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A New Analytically Regularizing Method for the Analysis of the Scattering by a Hollow Finite-Length PEC Circular Cylinder

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Abstract—In this paper, a new analytically regularizing method, based on Helmholtz decomposition and Galerkin method, for the analysis of the electromagnetic scattering by a hollow finite-length perfectly electrically conducting (PEC) circular cylinder is presented. After expanding the involved functions in cylindrical harmonics, the problem is formulated as an electric field integral equation (EFIE) in a suitable vector transform (VT) domain such that the VT of the surface curl-free and divergence-free contributions of the surface current density, adopted as new unknowns, are scalar functions. A fast convergent second-kind Fredholm infinite matrix-operator equation is obtained by means of Galerkin method with suitable expansion functions reconstructing the expected physical behaviour of the unknowns. Moreover, the elements of the scattering matrix are efficiently evaluated by means of analytical asymptotic acceleration technique.

1. INTRODUCTION

Integral equation formulations, in both spatial and spectral domains, are very frequently adopted when dealing with the analysis of the electromagnetic scattering by finite objects in non-shielded media [1–38]. EFIE formulation is particularly suited for the analysis of the scattering by PEC surfaces, assuming the surface current density as unknown [39]. However, due to the first-kind nature of such kind of integral equations with a singular kernel, the approximate solutions obtained by using general discretization schemes cannot converge to the solution of the problem, and the sequence of condition numbers of truncated system can be divergent [40, 41].

Methods to overcome the problems mentioned above are collectively called methods of analytical regularization [42]. They are based on the conversion of a first-kind integral equation in a second-kind Fredholm integral equation (at which Fredholm theory can be applied [43]), which can be done by inverting the most singular part of the integral operator (containing the leading singularities of the kernel). It is interesting to observe that the regularizing procedure is in general neither trivial nor unique, and that the computational cost of numerical algorithm is strictly related to the selected regularizing scheme. As a matter of fact, different solutions have been proposed in the literature, ranging from the explicit inversion of the most singular part of the integral operator [14, 22–24, 29] to the adoption of a Nystrom-type discretization scheme [17, 25, 30].

Moreover, Galerkin method with a complete set of expansion functions which makes the most singular part of the integral operator invertible with a continuous two-side inverse immediately results in a regularized discretization scheme. This approach has been implemented in [4–6, 9, 12, 13, 15, 16, 18, 20, 21, 26–28, 31–38] devoted to the analysis of propagation, radiation, and scattering by PEC/dielectric polygonal cross-section cylinders and canonical shape PEC surfaces, by choosing basis functions reconstructing the expected physical behaviour of the fields. As a consequence, few expansion functions are needed to achieve accurate results, and the convolution integrals are reduced

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to algebraic products. Moreover, suitable techniques have been developed to make the matrix coefficients rapidly converging integrals.

In this paper, the analysis of the electromagnetic scattering by a hollow finite-length PEC circular cylinder is carried out by means of a new analytically regularizing method based on Helmholtz decomposition and Galerkin method. After representing all the involved functions as superposition of cylindrical harmonics, the problem is formulated as an EFIE in a suitable VT domain such that the VT of the surface curl-free and divergence-free contributions of the surface current density, adopted as new unknowns, are scalar functions. The obtained integral equation is discretized by means of Galerkin method with a suitable choice of the expansion basis. First of all, the expansion functions used form a complete set of orthogonal eigenfunctions of the most singular part of the integral operator leading to a second-kind Fredholm infinite matrix-operator equation. Secondly, they reconstruct the expected physical behaviour of the surface current density, i.e., few expansion functions are needed to achieve highly accurate results. Moreover, they have closed-form spectral domain counterparts and, hence, the elements of the scattering matrix result to be single improper integrals of oscillating functions efficiently evaluable by means of analytical asymptotic acceleration technique.

This paper is organized as follows. In Sections 2 and 3, the formulation and proposed solution of the problem are presented. The fast convergence of the method and comparisons with the literature and the commercial software CST-MWS are shown in Section 4, while the conclusions are summarized in Section 5.

2. FORMULATION OF THE PROBLEM

The geometry of the problem, depicted in Figure 1, shows a hollow finite-length PEC circular cylinder (of radius a and length $2b$) in vacuum.

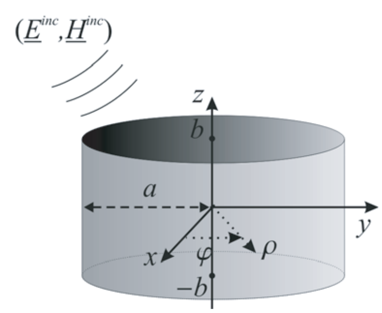

Figure 1. Geometry of the problem.

Let us denote ε_0, μ_0 and $k_0 = 2\pi/\lambda = \omega\sqrt{\varepsilon_0\mu_0}$ as the dielectric permittivity, magnetic permeability and wavenumber of the vacuum, respectively, where λ is the wavelength and ω the angular frequency. A Cartesian coordinate system (x, y, z) and a cylindrical coordinate system (ρ, φ, z) with $x = \rho\cos\varphi$ and $y = \rho\sin\varphi$ are introduced so that the z axis coincides with the cylinder axis, and the origin is at the centre of the cylinder itself. An incident field $(\underline{E}^{inc}(\underline{r}), \underline{H}^{inc}(\underline{r}))$, where $\underline{r} = x\hat{x} + y\hat{y} + z\hat{z}$, induces a surface current density $\underline{J}(\varphi, z) = J_\varphi(\varphi, z)\hat{\varphi} + J_z(\varphi, z)\hat{z}$ on the scatterer surface which, in turn, generates a scattered field $(\underline{E}^{sc}(\underline{r}), \underline{H}^{sc}(\underline{r}))$.

The revolution symmetry of the problem at hand with respect to the z axis suggests to expand all the involved functions in cylindrical harmonics, i.e.,

$$f(\rho, \varphi, z) = \sum_{n=-\infty}^{+\infty} f^{(n)}(\rho, z)\, e^{jn\varphi}. \tag{1}$$

Due to the orthogonality of the cylindrical harmonics, in the following we can refer directly to the general (n-th) harmonic whenever possible.

By imposing the tangential component of the n-th harmonic of the electric field to be vanishing on the cylinder surface, an EFIE is obtained

$$\underline{\mathbf{E}}^{sc(n)}(a, z) = -\underline{\mathbf{E}}^{inc(n)}(a, z) \tag{2}$$

for $|z| \leq b$, where [6]

$$\underline{\mathbf{E}}^{inc(n)}(a,z) = \begin{pmatrix} E_{\varphi}^{inc(n)}(a,z) \\ E_{z}^{inc(n)}(a,z) \end{pmatrix}, \tag{3}$$

$$\underline{\mathbf{E}}^{sc(n)}(a,z) = \int_{-\infty}^{+\infty} \underline{\underline{\tilde{\mathbf{G}}}}^{(n)}(u)\,\underline{\tilde{\mathbf{J}}}^{(n)}(u)\,e^{-juz}du, \tag{4}$$

the following definition of Fourier transform with respect to the z axis

$$\underline{\tilde{\mathbf{J}}}^{(n)}(u) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underline{\mathbf{J}}^{(n)}(z)\,e^{juz}dz \tag{5}$$

has been introduced,

$$\underline{\mathbf{J}}^{(n)}(z) = \begin{pmatrix} J_{\varphi}^{(n)}(z) \\ J_{z}^{(n)}(z) \end{pmatrix}, \tag{6}$$

$$\underline{\underline{\tilde{\mathbf{G}}}}^{(n)}(u) = -\frac{\pi}{2\omega\varepsilon_0 a}\begin{pmatrix} k_0^2a^2\tilde{A}^{(n)}(u) - n^2\tilde{B}^{(n)}(u) & nua\tilde{B}^{(n)}(u) \\ nua\tilde{B}^{(n)}(u) & \left(k_0^2a^2 - u^2a^2\right)\tilde{B}^{(n)}(u) \end{pmatrix}, \tag{7}$$

$$\tilde{A}^{(n)}(u) = \frac{\tilde{B}^{(n-1)}(u) + \tilde{B}^{(n+1)}(u)}{2}, \tag{8}$$

$$\tilde{B}^{(n)}(u) = J_n\left(a\sqrt{k_0^2 - u^2}\right)H_n^{(2)}\left(a\sqrt{k_0^2 - u^2}\right), \tag{9}$$

$J_\nu(\cdot)$ is the Bessel function of first kind and order ν, and $H_\nu^{(2)}(\cdot)$ is the Hankel function of second kind and order ν [44].

Now, an alternative expression for the tangential component of the n-th harmonic of the scattered electric field on the cylinder surface (useful for what will be done later) can be obtained by introducing the following vector transform pair of order n (VT_n)

$$\underline{\tilde{\tilde{\mathbf{J}}}}^{(n)}(u) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underline{\underline{\tilde{\mathbf{T}}}}^{(n)}(u)\underline{\mathbf{J}}^{(n)}(z)\,e^{jzu}dz \;\Leftrightarrow\; \underline{\mathbf{J}}^{(n)}(z) = \int_{-\infty}^{+\infty} \underline{\underline{\tilde{\mathbf{T}}}}^{(n)}(u)\underline{\tilde{\tilde{\mathbf{J}}}}^{(n)}(u)\,e^{-jzu}du, \tag{10}$$

where

$$\underline{\underline{\tilde{\mathbf{T}}}}^{(n)}(u) = \frac{1}{\sqrt{n^2 + u^2a^2}}\begin{pmatrix} -n & ua \\ ua & n \end{pmatrix} \tag{11}$$

and the following relation can be readily established

$$\underline{\tilde{\tilde{\mathbf{J}}}}^{(n)}(u) = \underline{\underline{\tilde{\mathbf{T}}}}^{(n)}(u)\,\underline{\tilde{\mathbf{J}}}^{(n)}(u) \;\Leftrightarrow\; \underline{\tilde{\mathbf{J}}}^{(n)}(u) = \underline{\underline{\tilde{\mathbf{T}}}}^{(n)}(u)\,\underline{\tilde{\tilde{\mathbf{J}}}}^{(n)}(u), \tag{12}$$

which allows us to conclude that the VT_n of a vector function exists if and only if the Fourier transform of the same function exists.

Hence, by substituting Eq. (12) in Eq. (4) it is simple to obtain

$$\underline{\mathbf{E}}^{sc(n)}(a,z) = \int_{-\infty}^{+\infty} \underline{\underline{\tilde{\mathbf{T}}}}^{(n)}(u)\,\underline{\underline{\tilde{\tilde{\mathbf{G}}}}}^{(n)}(u)\,\underline{\tilde{\tilde{\mathbf{J}}}}^{(n)}(u)\,e^{-jzu}du, \tag{13}$$

where

$$\begin{aligned}\underline{\underline{\tilde{\tilde{\mathbf{G}}}}}^{(n)}(u) &= \frac{1}{n^2 + u^2a^2}\begin{pmatrix} \tilde{\tilde{G}}_{CC}^{(n)}(u) & \tilde{\tilde{G}}_{CD}^{(n)}(u) \\ \tilde{\tilde{G}}_{DC}^{(n)}(u) & \tilde{\tilde{G}}_{DD}^{(n)}(u) \end{pmatrix} \\ &= -\frac{\pi}{2\omega\varepsilon_0 a}\begin{pmatrix} k_0^2a^2 - n^2 - u^2a^2 & 0 \\ 0 & k_0^2a^2 \end{pmatrix}\tilde{B}^{(n)}(u) - \frac{\pi\omega\mu_0 a}{2}\begin{pmatrix} n^2 & -nua \\ -nua & u^2a^2 \end{pmatrix}\frac{\tilde{A}^{(n)}(u) - \tilde{B}^{(n)}(u)}{n^2 + u^2a^2}. \end{aligned} \tag{14}$$

3. PROPOSED SOLUTION

In this section, starting from the integral equation (2), a second-kind Fredholm infinite matrix-operator equation is obtained by means of the introduction of new unknowns via Helmholtz decomposition and the discretization of the integral equation with a suitable choice of the expansion basis in a Galerkin scheme.

3.1. Change of the Unknowns

The physical behaviour of the components of the surface current density can be readily established starting from Maxwell's equations and Meixner's theory [45]

$$J_t\left(\varphi,z\right)=\begin{cases}\left[1-\left(z/b\right)^2\right]^{p_t}\bar{J}_t\left(\varphi,z\right) & |z|<b\\ 0 & |z|>b\end{cases},\quad \varphi\in[0,2\pi) \tag{15}$$

for $t\in\{\varphi,z\}$, where $p_\varphi=-1/2$, $p_z=1/2$, and it is supposed to be $\bar{J}_t(\varphi,z)\in C^{(1)}([0,2\pi)\times[-b,b])$. Hence, by means of Helmholtz decomposition, the surface current density can be written almost everywhere with the superposition of a surface curl-free contribution $\underline{J}_C(\varphi,z)=\nabla_s\phi_C(\varphi,z)$ and a surface divergence-free contribution $\underline{J}_D(\varphi,z)=-\hat{\rho}\times\nabla_s\phi_D(\varphi,z)$, which we assume as new unknowns, where $\nabla_s=\hat{\varphi}\frac{1}{a}\frac{\partial}{\partial\varphi}+\hat{z}\frac{\partial}{\partial z}$, and $\phi_C(\varphi,z)$ and $\phi_D(\varphi,z)$ are suitable potential functions [46].

In order to characterize the functional spaces to which the surface curl-free and divergence-free contributions belong, it is convenient to refer to the general cylindrical harmonic. As a matter of fact, the behaviour in Eq. (15) can be stated even for the components of the n-th harmonic of the surface current density (due to the orthogonality of cylindrical harmonics), i.e.,

$$J_t^{(n)}\left(z\right)=\begin{cases}\left[1-\left(z/b\right)^2\right]^{p_t}\bar{J}_t^{(n)}\left(z\right) & |z|<b\\ 0 & |z|>b\end{cases} \tag{16}$$

for $t\in\{\varphi,z\}$, which are simply related to the n-th harmonic of the potential functions by means of the following relations

$$J_\varphi^{(n)}\left(z\right)=j\frac{n}{a}\phi_C^{(n)}\left(z\right)+\frac{d}{dz}\phi_D^{(n)}\left(z\right), \tag{17}$$

$$J_z^{(n)}\left(z\right)=\frac{d}{dz}\phi_C^{(n)}\left(z\right)-j\frac{n}{a}\phi_D^{(n)}\left(z\right). \tag{18}$$

Let us consider the case for $n\neq 0$. If $\phi_T^{(n)}(z)$ for $T\in\{C,D\}$ is required to approach zero at infinite, from Eqs. (16), (17) and (18) it is simple to conclude that

$$\phi_T^{(n)}\left(z\right)\propto e^{-\frac{|n||z|}{a}} \tag{19}$$

for $|z|>b$. Now, from Eq. (A1), it is not difficult to understand that the following functions have the behaviour in Eq. (19)

$$\hat{\phi}_C^{(n)}\left(z\right)=-\gamma_\varphi^{(n)}\int_{-\infty}^{+\infty}J_0\left(bu\right)\frac{ne^{-juz}du}{n^2+u^2a^2}+\gamma_z^{(n)}\int_{-\infty}^{+\infty}J_1\left(bu\right)\frac{e^{-juz}du}{n^2+u^2a^2}, \tag{20}$$

$$\hat{\phi}_D^{(n)}\left(\rho\right)=\gamma_z^{(n)}\int_{-\infty}^{+\infty}\frac{J_1\left(bu\right)}{ua}\frac{ne^{-juz}du}{n^2+u^2a^2}+\gamma_\varphi^{(n)}\int_{-\infty}^{+\infty}J_0\left(bu\right)\frac{uae^{-juz}du}{n^2+u^2a^2}. \tag{21}$$

We want to verify that such functions can be considered as particular potential functions. Indeed, by substituting Eqs. (20) and (21) in Eqs. (17) and (18), and remembering Eq. (A2), it is simple to conclude that

$$j\frac{n}{a}\hat{\phi}_C^{(n)}(z)+\frac{d}{dz}\hat{\phi}_D^{(n)}(z)=-j\frac{1}{a}\gamma_\varphi^{(n)}\int_{-\infty}^{+\infty}J_0\left(bu\right)e^{-juz}du=-j\frac{1}{a}\gamma_\varphi^{(n)}\begin{cases}\frac{2}{b}\left(1-z^2/b^2\right)^{-1/2} & |z|<b\\ 0 & |z|>b\end{cases},\quad(22)$$

$$\frac{d}{dz}\hat{\phi}_C^{(n)}(z)-j\frac{n}{a}\hat{\phi}_D^{(n)}(z)=-j\frac{1}{a}\gamma_z^{(n)}\int_{-\infty}^{+\infty}\frac{J_1\left(bu\right)}{au}e^{-juz}du=-j\frac{1}{a}\gamma_z^{(n)}\begin{cases}\frac{2}{a}\left(1-z^2/b^2\right)^{1/2} & |z|<b\\ 0 & |z|>b\end{cases},\quad(23)$$

which agrees with the physical behaviour prescribed for the components of the n-th harmonic of the surface current density.

Therefore, according to Eqs. (16), (17), (18) and (19), a general expression for $\phi_T^{(n)}(\rho)$ can be

$$\phi_T^{(n)}\left(z\right)=\left(1-\delta_{n,0}\right)\hat{\phi}_T^{(n)}\left(z\right)+\begin{cases}\left(1-z^2/b^2\right)^{p_T}\bar{\phi}_T^{(n)}\left(z\right) & |z|<b\\ 0 & |z|>b\end{cases},\tag{24}$$

where $\delta_{n,m}$ is the Kronecker delta, $p_C = 3/2$, $p_D = 1/2$, and $\bar{\phi}_T^{(n)}(z) \in C^2([-b,b])$ from which the following behaviour for the surface curl-free and divergence-free contributions can be immediately deduced

$$\begin{aligned}\underline{\mathbf{J}}_C^{(n)}\left(z\right)&=\begin{pmatrix}j\frac{n}{a}\\ \frac{d}{dz}\end{pmatrix}\left\{-\gamma_\varphi^{(n)}\int_{-\infty}^{+\infty}J_0\left(bu\right)\frac{ne^{-juz}du}{n^2+u^2a^2}+\gamma_z^{(n)}\int_{-\infty}^{+\infty}J_1\left(bu\right)\frac{e^{-juz}du}{n^2+u^2a^2}\right.\\&\quad\left.+\begin{cases}\left[1-(z/b)^2\right]^{3/2}\bar{\phi}_C^{(n)}\left(z\right) & |z|<b\\ 0 & |z|>b\end{cases}\right\},\end{aligned}\tag{25}$$

$$\begin{aligned}\underline{\mathbf{J}}_D^{(n)}\left(z\right)&=\begin{pmatrix}\frac{d}{dz}\\ -j\frac{n}{a}\end{pmatrix}\left\{\gamma_z^{(n)}\int_{-\infty}^{+\infty}\frac{J_1\left(bu\right)}{ua}\frac{ne^{-juz}du}{n^2+u^2a^2}+\gamma_\varphi^{(n)}\int_{-\infty}^{+\infty}J_0\left(bu\right)\frac{uae^{-juz}du}{n^2+u^2a^2}\right.\\&\quad\left.+\begin{cases}\left[1-(z/b)^2\right]^{1/2}\bar{\phi}_D^{(n)}\left(z\right) & |z|<b\\ 0 & |z|>b\end{cases}\right\}.\end{aligned}\tag{26}$$

It is simple to understand that expressions (25) and (26) can be immediately extended to the case for $n = 0$ by means of analytical continuation.

3.2. Discretization of the Integral Equation

Weiestrass approximation theorem allows us to state that the function $\bar{\phi}_T^{(n)}(z)$, defined for $|z| \leq b$, can be represented by a uniformly convergent series of polynomials. Gegenbauer polynomials $\{C_h^{(p_T+1/2)}(z/b)\}_{h=0}^{+\infty}$ are best suited for the case at hand since they form an orthogonal basis in the weighted Hilbert space $L^2_{p_T}([-b,b])$ with inner product

$$\langle f,g\rangle=\int_{-b}^{b}\left[1-(z/b)^2\right]^{p_T}f\left(z\right)g^*\left(z\right)dz.\tag{27}$$

Hence, suitable expansion basis reconstructing the physical behaviour of the n-th harmonic of the surface curl-free and divergence-free contributions are the following

$$\underline{\mathbf{J}}_T^{(n)}\left(z\right)=\sum_{h=-2}^{+\infty}\gamma_T^{(n,h)}\underline{\mathbf{f}}_T^{(n,h)}\left(z\right)\tag{28}$$

for $T \in \{C, D\}$, where $\gamma_C^{(n,-2)} = \gamma_D^{(n,-1)} = \gamma_\varphi^{(n)}$, $\gamma_C^{(n,-1)} = \gamma_D^{(n,-2)} = \gamma_z^{(n)}$,

$$\underline{\mathbf{f}}_C^{(n,h)}(z) = \begin{pmatrix} j\dfrac{n}{a} \\ \dfrac{d}{dz} \end{pmatrix} f_C^{(n,h)}(z), \tag{29}$$

$$\underline{\mathbf{f}}_D^{(n,h)}(z) = \begin{pmatrix} \dfrac{d}{dz} \\ -j\dfrac{n}{a} \end{pmatrix} f_D^{(n,h)}(z), \tag{30}$$

and the expressions of $f_T^{(n,h)}(z)$ are in Eqs. (A4)–(A9), so that the sequence $\{\gamma_T^{(n,h)}\}_{h=0}^{+\infty}$ belongs to the space $l_{p_T}^2 = \{x_h : \sum_{h=0}^{+\infty} \frac{|x_h|^2 h!}{(h+2p_T)!} < \infty\}$. Since $\frac{h!}{\Gamma(h+2p_T)!} \overset{h\to+\infty}{\sim} \frac{1}{h^{2p_T}}$, it is simple to note that $l^2 = l_0^2 \subset l_{p_D}^2 \subset l_{p_C}^2$.

Remembering Eq. (A2), VT$_n$ of Eq. (28) can be expressed in closed form

$$\tilde{\underline{\mathbf{J}}}_C^{(n)}(u) = -j\frac{\sqrt{n^2+u^2a^2}}{a}\begin{pmatrix} \tilde{\phi}_C^{(n)}(u) \\ 0 \end{pmatrix}, \tag{31}$$

$$\tilde{\underline{\mathbf{J}}}_D^{(n)}(u) = -j\frac{\sqrt{n^2+u^2a^2}}{a}\begin{pmatrix} 0 \\ \tilde{\phi}_D^{(n)}(u) \end{pmatrix}, \tag{32}$$

where

$$\tilde{\phi}_T^{(n)}(u) = \sum_{h=-2}^{+\infty} \gamma_T^{(n,h)} \tilde{f}_T^{(n,h)}(u), \tag{33}$$

and the expressions of $\tilde{f}_T^{(n,h)}(u)$ are in Eqs. (A10)–(A14).

By projecting the EFIE in Eq. (2) onto the expansion functions in Eqs. (29) and (30), the following linear system of algebraic equations can be readily obtained

$$\begin{bmatrix} \mathbf{M}_{CC}^{(n)} & \mathbf{M}_{CD}^{(n)} \\ \mathbf{M}_{DC}^{(n)} & \mathbf{M}_{DD}^{(n)} \end{bmatrix} \begin{bmatrix} \gamma_C^{(n)} \\ \gamma_D^{(n)} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_C^{(n)} \\ \mathbf{b}_D^{(n)} \end{bmatrix}, \tag{34}$$

where the elements of the symmetric scattering matrix (due to reciprocity) are in Eqs. (A15)–(A19), while the unknown coefficients and the constant terms are, respectively,

$$\left(\gamma_T^{(n)}\right)_{-1} = \gamma_T^{(n,-1)}, \tag{35}$$

$$\left(\gamma_T^{(n)}\right)_h = \gamma_T^{(n,h)}, \tag{36}$$

$$\left(\mathbf{b}_T^{(n)}\right)_{-1} = b_T^{(n,-1)} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} \left\{\left[\underline{\mathbf{f}}_T^{(n,-1)}(z)\right]^H + \left[\underline{\mathbf{f}}_{\bar{T}}^{(n,-2)}(z)\right]^H\right\} \underline{\mathbf{E}}^{inc(n)}(a,z)\,dz, \tag{37}$$

$$\left(\mathbf{b}_T^{(n)}\right)_k = b_T^{(n,k)} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} \left[\underline{\mathbf{f}}_T^{(n,k)}(z)\right]^H \underline{\mathbf{E}}^{inc(n)}(a,z)\,dz \tag{38}$$

for $k, h \geq 0$.

3.3. Second-kind Fredholm Nature of the Infinite Matrix-Operator Equation

The general element of the scattering matrix can be written as

$$\left(\mathbf{M}_{TS}^{(n)}\right)_{k,h} = \int_{-\infty}^{+\infty} \tilde{K}_{TS}^{(n,k,h)}(u)\, J_{k+p_T+1/2}(bu)\, J_{h+p_S+1/2}(bu)\,du \tag{39}$$

for $k, h \geq -1$, where the kernels $\tilde{K}_{TS}^{(n,k,h)}(u)$ are in Eqs. (A20)–(A28). Observing that

$$\tilde{A}^{(n)}(u), \tilde{B}^{(n)}(u) \overset{|u|\to+\infty}{\sim} j\frac{1}{\pi |u| a} + O\left(\frac{1}{|u|^3}\right), \tag{40}$$

$$\tilde{A}^{(n)}(u) - \tilde{B}^{(n)}(u) \overset{|u|\to+\infty}{\sim} -j\frac{1}{2\pi |u|^3 a^3} + O\left(\frac{1}{|u|^5}\right), \tag{41}$$

the asymptotic behaviour of the kernel of the integral in Eq. (39) reported in Eqs. (A29)–(A33) can be readily established.

Formulas (A29), (A30), (A31) and the Weber-Schafheitlin discontinuous integral in Eq. (A3) allow us to conclude that

$$\int_{-\infty}^{+\infty} \tilde{K}_{CC,\infty}^{(n,k,h)}(u)\, J_{k+2}(bu)\, J_{h+2}(bu)\, du = \begin{cases} j\dfrac{1}{\omega\varepsilon_0 a^4} & h = k \\ 0 & h \neq k \end{cases} \tag{42}$$

for $k, h \geq -1$,

$$\int_{-\infty}^{+\infty} \tilde{K}_{CD,\infty}^{(n,k,-1)}(u)\, J_{k+2}(bu)\, J_0(bu)\, du = 0 \tag{43}$$

for $k \geq 0$,

$$\int_{-\infty}^{+\infty} \tilde{K}_{DD,\infty}^{(n,k,h)}(u)\, J_{k+1}(bu)\, J_{h+1}(bu)\, du = \begin{cases} -j\dfrac{\omega\mu_0}{a^2} & h = k \\ 0 & h \neq k \end{cases} \tag{44}$$

for $k, h \geq -1$ and $k + h \geq -1$, where $\tilde{K}_{TS,\infty}^{(n,k,h)}(u)$ denotes the zero-order asymptotic behaviour of the general kernel. Hence, the matrix Equation (34) can be rewritten as follows

$$\mathbf{x}^{(n)} + \mathbf{A}^{(n)}\mathbf{x}^{(n)} = \mathbf{b}^{(n)}, \tag{45}$$

where

$$\mathbf{A}^{(n)} = \begin{bmatrix} -j\omega\varepsilon_0 a^4 \breve{\mathbf{M}}_{CC}^{(n)} & -ja^3\sqrt{\dfrac{\varepsilon_0}{\mu_0}}\breve{\mathbf{M}}_{CD}^{(n)} \\ ja^3\sqrt{\dfrac{\varepsilon_0}{\mu_0}}\breve{\mathbf{M}}_{DC}^{(n)} & j\dfrac{a^2}{\omega\mu_0}\breve{\mathbf{M}}_{DD}^{(n)} \end{bmatrix}, \tag{46}$$

$$\mathbf{x}^{(n)} = \begin{bmatrix} \dfrac{1}{a^2\sqrt{\omega\varepsilon_0}}\gamma_C^{(n)} \\ \dfrac{\sqrt{\omega\mu_0}}{a}\gamma_D^{(n)} \end{bmatrix}, \tag{47}$$

$$\mathbf{b}^{(n)} = \begin{bmatrix} -ja^2\sqrt{\omega\varepsilon_0}\mathbf{b}_C^{(n)} \\ j\dfrac{a}{\sqrt{\omega\mu_0}}\mathbf{b}_D^{(n)} \end{bmatrix}, \tag{48}$$

and the elements of the matrix operator $\mathbf{A}^{(n)}$ are in Eqs. (A34)–(A39).

Now, the kernel of the general element of the matrix operator $\mathbf{A}^{(n)}$, $\tilde{\tilde{K}}_{TS}^{(n,h,k)}(u)$, decays asymptotically at least as $1/|u|^3$ (with the exception of the kernel of $(\breve{\mathbf{M}}_{DD}^{(n)})_{-1,-1}$). Therefore, observing from Eq. (A3) that

$$\int_{0}^{+\infty} \frac{J_{h+\nu}^2(\alpha t)}{t^\beta} dt \overset{h\to+\infty}{\sim} O\left(\frac{1}{h^\beta}\right) \tag{49}$$

for $\Re\{\beta\} > 0$ and $\alpha > 0$, and using the Cauchy-Bunjakovskii inequality, it is possible to state that

$$\begin{aligned}
&\left|\int_{-\infty}^{+\infty} \tilde{\tilde{K}}_{TS}^{(n,k,h)}(u)\, J_{k+p_T+1/2}(bu)\, J_{h+p_S+1/2}(bu)\, du\right|^2 \\
&\leq \int_{-\infty}^{+\infty} \left|\tilde{\tilde{K}}_{TS}^{(n,k,h)}(u)\right|^2 |u|^{5-\varepsilon} du \cdot \int_{-\infty}^{+\infty} \frac{J^2_{k+p_T+1/2}(bu)}{|u|^{(5-\varepsilon)/2}} du \cdot \int_{-\infty}^{+\infty} \frac{J^2_{h+p_S+1/2}(bu)}{|u|^{(5-\varepsilon)/2}} du \\
&= \alpha_{TS}^{(n)} \underbrace{(k+p_T+1/2)\int_{-\infty}^{+\infty} \frac{J^2_{k+p_T+1/2}(bu)}{|u|^{(5-\varepsilon)/2}} du}_{\overset{k\to+\infty}{\sim} O\left(\frac{1}{k^{(3-\varepsilon)/2}}\right)} \cdot \underbrace{(h+p_S+1/2)\int_{-\infty}^{+\infty} \frac{J^2_{h+p_S+1/2}(bu)}{|u|^{(5-\varepsilon)/2}} du}_{\overset{h\to+\infty}{\sim} O\left(\frac{1}{h^{(3-\varepsilon)/2}}\right)}
\end{aligned} \tag{50}$$

for $k + p_T + 1/2, h + p_S + 1/2 > (3 - \varepsilon)/4$, where $\alpha_{TS}^{(n)}$ is a bounded parameter and $0 < \varepsilon < 1$, from which it is immediate to establish that

$$\sum_{p,q=0}^{+\infty} \left|\left(\mathbf{A}^{(n)}\right)_{p,q}\right|^2 < \infty. \tag{51}$$

The condition in Eq. (51) is a sufficient condition for the compactness of the matrix-operator $\mathbf{A}^{(n)}$ in l^2. If $\mathbf{b}^{(n)} \in l^2$ then $\mathbf{x}^{(n)} \in l^2 \subset l^2_{p_D} \subset l^2_{p_C}$ since the scattering matrix is asymptotically diagonal, and the matrix Equation (45) is a Fredholm equation of the second kind in the space l^2 which can be solved via truncation method.

4. NUMERICAL RESULTS

In order to show the accuracy and efficiency of the presented technique, let us assume as incident field a plane wave impinging onto the scatterer surface, i.e., $\underline{E}^{inc}(\underline{r}) = \underline{E}_0 e^{-j\underline{k}\cdot\underline{r}}$ where $\underline{k} = -k_0(\sin\bar{\vartheta}\cos\bar{\varphi}\hat{x} + \sin\bar{\vartheta}\sin\bar{\varphi}\hat{y} + \cos\bar{\vartheta}\hat{z})$.

It is shown that the elements of the constant term in Eqs. (37) and (38) become

$$b_C^{(n,-1)} = -e^{-j\bar{\varphi}n} j^n \frac{J_1\left(k_0 b\cos\bar{\vartheta}\right)}{k_0 a\cos\bar{\vartheta}} J_n\left(k_0 a\sin\bar{\vartheta}\right) E_{0z}^{inc}, \tag{52}$$

$$\begin{aligned}
b_D^{(n,-1)} = &-e^{-j\bar{\varphi}n} j^n J_0\left(k_0 b\cos\bar{\vartheta}\right)\left[\frac{1}{2}J_{n-1}\left(k_0 a\sin\bar{\vartheta}\right)\left(E_{0x}^{inc} - jE_{0y}^{inc}\right)e^{j\bar{\varphi}}\right. \\
&\left.+\frac{1}{2}J_{n+1}\left(k_0 a\sin\bar{\vartheta}\right)\left(E_{0x}^{inc} + jE_{0y}^{inc}\right)e^{-j\bar{\varphi}}\right],
\end{aligned} \tag{53}$$

$$\begin{aligned}
b_C^{(n,k)} = &\, e^{-j\bar{\varphi}n} j^n \tilde{f}_C^{(n,k)}\left(-k_0\cos\bar{\vartheta}\right)\cdot\left[\frac{n}{2}J_{n-1}\left(k_0 a\sin\bar{\vartheta}\right)\left(E_{0x}^{inc} - jE_{0y}^{inc}\right)e^{j\bar{\varphi}}\right. \\
&\left.+\frac{n}{2}J_{n+1}\left(k_0 a\sin\bar{\vartheta}\right)\left(E_{0x}^{inc} + jE_{0y}^{inc}\right)e^{-j\bar{\varphi}} + k_0 a\cos\bar{\vartheta} J_n\left(k_0 a\sin\bar{\vartheta}\right)E_{0z}^{inc}\right],
\end{aligned} \tag{54}$$

$$\begin{aligned}
b_D^{(n,k)} = &\, e^{-j\bar{\varphi}n} j^n \tilde{f}_D^{(n,k)}\left(-k_0\cos\bar{\vartheta}\right)\cdot\left[\frac{k_0 a\cos\bar{\vartheta}}{2}J_{n-1}\left(k_0 a\sin\bar{\vartheta}\right)\left(E_{0x}^{inc} - jE_{0y}^{inc}\right)e^{j\bar{\varphi}}\right. \\
&\left.+\frac{k_0 a\cos\bar{\vartheta}}{2}J_{n+1}\left(k_0 a\sin\bar{\vartheta}\right)\left(E_{0x}^{inc} + jE_{0y}^{inc}\right)e^{-j\bar{\varphi}} - nJ_n\left(k_0 a\sin\bar{\vartheta}\right)E_{0z}^{inc}\right]
\end{aligned} \tag{55}$$

for $k \geq 0$.

The asymptotic expansion for large order of the Bessel functions of the first kind [44] immediately leads to the following asymptotic behaviour

$$\tilde{f}_T^{(n,h)}\left(-k_0\sin\bar{\vartheta}\right) \overset{h\to+\infty}{\sim} \frac{1}{\sqrt{2\pi}\left(-k_0 a\sin\bar{\vartheta}\right)^{p_T+1/2}}\left(-\frac{ebk_0\sin\bar{\vartheta}}{2h}\right)^h \tag{56}$$

from which it is simple to conclude that $\mathbf{b}^{(n)} \in l^2$.

The elements of the scattering matrix are single improper integrals of oscillating functions. In order to speed up the convergence of such kind of integrals, the analytical asymptotic acceleration technique detailed in [38] is adopted. Thereby, more than 100 integrals per second are evaluated by using a Matlab code implementing adaptive Gauss-Legendre quadrature routine on a PC equipped with an Intel Core 2 Quad CPU Q9550 2.83 GHz, 3.25 GB RAM, running Windows XP.

On the other hand, the symmetries of the scattering matrix allow a dramatic reduction of the number of integrals to be numerically evaluated. As a matter of fact, the number of integrals to be computed is $N(2M^2 + M - 1)$ (despite the overall number of matrix coefficients is $4(2N-1)M^2$), where $2N-1$ is the number of cylindrical harmonics considered for the surface current density and M the number of expansion functions used for each contribution (i.e., surface curl-free and divergence-free contributions) of each harmonic.

The following normalized truncation error is introduced

$$\mathrm{err}\,(M) = \sqrt{\sum_{n=-N+1}^{N-1} \left\| \mathbf{x}_{M+1}^{(n)} - \mathbf{x}_M^{(n)} \right\|^2 \Bigg/ \sum_{n=-N+1}^{N-1} \left\| \mathbf{x}_M^{(n)} \right\|^2} \tag{57}$$

where $\|\cdot\|$ is the usual Euclidean norm and $\mathbf{x}_M^{(n)}$ the vector of the expansion coefficients evaluated by using M expansion functions for each contribution of the n-th harmonic, and the number of cylindrical harmonics to be used is estimated following the same line of reasoning reported in [47].

In Figure 2, the normalized truncation error for cylinders of different radii ($a = \lambda/2, \lambda, 2\lambda$) when a

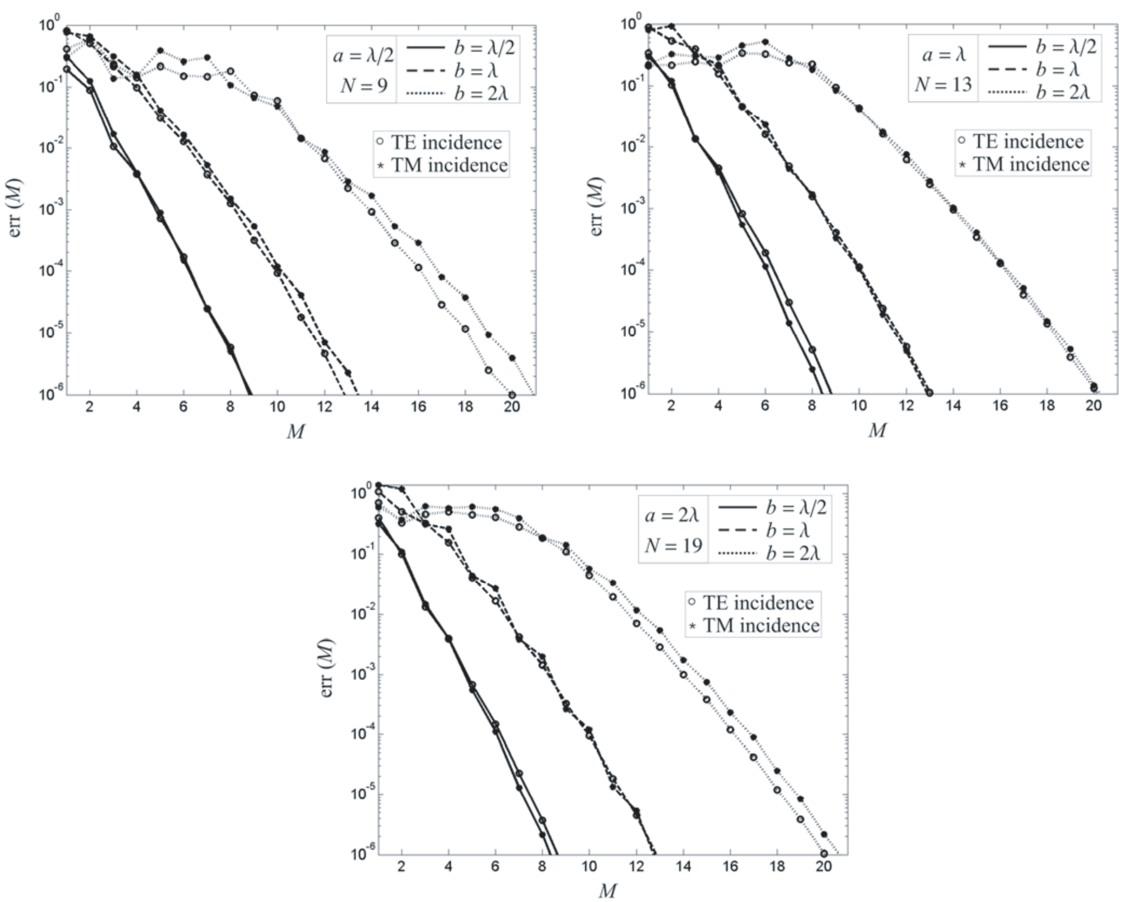

Figure 2. Normalized truncation error for cylinders of different radii and lengths, for TE and TM incidence. $\bar{\vartheta} = 45$ deg., $\bar{\varphi} = 0$ deg. and $|\underline{H}_0| = 1$ A/m.

TE ($\underline{E}_0 = E_0\hat{y}$) or a TM ($\underline{H}_0 = H_0\hat{y}$) incident plane wave (with respect to the z axis) impinges onto the scatterer surface with $\bar{\vartheta} = 45\,\text{deg.}$, $\bar{\varphi} = 0\,\text{deg.}$ and $|\underline{H}_0| = \sqrt{\varepsilon_0/\mu_0}|\underline{E}_0| = 1\,\text{A/m}$, is shown as a function of M assuming $N = 9$, 13, 19 respectively. The convergence is of exponential type in all the

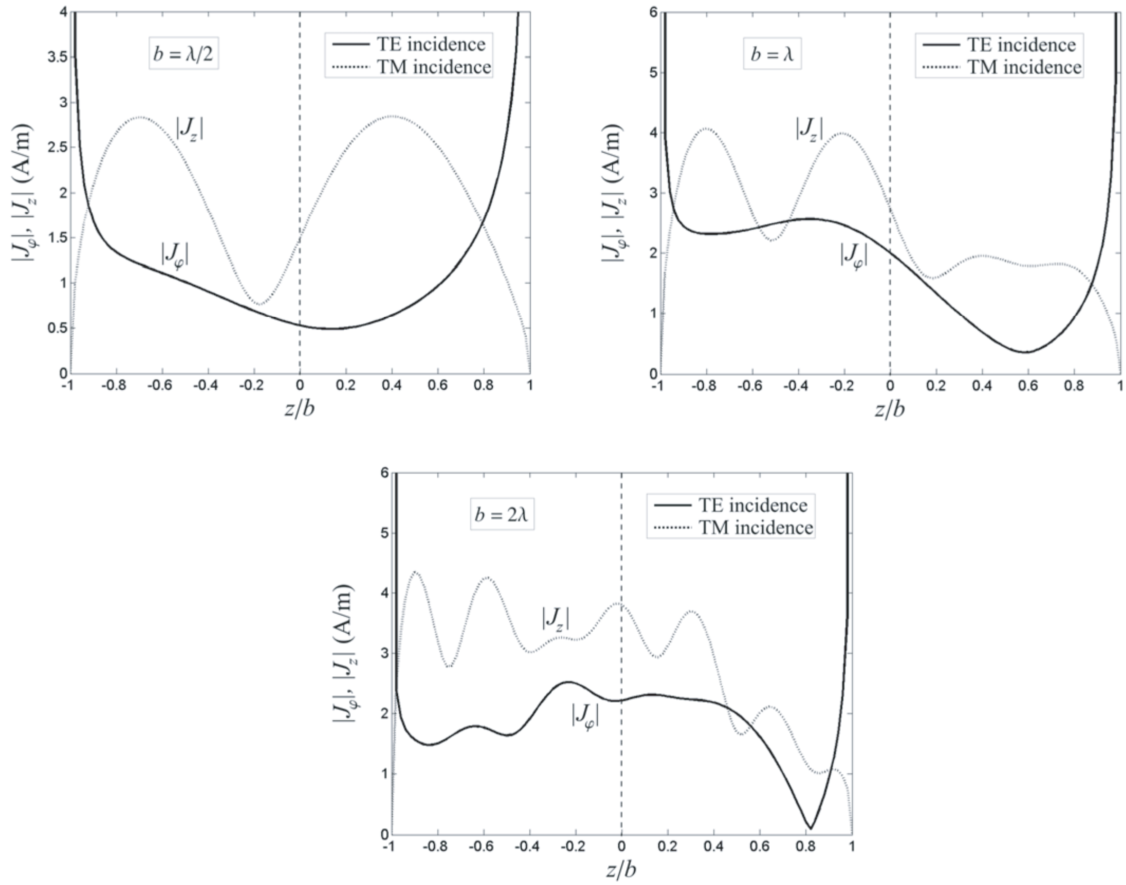

Figure 3. Components of the surface current density on circular cylinders of different lengths, for TE and TM incidence. $a = \lambda$, $\bar{\vartheta} = 45\,\text{deg.}$, $\bar{\varphi} = 0\,\text{deg.}$, $|\underline{H}_0| = 1\,\text{A/m}$ and $\varphi = 0\,\text{deg.}$.

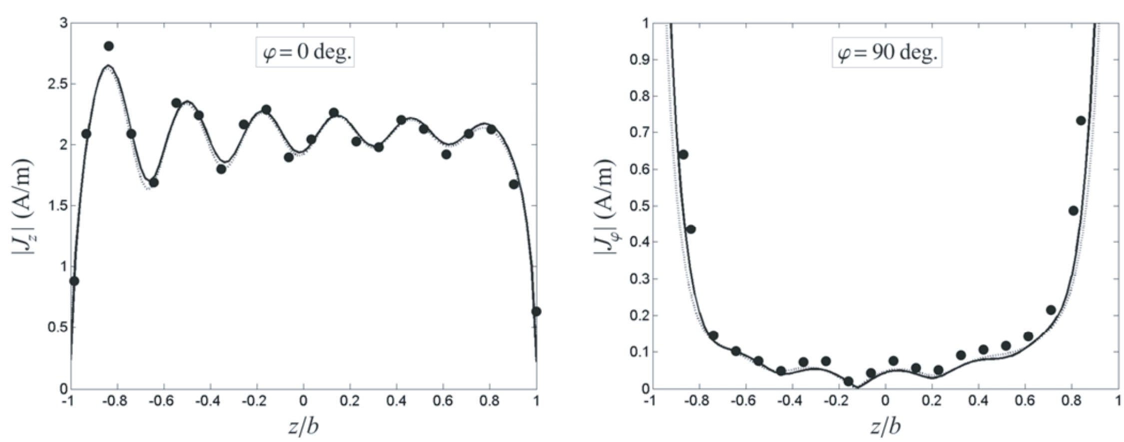

Figure 4. Components of the surface current density on a circular cylinder of finite length. Solid lines: this method; circles: data from [6]; dotted lines: CST-MWS. $k_0a = 1$, $b = 10a$, $\bar{\vartheta} = 0\,\text{deg.}$, $\underline{H}_0 = 1\hat{y}\,\text{A/m}$.

examined cases. Moreover, an error less than 10^{-3} can be achieved for $M = 5$, 9, 15 respectively, while $M = 9$, 14, 21, respectively, allow us to obtain an error even less than 10^{-6}. The computation time for the examined cases ranges from 5 secs to 3 mins. In Figure 3, just for the sake of completeness, the components of the surface current density for the case $a = \lambda$ are plotted as a function of z/b for $\varphi = 0$ deg.

Comparisons with the literature and the commercial software CST-MWS are shown. In Figure 4, the components of the surface current density on a cylinder, with $k_0 a = 1$ and $b = 10a$ for an incident plane wave with $\bar{\vartheta} = 0$ deg and $\underline{H}_0 = 1\hat{y}$ A/m, are plotted as functions of z/b and compared with the results presented in [6], obtained by means of an EFIE formulation for the surface current density discretized by means of an analytically regularizing procedure consisting in Galerkin method with expansion functions reconstructing the physical behaviour of the unknown surface current density, and the results obtained by means of CST-MWS. Assuming $N = 2$ (only the cylindrical harmonics for $n = \pm 1$ contribute to reconstruct the surface current density), $M = 16$ has to be chosen in order to achieve a normalized truncation error less than 10^{-3}. However, an error less than 10^{-6} can be achieved for $M = 20$ with a computation time of about 10 secs. As can be seen, the agreement with the literature is quite good, while the obtained results agree very well with the ones obtained by using CST-MWS. However, CST-MWS requires 4.5 million mesh-cells and a computation time of about 20 mins, while a coarser mesh would lead to unsatisfactory results.

5. CONCLUSIONS

In this paper, a new analytically regularizing method, based on Helmholtz decomposition and Galerkin method, for the analysis of the electromagnetic scattering by a hollow finite-length perfectly electrically conducting (PEC) circular cylinder is presented. As shown in the numerical results section, the presented method is very accurate and efficient in terms of computation time and storage requirement. We are working on the generalization of the method when layered media and closed PEC or dielectric cylinders are involved for which different edge behaviour for the fields have to be considered on the wedges.

APPENDIX A.

- Remarkable identities [48]:

$$\int_{-\infty}^{+\infty} \frac{J_{h+\nu}(bu)}{(ua)^{\nu}} \frac{a^2 e^{-juz} du}{n^2 + u^2 a^2} = \frac{(-j sgn(z))^h \pi a}{|n|^{\nu+1}} I_{h+\nu}\left(\frac{|n|\, b}{a}\right) e^{-\frac{|n||z|}{a}} \tag{A1}$$

for $|z| > b$, where $J_\nu(\cdot)$ is the Bessel function of first kind and order ν, sgn($\cdot$) is the signum function and $I_\nu(\cdot)$ is the modified Bessel function of first kind and order ν [44],

$$\int_{-\infty}^{+\infty} \frac{J_{h+\nu}(bu)}{(bu)^{\nu}} e^{-juz} du = \begin{cases} \dfrac{h! 2^{\nu} \Gamma(\nu)}{j^h b \Gamma(h+2\nu)} \left[1 - \left(\dfrac{z}{b}\right)^2\right]^{\nu-1/2} C_h^{(\nu)}\left(\dfrac{z}{b}\right) & |z| < b \\ 0 & |z| > b \end{cases}, \tag{A2}$$

where $\Gamma(\cdot)$ is the Gamma function and $C_h^{(\nu)}(\cdot)$ is the Gegenbauer polynomial of order h and parameter ν [44],

$$\int_{0}^{+\infty} J_\nu(\alpha t) J_\mu(\alpha t) \frac{dt}{t^\beta} = \frac{\alpha^{\beta-1} \Gamma(\beta) \Gamma\left(\dfrac{\nu+\mu-\beta+1}{2}\right)}{2^\beta \Gamma\left(\dfrac{-\nu+\mu+\beta+1}{2}\right) \Gamma\left(\dfrac{\nu+\mu+\beta+1}{2}\right) \Gamma\left(\dfrac{\nu-\mu+\beta+1}{2}\right)} \tag{A3}$$

for $\Re\{\nu + \mu + 1\} > \Re\{\beta\} > 0$ and $\alpha > 0$.

- Expansion functions in the spatial domain:

$$f_C^{(n,-2)}(z) = -\int_{-\infty}^{+\infty} J_0(bu)\frac{ne^{-juz}du}{n^2+u^2a^2}, \tag{A4}$$

$$f_C^{(n,-1)}(z) = \int_{-\infty}^{+\infty} J_1(bu)\frac{e^{-juz}du}{n^2+u^2a^2}, \tag{A5}$$

$$f_D^{(n,-2)}(z) = \int_{-\infty}^{+\infty} \frac{J_1(bu)}{ua}\frac{ne^{-juz}du}{n^2+u^2a^2}, \tag{A6}$$

$$f_D^{(n,-1)}(z) = \int_{-\infty}^{+\infty} J_0(bu)\frac{uae^{-juz}du}{n^2+u^2a^2}, \tag{A7}$$

$$f_T^{(n,h)}(z) = \begin{cases} \left[1-(z/b)^2\right]^{p_T}\xi_T^{(h)}C_h^{(p_T+1/2)}\left(\frac{z}{b}\right) & |z|\le b \\ 0 & |z|>b \end{cases}, \tag{A8}$$

$$\xi_T^{(h)} = (-j)^h\frac{2^{p_T+1/2}b^{p_T-1/2}h!\sqrt{h+p_T+1/2}}{a^{p_T+1/2}(h+2p_T)!} \tag{A9}$$

for $h \geq 0$.

- Expansion functions in the spectral domain:

$$\tilde{f}_C^{(n,-2)}(u) = -\frac{nJ_0(bu)}{n^2+u^2a^2}, \tag{A10}$$

$$\tilde{f}_C^{(n,-1)}(u) = \frac{J_1(bu)}{n^2+u^2a^2}, \tag{A11}$$

$$\tilde{f}_D^{(n,-2)}(u) = \frac{nJ_1(bu)}{ua(n^2+u^2a^2)}, \tag{A12}$$

$$\tilde{f}_D^{(n,-1)}(u) = \frac{uaJ_0(bu)}{n^2+u^2a^2}, \tag{A13}$$

$$\tilde{f}_T^{(n,h)}(u) = \sqrt{h+p_T+1/2}\frac{J_{h+p_T+1/2}(bu)}{(ua)^{p_T+1/2}} \tag{A14}$$

for $h \geq 0$.

- Elements of the scattering matrix in (34):

$$\left(\mathbf{M}_{TT}^{(n)}\right)_{-1,-1} = \left(\mathbf{M}_{TT}^{(-n)}\right)_{-1,-1} = \left(\bar{\mathbf{M}}_{TT}^{(n)}\right)_{-1,-1} + 2\left(\bar{\mathbf{M}}_{T\bar{T}}^{(n)}\right)_{-1,-2} + \left(\bar{\mathbf{M}}_{\bar{T}\bar{T}}^{(n)}\right)_{-2,-2}, \tag{A15}$$

$$\left(\mathbf{M}_{T\bar{T}}^{(n)}\right)_{-1,-1} = 0, \tag{A16}$$

$$\begin{aligned}\left(\mathbf{M}_{TS}^{(n)}\right)_{-1,h} &= \left(\mathbf{M}_{ST}^{(n)}\right)_{h,-1} = (-1)^{p_T+p_S+1}\left(\mathbf{M}_{TS}^{(-n)}\right)_{-1,h} = (-1)^{p_T+p_S+1}\left(\mathbf{M}_{ST}^{(-n)}\right)_{h,-1} \\ &= \left(\bar{\mathbf{M}}_{TS}^{(n)}\right)_{-1,h} + \left(\bar{\mathbf{M}}_{\bar{T}S}^{(n)}\right)_{-2,h},\end{aligned} \tag{A17}$$

$$\begin{aligned}\left(\mathbf{M}_{TS}^{(n)}\right)_{k,h} &= \left(\mathbf{M}_{ST}^{(n)}\right)_{h,k} = (-1)^{p_T+p_S+1}\left(\mathbf{M}_{TS}^{(-n)}\right)_{k,h} = (-1)^{p_T+p_S+1}\left(\mathbf{M}_{ST}^{(-n)}\right)_{h,k} \\ &= \left(\bar{\mathbf{M}}_{TS}^{(n)}\right)_{k,h},\end{aligned} \tag{A18}$$

for $k, h \geq 0$, $T, \bar{T}, S, \bar{S} \in \{C, D\}$, $\bar{T} \neq T$, $\bar{S} \neq S$, being

$$\left(\bar{\mathbf{M}}_{TS}^{(n)}\right)_{k,h} = \frac{1}{a^2} \int_{-\infty}^{+\infty} \tilde{f}_T^{(n,k)}(u)\, \tilde{\bar{G}}_{TS}^{(n)}(u)\, \tilde{f}_S^{(n,h)}(u)\, du. \tag{A19}$$

- Kernels of the integrals in (39):

$$\tilde{K}_{CC}^{(n,-1,-1)}(u) = -\frac{\pi\left(k_0^2 - u^2\right)\tilde{B}^{(n)}(u)}{\omega\varepsilon_0 a^3 u^2}, \tag{A20}$$

$$\tilde{K}_{CC}^{(n,-1,h)}(u) = \tilde{K}_{CC}^{(n,h,-1)}(u) = -\frac{\pi\left(k_0^2 a^2 - n^2 - a^2 u^2\right)\tilde{B}^{(n)}(u)}{\omega\varepsilon_0 a^5 u^2}, \tag{A21}$$

$$\tilde{K}_{CC}^{(n,k,h)}(u) = \tilde{K}_{CC}^{(n,h,k)}(u) = -\frac{\pi\left[\left(k_0^2 a^2 - n^2 - a^2 u^2\right)\left(n^2 + a^2 u^2\right)\tilde{B}^{(n)}(u) + n^2 k_0^2 a^2\left(\tilde{A}^{(n)}(u) - \tilde{B}^{(n)}(u)\right)\right]}{\omega\varepsilon_0 a^7 u^4}, \tag{A22}$$

$$\tilde{K}_{CD}^{(n,k,-1)}(u) = \tilde{K}_{DC}^{(n,-1,k)}(u) = \frac{\pi n\left[k_0^2 a^2 \tilde{A}^{(n)}(u) - \left(n^2 + a^2 u^2\right)\tilde{B}^{(n)}(u)\right]}{\omega\varepsilon_0 a^5 u^2}, \tag{A23}$$

$$\tilde{K}_{CD}^{(n,-1,h)}(u) = \tilde{K}_{DC}^{(n,h,-1)}(u) = -\frac{\pi n\omega\mu_0 \tilde{B}^{(n)}(u)}{a^3 u^2}, \tag{A24}$$

$$\tilde{K}_{CD}^{(n,k,h)}(u) = \tilde{K}_{DC}^{(n,h,k)}(u) = \frac{\pi n\omega\mu_0\left(\tilde{A}^{(n)}(u) - \tilde{B}^{(n)}(u)\right)}{a^3 u^2}, \tag{A25}$$

$$\tilde{K}_{DD}^{(n,-1,-1)}(u) = -\frac{\pi\left(k_0^2 a^2 \tilde{A}^{(n)}(u) - n^2 \tilde{B}^{(n)}(u)\right)}{\omega\varepsilon_0 a^3}, \tag{A26}$$

$$\tilde{K}_{DD}^{(n,-1,h)}(u) = \tilde{K}_{DD}^{(n,h,-1)}(u) = -\frac{\pi\omega\mu_0 \tilde{A}^{(n)}(u)}{a}, \tag{A27}$$

$$\tilde{K}_{DD}^{(n,k,h)}(u) = \tilde{K}_{DD}^{(n,h,k)}(u) = -\frac{\pi\left[k_0^2\left(n^2 + a^2 u^2\right)\tilde{B}^{(n)}(u) + u^2 k_0^2 a^2\left(\tilde{A}^{(n)}(u) - \tilde{B}^{(n)}(u)\right)\right]}{\omega\varepsilon_0 a^3 u^2}, \tag{A28}$$

for $k, h \geq 0$.

- Asymptotic behaviour of the kernels in (A20)–(A28)

$$\tilde{K}_{CC}^{(n,k,h)}(u) = \tilde{K}_{CC}^{(n,h,k)}(u) \overset{|u|\to+\infty}{\sim} j\frac{\sqrt{(k+2)(h+2)}}{\omega\varepsilon_0 |u| a^4}, \tag{A29}$$

$$\tilde{K}_{DD}^{(n,k,h)}(u) = \tilde{K}_{DD}^{(n,h,k)}(u) \overset{|u|\to+\infty}{\sim} j\frac{n^2\delta_{k+h,-2} - k_0^2 a^2\sqrt{(k+\delta_{k,-1}+1)(h+\delta_{h,-1}+1)}}{\omega\varepsilon_0 |u| a^4} \tag{A30}$$

for $k, h \geq -1$,

$$\tilde{K}_{CD}^{(n,k,-1)}(u) = \tilde{K}_{DC}^{(n,-1,k)}(u) \overset{|u|\to+\infty}{\sim} -j\frac{n\sqrt{k+2}}{\omega\varepsilon_0 |u| a^4}, \tag{A31}$$

$$\tilde{K}_{CD}^{(n,-1,h)}(u) = \tilde{K}_{DC}^{(n,h,-1)}(u) \overset{|u|\to+\infty}{\sim} -j\frac{n\omega\mu_0\sqrt{h+1}}{a^4 |u|^3}, \tag{A32}$$

$$\tilde{K}_{CD}^{(n,k,h)}(u) = \tilde{K}_{DC}^{(n,h,k)}(u) \overset{|u|\to+\infty}{\sim} -j\frac{n\omega\mu_0\sqrt{(k+2)(h+1)}}{2a^6 |u|^5} \tag{A33}$$

for $k, h \geq 0$.

- Elements of the matrix operator in (46):

$$\left(\breve{\mathbf{M}}_{CC}^{(n)}\right)_{k,h} = \left(\breve{\mathbf{M}}_{CC}^{(n)}\right)_{h,k} = \left(\breve{\mathbf{M}}_{CC}^{(-n)}\right)_{k,h} = \left(\breve{\mathbf{M}}_{CC}^{(-n)}\right)_{h,k} = \int_{-\infty}^{+\infty} \left[\tilde{K}_{CC}^{(n,k,h)}(u) - \tilde{K}_{CC,\infty}^{(n,k,h)}(u)\right] J_{k+2}(bu)\, J_{h+2}(bu)\, du \tag{A34}$$

for $k, h \geq -1$,

$$\left(\breve{\mathbf{M}}_{CD}^{(n)}\right)_{-1,-1} = \left(\breve{\mathbf{M}}_{DC}^{(n)}\right)_{-1,-1} = 0, \tag{A35}$$

$$\left(\breve{\mathbf{M}}_{CD}^{(n)}\right)_{k,-1} = \left(\breve{\mathbf{M}}_{DC}^{(n)}\right)_{-1,k} = -\left(\breve{\mathbf{M}}_{CD}^{(-n)}\right)_{k,-1} = -\left(\breve{\mathbf{M}}_{DC}^{(-n)}\right)_{-1,k} = \int_{-\infty}^{+\infty} \left[\tilde{K}_{CD}^{(n,k,-1)}(u) - \tilde{K}_{CD,\infty}^{(n,k,-1)}(u)\right] J_{k+2}(bu)\, J_0(bu)\, du \tag{A36}$$

for $k \geq 0$,

$$\left(\breve{\mathbf{M}}_{CD}^{(n)}\right)_{k,h} = \left(\breve{\mathbf{M}}_{DC}^{(n)}\right)_{h,k} = -\left(\breve{\mathbf{M}}_{CD}^{(-n)}\right)_{k,h} = -\left(\breve{\mathbf{M}}_{DC}^{(-n)}\right)_{h,k} = \left(\mathbf{M}_{CD}^{(n)}\right)_{k,h} \tag{A37}$$

for $k \geq -1$ and $h \geq 0$,

$$\left(\breve{\mathbf{M}}_{DD}^{(n)}\right)_{-1,-1} = \left(\breve{\mathbf{M}}_{DD}^{(-n)}\right)_{-1,-1} = \left(\mathbf{M}_{DD}^{(n)}\right)_{-1,-1} + j\frac{\omega\mu_0}{a^2}, \tag{A38}$$

$$\left(\breve{\mathbf{M}}_{DD}^{(n)}\right)_{k,h} = \left(\breve{\mathbf{M}}_{DD}^{(n)}\right)_{h,k} = \left(\breve{\mathbf{M}}_{DD}^{(-n)}\right)_{k,h} = \left(\breve{\mathbf{M}}_{DD}^{(-n)}\right)_{h,k} = \int_{-\infty}^{+\infty} \left[\tilde{K}_{DD}^{(n,k,h)}(u) - \tilde{K}_{DD,\infty}^{(n,k,h)}(u)\right] J_{k+1}(bu)\, J_{h+1}(bu)\, du \tag{A39}$$

for $k, h \geq -1$ and $k + h \geq -1$.

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