# Modeling of a Ku-Band Rectangular Ferrite-Loaded Waveguide Based on Left-Handed Metamaterial

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Abstract—This paper presents the modeling and simulation of a new rectangular ferrite-loaded waveguide based on left-handed metamaterial (LHM) unit cells at  $K_u$ -band. The structure has an  $8 \times 8$  unit cell configuration, whose negative permittivity and negative permeability are achieved by metallic wires array and ferrite medium, respectively. The equivalent circuit model and transmission parameter matrix for the unit cell are presented based on microwave two-port network theory. The operating frequency is in the  $TE_{10}$  single mode range at 12.97–15.90 GHz where magnetic and electric resonances are coupled simultaneously. The finite-element method (FEM) based simulation software HFSS has been used to set original model and optimized model with vacuum layers for decoupling. Analysis of 3D electromagnetic waves propagation and scattering parameters demonstrate the backward wave property of the optimized waveguide. Negative propagation constant and negative index of refraction are calculated based on a method for extracting effective parameters of LHM. The proposed structure has scalability, double negative, and broad-band operation characteristics in the electromagnetic paradigm.

# 1. INTRODUCTION

Metamaterials are generally defined as artificial structures that have unusual electromagnetic properties which do not exist in conventional materials. They are usually made from repeating patterns and have smaller size than the operating wavelengths. Metamaterials derive their extraordinary properties not from basic composed materials such as metals or plastics, but from their precisely designed geometry, size and arrangement. Appropriately designed metamaterials give scientists and engineers new inspiration to manipulate electromagnetic waves by blocking, absorbing, or bending. Left-handed metamaterial (LHM) with simultaneous negative magnetic permeability and negative electric permittivity exhibits a negative index of refraction, which was first deeply studied by Veselago in 1968 [1]. He showed that the phase velocity could be anti-parallel to the direction of pointing vector in LHM, which is contrary to wave propagation in naturally-occurring materials.

Over the past 20 years, there has been great interest in LHM research since Pendry et al. proposed periodic fine metallic wires array as negative permittivity implementation structure in 1996 [2] and split ring resonators (SRRs) as negative permeability implementation structure [3]. Smith et al. firstly validated the unusual phenomena of LHM through microwave experiments in 2000 [4]. Their work has greatly enhanced the research interest of LHM within worldwide, and the wire-SRRs construct has become a classic model for LHM structures widely used in the research field of antennas [5–7], waveguides [8–12] and lenses [13]. For giving rise to negative permeability, researchers introduce ferrite medium to replace SRR structure. Chui and Hu investigated the possibility of preparing LHM in metallic magnetic granular composites [14], and Dewar replaced the array of SRRs with a

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nonconducting ferrimagnet to extend operating frequency into the far infrared and make LHM smaller with low losses [15] in 2002. Ferrite medium has nonreciprocal and tunable characteristics depending on the applied dc magnetic field and can be a medium with negative permeability and show gyrotropic characteristics [16]. It is worth emphasizing that the tunable, low-loss and broad band nature of ferrite LHM structures are explored, and such structures are vastly more practical in shape and size than the wire-SRR construct [17]. Since then, the wire-ferrite construct provides a means to fabricate low-loss, tunable and practicable LHM structures. Due to its unique advantages, potential LHM applications including waveguides design [18–20], different antennas [21, 22] have been explored based on different wire-ferrite constructs.

In this paper, a ferrite-loaded waveguide with 64 LHM unit cells in  $K_u$ -band is introduced. For unit cell of the LHM, ferrite medium fabricates a cuboid structure with a single metallic wire embedded in the center position in the cuboid, and the metallic wires and ferrite medium give rise to negative permittivity and negative permeability respectively. Such a unit cell is very compact and easily arranged periodically. Therefore, the waveguide is composed of 64 cells with a phalanx arrangement. By adjusting the dimensions of unit cells and external magnetic field for ferrite, negative parameter frequency bands can be tuned to exhibit in  $K_u$ -band. The Left-handed properties of the proposed waveguide are verified by subsequent theoretical calculation and numerical simulation based on finite element method (FEM). Through analysis of the waveguide prototype, vacuum layers for decoupling are applied to optimize the structure. The scattering parameter curves and 3D transmission characteristics of electromagnetic waves clearly indicate a backward wave phenomenon, which confirm LH properties of the optimized waveguide.

## 2. THEORETIC ANALYSIS OF FERRITE-LOADED LHM

The ferrite-loaded LHM is a combination of fine metallic wires array and magnetized ferrite medium, which achieves negative permittivity and negative permeability, respectively. The desired left-handed frequency band is that the equivalent plasma frequency band of metallic wires array and equivalent frequency band of ferromagnetic resonance coincide.

# 2.1. Physical and Mathematical Models of Negative Permittivity

The plasma effect of frequency band of metal can be significantly reduced from visible and ultraviolet light band to microwave band in this structure. The permittivity of plasma has a typical Drude form as [2]:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\xi)} \tag{1}$$

In Equation (1),  $\omega_p$  is the plasma angular frequency;  $\xi$  is the damping coefficient, which is used to characterize the energy loss in the media;  $\omega$  is the angular frequency of incident electromagnetic wave. If the loss is ignored:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{2}$$

Equation (2) indicates that when  $\omega < \omega_p$ , the equivalent permittivity is negative. The plasma angular frequency of fine metallic wires array is [2]:

$$\omega_p^2 = \frac{2\pi c_0^2}{a^2 \ln\left(\frac{a}{r}\right)} \tag{3}$$

In Equation (3), a represents the spacing between two adjacent metallic wires in the array; r is the radius of single metallic wire; c is the velocity of light in vacuum. Equation (3) indicates that the equivalent plasma frequency is determined only by parameters of metallic wires array, that is to say, wires spacing and radius determine negative permittivity frequency band.

## 2.2. Physical and Mathematical Models of Negative Permeability

Ferrite medium is chosen as the implementation structure of negative permeability. In terms of dielectric property, ferrite has similar properties to magnetic medium, which has a dielectric loss tangent of  $10^{-3} \sim 10^{-4}$ , relative permittivity between  $5 \sim 25$  and relative permeability above 100 in microwave band. The magnetism of ferrite is caused by the electron spin with magnetic saturation of 200–5500, covering most of the microwave band. When it is far away from the resonance region, microwave energy can go through ferrite medium freely by fully interacting with spin electrons. Under the influence of an external magnetic field, the permeability of ferrite medium appears in a tensor form. In the Cartesian coordinate system, the external magnetic field is supposed to be along z axis. Ignoring damping loss, the permeability of ferrite medium is [23]:

$$\mathbf{m} = \begin{bmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

In Equation (4),

$$\mu = 1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2} \quad k = \frac{\omega_m \omega}{\omega_0^2 - \omega^2} \tag{5}$$

In Equation (5),  $\omega$  is the angular frequency of incident electromagnetic wave,  $\omega = \gamma H$  the angular frequency of ferrite precession resonance,  $\gamma = 1.759 \times 10^{11} \, \mathrm{rad/sT}$  the gyromagnetic ratio, H the amplitude of external magnetic field,  $\omega_m = \gamma M$  the characteristic frequency, and M the amplitude of ferrite saturation magnetization. Equation (5) can be written in the following form after substitution:

$$\mu = 1 + \frac{\gamma^2 H_0 M_0}{\gamma^2 H_0^2 - \omega^2} \quad k = \frac{\gamma M_0 \omega}{\gamma^2 H_0^2 - \omega^2} \tag{6}$$

The equivalent permeability of ferrite medium is:

$$\mu_{eff} = \frac{\mu^2 - k^2}{\mu} \tag{7}$$

Putting Equation (6) into Equation (7) obtains:

$$\mu_{eff} = \frac{\left(H_0 + M_0\right)^2 - \left(\frac{\omega}{\gamma}\right)^2}{H_0(H_0 + M_0) - \left(\frac{\omega}{\gamma}\right)^2} \tag{8}$$

If  $\mu_{eff} < 0$ , the angular frequency of incident electromagnetic wave is:

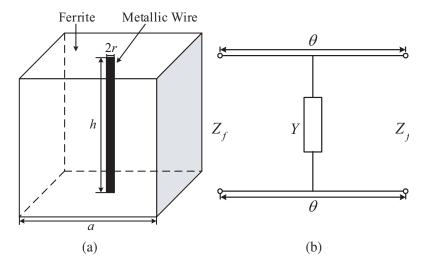
$$\gamma \sqrt{H_0^2 + H_0 M_0} < \omega < \gamma (M_0 + H_0) \tag{9}$$

When the angular frequency satisfies Equation (9), there will be a ferromagnetic resonance effect in the ferrite medium, which has an effect that microwave energy cannot pass, and equivalent negative permeability appears.

#### 2.3. The Ferrite-Loaded LHM Unit Cell

Due to the repeatability of fine metallic wires array and the uniformity of ferrite medium ferrite loaded LHM can theoretically be repeatable and uniform by combing them. Therefore, a ferrite loaded LHM unit cell structure is proposed, i.e., the smallest part of the ferrite loaded LHM waveguide. LHM structures with different sizes and different frequency bands can be obtained by periodically connecting a number of unit cells. A cylindrical fine metallic wire, with height of h and radius of r, is vertically inserted in the unit cell in the center position of a cuboid-shaped ferrite medium with height of h and side length of h, as shown in Figure 1(a).

The phase change of electromagnetic waves can be ignored if the working wavelength is 10 times greater than physical size of the unit cell. Therefore, it is an electrically small structure, and microwave



**Figure 1.** (a) Schematic diagram of the ferrite loaded LHM unit cell. (b) Equivalent circuit of the ferrite-loaded LHM unit cell.

network parameters is applicable to the cell. The ferrite-loaded LHM waveguide is composed by a concatenation of 64 unit cells, so it is suitable for understanding the transfer properties of LHM by establishing theoretical model based on transfer parameter A matrix. Equivalent circuits of single metallic wire and ferrite medium are expressed by parallel admittance form and transmission line form respectively according to microwave two-port network theory, and equivalent circuit of the unit cell is shown in Figure 1(b).

Transfer parameter matrix  $A_c$  of the unit cell can be expressed as [17]:

$$A_{c} = A_{w} \cdot A_{f}$$

$$= \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\beta_{f}a) & jZ_{f}\sin(\beta_{f}a) \\ j\sin(\beta_{f}a)/Z_{f} & \cos(\beta_{f}a) \end{bmatrix}$$
(10)

In Equation (10),  $A_w$  and  $A_f$  are the transfer parameter matrices of single wire and cuboid-shaped ferrite medium, respectively; a is simultaneously side length of ferrite cuboid and wires spacing between two adjacent unit cells;  $\beta_f$  is the phase propagation constant of ferrite medium  $\theta = \beta_f \times a$ ;  $Z_f$  is the characteristic impedance of ferrite medium.  $\beta_f$  and  $Z_f$  are expressed as:

$$\beta_f = \frac{2\pi}{\lambda} \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2} \qquad Z_f = Z_0 \sqrt{\frac{|\mu_{eff}|}{\varepsilon_r \left[1 - \left(\frac{\lambda}{2a}\right)^2\right]}}$$
(11)

In Equation (11),  $\lambda = 2\pi c_0/(\omega\sqrt{|\mu_{eff}|\varepsilon_r})$  represents working wavelength in the ferrite loaded LHM unit cell; Z is the characteristic impedance of media in vacuum, which approximately equals to 377  $\Omega$ ;  $\varepsilon_r$  is the relative permittivity of ferrite medium;  $\mu_{eff}$  can be calculated by Equation (8).

In Equation (10), Y is the admittance of single wire, which can be expressed as [17]:

$$Y = [R + j\omega(L_i + L_o)]^{-1}$$

$$= \left[\frac{h}{2\pi r}\sqrt{\frac{\omega\mu_0}{2\sigma}} + j\omega\left(\frac{h}{2\pi r}\sqrt{\frac{\mu_0}{2\omega\sigma}} + \mu_0 h \frac{\ln(a/r)}{2\pi}\right)\right]^{-1}$$

$$= \frac{2\pi r}{h} \cdot \left[\sqrt{\frac{\omega\mu_0}{2\sigma}} + j\omega\left(\sqrt{\frac{\mu_0}{2\omega\sigma}} + \mu_0 r \ln\frac{a}{r}\right)\right]^{-1}$$
(12)

In Equation (12), the impedance of single metallic wire includes resistance R and reactance  $\omega \times (L_i + L_o)$ . Due to the skin effect of current in microwave band, R approximately equals the

metal resistance with length of a, and cross-sectional area of  $2\pi r \sqrt{2/\omega\mu_0\sigma} \cdot \mu_0$  is the permeability in vacuum.  $\sigma$  is the conductivity of metal,  $L_i$  the self-inductance of single metallic wire, and  $L_o$  the mutual inductance between adjacent metal wires in two unit cells.

Transmission characteristics of electromagnetic energy velocity and phase velocity are mainly focused for electromagnetic properties of ferrite loaded LHM waveguide, so the scattering parameter matrix is more suitable than transfer parameter matrix to characterize electromagnetic properties in microwave band. Microwave energy can freely pass the general waveguides, but in the negative permittivity or negative magnetic permeability medium, microwave energy is almost completely blocked due to a strong dispersion effect [1]. For ferrite loaded LHM waveguides, microwave energy can pass it with a great power loss. Therefore, scattering parameter  $S_{21}$  is more suitable for judging properties of ferrite loaded LHM waveguides. Assuming that there are N ferrite LHM unit cells in the propagation direction of electromagnetic waves, the total transfer parameter matrix  $A_z$  is:

$$A_z = (A_c)^N = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$
 (13)

By converting relationship between scattering parameters and transfer parameters in microwave two-port network theory:

$$S = \frac{1}{a_{11} + a_{12} + a_{21} + a_{22}} \begin{bmatrix} a_{11} + a_{12} - a_{21} - a_{22} & 2 \det A_z \\ 2 & -a_{11} + a_{12} - a_{21} + a_{22} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$
(14)

In the waveguide,  $S_{21}$  are analyzed in theoretical left-handed frequency band to identify electromagnetic energy transmission. Furthermore, unusual left-handed properties of the proposed waveguide are observed and calculated by simulation based on FEM in the next section.

#### 3. PARAMETERS AND SIMULATION

To establish simulation models of the proposed ferrite loaded rectangular waveguide based on LHM unit cells, theoretical parameters are as follows:  $c=3\times10^8\,\mathrm{m/s},\,\mu_0=4\pi\times10^{-7}\,\mathrm{H/m};$  copper is chosen for metallic wires array and  $a=2\,\mathrm{mm},\,h=4\,\mathrm{mm},\,r=0.1\,\mathrm{mm},\,\sigma=5.9\times10^7\,\mathrm{S/m}.$  Thus, plasma frequency of metallic wires array  $f_p=34.57\,\mathrm{GHz}.$  As for the ferrite medium  $\varepsilon_r=5,\,\mu_r=1000,\,H=0.378\,\mathrm{T}$  magnetic saturation  $M=0.19\,\mathrm{T}$ , magnetic loss line width  $\Delta H=0.001\,\mathrm{T}.$  The ferromagnetic resonance frequency band is 12.97–15.90 GHz which can be calculated by Equation (9). Combining plasma frequency and ferromagnetic resonance frequency band, theoretical left-handed frequency band is determined in 12.97–15.90 GHz, and homologous working wavelength  $\lambda$  is 18.87–23.13 mm. The broad and narrow sides of the waveguide cross-section are w and h, respectively. In order to make the waveguide working in the TE<sub>10</sub> single mode area,  $w<\lambda<2w$  and w>2h should be satisfied simultaneously. Based on above parameters and condition,  $h=4\,\mathrm{mm}$  and  $w=16\,\mathrm{mm}$  are selected for the waveguide size. The LHM unit cell has a side length of  $a=2\,\mathrm{mm}$ ; therefore, 8 unit cells can be arranged in a line. An arrangement of  $8\times8$  phalanx form for ferrite loaded LHM unit cells is presented, and the size of waveguide is  $16\,\mathrm{mm}\times16\,\mathrm{mm}\times4\,\mathrm{mm}.$ 

A 3D simulation model of the waveguide is built using high frequency structure simulator (HFSS) based on FEM. Besides, an input wave port is set at one end and an output wave port at the other end of the structure, which have the same size with waveguide cross-section area. In the simulation model, operating frequency band to be solved is 12.97–15.90 GHz in  $K_u$ -band. The center solution frequency is 14.44 GHz, and step frequency is 0.05 GHz. As for incident waves in the simulation model, a plane wave is set as the excitation with electric field vector E, magnetic field vector H and wave vector K along z, x and y axis respectively, and coordinate position of the incident wave is (8 mm, 0 mm, 2 mm). The proposed waveguide model is shown in Figure 2(a), and the total number of mesh elements is 117  $\sim$  781. Dewar indicates that the plasma oscillation is destroyed if the medium surrounding the wires has a negative permeability [15]. Accordingly it may be necessary to add layers for decoupling between metallic wires and ferrite medium. Then, cylindrical vacuum decoupling layers are added outside every copper wires, and the central axis of layers coincides with copper wires. To determine the best value of decoupling layers radius, optimization of radius is made between 0.15–1.95 mm, and the step value is 0.05 mm. After 17 times of optimization analysis of  $S_{21}$  value, 0.6 mm is selected to be the

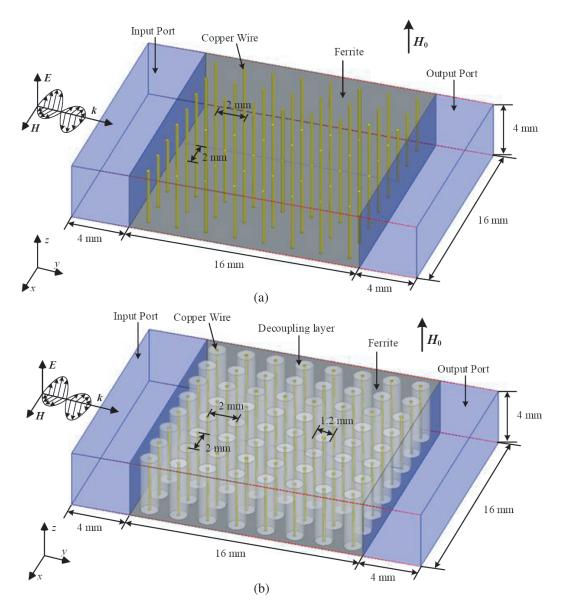


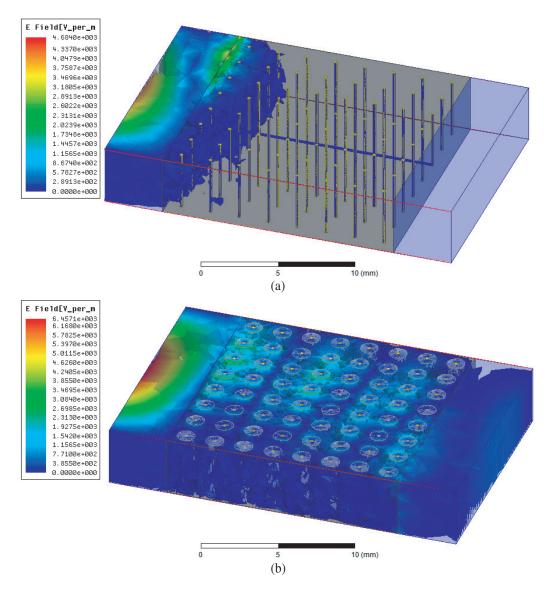
Figure 2. (a) Original ferrite loaded LHM waveguide structure. (b) Optimized ferrite-loaded LHM waveguide structure.

best value. Finally, the optimized waveguide structure is shown in Figure 2(b), and the total number of mesh elements is  $168 \sim 607$ .

In order to analyze propagation properties of electromagnetic waves in the models, a straight sampling line is drawn in the waveguide center position along y axis to observe electric field values and phase change. Two endpoints of the straight line are  $(8\,\mathrm{mm},\,4\,\mathrm{mm},\,2\,\mathrm{mm})$  and  $(8\,\mathrm{mm},\,20\,\mathrm{mm},\,2\,\mathrm{mm})$ . The detailed simulation results and analysis about left-handed properties of the proposed LHM waveguide are discussed in the next section.

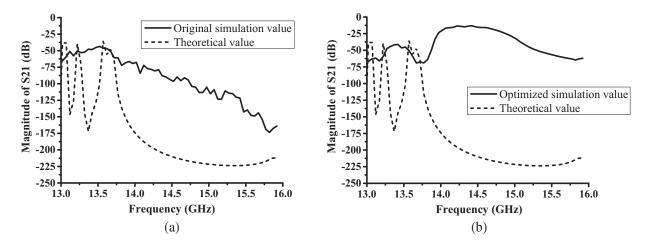
# 4. RESULTS AND ANALYSIS

3D full-wave electromagnetic waves propagation in the original and optimized ferrite loaded LHM waveguide are shown in Figure 3(a) and Figure 3(b). Operating frequency of the incident wave is 14.44 GHz, which is the center frequency of left-handed band, and initial phase is 0 degree. For the



**Figure 3.** (a) 3D electromagnetic wave propagation at 14.44 GHz in the original waveguide. (b) 3D electromagnetic wave propagation at 14.44 GHz in the optimized waveguide.

single negative medium, i.e., permittivity or permeability is negative, there is a great dispersion effect, so microwave energy is completely blocked. When ferrite medium and metallic wires are connected directly, electric resonance produced by copper wires array and magnetic resonance produced by ferrite medium affect each other, so the coupling effect of negative permittivity and negative permeability is extremely weak. The electromagnetic waves propagation distance can be seen intuitively in Figure 3(a). Incident waves mostly penetrate 2 mm into the original waveguide, and electromagnetic energy almost losses at the beginning part of the structure; therefore, field values on the sampling line approximately equal zero. For the purpose of analyzing propagation properties accurately, both theoretical  $S_{21}$  curve and simulation  $S_{21}$  curve are plotted in Figure 4(a). For complex 3D structures, it is difficult to accurately identify the theoretical calculation, and simulation based on FEM method may have less error than theoretical calculation method. The theoretical value of  $S_{21}$  is calculated by Equation (14), which is an approximate estimation of the theoretical model. Relatively, the simulation value of  $S_{21}$  is calculated by FEM method, so there is some disagreement between theoretical and experimental results. The LHM frequency band can be divided into two parts, i.e., fluctuation stage 12.97–13.75 GHz and stabilization stage 13.75–15.90 GHz. In the fluctuation stage, ferromagnetic resonance effect of ferrite medium is not



**Figure 4.** (a)  $S_{21}$  curve of the original waveguide. (b)  $S_{21}$  curve of the optimized waveguide.

stable, thus the theoretical calculation has great volatility. There are obviously maximum and minimum points in MATLAB programming results. When the frequency is higher than  $13.75\,\mathrm{GHz}$ , ferromagnetic resonance effect is strong enough, and a conclusion is drawn from both theoretical and experimental results that the higher the working frequencies are, the greater the loss of electromagnetic energy is in the left-handed band. As for simulation results, the maximum value is  $-43.56\,\mathrm{dB}$ , and average value is  $-93.08\,\mathrm{dB}$ . Obviously, theoretical and simulated results of the original waveguide validate Dewar's view mentioned in Section 3.

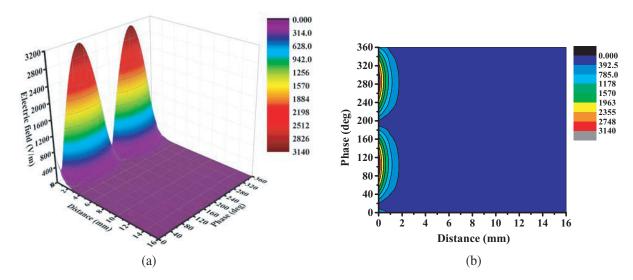
In the optimized waveguide, significantly different 3D electromagnetic waves propagations are observed in Figure 3(b). Firstly, the electromagnetic waves can pass through the whole structure to the output port. This is a basic feature of LHM that electromagnetic energy can pass. Figure 4(b) shows theoretical and simulated  $S_{21}$  curves of the optimized waveguide, and the maximum value of simulation is  $-13.03\,\mathrm{dB}$ , and average value is  $-41.61\,\mathrm{dB}$ . It is necessary to emphasize that  $S_{21}$  value is greater than  $-25\,\mathrm{dB}$  in 13.97– $14.87\,\mathrm{GHz}$ , which is a nearly 900 MHz band in  $K_u$ -band. When the frequency is higher than  $13.75\,\mathrm{GHz}$ , simulation curve has an opposite trend with theoretical results, which demonstrates that the proposed waveguide has good transmission characteristics of left-handed properties after optimization.

The cross product of  $\mathbf{E}$  and  $\mathbf{H}$  for a plane wave in conventional materials gives the directions of both propagation and energy flow. The electric field  $\mathbf{E}$ , magnetic field  $\mathbf{H}$  and wave vector  $\mathbf{k}$  follow a right-hand rule. In contrast, the left-handed medium has negative values of  $\varepsilon$  and  $\mu$ , and  $\mathbf{E} \times \mathbf{H}$  still gives the direction of energy flow, but the phase velocity propagates in the opposite direction. In the proposed waveguide, incident electromagnetic waves propagate from left side to the right side along positive y axis, i.e., the energy flow is along positive y axis. To validate the backward wave property, it should be proved that phase velocity is along negative y axis. To confirm the phase velocity  $v_p$  in the waveguide, it is important to ensure the relationship between wave phase  $\varphi$ , angular frequency  $\omega$  and propagation constant  $\beta$ . The relationship is:

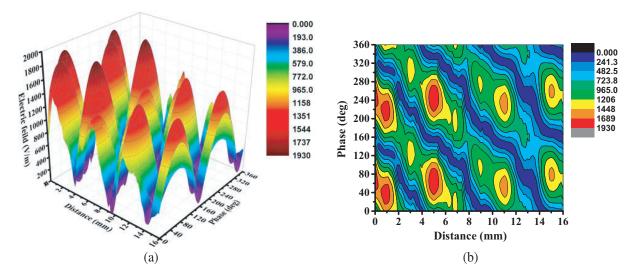
$$v_p = \frac{\omega}{\beta} = \frac{\mathrm{d}y}{\mathrm{d}t} \tag{15}$$

$$\beta = \frac{\mathrm{d}\varphi}{\mathrm{d}y} = \frac{\omega \cdot \mathrm{d}t}{\mathrm{d}y} \tag{16}$$

Equation (16) indicates that  $\beta$  equals the slope of  $\varphi y$  curve, which can be judged from  $\varphi y$  plot. 3D electric field on the sampling line is shown in Figure 5 and Figure 6. Distance (length of sampling line), Phase and Electric field are along x axis, y axis and z axis, respectively. Figure 5(a) shows that there is a sharp decrease of electric field values along the distance direction in the original waveguide, and electromagnetic energy is almost inhibited at the beginning of the structure. A magnitude contour plot of electric field is shown in Figure 5(b), which clearly indicates that microwave energy is limited within 2 mm. In the optimized waveguide, 3D electric field on the sampling line is shown in Figure 6(a).



**Figure 5.** (a) 3D field relationship between Distance, Phase and Electric field on sampling line of the original waveguide. (b) Electric field contour plot between Distance, Phase and Electric field on sampling line of the original waveguide.



**Figure 6.** (a) 3D field relationship between Distance, Phase and Electric field on sampling line of the optimized waveguide. (b) Electric field contour plot between Distance, Phase and Electric field on sampling line of the optimized waveguide.

Similarly, it can be seen intuitively that electromagnetic wave propagates along the distance direction. The contour plot in Figure 6(b) shows distinguished results that minimum values of electric field form four blue strips which are approximately linear and parallel. More importantly, every strip exhibits a negative slope, which means that propagation constant  $\beta$  is negative. Therefore, phase velocity  $v_p$  in the optimized waveguide is negative.

To calculate the approximation of propagation constant  $\beta$  and index of refraction n, four blue strips are approximately regarded as four straight lines. Four groups of endpoint values on straight lines are shown in Table 1, where Lines 1–4 are corresponding to the four strips from left to right in Figure 6(b).  $\beta$  is calculated by Equation (16), and n is calculated by:

$$n = -\sqrt{\left(\frac{\lambda_0}{2w}\right)^2 + \left(\frac{\lambda_0}{2\pi} \cdot \beta\right)^2} \tag{17}$$

In Equation (17),  $\lambda$  is the free space wavelength at 14.44 GHz, which equals 20.78 mm. w is 16 mm, which is the broad side size of the waveguide cross-section. The detailed results of  $\beta$  and n are shown in Table 1, and average values of  $\beta$  and n are 619.87 and 2.15, respectively. There is a decreasing trend for  $\beta$  and n, and the reason may be that as the transfer distance increases, magnitude of microwave energy declines, which has a weakening effect on left-handed properties in the waveguide.

	Distance	Phase	${ m E}$	Distance	Phase	${ m E}$	Q	20
	(mm)	(deg)	(V/m)	(mm)	(deg)	(V/m)		n
Line 1	4.0	0	116.36	0	160	95.21	-698.13	-2.40
Line 2	9.6	0	0	0	340	95.21	-618.14	-2.15

4.0

9.6

360

360

116.36

0

-604.15

-559.05

-2.10

-1.96

Table 1. Endpoint values on four straight lines.

0

155

116.01

84.35

# 5. CONCLUSIONS

14.4

16.0

Line 3

Line 4

We propose a ferrite-loaded rectangular waveguide based on LHM unit cells with an  $8 \times 8$  phalanx arrangement. The waveguide can achieve negative propagation constant and negative refractive index at  $K_u$ -band of microwave spectra. A method for extracting effective parameters of LHM is proposed. Negative  $\beta$  and n are calculated based on this method. The backward wave properties of the proposed structure are verified by both theoretical analysis and numerical simulations. This structure can be used for broad-band, double negative or near zero refractive index applications, and may find its potential application for satellite communication.

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