Progress In Electromagnetics Research, Vol. 158, 1–6, 2017

On the Possibility of a Perfect Power Combiner

Sailing He[1, 2, *] and Kexin Liu[1, 2]

Abstract—By reductio ad absurdum, we show that a perfect power combiner of single-mode waveguides is impossible for incoherent input waves of the same frequency and same polarization as it is against the law of conservation of energy. The inevitable 3 dB loss of a three-port power combiner is explained physically. An incoherent power combiner of nearly 100% efficiency can be realized only if the two input fields have different wavelengths, have different polarizations, or are of orthogonal modes.

1. INTRODUCTION

A power combiner is a basic key component for many applications, e.g., high-power electromagnetic radiation. They are widely used in applications that require power higher than the capacity of a single electromagnetic source. They can combine electromagnetic waves from multiple sources to a single output port. These power combiners are commercially available at different frequency ranges (see e.g., [1] for multiple-way power combiners). They are well matched in impedance (with minimal local reflection) at each port, and they are also reciprocal components, so they can also serve as power dividers or splitters. However, these multiport networks of components suffer some power loss. For example, the 2-way Wilkinson power combiner is able to combine two coherent in-phase input waves, but for two incoherent waves,it needs a resistor to dissipate the undesired wave propagation between the two input ports [2, 3]. During many years' teaching of fiber optical communication technologies, the first author of this article has found that nearly all the students believed that an ideal 1×2 splitter can be utilized in the reversed direction as a perfect combiner, and no student could show why such a perfect combiner is impossible. After being told this is impossible, many students tended to attribute this to reciprocity, instead of the law of conservation of energy. Thus, in this paper, we want to demonstrate a solution to this puzzle, which seems quite classical.

In this paper, we first use an S matrix to analyze a simple three-port system, which shows the physical mechanism of combining two coherent powers. We then demonstrate the impossibility of achieving a perfect combiner in a passive system by using the law of conservation of energy. Furthermore, we discuss feasible systems that can combine electromagnetic waves by utilizing frequency, polarization, or mode multiplexing.

2. ANALYSIS OF A DEVICE SYSTEM CONSISTING OF THREE SINGLE-MODE PORTS

In this section, we assume that all waveguides are of single mode. Consider a device system with N ports. Let a_i denote the complex amplitude of the wave travelling into (i.e., entering) the device system through port i, and let b_i denote the amplitude of the wave leaving (propagating in the reverse direction)

Received 23 September 2016, Accepted 10 November 2016, Scheduled 29 November 2016
* Corresponding author: Sailing He (sailing@kth.se).
[1] Department of Electromagnetic Engineering, School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm S-100 44, Sweden. [2] Centre for Optical and Electromagnetic Research, State Key Laboratory of Modern Optical Instrumentation, JORCEP (Sino-Swedish Joint Research Center of Photonics), Zhejiang University, Hangzhou 310058, China.

the device system through port i, $i = 1, 2, 3, \dots, N$. We can define a scattering matrix S (related to some complex field, not field intensity) as follows:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_N \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{1N} \\ S_{21} & S_{22} & S_{23} & S_{2N} \\ S_{31} & S_{32} & S_{33} & S_{3N} \\ S_{N1} & S_{N2} & S_{N3} & S_{NN} \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_N \end{pmatrix} \equiv S \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_N \end{pmatrix} \tag{1}$$

We take a three-port reciprocal system (the simplest and most common case) as an example. Fig. 1 gives the schematic diagram for such a power splitter.

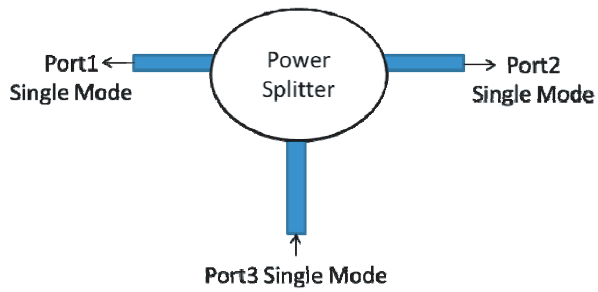


Figure 1. A schematic diagram for a power splitter.

We make the following assumptions:

(i) Every port has only one mode.
(ii) The system is reciprocal, i.e., $S^T = S$, where superscript T denotes the transpose of a matrix.
(iii) All the materials of the system are lossless, such that the total out-going power is equal to the total entering power, i.e., $S^{*T} S = I$, where the superscript * denotes the complex conjugate.
(iv) The power is equally divided without reflection.
(v) The output phases at Port 1 and Port 2 (with respect to the input phase at Port 3) are the same.

According to assumptions (i), (iv) (i.e., the power, which is proportional to the square of the absolute value of the field magnitude, is conserved),and (v), we have:

$$\begin{cases} S_{13} = S_{23} = \dfrac{\sqrt{2}}{2} \\ S_{33} = 0 \end{cases} \tag{2}$$

According to assumption (ii), we have:

$$\begin{cases} S_{31} = S_{32} = \dfrac{\sqrt{2}}{2} \\ S_{21} = S_{12} \end{cases} \tag{3}$$

According to assumption (iii), and using the above results, it follows that:

$$\begin{cases} |S_{11}|^2 + |S_{21}|^2 + \dfrac{1}{2} = 1 \\ S_{11} S_{12}^* + S_{21} S_{22}^* + \dfrac{1}{2} = 0 \\ S_{11} + S_{21} = 0 \\ S_{22} + S_{12} = 0 \end{cases} \tag{4}$$

The general solution to equation system (4) for four unknowns is:

$$S_{12} = S_{21} = -S_{11} = -S_{22} = \frac{1}{2} \exp(i\varphi).$$

Therefore, we obtain the following S matrix for the above 3-port system:

$$S = \begin{bmatrix} -\frac{1}{2}\exp(i\varphi) & \frac{1}{2}\exp(i\varphi) & \frac{\sqrt{2}}{2} \\ \frac{1}{2}\exp(i\varphi) & -\frac{1}{2}\exp(i\varphi) & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix} \tag{5}$$

Now, we will use the above system as a power combiner (instead of a power splitter; note that both the power splitter and combiner satisfy system (1) with the same S matrix). For the power part (of unit intensity) that is inputted at Port 1 (i.e., $a_1 = 1$, and $a_2 = a_3 = 0$), it follows from Eq. (5) that 25% is reflected at Port 1 (as $|S_{11}|^2 = 1/4$) and 25% is transmitted to Port 2 (as $|S_{21}|^2 = 1/4$), while only 50% of the total power is transferred to Port 3 (as $|S_{31}|^2 = 1/2$). This is also illustrated in Fig. 2, where the field distribution is calculated in commercial software CST. The standing wave on the left straight waveguide leading to Port 1 is due to the interference of the input right-going wave and the left-going reflected wave (due to the reflection at the junction). Similarly, for the power part (of unit intensity) that is inputted at Port 2 (i.e., $a_2 = 1$, and $a_1 = a_3 = 0$), it follows from Eq. (5) that 25% is reflected at Port 2 (as $|S_{22}|^2 = 1/4$) and 25% is transmitted to Port 1 (as $|S_{12}|^2 = 1/4$), while only 50% of the total power is transferred to Port 3 (as $|S_{32}|^2 = 1/2$). Therefore, due to the linearity of system (1), for a power combiner with simultaneous inputs with 50% non-coherent power input at each of ports(Port 1 and Port 2), we would obtain only 50% (instead of a total of 100%) of the total power output at Port 3 (the remaining 50% is outputted at undesired Ports 1 and 2), i.e., the combiner inevitably suffers 3 dB loss.

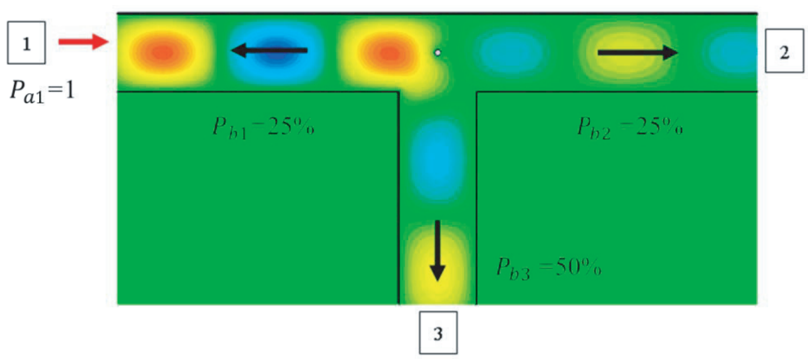


Figure 2. Field analysis for a three-port system when the input is from Port 1. The field distribution is calculated in commercial software CST.

3. DISCUSSION

The above conclusion is not valid when the input waves at Ports 1 and 2 (called input wave 1 and input wave 2) are coherent. For example, for a coherent input field, we can set the two input fields to have the same phase at Ports 1 and 2. Then, it follows from Eq. (5) that

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{bmatrix} -\frac{1}{2}\exp(i\varphi) & \frac{1}{2}\exp(i\varphi) & \frac{\sqrt{2}}{2} \\ \frac{1}{2}\exp(i\varphi) & -\frac{1}{2}\exp(i\varphi) & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{6}$$

We have $b_1 = b_2 = 0$, $b_3 = \sqrt{2}$. This means that the total power input from Ports 1 and 2 is totally outputted from Port 3 in the case of coherent field inputs. This can be explained by the destructive

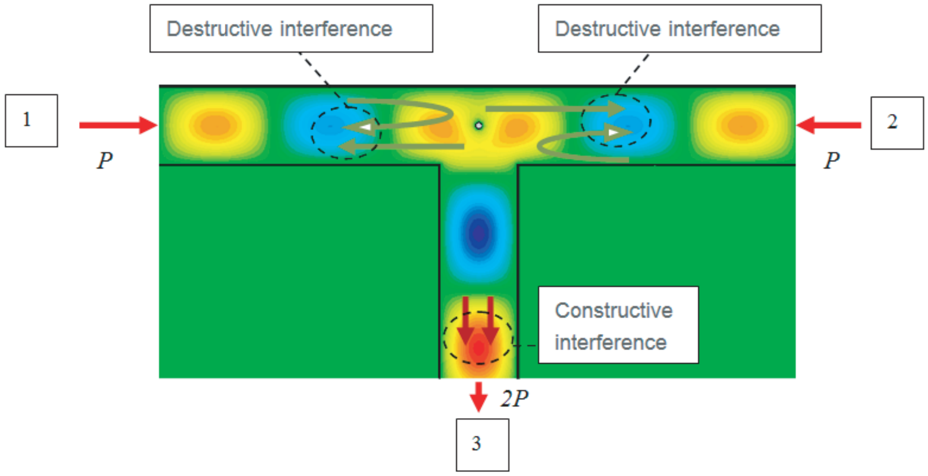


Figure 3. Field analysis for a three-port system when in-phase coherent powers are input from Ports 1 and 2. The field distribution is calculated in commercial software CST.

interference at Port 1 (between the left-going reflection of input wave 1 and the left-going transmission of input wave 2) and Port 2 (between the right-going reflection of input wave 2 and the right-going transmission of input wave 1) and constructive interference at Port 3, as shown in Fig. 3.

4. IMPOSSIBILITY OF A PERFECT THREE-PORT POWER COMBINER OF SINGLE-MODE WAVEGUIDES

The above analysis gives us a good example in which 3 dB loss of a three-port power combiner is physically explained. One may naturally wonder whether it is possible to achieve a perfect power combiner of three-port system with single-mode input/output waveguides. We can show this is impossible by the method of reductio ad absurdum.

If such a perfect combiner exists as shown in Fig. 4, then the following must be true:

(i) All the power from Port 1 goes to Port 3.

(ii) All the power from Port 2 also goes to Port 3.

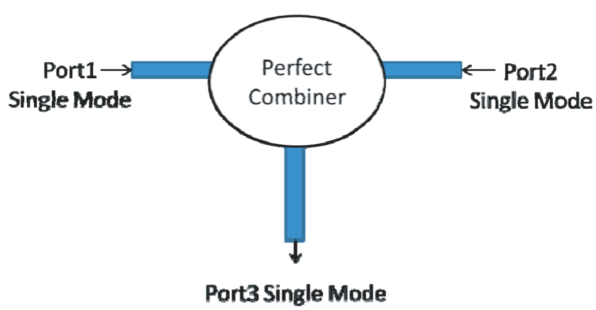


Figure 4. A perfect combiner.

First, we assume that the three single-mode input and output waveguides have the same cross-sectional size. Then we have the following S matrix:

$$S = \begin{bmatrix} 0 & 0 & S_{13} \\ 0 & 0 & S_{23} \\ \exp(i\varphi_1) & \exp(i\varphi_2) & S_{33} \end{bmatrix} \tag{7}$$

where φ_1 is the phase difference of the wave at Port 3 with respect to the input wave at Port 1, and φ_2 is the phase difference of the wave at Port 3 with respect to the input wave at Port 2.

From the above S matrix,it can be seen that:

$$S^{*T}S = \begin{bmatrix} 1 & \exp(i\varphi_2 - i\varphi_1) & S_{33}\exp(-i\varphi_1) \\ \exp(i\varphi_1 - i\varphi_2) & 1 & S_{33}\exp(-i\varphi_2) \\ S_{33}^*\exp(i\varphi_1) & S_{33}^*\exp(i\varphi_2) & |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 \end{bmatrix} \neq I$$

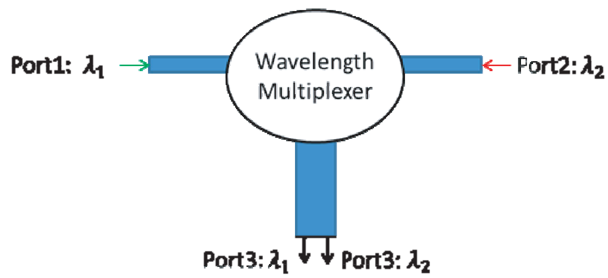
This indicates that total energy is not conserved in the system. For example, if we intentionally choose some specific coherent field inputs at Port 1 and Port 2 with $a_1 = E_0\exp(-i\varphi_1)$, $a_2 = E_0\exp(-i\varphi_2)$ as a particular case of power combination, we can obtain:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{bmatrix} 0 & 0 & S_{13} \\ 0 & 0 & S_{23} \\ \exp(i\varphi_1) & \exp(i\varphi_2) & S_{33} \end{bmatrix} \begin{pmatrix} E_0\exp(-i\varphi_1) \\ E_0\exp(-i\varphi_2) \\ 0 \end{pmatrix}$$

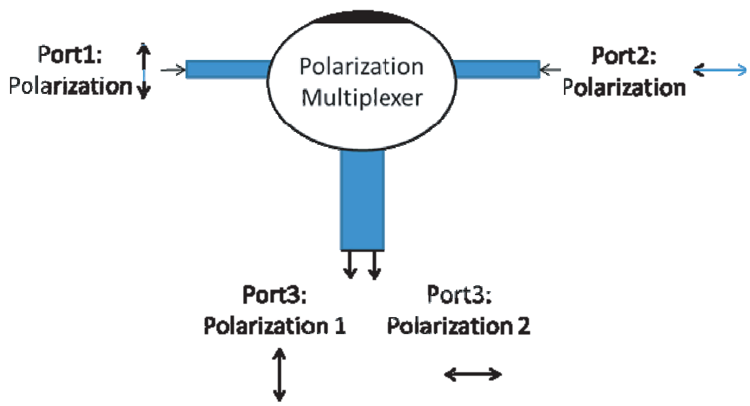
This would lead to $b_3 = 2E_0$.

From the perspective of power, the input power at Port 1 or Port 2 is $P_1 = P_2 = \eta|E_0|^2 \equiv P$, where η is related to $\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}$ and the waveguide cross-section size. However, the output power at Port 3

(a)



(b)



(c)

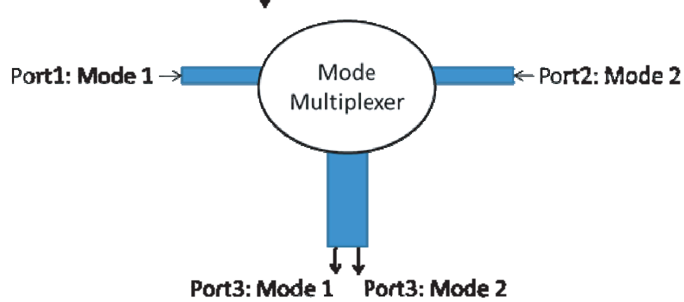


Figure 5. Perfect power combiners for waves of (a) different wavelengths; (b) different polarizations; and (c) different modes.

becomes:

$$P_3 = \eta \left| E_0 + E_0 \right|^2 = 4P > P_1 + P_2.$$

The law of conservation of energy is broken for such a passive system. If the three single-mode input and output waveguides have different cross-sectional sizes, we can always adiabatically extend (to the outside) all the three waveguides to a single-mode waveguide of the same cross-section size and then obtain the same conclusion (broken the law of conservation of energy) for the extended passive system with three new input/output single-mode waveguides of the same cross-section size. Note that the condition of reciprocity has not been used in the above proof. No matter what values S_{13}, S_{13} and S_{33} take, the conclusion still holds. This means that the conclusion applies to both reciprocal (in which $S_{13} = \exp(i\varphi_1)$ and $S_{23} = \exp(i\varphi_2)$) and non-reciprocal systems. Therefore, a passive system cannot give a perfect power combination when the input and output waveguides are all of single mode.

5. COMBINATION THROUGH DIFFERENT CHANNELS

From the above proof, it is evident that what makes a perfect combiner of single-mode waveguide type impossible is that the two fields can be added coherently. We know that the requirement for two fields to be added coherently is that the two fields should satisfy the following conditions:

(i) same frequency;
(ii) same polarization;
(iii) same mode.

Therefore, we can make a power combiner of 100% efficiency if we let the two input fields have different wavelengths (i.e., the well-known wavelength-division-multiplexing technology, see, e.g., [4]), or have different polarizations (i.e., polarization multiplexing, see, e.g., [5]), or be of two modes orthogonal to each other (i.e., spatial mode multiplexing, see, e.g., [6]). Fig. 5 gives the diagrams for these kinds of power combiners of nearly 100% efficiency for waves of different wavelengths, or different polarizations, or different modes.

In summary, we have solved the classical puzzle of the perfect power combiner in this short article. When an ideal 1×2 splitter is utilized in the reversed direction as a power combiner, it always suffers at least a 3 dB loss. A perfect power combiner is impossible for incoherent waves of the same frequency, same polarization, and same mode, as it is against the law of conservation of energy.

ACKNOWLEDGMENT

This work is partially supported by the Swedish VR grant (# 621-2011-4620) and AOARD.

REFERENCES

1. http://www.pulsarmicrowave.com/products/power_dividers/power_dividers.htm.
2. Wilkinson, E., "An N-way hybrid power divider," *IRE Transactions on Microwave Theory and Techniques*, Vol 8, 116–118, January 1960.
3. Pozar, D. M., *Microwave Engineering*, 4th Edition, Ch. 7, John Wiley & Sons, New York, 2012.
4. Mynbaev, D. K. and L. L. Scheiner, *Fiber-Optic Communications Technology*, 1st Edition, Ch. 13, Prentice Hall, 2000.
5. Chang, C., J. Sun, Z. F. Xiong, L. T. Guo, Y. S. Liu, Z. Q. Zhang, and X. L. Wu, "A compact two-way high-power microwave combiner," *Review of Scientific Instruments*, Vol. 85, No. 8, 084704, 2014.
6. Dai, D., J. Wang, and S. He, "Silicon multimode photonic integrated devices for on-chip mode-division-multiplexed optical interconnects," *Progress In Electromagnetics Research*, Vol. 143, 773–819, 2013.