod with a length varying from 10 mm to 500 mm. Thus, the l/d ratio varies from 1 to 50. In order to use an available VNA for the measurements and to keep the impedance inductive for very high μ, the simulation has been performed 350 kHz. Considering this frequency, the boundary box should have a huge size, but since we are interested only in the impedance of the coils, a $1 \times 1 \times 1$ meter box has been defined around the model and provided stable impedance response with an error of about 1% compared to a $30 \times 30 \times 30$ m bounding box. In order to decrease further the simulation time, a simplified model of winding has been introduced having a square cross-section wire with a revolution approximated with 12 segments.

The difference of the inductance L between a simplified model (Fig. 2(a)) and an accurate model (Fig. 2(b)), having an octagonal cross-section and 36 segments per turn, have been evaluated and also compared to a theoretical value based on the work of Maxwell, Lorentz and Rosa [6, 7]:

$$L = L_s - 4\pi a N(A + B) \tag{4}$$

with N the number of turns, A and B tabulated correction factors and L_s are defined as follows:

$$L_s = aN^2 \left(\frac{8\pi}{3} \left(\sqrt{1 + \frac{b^2}{4a^2}} \left(\frac{4a^2}{b^2} - 1 \right) E + \sqrt{1 + \frac{b^2}{4a^2}} F - \frac{4a^2}{b^2} \right) \right) \tag{5}$$

with E and F elliptic integrals defined by general Equations (6) and (7) obtained from the original ones

described in [6].

$$
\begin{aligned}
F = {} & \ln\left(\frac{4}{k'}\right) + ln\left(\frac{4}{k'}\right) \sum_{n=1}^{\infty} \left(k'^{2n} \left[\frac{(2n)!}{2^{2n}(n!)^2} \right]^2 \right. \\
& \left. \times \left(1 - ln\left(\frac{4}{k'}\right)^{-1} \sum_{m=1}^{n} \frac{2}{(2m-1)(2m)} \right) \right)
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
E = {} & 1 + \ln\left(\frac{4}{k'}\right) \sum_{n=1}^{\infty} \left(k'^{2n} \left[\frac{(2(n-1))!}{2^{2(n-1)}((n-1)!)^2} \right] \left[\frac{(2n)!}{2^{2n}(n!)^2} \right] \right. \\
& \left. \times \left(1 - ln\left(\frac{4}{k'}\right)^{-1} \left(\sum_{m=1}^{n} \frac{2}{(2m-1)(2m)} - \sum_{m=n}^{n} \frac{1}{(2m-1)(2m)} \right) \right) \right)
\end{aligned}
\tag{7}
$$

with $k' = \sqrt{1 - k^2}$ and Lorentz module $k = \frac{2a}{4a^2 + b^2}$, where a and b are respectively the radius and length of the solenoid.

As a result (Table 1), an error of 3.8% is noticed when the simplified model is used compared to the accurate model. From our point of view, this difference is still acceptable considering the simulation time gain.

Table 1. Comparison of the inductance between an accurate model, a simplified model and the theoretical value from the Equation (4) considering simulated value of the μ_{rod}.

	Accurate model	Simplified model	Theoretical value
$50 \times 10\,\text{mm}$, l/d ratio 5, 40 turns, $\mu = 100$	$41.56\,\mu\text{H}$	$43.14\,\mu\text{H}$	$41.14\,\mu\text{H}$
Simulation time	$27\,\text{min}\ 1\,\text{sec}$	$5\,\text{min}\ 55\,\text{sec}$	-

3. SIMULATED RESULTS AND COMPARISON TO MEASUREMENTS

μ_{rod} has been calculated from the imaginary part of the impedance of the simulated coils by using the following equation:

$$
\mu_{rod} = \frac{L_{rod}}{L_{air}}
\tag{8}
$$

where L_{rod} and L_{air} are the inductances of the winding around a ferrite rod and without it respectively, and defined by:
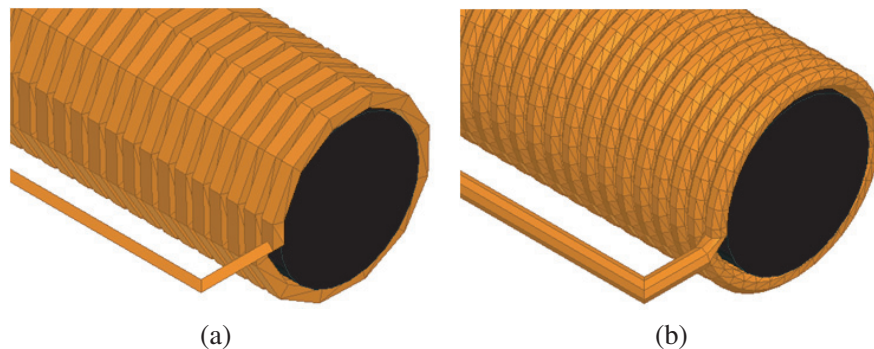
$$
L = \frac{\Im(Z)}{2\pi f}
\tag{9}
$$

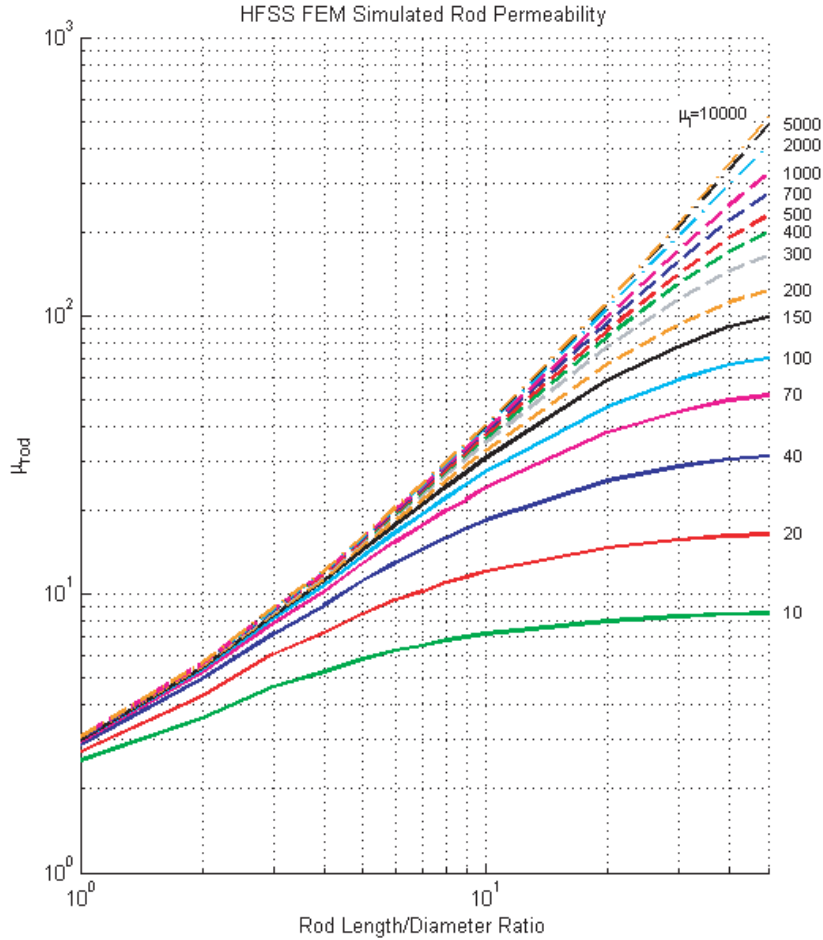**Figure 2.** Simulated model. (a) Simplified model. (b) Accurate model.

Figure 3. Simulated result of the effective permeability μ_{rod} for different l/d ratios and μ.

where $\Im(Z)$ is the reactance of the coil and f the operating frequency.

The obtained results are shown in Fig. 3. The overall simulation time considering the described simplified model has taken more than 50 hours on a 12 core Intel Xeon computer with a 96 Gb of RAM. Compared to Snelling solution (Fig. 1) we can see that for small l/d ratios of ferrites, both graphs provide very close results. However, when l/d becomes larger (more than 10), the simulated results are smaller than those provided by the analytical solution.

In the last step, we compare these results to the measurement for different μ and l/d ratios. For the experimental case, μ_{rod} is deduced using Equations (8) and (9). For each configuration, we have realized a winding with a ferrite core in order to get L_{rod}. The same winding has then been measured with a PVC core to obtain L_{air}. An example of measured winding is presented in Fig. 5. The measurement was performed with an HP8714C VNA at 350 kHz where we were interested in the imaginary part of the impedance of the coil. In Table 2, we compare measured results to the simulation and existing solutions for four configurations:

1) $l = 45$ mm, $d = 8$ mm, l/d ratio $= 5.6$, 33 turns, type Fair-Rite 61, $\mu = 125$.
2) $l = 157$ mm, $d = 8$ mm, l/d ratio $= 19.6$, 122 turns, type 150 VH, $\mu = 150$.
3) $l = 200$ mm, $d = 10$ mm, l/d ratio $= 20$, 152 turns, type 400 HH, $\mu = 400$.
4) $l = 100$ mm, $d = 10$ mm, l/d ratio $= 10$, 77 turns, type 400 HH, $\mu = 400$.

Table 2. Comparison of μ_{rod} between the graphs, the equations and the measurement for different dimensions of ferrites, length/diameter ratio and material permeabilities and associated errors compared to the measurement.

Configuration	Measured	Analytical method (Fig. 1)	Numerical method (Fig. 3)
1)	18	25 (+39%)	16.5 (−8.3%)
2)	43	87 (+102%)	60 (+39%)
3)	71	150 (+113%)	81 (+14%)
4)	33.6	55 (+64%)	36 (+7.1%)

Configuration	Measured	Equation (2)	Equation (3)
1)	18	19.2 (+6.7%)	25 (+39%)
2)	43	83.6 (+94%)	88.4 (+106%)
3)	71	130.8 (+84%)	125.2 (+76%)
4)	33.6	52 (+55%)	51.6 (+54%)

From Table 2 and Fig. 4, we can see that the analytical solution in all cases is exaggerated by sometimes more than 100% compared to the measurement and grows with the l/d ratio. Even if the tolerance given by the manufacturers of the ferrite rods on μ is generally around 20%, such an error cannot be explained and should come from the demagnetizing factor D used in Equation (1). On the other hand, the simulated result provides a better agreement with the measurement which does not exceed 15% in most cases, and 40% in the worst case which could be explained by probably extended tolerance of the 150 VH rod.

Considering the equations, they provide better results than analytical graph but still, important compared to the measurement. The difference, in this case, could probably come from some particular conditions of measurement (feeding point, frequency, measurement device) or an erroneous extrapolation provided by equations.
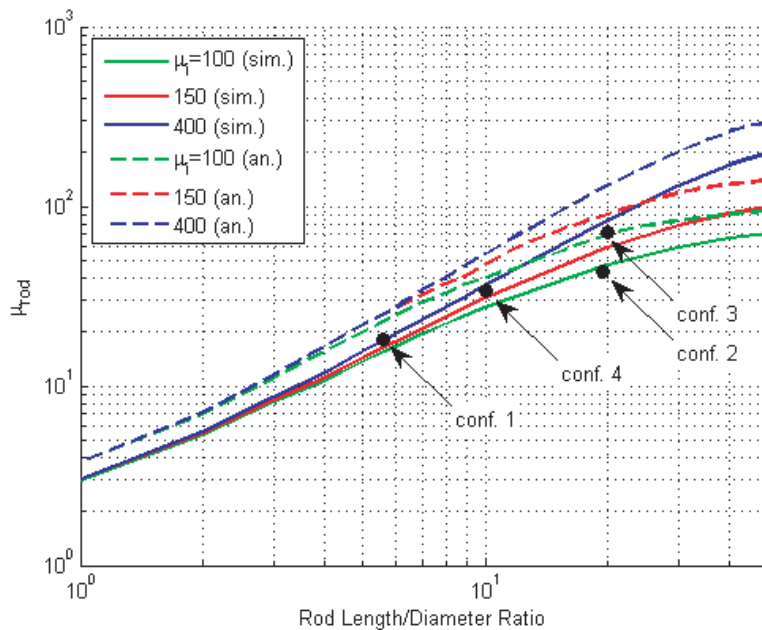

Figure 4. Measured configurations (dots) compared to analytical (an.) and simulation results (sim.).
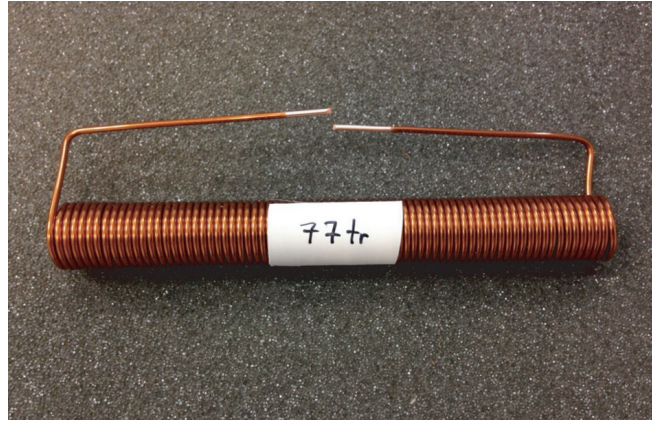
Figure 5. Measured coil composed of 77 turns around a 400 HH ferrite rod with the l/d ratio of 10 and material permeability of 400.

4. CONCLUSION AND DISCUSSION

In this paper, we have provided simulated results for an effective magnetic permeability of rod-shaped ferrites with a circular cross-section. The results cover ferrites with l/d ratios up to 50 which is usually enough considering commercially available rods. Obtained results have been compared to the measurement with a good agreement. A comparison to existing analytical graph and equations is also provided showing some important differences which concern at least the particular type of winding used in the paper.

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