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Three-Dimensional Analysis of Ferrite-Loaded Waveguide Discontinuity by Transverse Operator Method Combined with Mode-Matching Method

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Abstract—In this paper, a rigorous study of rectangular waveguide partially filled with longitudinally magnetized ferrite is presented. The propagation constant as a function of frequency is first obtained using Transverse Operator Method. Through this study, we try to show the existence of the complex modes in this type of structures. In this case, the interface between an air filled rectangular waveguide and E-plane ferrite slab loaded rectangular waveguide represents a discontinuity problem. This analysis is based on a combination of the Transverse Operator Method and the Mode Matching Method to determine the scattering parameters. This proposed approach is validated by comparing the presented results with the published data and numerical ones obtained from commercial software.

1. INTRODUCTION

Ferrites are basic materials in the realization of non-reciprocal and control devices such as isolators, circulators and phase shifters. However, the analysis of a structure that has magnetized ferrite is usually very complex, and in most cases, it is not possible to find an analytical solution. The reason is the anisotropic properties of the ferrite whose permeability is a tensor which depends on the operating frequency and DC-bias field. The complexity of the electromagnetic fields in these devices suggests the use of numerical techniques for their analysis.

It is known that the 3D discontinuity problems of rectangular waveguides loaded with dielectric have been computed by many researchers. The special case of a full-height E-plane dielectric slab located in the center of a rectangular waveguide was studied by [1–4]. Other geometries such as two slabs placed symmetrically with respect to the waveguide center and one asymmetrically located slab were analyzed by [5, 6], respectively. Moreover, Gardiol [7] analyzed the H-plane dielectric slab-loaded waveguide. However, there is a lack of published results for 3D guiding structures such as discontinuities that contain ferrites and especially those magnetized longitudinally. The structures that have been analyzed are limited to those containing slabs of magnetized ferrite in the transverse direction, which fill the entire height of the waveguide [8–10]. In this research, first, Transverse Operator Method (TOM) is applied to study the dispersion characteristics in rectangular waveguides which contain longitudinally magnetized ferrite (LMF) and exploit the complex modes [11]. Then, we combine the last method with Mode Matching method (MM) for analyzing the 3D discontinuity problem. It is an original result which has not been published in literature. Numerical results are compared with other methods and with those obtained by Ansoft's HFSS [12].

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2. TRANSVERSE OPERATOR METHOD (TOM)

The permeability of longitudinally magnetized ferrite is expressed by the tensor of Polder:

$$\bar{\bar{\mu}}_f = \mu_0 \cdot \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} = \mu_0 \cdot \bar{\bar{\mu}}_{rf} \tag{1}$$

where μ, κ and μ_{rz} are real quantities. For a partial magnetization of ferrite with z direction to a value of magnetization M, and the non-zero elements of this tensor are given by the following empirical expressions [13, 14]:

$$\mu = \mu_d + (1-\mu_d) \cdot \left(\frac{4\pi \cdot M}{4\pi \cdot M_S}\right)^{3/2} \tag{2}$$

$$\kappa = \gamma \cdot \frac{4\pi \cdot M}{\omega} \tag{3}$$

$$\mu_{rz} = \mu_d^{\left(1-\left(\frac{4\pi \cdot M}{4\pi \cdot M_S}\right)^{5/2}\right)} \tag{4}$$

ω is the work pulsation, γ the gyromagnetic ratio, $4\pi M_s$ the magnetization at saturation and $4\pi M$ the magnetization which is lower than saturation. In Eq. (4), μ_d denotes the relative permeability of the ferrite for the completely demagnetized state, and it is given by:

$$\mu_d = \frac{1}{3} \cdot \left[1 + 2 \cdot \sqrt{1 - \left(\frac{\gamma 4\pi \cdot M_S}{\omega}\right)^2}\right] \tag{5}$$

Using the Transverse Operator Method in a guide partially filled with magnetized longitudinally ferrite which is represented by Figure 1, the Maxwell equations can be written:

$$rot\,\vec{E} = -j\omega\mu_0\bar{\bar{\mu}}_{rf}\,\vec{H} \tag{6}$$

$$rot\,\vec{H} = j\omega\varepsilon_0\varepsilon_{rf}\,\vec{E} \tag{7}$$

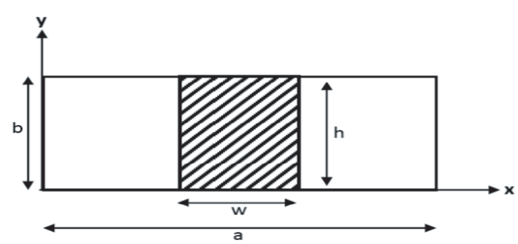

Figure 1. Transverse section of the rectangular waveguide loaded with ferrite slab.

Considering the propagation in the OZ direction, the following relationship can be obtained by eliminating the longitudinal field components from Maxwell's equations [15]:

$$L_t\phi = j\eta\frac{\partial\phi}{\partial z} \tag{8}$$

with Φ and $\hat{L}_t$ respectively the transverse field vector and the transverse operator which are defined by:

$$\Phi(x, y, z) = \Phi(x, y) \cdot \exp(-jk_z z) \tag{9}$$

$$\hat{L}_t = \begin{bmatrix} \hat{L}_{11} & 0 \\ 0 & \hat{L}_{22} \end{bmatrix} = \begin{bmatrix} k_0\varepsilon_r - 1/k_0\partial_t[1/\mu_{rz}\partial_t^+] & 0 \\ 0 & k_0\bar{\bar{\mu}}_{tt} - 1/k_0\partial_t[1/\varepsilon_r\partial_t^+] \end{bmatrix} \tag{10}$$

where

$$k_0 = \omega\sqrt{\varepsilon_0\mu_0}\,; \quad \partial_t = \begin{pmatrix} \partial_y \\ -\partial_x \end{pmatrix}; \quad \partial_t^+ = \begin{pmatrix} -\partial_y & \partial_x \end{pmatrix}; \quad \eta = \begin{pmatrix} 0 & \eta_0 \\ \eta_0 & 0 \end{pmatrix};$$
$$\eta_0 = \begin{pmatrix} 0 & -j \\ j & 0 \end{pmatrix}; \quad \bar{\bar{\mu}}_{tt} = \begin{bmatrix} \mu & -j\kappa \\ j\kappa & \mu \end{bmatrix}. \tag{11}$$

Furthermore, by eliminating the transverse magnetic fields from the above equation, we can obtain a new formulation which only shows the transverse electric fields:

$$\hat{L}' E_T = k_z^2 E_T \tag{12}$$

with

$$\hat{L}' = k_0^2 \varepsilon_r \bar{\bar{\mu}}_{tt} - \varepsilon_r \partial_t \frac{1}{\varepsilon_r} \partial_t^+ - \eta_0 \partial_t \frac{1}{\mu_{rz}} \partial_t^+ \eta_0 \bar{\bar{\mu}}_{tt} \tag{13}$$

The expressions of the permittivity and permeability can be written as follows:

$$\varepsilon_r(x) = 1 + (\varepsilon_{r2} - 1)\, U(x) \tag{14}$$
$$\mu_{rz}(x) = 1 + (\mu_{rz2} - 1)\, U(x) \tag{15}$$

where

$$U(x) = U(x - (a/2 - w/2)) - U(x - (a/2 + w/2)) \tag{16}$$

Let's consider $U(x)$ the Heaviside function. The Delta Dirac function δ will appear during the derivation.

Below is the decomposition of the transverse field $\vec{E}_T$ in a complete base which verifies the boundary conditions:

$$E_x = \sum_{m,\,n=0}^{\infty} E_{xmn} \cos\left(\frac{m\pi\, x}{a}\right) \sin\left(\frac{n\pi\, y}{b}\right) \tag{17}$$

$$E_y = \sum_{m,\,n=0}^{\infty} E_{ymn} \sin\left(\frac{m\pi\, x}{a}\right) \cos\left(\frac{n\pi\, y}{b}\right) \tag{18}$$

The application of Galerkin's method leads to the eigenvalue equation:

$$G E_T = k_z^2 E_T \tag{19}$$

G is a square matrix of rectangular waveguide partially filled with LMF. The dimension of G is $(2N \times 2N)$ with N being the number of modes. The eigenvalues and eigenvectors of G are respectively the propagation constant k_z and the development coefficients of the guide field. After obtaining all of the field components (E_x, E_y, H_x, H_y) in the ferrite slab region, we can apply the Mode Matching Method to study the 3D-discontinuity problem.

3. MODE MATCHING METHOD (MM)

Figure 2 presents the geometry of a rectangular waveguide loaded with a section of E-plane ferrite slab.

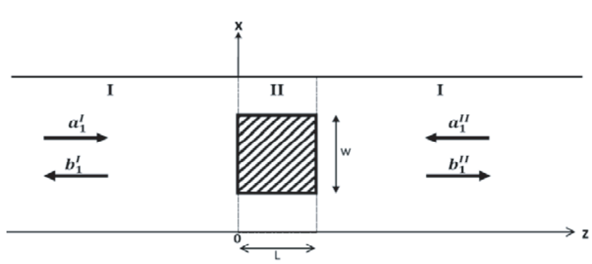

Figure 2. Longitudinal section of the rectangular waveguide loaded with ferrite slab.

Consider the simple junction at $z = 0$. Then, matching the transverse field components in regions I and II, we obtain:

$$\sum_{m=1}^{N_1} A_m^I \left(a_m^I + b_m^I\right) e_m^I = \sum_{p=1}^{N_2} A_p^{II} \left(a_p^{II} + b_p^{II}\right) e_p^{II} \tag{20}$$

$$\sum_{m=1}^{N_1} B_m^I \left(a_m^I - b_m^I\right) h_m^I = \sum_{p=1}^{N_2} B_p^{II} \left(-a_p^{II} + b_p^{II}\right) h_p^{II} \tag{21}$$

where A and B are complex coefficients determined from the normalization of the power.

e_m^I (respectively h_m^I) represents the mth orthonormal basis vector mode of electric fields (magnetic fields) in the empty waveguide (I). As for e_p^I (respectively h_p^I), it represents the pth orthonormal basis vector mode of electric fields (magnetic fields) in the ferrite filled waveguide (II) determined in the previous section by TOM.

The generalized Scattering Matrix of the simple discontinuity is then found by applying the Galerkin's method:

$$S = \begin{bmatrix} U & M_1 \\ -M_2 & U \end{bmatrix}^{-1} \begin{bmatrix} U & M_1 \\ M_2 & -U \end{bmatrix} \tag{22}$$

with U being the identity matrix, M_1 and M_2 (of dimension respectively $(N_1 \times N_2)$ and $(N_2 \times N_1)$) being matrices of general terms:

$$M_{1ij} = \frac{B_j^{II}}{B_i^I} \left\langle h_j^{II} \left| h_i^I \right.\right\rangle \tag{23}$$

$$M_{2ij} = \frac{A_i^I}{A_j^{II}} \left\langle e_i^I \left| e_j^{II} \right.\right\rangle \tag{24}$$

so that we can conclude the Generalized Scattering Matrix of double discontinuity:

$$S = \begin{bmatrix} S_{11}^I + S_{12}^I D E S_{22}^{II} D S_{21}^I & S_{12}^I D E S_{21}^{II} \\ S_{12}^I D \left[U + S_{22}^I D E S_{22}^{II} D\right] S_{21}^I & S_{11}^{II} + S_{12}^{II} D S_{22}^I D E S_{21}^{II} \end{bmatrix} \tag{25}$$

With

$$E = \left[U - S_{22}^{II} D S_{22}^I D\right]^{-1} \tag{26}$$

U is the identity matrix and D the matrix of transmission line which is defined by:

$$D = \begin{bmatrix} e^{-\gamma_1 L} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & e^{-\gamma_N L} \end{bmatrix} \tag{27}$$

γ_i is the propagation constant of the ith mode of the central guide, N the number of modes in the same guide and L the length of the middle waveguide.

4. RESULTS AND DISCUSSIONS

The calculations of the proposed model are performed in MATLAB. In order to validate the formulation described in the previous sections and the written computer program, we first consider a double junction between an air filled rectangular waveguide and an E-plane dielectric slab loaded waveguide (dielectric slab is a special ferrite material with $H_0 = M_s = 0$) as shown in Figure 1. The rectangular waveguide has a width of $a = 22.86\,\text{mm}$ and height of $b = 10.16\,\text{mm}$, and the inserted dielectric material slab has a dimension of $w = 12\,\text{mm}$ and relative permittivity $\varepsilon_r = 8.2$.

Figure 3 shows the variation of the magnitude of the reflection coefficient as a function of a frequency. It can be seen as an excellent agreement between our numerical results and those of Chen et al. [16], and the outcome is satisfactory. We confirm the validity of our numerical model.

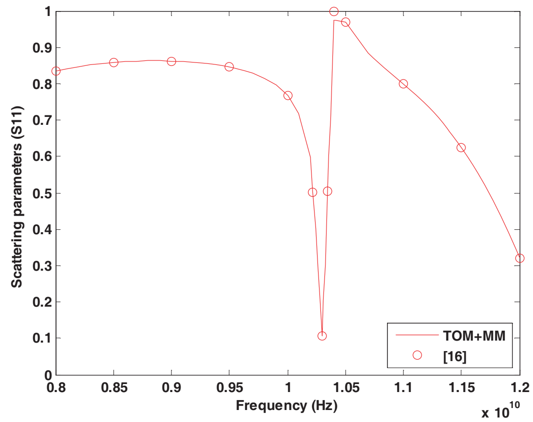

Figure 3. Magnitude of S_{11} versus frequency for the dielectric post discontinuity problem with length $L = 9\,\mathrm{mm}$.

Table 1. Characteristics of Ferrite TT1- 414 (Trans-tech).

ε_{rf}	$4\pi\mathrm{M}_s$ (Gauss)	ΔH (Oe)	g
11.3	750	120	1.98

Now, let's consider the guide of Figure 2 loaded with ferrite TT1-414 slab, whose characteristics are mentioned in Table 1.

At first, we calculate the normalized propagation constant as a function of frequency in the band [4–5.5] GHz. The stability of convergence of the propagation constant is obtained from 12 modes. Figure 4(a) shows the comparison between ferrite magnetized longitudinally with $M = 0.7M_s$ and demagnetized ferrite $M = 0$ for the first four modes (TE_{10}, TE_{20}, TE_{01} and TM_{11}) by the TOM.

Figure 4(b) shows the phenomena of bifurcation of the modes (see 6th mode) due to the anisotropy of ferrite. In Figure 4(c) we show the existence of the complex modes in these types of structures. However, the total complex modes in positive z and negative z are cancelled. These modes depend on the dimensions of the guide and the value of magnetization.

Combined with the TOM, the MM method is used to analyze a three-dimensional discontinuity problem in the rectangular waveguide loaded with a complete-height ferrite slab placed in the middle of the broad sides as shown in Figure 2. The electromagnet wave propagates along the Z axis. The ferrite is saturated with an internal DC magnetic field of 3200 Oe applied along Z-direction.

In Figure 5, the input reflection coefficient S_{11} in decibels is shown as a function of frequency. Our numerical results are compared with those calculated using commercial software HFSS. Thus, we obtain an excellent agreement.

The convergence behavior of our mode matching method is examined in Table 1 where the scattering parameters of the waveguide junction in Figure 5 are checked with different numbers of modes retained. A relationship exists between the ratio of the number of modes taken in each of the guides and the ratio of the guide's dimensions [17–19]. There is an equal number of modes in both the air filled and the ferrite loaded waveguides. From Table 2, we can conclude that convergent results can be obtained when the number of eigenmodes considered is greater than 15.

Table 2. Convergence behavior of the reflection coefficient with respect to the number of modes taken in both waveguides ($f = 10\,\mathrm{GHz}$).

Number of eigenmodes	5	10	15	20	30	50
$\lvert S_{11}\rvert$	0.8249	0.8231	0.8270	0.8270	0.8269	0.8270

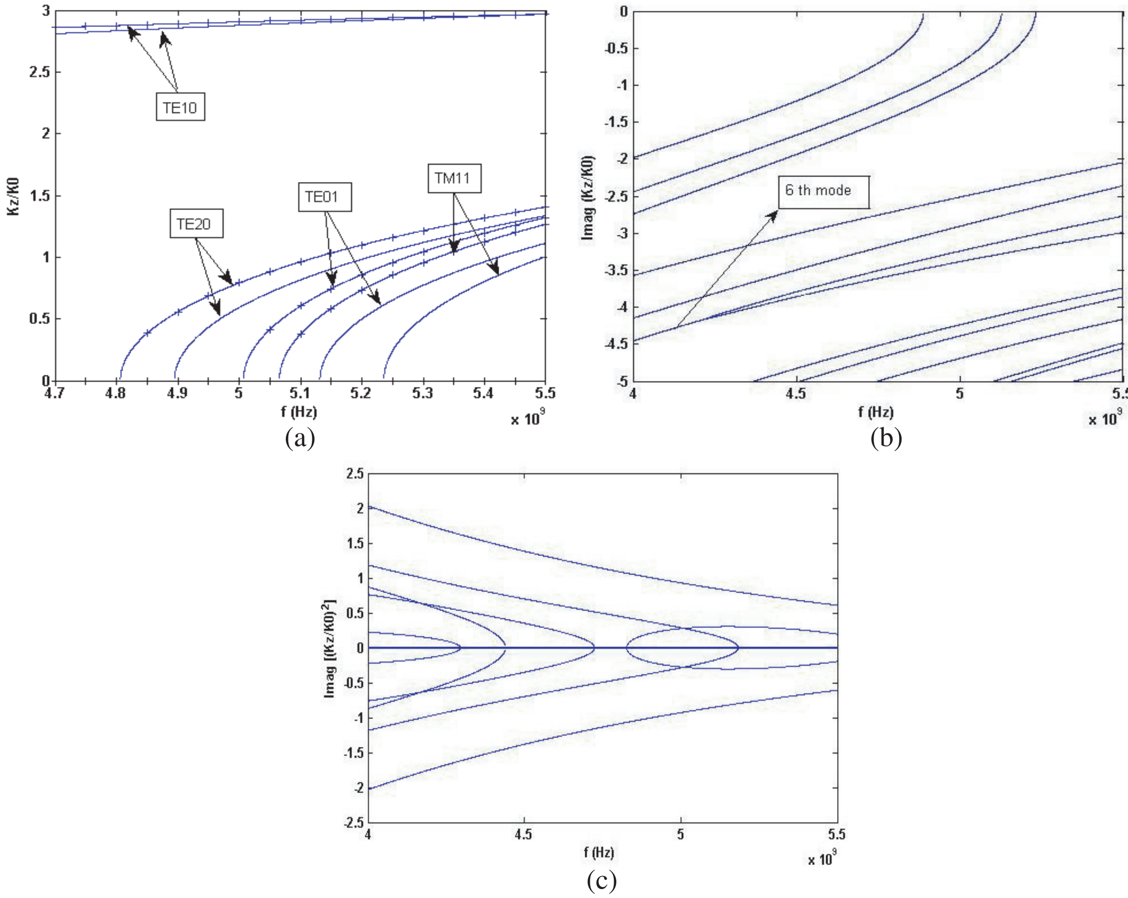

Figure 4. (a) The variation of the normalized propagation constant as a function of frequency: ______: Our results using the TOM for a magnetized ferrite. + + + + + +: Our results using the TOM for a demagnetized ferrite. (b) The variation of the imaginary part of (k_z/k_0) as a function of frequency: Our results using the TOM for $M/M_s = 0.7$. (c) The variation of the imaginary part of $[(k_z/k_0)^2]$ as a function of frequency: Our results using the TOM for $M/M_s = 0.7$.

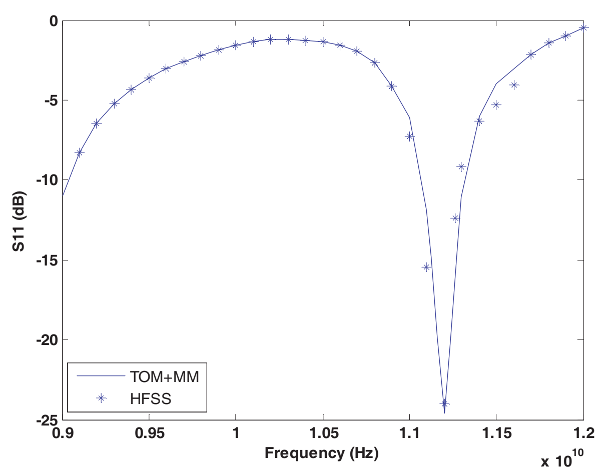

Figure 5. Reflection coefficients for a WR90 waveguide with a ferrite TT1-414 slab of width $w = 12\,\mathrm{mm}$, height $h = 10.16\,\mathrm{mm}$ and length $L = 9\,\mathrm{mm}$.

5. CONCLUSION

A rigorous analysis based on the Transverse Operator Method in the case of the guides of rectangular waves partially filled with longitudinally magnetized ferrite is presented. Higher order complex modes have been obtained. The Mode Matching method and Transverse Operator Method are combined in such a way to provide a solution. This combination has been successfully used to obtain scattering parameters of the discontinuity between an air filled waveguide and an E-plane ferrite slab loaded waveguide. Good agreement is observed between our numerical results and those simulated by Ansoft's HFSS. The present technique can be easily extended to the case of nonreciprocal differential phase shifter and filter devices loaded with LMF. More effort can be employed in the future work which will contain complex structures to improve the calculation time and memory size requirements.

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