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# Analysis and Design of E-CRLH TL Characteristics with New Closed-Form Solutions

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**Abstract**—In this study, new closed-form solutions are presented for deriving inductance and capacitance elements of the extended-composite right/left-handed transmission line (E-CRLH TL) unit cell from the cutoff frequencies of right-handed (RH) and left-handed (LH) bands. The characteristics of the E-CRLH TL are investigated for unbalanced, balanced, and mixed cases. The dispersion diagram, Bloch impedance, $S$-parameters are analyzed by the TL, circuit theories and the Bloch-Floquet theorem. Lastly, the usefulness of our method has been shown in detail by designing the desired characteristics for various cases.

## 1. INTRODUCTION

In recent years, metamaterials have drawn much attention in both theoretical and experimental studies, because of the unique electromagnetic properties not available in nature. Metamaterials have been applied in a large number of microwave applications such as antennas, filters, power dividers and directional couplers. With the metamaterial transmission line approach, a composite-right/left-handed transmission line (CRLH TL) is introduced in 2006 [1]. The CRLH TL exhibits a LH band at low frequencies and a RH band at high frequencies. At the zeroth order resonance, the CRLH TL also includes an unusual characteristic, namely the resonant frequency is independent of the physical size of the structure. After the CRLH TL had been reported, a dual-composite right/left-handed transmission line (D-CRLH TL) has been proposed [2, 3]. Dispersion diagram of a D-CRLH TL shows two RH and one LH bands [4]. More recently, an extended-composite right/left-handed transmission line (E-CRLH TL) has been combined from CRLH and D-CRLH TLs to get more involved band structures [5, 6]. This TL is also known as a generalized negative refraction index transmission line (NRI TL) [7]. The E-CRLH TL has recently been found in dual-band filters and quad-band power dividers [6, 8].

Most microwave application designs with E-CRLH TL are based on the controllability of the dispersion diagram, and the impedance matching consideration of the Bloch impedance. Therefore, it is important to find appropriate L-C elements to match the requirements. Previous studies for designing E-CRLH TL have presented balanced cases and seldom mentioned unbalanced cases. Reference [9] presented a possible solution for a balanced case to achieve a desired phase at four specified frequencies. A balanced E-CRLH TL with arbitrary phase shifts at four arbitrary frequencies is reported in [10] by using the results of homogeneous E-CRLH medium. Homogeneous E-CRLH medium may be useful for idealization but not for the case of a practical E-CRLH TL lumped implementation, since the unit-cells will sometimes cascade periodically to build effectively the corresponding uniform TL structure [5].

The objective of this study is to show a novel procedure for designing the E-CRLH TL in unbalanced, balanced, and mixed cases. By solving a set of equations explicitly, one can easily design a desired dispersion diagram and control the Bloch impedance. In addition, scattering parameters of a periodic E-CRLH TL unit-cells network have been investigated carefully to show a complete view about the applicability of E-CRLH TLs.

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This paper is organized as follows. E-CRLH TL is analyzed first in Section 2, and new closed-form solutions for designing E-CRLH TL are presented. Numerical results are given to demonstrate the effectiveness of our approach in Section 3. Section 4 shows more the results at a desired phase characteristic. Conclusions are made in Section 5.

## 2. E-CRLH TL ANALYSIS

The equivalent circuit of an E-CRLH TL unit cell [5, 7] is shown in Fig. 1. In the horizontal branch, a series $L_1$-$C_1$ resonator connects in series with a parallel $L_2$-$C_2$ resonator. The vertical branch contains a parallel $L_3$-$C_3$ resonator in shunt with a series $L_4$-$C_4$ resonator. From this equivalent circuit model, the fundamental characteristics of this transmission line are straight forwardly analyzed by a standard TL and circuit theories. The impedance $Z_h$ of the horizontal branch and the admittance $Y_v$ of the vertical branch are given respectively by

$$Z_h = \frac{jL_1(\omega^2 - \omega_{Z01}^2)(\omega^2 - \omega_{Z02}^2)}{\omega(\omega^2 - \omega_{Z\infty}^2)}, \tag{1}$$

$$Y_v = \frac{jC_3(\omega^2 - \omega_{Y01}^2)(\omega^2 - \omega_{Y02}^2)}{\omega(\omega^2 - \omega_{Y\infty}^2)}, \tag{2}$$

where

$$\omega_{Z\infty}^2 = \frac{1}{L_2C_2}, \quad \omega_{Y\infty}^2 = \frac{1}{L_4C_4}, \tag{3}$$

$$\omega_{Z01}^2 = \frac{B_1 + \sqrt{B_1^2 - 4A_1}}{2}, \quad \omega_{Z02}^2 = \frac{B_1 - \sqrt{B_1^2 - 4A_1}}{2}, \tag{4}$$

$$\omega_{Y01}^2 = \frac{B_2 + \sqrt{B_2^2 - 4A_2}}{2}, \quad \omega_{Y02}^2 = \frac{B_2 - \sqrt{B_2^2 - 4A_2}}{2}, \tag{5}$$

$$A_1 = \frac{1}{L_1C_1L_2C_2}, \quad B_1 = \frac{1}{L_1C_1} + \frac{1}{L_2C_2} + \frac{1}{L_1C_2}, \tag{6}$$

$$A_2 = \frac{1}{L_3C_3L_4C_4}, \quad B_2 = \frac{1}{L_3C_3} + \frac{1}{L_4C_4} + \frac{1}{L_4C_3}. \tag{7}$$

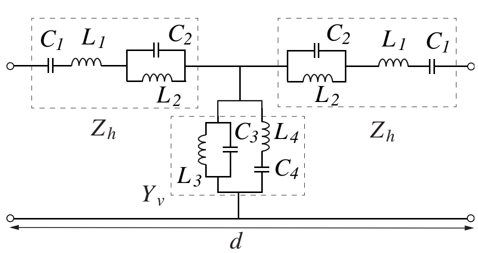


**Figure 1.** Equivalent circuit of an E-CRLH TL unit cell [5, 7].

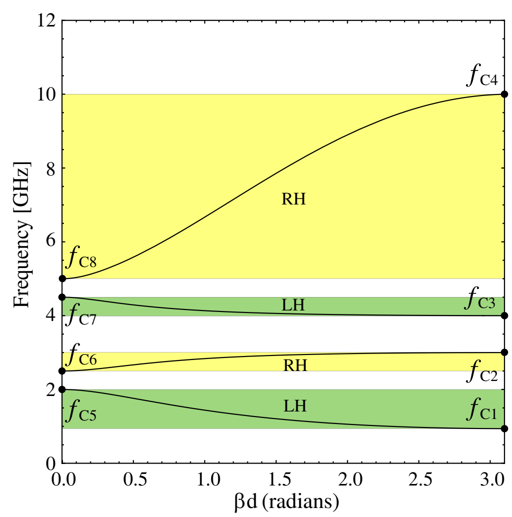


**Figure 2.** A typical dispersion diagram of E-CRLH TL in an unbalanced case.

By applying the periodic boundary conditions related with Bloch-Floquet theorem [11] to the unit cell, the dispersion relation is obtained as

$$\cos(\beta d) = 1 + Z_h Y_v, \tag{8}$$

where $\beta$ is the propagation constant for the Bloch waves and $d$ the length of the unit cell. The possible bands of the E-CRLH TL are shown by the dispersion diagram, which can be plotted from Eq. (8). An example of the dispersion diagram of the unbalanced E-CRLH TL is depicted in Fig. 2. The dispersion diagram shows two RH bands ($f_{C8}$, $f_{C4}$), ($f_{C6}$, $f_{C2}$), two LH bands ($f_{C1}$, $f_{C5}$), ($f_{C3}$, $f_{C7}$). The balanced E-CRLH TL is obtained when $f_{C5} = f_{C6}$ and $f_{C7} = f_{C8}$. The cutoff frequencies $f_{C1} \sim f_{C8}$ are calculated from the conditions: $\cos(\beta d) = \pm 1$.

The Bloch impedance is a quantity to use for the impedance matching. The Bloch impedance of the proposed metamaterial transmission line may be approximately calculated by the expression [7, 12]:

$$Z_B = \sqrt{\frac{2Z_h}{Y_v}} = \sqrt{\frac{2L_1(\omega^2 - \omega_{Z01}^2)(\omega^2 - \omega_{Z02}^2)(\omega^2 - \omega_{Y\infty}^2)}{C_3(\omega^2 - \omega_{Y01}^2)(\omega^2 - \omega_{Y02}^2)(\omega^2 - \omega_{Z\infty}^2)}}. \tag{9}$$

In this study, a different approach is proposed to determine lumped elements of the equivalent circuit. Required L-C elements of the E-CRLH TL unit cell are calculated from the cutoff frequencies $f_{C1} \sim f_{C8}$ of a desired dispersion diagram. So the proposed procedure can be used for both an unbalanced case and a balanced case.

Let us begin with a general unbalanced case. From the condition $\cos(\beta d) = 1$, Eq. (8) becomes $Z_h Y_v = 0$, then the cutoff frequencies $f_{C5} \sim f_{C8}$ are determined from $\omega_{Z01}$, $\omega_{Z02}$, $\omega_{Y01}$, $\omega_{Y02}$. Accordingly, six cases are available:

$$\omega_{Z02} \le \omega_{Y02} < \omega_{Z01} \le \omega_{Y01}, \tag{10}$$
$$\omega_{Z02} \le \omega_{Z01} < \omega_{Y02} \le \omega_{Y01}, \tag{11}$$
$$\omega_{Z02} \le \omega_{Y02} < \omega_{Y01} \le \omega_{Z01}, \tag{12}$$
$$\omega_{Y02} \le \omega_{Z02} < \omega_{Y01} \le \omega_{Z01}, \tag{13}$$
$$\omega_{Y02} \le \omega_{Y01} < \omega_{Z02} \le \omega_{Z01}, \tag{14}$$
$$\omega_{Y02} \le \omega_{Z02} < \omega_{Z01} \le \omega_{Y01}, \tag{15}$$

provided $\Delta_1 = B_1^2 - 4A_1 \ge 0$, and $\Delta_2 = B_2^2 - 4A_2 \ge 0$.

From the condition $\cos(\beta d) = -1$, the cutoff frequencies $f_{C1} \sim f_{C4}$ are calculated from:

$$Z_h Y_v + 2 = 0, \tag{16}$$

which is a fourth order equation with respect to $\omega^2$ and has four roots $(\omega_{C1}^2, \omega_{C2}^2, \omega_{C3}^2, \omega_{C4}^2)$. From the relation between these roots and the coefficients of Eq. (16), one gets:

$$\omega_{Z01}^2 \omega_{Z02}^2 \omega_{Y01}^2 \omega_{Y02}^2 = \omega_{C1}^2 \omega_{C2}^2 \omega_{C3}^2 \omega_{C4}^2, \tag{17}$$

$$\omega_{Z01}^2 + \omega_{Z02}^2 + \omega_{Y01}^2 + \omega_{Y02}^2 + \frac{2}{L_1 C_3} = \omega_{C1}^2 + \omega_{C2}^2 + \omega_{C3}^2 + \omega_{C4}^2, \tag{18}$$

$$\begin{aligned} & \omega_{Z01}^2 \omega_{Z02}^2 + \omega_{Y01}^2 \omega_{Y02}^2 + (\omega_{Z01}^2 + \omega_{Z02}^2)(\omega_{Y01}^2 + \omega_{Y02}^2) + \frac{2}{L_1 C_3}(\omega_{Z\infty}^2 + \omega_{Y\infty}^2) \\ = \ & \omega_{C1}^2 \omega_{C2}^2 + \omega_{C2}^2 \omega_{C3}^2 + \omega_{C3}^2 \omega_{C4}^2 + \omega_{C4}^2 \omega_{C1}^2 + \omega_{C4}^2 \omega_{C2}^2 + \omega_{C1}^2 \omega_{C3}^2, \end{aligned} \tag{19}$$

$$\begin{aligned} & (\omega_{Z01}^2 + \omega_{Z02}^2)\omega_{Y01}^2 \omega_{Y02}^2 + (\omega_{Y01}^2 + \omega_{Y02}^2)\omega_{Z01}^2 \omega_{Z02}^2 + \frac{2}{L_1 C_3}\omega_{Z\infty}^2 \omega_{Y\infty}^2 \\ = \ & \omega_{C1}^2 \omega_{C2}^2 \omega_{C3}^2 + \omega_{C2}^2 \omega_{C3}^2 \omega_{C4}^2 + \omega_{C3}^2 \omega_{C4}^2 \omega_{C1}^2 + \omega_{C4}^2 \omega_{C1}^2 \omega_{C2}^2. \end{aligned} \tag{20}$$

From the magnitude relations in Eqs. (10)–(15), one gets respectively, as

$$\omega_{C5} = \omega_{Z02},\ \omega_{C6} = \omega_{Y02},\ \omega_{C7} = \omega_{Z01},\ \omega_{C8} = \omega_{Y01}, \tag{21}$$
$$\omega_{C5} = \omega_{Z02},\ \omega_{C6} = \omega_{Z01},\ \omega_{C7} = \omega_{Y02},\ \omega_{C8} = \omega_{Y01}, \tag{22}$$
$$\omega_{C5} = \omega_{Z02},\ \omega_{C6} = \omega_{Y02},\ \omega_{C7} = \omega_{Y01},\ \omega_{C8} = \omega_{Z01}, \tag{23}$$
$$\omega_{C5} = \omega_{Y02},\ \omega_{C6} = \omega_{Z02},\ \omega_{C7} = \omega_{Y01},\ \omega_{C8} = \omega_{Z01}, \tag{24}$$
$$\omega_{C5} = \omega_{Y02},\ \omega_{C6} = \omega_{Y01},\ \omega_{C7} = \omega_{Z02},\ \omega_{C8} = \omega_{Z01}, \tag{25}$$
$$\omega_{C5} = \omega_{Y02},\ \omega_{C6} = \omega_{Z02},\ \omega_{C7} = \omega_{Z01},\ \omega_{C8} = \omega_{Y01}. \tag{26}$$

From Eqs. (3)–(5), (17)–(20), one gets two solutions in the case of Eq. (21) for L-C elements, if $L_1$ is given.

$$\begin{aligned}
&\text{Solution 1 :} && \text{Solution 2 :}\\
&C_1=\frac{B+\sqrt{B^2-4A}}{2x_5x_7L_1}, && C_1=\frac{B-\sqrt{B^2-4A}}{2x_5x_7L_1},\\
&C_2=\left[\left(x_5+x_7-\frac{2x_5x_7}{B+\sqrt{B^2-4A}}-\frac{B+\sqrt{B^2-4A}}{2}\right)L_1\right]^{-1}, && C_2=\left[\left(x_5+x_7-\frac{2x_5x_7}{B-\sqrt{B^2-4A}}-\frac{B-\sqrt{B^2-4A}}{2}\right)L_1\right]^{-1},\\
&L_2=\frac{2}{(B+\sqrt{B^2-4A})C_2}, && L_2=\frac{2}{(B-\sqrt{B^2-4A})C_2},\\
&C_3=\frac{2}{[(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)]L_1}, && C_3=\frac{2}{[(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)]L_1},\\
&L_3=\frac{B-\sqrt{B^2-4A}}{2x_6x_8C_3}, && L_3=\frac{B+\sqrt{B^2-4A}}{2x_6x_8C_3},\\
&L_4=\left[\left(x_6+x_8-\frac{2x_6x_8}{B-\sqrt{B^2-4A}}-\frac{B-\sqrt{B^2-4A}}{2}\right)C_3\right]^{-1}, && L_4=\left[\left(x_6+x_8-\frac{2x_6x_8}{B+\sqrt{B^2-4A}}-\frac{B+\sqrt{B^2-4A}}{2}\right)C_3\right]^{-1},\\
&C_4=\frac{2}{(B-\sqrt{B^2-4A})L_4}, && C_4=\frac{2}{(B+\sqrt{B^2-4A})L_4},
\end{aligned} \tag{27}$$

where $A$ and $B$ are calculated as follows:

$$A=\frac{D-[x_5x_7(x_6+x_8)+x_6x_8(x_5+x_7)]}{(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)}, \tag{28}$$

$$B=\frac{E-[x_5x_7+x_6x_8+(x_5+x_7)(x_6+x_8)]}{(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)}, \tag{29}$$

$$D=x_1x_2x_3+x_2x_3x_4+x_1x_2x_4+x_1x_3x_4, \tag{30}$$

$$E=x_1x_2+x_2x_3+x_3x_4+x_4x_1+x_1x_3+x_2x_4, \tag{31}$$

$$x_i=\omega_{Ci}^2=(2\pi f_{Ci})^2, \quad i=1,2,\ldots,8. \tag{32}$$

In addition, the following conditions should be satisfied:

$$\Delta=B^2-4A\geq 0, \tag{33}$$

$$L_1,C_1,L_2,C_2,L_3,C_3,L_4,C_4>0. \tag{34}$$

When $\Delta=0$, two solutions degenerate. Two solutions in the cases of Eqs. (22)–(26) can be obtained easily from two solutions in the case of Eq. (21) (see Appendix A).

In order to design an E-CRLH TL, cutoff frequencies $f_{C1}\sim f_{C8}$ and inductance $L_1$ are set as design parameters. From Eq. (17), one gets for positive cutoff frequencies

$$f_{C1}f_{C2}f_{C3}f_{C4}=f_{C5}f_{C6}f_{C7}f_{C8}. \tag{35}$$

Accordingly, the cutoff frequencies $f_{C1}\sim f_{C8}$ are not independent. Our design method can be described in the following steps:

1) Select seven of eight cutoff frequencies, the other is determined from Eq. (35).
2) By setting inductance $L_1$, other elements will be calculated from Eq. (27).
3) Test the conditions in Eqs. (33), (34).

In a balanced case, two solutions for Eqs. (23), (24) and (26) are the same as the two solutions for Eq. (21) since $x_5=x_6$ and $x_7=x_8$. Thus one may get maximum 6 solutions for a balanced case and maximum 12 solutions for an unbalanced case with fixed design parameters ($f_{C1}\sim f_{C8}$, $L_1$). The variety of the solutions gives a chance to choose suitable L-C elements. From Eqs. (9) and (27), the Bloch impedance becomes a function of $L_1$ with fixed cutoff frequencies $f_{C1}\sim f_{C8}$. So the Bloch impedance level is controlled by changing the value of $L_1$. Therefore, the desired dispersion diagram and the Bloch impedance are designed easily by using the proposed closed-form solutions. In the next section, some numerical examples will be presented for various cases.

## 3. NUMERICAL RESULTS

### 3.1. Unbalanced Case

In order to check the validity of our method, let us first start with an unbalanced case, where two sets of the short-circuited frequencies of the horizontal branch and the open-circuited frequencies of the vertical branch are different. For example, let us find a desired dispersion diagram in an unbalanced case with design frequencies: $f_{C2} = 3.000\,\text{GHz}$, $f_{C3} = 4.000\,\text{GHz}$, $f_{C4} = 10.00\,\text{GHz}$, $f_{C5} = 2.000\,\text{GHz}$, $f_{C6} = 2.500\,\text{GHz}$, $f_{C7} = 4.500\,\text{GHz}$, $f_{C8} = 5.000\,\text{GHz}$. Then from Eq. (35), one gets $f_{C1} = 0.9375\,\text{GHz}$.

By setting $L_1 = 1.50\,\text{nH}$ and applying our closed-form solutions, totally eight solutions for Eqs. (21), (23), (24), and (26) are listed in Table 1. Solutions for Eqs. (22) and (25) do not satisfy the condition in Eq. (34) because $C_2$, $L_2$, $C_4$, and $L_4$ are found to be negative. Eight solutions have exactly the same dispersion diagram characteristic in Fig. 3(a). Curves connect the desired cutoff frequencies smoothly, as expected. From the dispersion diagram of the unbalanced case, one may be able to design four pass-band (two RH and two LH bands) characteristics. Three gaps exist between these RH and LH bands. The Bloch impedances for Eqs. (21), (23), (24), and (26) are plotted in Figs. 3(b), (c), (d), and (e), respectively. These figures show three stop-bands corresponding to three gaps in the dispersion diagram at ($2.000 \sim 2.500\,\text{GHz}$), ($3.000 \sim 4.000\,\text{GHz}$), and ($4.500 \sim 5.000\,\text{GHz}$). At the high-frequency band ($> 5.000\,\text{GHz}$) or the low-frequency band ($< 2.000\,\text{GHz}$), impedance matching may be facilitated. However, the Bloch impedances change dramatically at the middle-frequency bands ($2.500 \sim 3.000\,\text{GHz}$) and ($4.000 \sim 4.500\,\text{GHz}$). In these bands, it is not easy for the impedance matching.

**Table 1.** The solutions in an unbalanced case with $f_{C1} = 0.9375$ GHz, $f_{C2} = 3.000\,\text{GHz}$, $f_{C3} = 4.000\,\text{GHz}$, $f_{C4} = 10.00\,\text{GHz}$, $f_{C5} = 2.000\,\text{GHz}$, $f_{C6} = 2.500\,\text{GHz}$, $f_{C7} = 4.500\,\text{GHz}$, $f_{C8} = 5.000\,\text{GHz}$.

| Eqs. | Sol. | $L_1$ [nH] | $C_1$ [pF] | $C_2$ [pF] | $L_2$ [nH] | $C_3$ [pF] | $L_3$ [nH] | $L_4$ [nH] | $C_4$ [pF] |
|---|---|---|---|---|---|---|---|---|---|
| (21) | 1 | 1.50 | 3.21 | 4.68 | 0.352 | 0.480 | 3.25 | 9.80 | 0.269 |
| | 2 | 1.50 | 2.00 | 2.72 | 0.969 | 0.480 | 5.20 | 9.24 | 0.178 |
| (23) | 1 | 1.50 | 2.60 | 2.37 | 0.694 | 0.480 | 4.01 | 14.2 | 0.186 |
| | 2 | 1.50 | 1.62 | 1.88 | 1.40 | 0.480 | 6.41 | 18.3 | 0.0902 |
| (24) | 1 | 1.50 | 1.66 | 2.96 | 0.557 | 0.480 | 6.26 | 8.50 | 0.310 |
| | 2 | 1.50 | 1.04 | 3.14 | 0.840 | 0.480 | 10.0 | 14.6 | 0.112 |
| (26) | 1 | 1.50 | 2.05 | 5.84 | 0.282 | 0.480 | 5.07 | 5.88 | 0.448 |
| | 2 | 1.50 | 1.28 | 4.54 | 0.581 | 0.480 | 8.12 | 7.41 | 0.222 |

While a proper method for deriving the parameters of the unbalanced E-CRLH TL is not yet reported, our closed-form solutions clearly have effectiveness for deriving this case. In addition, the ability to select flexibly the cutoff frequencies $f_{C1} \sim f_{C8}$ is beneficial for designing the resonant frequencies of a multi-band antenna, and building the pass- or stop-bands of a filter.

### 3.2. Mixed Case

Tri-pass band characteristics can also be realizable by choosing $f_{C5} = f_{C6}$ or $f_{C7} = f_{C8}$. This means that one set of the short-circuited frequencies of the horizontal branch and the open-circuited frequencies of the vertical branch are equal (balanced case) while another set of those is different (unbalanced case). This mixed case of the E-CRLH TL has not been mentioned in previous papers. A numerical example for $f_{C5} = f_{C6}$ is presented for this case. Design parameters are set to be $f_{C1} = 0.7500\,\text{GHz}$, $f_{C2} = 3.000\,\text{GHz}$, $f_{C3} = 4.000\,\text{GHz}$, $f_{C4} = 10.00\,\text{GHz}$, $f_{C5} = f_{C6} = 2.000\,\text{GHz}$, $f_{C7} = 4.500\,\text{GHz}$, $f_{C8} = 5.000\,\text{GHz}$, and $L_1 = 1.50\,\text{nH}$. Possible meaningful solutions are listed in Table 2.

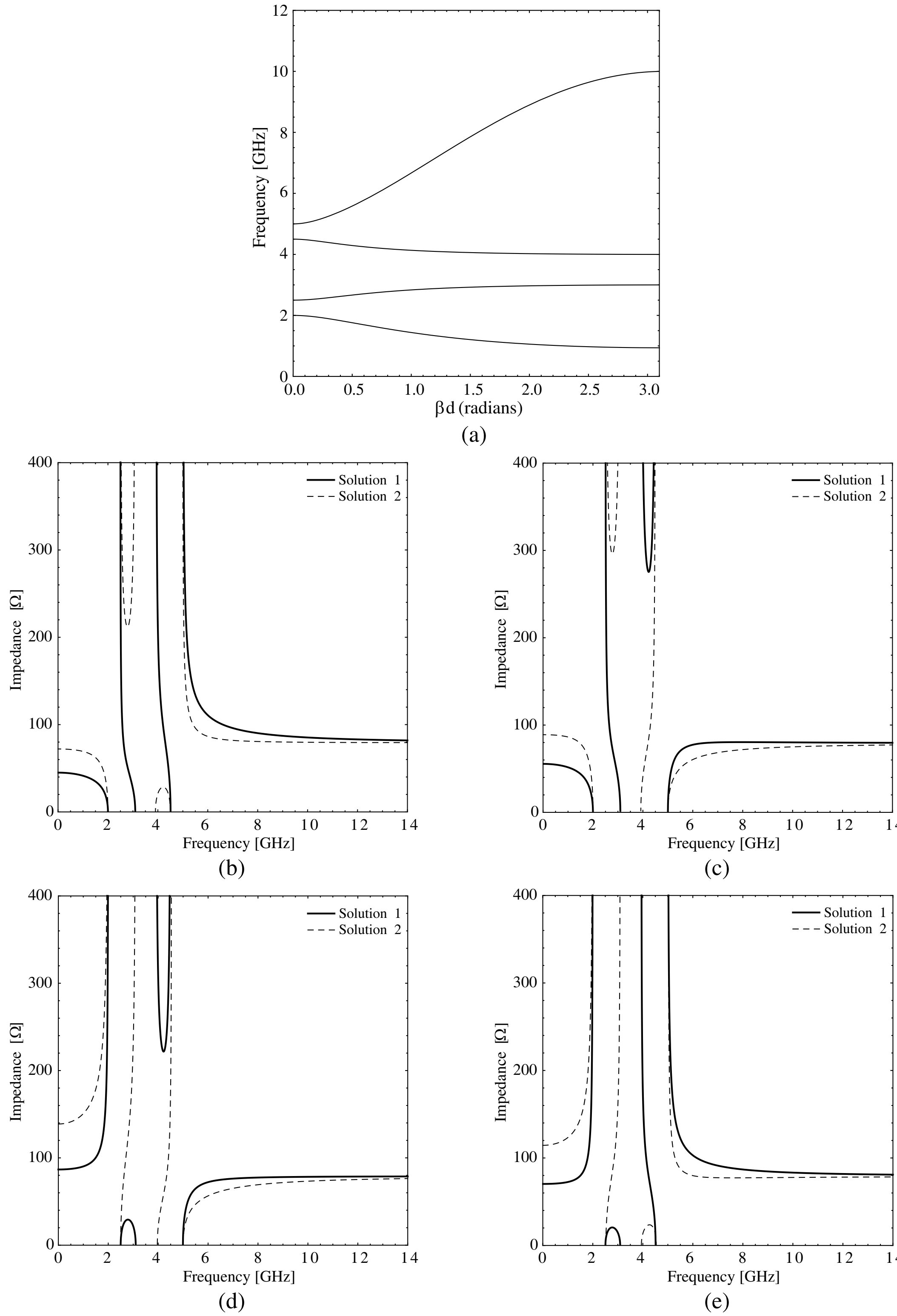


**Figure 3.** Dispersion diagram and Bloch impedances in an unbalanced case with the L-C elements in Table 1. (a) Dispersion diagram. (b) Bloch impedance for Eq. (21). (c) Bloch impedance for Eq. (23). (d) Bloch impedance for Eq. (24). (e) Bloch impedance for Eq. (26).

**Table 2.** The solutions in a mixed case with $f_{C1} = 0.7500\,\text{GHz}$, $f_{C2} = 3.000\,\text{GHz}$, $f_{C3} = 4.000\,\text{GHz}$, $f_{C4} = 10.00\,\text{GHz}$, $f_{C5} = f_{C6} = 2.000\,\text{GHz}$, $f_{C7} = 4.500\,\text{GHz}$, $f_{C8} = 5.000\,\text{GHz}$.

| Eqs. | Sol. | $L_1$ [nH] | $C_1$ [pF] | $C_2$ [pF] | $L_2$ [nH] | $C_3$ [pF] | $L_3$ [nH] | $L_4$ [nH] | $C_4$ [pF] |
|---|---|---|---|---|---|---|---|---|---|
| (21), (26) | 1 | 1.50 | 3.17 | 4.53 | 0.368 | 0.467 | 5.49 | 6.03 | 0.415 |
| | 2 | 1.50 | 2.11 | 2.76 | 0.908 | 0.467 | 8.24 | 7.51 | 0.222 |
| (23), (24) | 1 | 1.50 | 2.57 | 2.34 | 0.714 | 0.467 | 6.77 | 8.85 | 0.283 |
| | 2 | 1.50 | 1.71 | 1.88 | 1.33 | 0.467 | 10.2 | 14.6 | 0.115 |

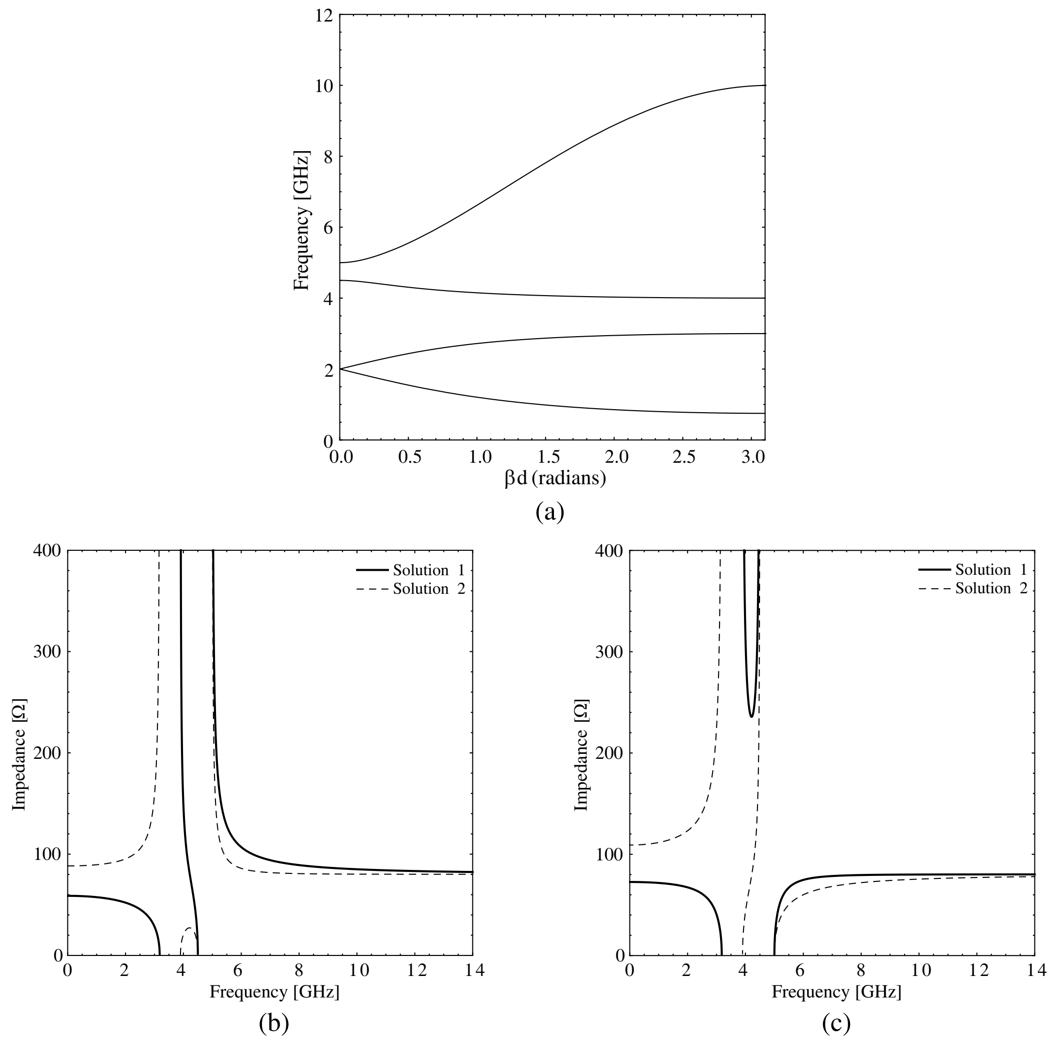


**Figure 4.** Dispersion diagram and Bloch impedances in a mixed case with the L-C elements in Table 2. (a) Dispersion diagram. (b) Bloch impedance for Eqs. (21) and (26). (c) Bloch impedance for Eqs. (23) and (24).

The dispersion diagram of these solutions is plotted in Fig. 4(a). The Bloch impedances are calculated in Fig. 4(b) for Eqs. (21) and (26), and in Fig. 4(c) for Eqs. (23) and (24). Two stop-bands at ($3.000 \sim 4.000\,\text{GHz}$) and ($4.500 \sim 5.000\,\text{GHz}$) are shown in these figures. At the middle-frequency band ($4.000 \sim 4.500\,\text{GHz}$) and the high-frequency band ($> 5.000\,\text{GHz}$), the Bloch impedances of two solutions show similar frequency behaviors with the unbalanced case in Figs. 3(b) and (c). On the other hand, the Bloch impedances exhibit a complementary behavior at the low-frequency band (< 3.000 GHz).

### 3.3. Balanced Case

For a balanced case with design parameters: $f_{C1} = 0.7500\,\text{GHz}$, $f_{C2} = 3.000\,\text{GHz}$, $f_{C3} = 4.000\,\text{GHz}$, $f_{C4} = 9.000\,\text{GHz}$, $f_{C5} = f_{C6} = 2.000\,\text{GHz}$, $f_{C7} = f_{C8} = 4.500\,\text{GHz}$, and $L_1 = 1.50\,\text{nH}$, two solutions for Eqs. (21), (23), (24), and (26) are shown in Table 3. Physically meaningful solutions also do not exist for Eqs. (22) and (25). Fig. 5 presents the dispersion diagram and Bloch impedances for this case. The dispersion diagram shows a dual-pass band characteristic at ($0.7500 \sim 3.000\,\text{GHz}$) and ($4.000 \sim 9.000\,\text{GHz}$). The Bloch impedance exhibits a complementary behavior and has one stop-band corresponding to one gap in the dispersion diagram at ($3.000 \sim 4.000\,\text{GHz}$).

**Table 3.** The solutions in a balanced case with $f_{C1} = 0.7500\,\text{GHz}$, $f_{C2} = 3.000\,\text{GHz}$, $f_{C3} = 4.000\,\text{GHz}$, $f_{C4} = 9.000\,\text{GHz}$, $f_{C5} = f_{C6} = 2.000\,\text{GHz}$, $f_{C7} = f_{C8} = 4.500\,\text{GHz}$.

| Eqs. | Sol. | $L_1$ [nH] | $C_1$ [pF] | $C_2$ [pF] | $L_2$ [nH] | $C_3$ [pF] | $L_3$ [nH] | $L_4$ [nH] | $C_4$ [pF] |
|---|---|---|---|---|---|---|---|---|---|
| (21), (23) | 1 | 1.50 | 3.24 | 4.83 | 0.338 | 0.582 | 5.34 | 7.07 | 0.361 |
| (24), (26) | 2 | 1.50 | 2.07 | 2.74 | 0.931 | 0.582 | 8.35 | 12.4 | 0.131 |

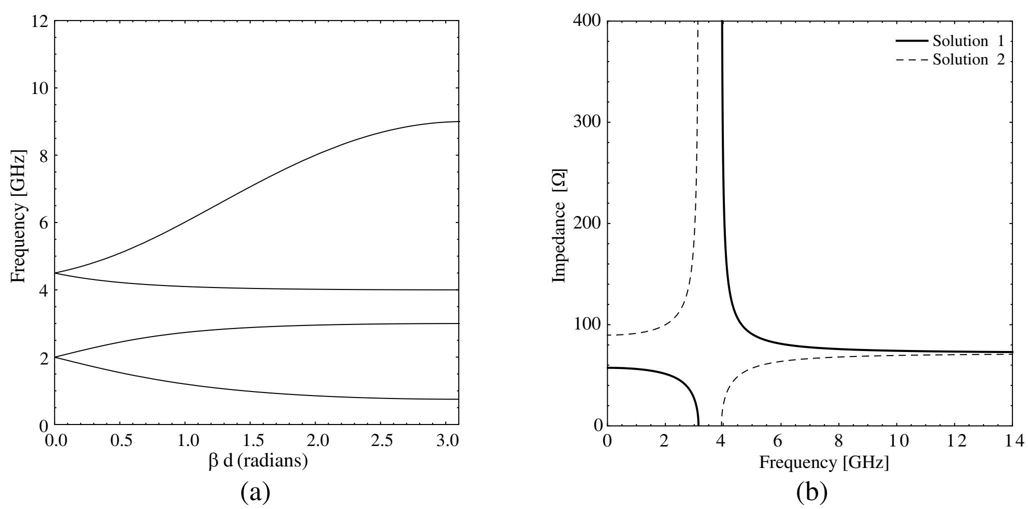


**Figure 5.** Dispersion diagram and Bloch impedance in a balanced case with the L-C elements in Table 3. (a) Dispersion diagram. (b) Bloch impedance.

### 3.4. A Special Case for a Constant Bloch Impedance

A constant Bloch impedance allows an easy broadband impedance matching with other circuits and is preferable for many practical applications. From Eq. (9), one gets a constant Bloch impedance

$$Z_B = \sqrt{2L_1/C_3}, \tag{36}$$

by setting:

$$\omega_{Z01} = \omega_{Y01}, \quad \omega_{Z02} = \omega_{Y02}, \quad \omega_{Z\infty} = \omega_{Y\infty}. \tag{37}$$

This is a special case of the balanced E-CRLH TL for Eqs. (21), (23), (24), and (26). It leads to $\Delta = 0$, and two solutions degenerate. Thus L-C elements become

$$C_1 = \frac{B}{2x_5x_7L_1}, \tag{38}$$

$$C_2 = \left[\left(x_5 + x_7 - \frac{2x_5x_7}{B} - \frac{B}{2}\right) L_1\right]^{-1}, \tag{39}$$

$$L_2 = \frac{2}{BC_2}, \tag{40}$$

$$C_3 = \frac{2}{\left[(x_1 + x_2 + x_3 + x_4) - (x_5 + x_6 + x_7 + x_8)\right] L_1}, \tag{41}$$

$$L_3 = \frac{B}{2x_6x_8C_3}, \tag{42}$$

$$L_4 = \left[\left(x_6 + x_8 - \frac{2x_6x_8}{B} - \frac{B}{2}\right) C_3\right]^{-1}, \tag{43}$$

$$C_4 = \frac{2}{BL_4}, \tag{44}$$

where

$$x_5 = x_6, \quad x_7 = x_8, \quad x_5x_7 = x_6x_8 = \sqrt{x_1x_2x_3x_4}, \quad x_5 + x_7 = x_6 + x_8 = T, \tag{45}$$

and $T$ is calculated from the following equation:

$$\begin{aligned} &T^4 - \left(12\sqrt{x_1x_2x_3x_4} + 2E\right) T^2 + \left[8\left(x_1 + x_2 + x_3 + x_4\right)\sqrt{x_1x_2x_3x_4} + 8D\right] T \\ &+ \left(E - 2\sqrt{x_1x_2x_3x_4}\right)^2 - 4D(x_1 + x_2 + x_3 + x_4) = 0. \end{aligned} \tag{46}$$

In this case, cutoff frequencies $f_{C5} \sim f_{C8}$ are calculated from $f_{C1} \sim f_{C4}$ because of the setting $\omega_{Z\infty} = \omega_{Y\infty}$. Equation (46) may have four possible roots. However, only the real and positive roots are chosen to satisfy the condition in Eq. (34). From Eqs. (36) and (41), the relation between $L_1$ and $Z_B$ is

$$L_1 = \frac{Z_B}{\sqrt{(x_1 + x_2 + x_3 + x_4) - (x_5 + x_6 + x_7 + x_8)}}. \tag{47}$$

Inductance $L_1$ may be determined from a desired $Z_B$ for an impedance matching. For example, let us design a desired dispersion diagram in a special case from design parameters: $Z_B = 50.00\,\Omega$, $f_{C1} = 0.7500\,\text{GHz}$, $f_{C2} = 3.000\,\text{GHz}$, $f_{C3} = 4.000\,\text{GHz}$, and $f_{C4} = 9.000\,\text{GHz}$. With these design parameters, one gets four roots of $T$ ($T_1 = -3.435 \times 10^{21}$, $T_2 = 4.145 \times 10^{20}$, $T_3 = 1.954 \times 10^{21}$, $T_4 = 1.066 \times 10^{21}$) from Eq. (46). Only $T = 1.066 \times 10^{21}$ satisfies the condition. Then L-C elements are calculated from Eqs. (38)–(45), and (47), and are shown in Table 4. The cutoff frequencies $f_{C5} \sim f_{C8}$ are given as $f_{C5} = f_{C6} = 1.854\,\text{GHz}$, $f_{C7} = f_{C8} = 4.854\,\text{GHz}$ from Eq. (45).

Figure 6 presents dispersion diagram and Bloch impedance of the designed E-CRLH TL in this special case. One should notice that the gap between $f_{C2}$ and $f_{C3}$ is unavoidable due to the band-stop nature of the D-CRLH TL [5].

**Table 4.** The solutions in a special case with $Z_B = 50.00\,\Omega$, $f_{C1} = 0.7500\,\text{GHz}$, $f_{C2} = 3.000\,\text{GHz}$, $f_{C3} = 4.000\,\text{GHz}$, $f_{C4} = 9.000\,\text{GHz}$.

| Eqs. | Sol. | $L_1$ [nH] | $C_1$ [pF] | $C_2$ [pF] | $L_2$ [nH] | $C_3$ [pF] | $L_3$ [nH] | $L_4$ [nH] | $C_4$ [pF] |
|---|---|---|---|---|---|---|---|---|---|
| (21), (23) (24), (26) | 1, 2 | 1.10 | 3.63 | 2.92 | 0.682 | 0.878 | 4.53 | 3.65 | 0.546 |

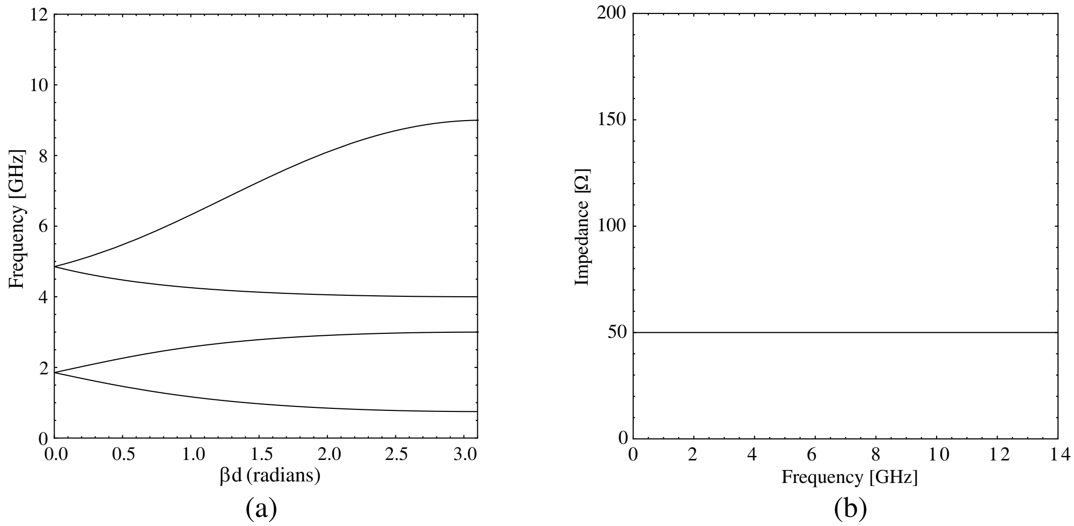


**Figure 6.** Dispersion diagram and Bloch impedance in a special case with the L-C elements in Table 4. (a) Dispersion diagram. (b) Bloch impedance.

### 3.5. Scattering Parameters

Scattering parameters of the E-CRLH TLs for various cases in the previous subsections are calculated to check the impedance matching for a network connection. A periodic E-CRLH TL unit cells network is shown in Fig. 7 with standard $50.00\,\Omega$ impedances in two ports, and $N$ is the number of unit cells. L-C elements in each cell are given from Solution 1 of Eq. (21) in Tables 1, 2, and 3 for an unbalanced case, a mixed case, and a balanced case, respectively, and in Table 4 for a special case.

An important parameter in design process is the number of unit cells. Effect of the number of unit cells is illustrated in Fig. 8 by calculating $S_{21}$ characteristic for different number $N$. A similar effect with the investigation of the CRLH TL unit cells network in [1] can be seen here. A small number of unit cells provides weak-slope edges of the bands, while the cutoffs are sharp as the number of cells is increased. The balanced and special cases have dual-band characteristics even designed with one cell. The highlight point occurs at $N = 1$ in the unbalanced and mixed cases. These cases show dual-band properties. One needs a larger number of the unit cells to accomplish the quad-band and tri-band properties. From our calculation, $N$ should be selected to be larger than 3 for better filter performance.

With $N = 10$, computed $S_{11}$ and $S_{21}$ of these designed E-CRLH TLs are presented in Fig. 9. $S_{11}$ and $S_{21}$ show the impedance matching and filtering characteristics. For practical applications, pass-bands are determined at frequencies in which $|S_{21}| = -3\,\text{dB}$. These frequencies are slightly different from the cutoff frequencies $f_{C1} \sim f_{C8}$ of RH and LH bands. For example, the pass-bands of the designed E-CRLH TL in the unbalanced case are ($0.954 \sim 1.915$ GHz), ($2.546 \sim 2.993$ GHz), ($4.001 \sim 4.517$ GHz) and ($5.361 \sim 9.884$ GHz). Therefore, the bandwidth of RH and LH bands in the dispersion diagram should be designed large enough to get desired pass-bands in $S$-parameters.

To avoid complexity, $S$-parameters of the E-CRLH TL in the unbalanced case have not been characterized [5, 9, 10]. By using our closed-form solutions, Fig. 9(a) confirms that the E-CRLH TL in

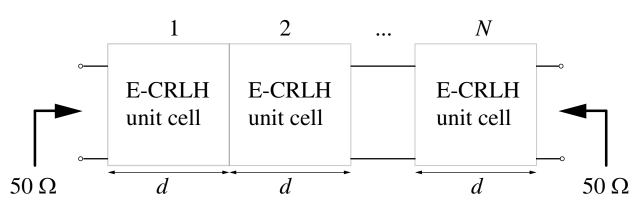


**Figure 7.** A periodic E-CRLH TL unit-cells network.

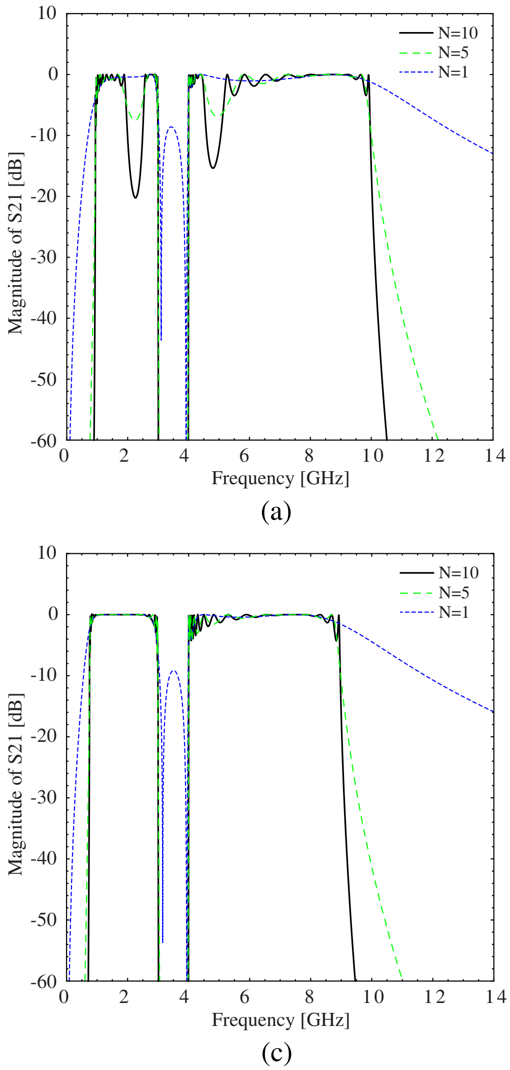


**Figure 8.** $S_{21}$ characteristic for different number $N$ of unit cells, $N = 1, 5$, and 10. (a) An unbalanced case with the L-C elements from Solution 1 of Eq. (21) in Table 1. (b) A mixed case with the L-C elements from Solution 1 of Eq. (21) in Table 2. (c) A balanced case with the L-C elements from Solution 1 of Eq. (21) in Table 3. (d) A special case with the L-C elements in Table 4.

the unbalanced case can be applied easily in quad-band applications while the mixed case is preferred for tri-band applications as can be seen from Fig. 9(b). On the other hand, this section presents the results for one solution of these case, while eight and four solutions for L-C elements are available from Tables 1 and 2. Since each solution has a distinct Bloch impedance for the impedance matching consideration, one may choose the best solution depending on the performance of $S$-parameters. Both the balanced case and special case are suitable for dual-band applications from Figs. 9(c) and (d). The main difference between the balanced case and special case are the performance of $S_{11}$ and bandwidth of RH, LH bands. With the constant Bloch impedance, the special case has better $S_{11}$ characteristics than the balanced case. However, the balanced case is easier to control the bandwidth of RH and LH bands because of a flexibility of selecting $f_{C5} \sim f_{C8}$, while the cutoff frequencies $f_{C5} \sim f_{C8}$ of the special case are calculated from $f_{C1} \sim f_{C4}$ in Eqs. (45) and (46). The bandwidth of RH and LH bands is useful for positive/negative index artificial lenses.

In order to evaluate our method for designing $S$-parameters of E-CRLH TL, one may compare it with previous results. Reference [5] already presented $S$-parameters of a balanced E-CRLH TL

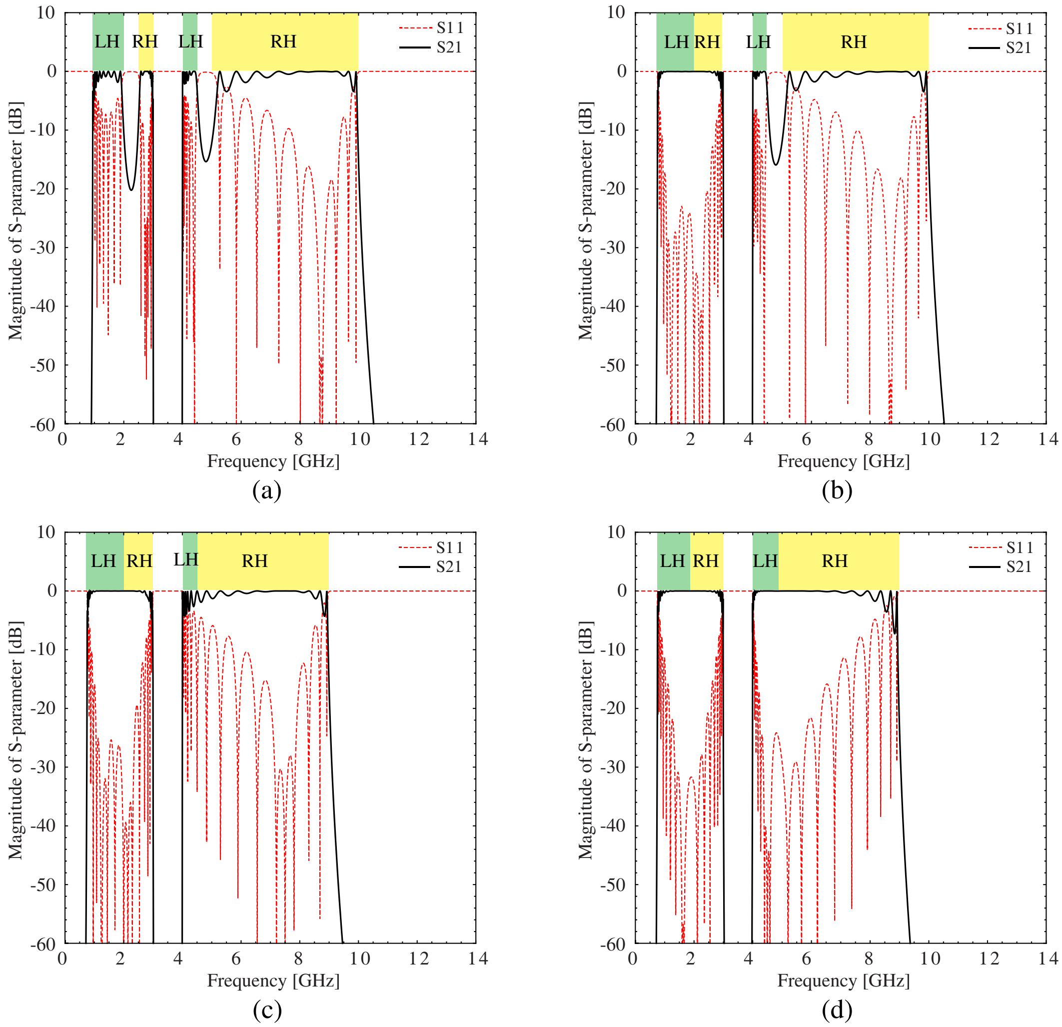


**Figure 9.** Computed $S$-parameters of E-CRLH TL for 10 cells network, $S_{11}$ (dashed line), $S_{21}$ (solid line). (a) An unbalanced case with the L-C elements from Solution 1 of Eq. (21) in Table 1. (b) A mixed case with the L-C elements from Solution 1 of Eq. (21) in Table 2. (c) A balanced case with the L-C elements from Solution 1 of Eq. (21) in Table 3. (d) A special case with the L-C elements in Table 4. Shading areas indicate RH and LH bands.

consisting of 10 unit-cells. This case is similar to a special case with a constant Bloch impedance, since the balanced case of homogeneous E-CRLH medium in [5] is associated with a constant characteristic impedance. Our closed-form solutions in the special case can be used to calculate L-C elements from cutoff frequencies $f_{C1} \sim f_{C4}$ and Bloch impedance. The design parameters for this case are $f_{C1} = 0.726\,\mathrm{GHz}$, $f_{C2} = 1.953\,\mathrm{GHz}$, $f_{C3} = 2.351\,\mathrm{GHz}$, $f_{C4} = 6.311\,\mathrm{GHz}$ (these cutoff frequencies are chosen approximately from Fig. 6 of [5]), and $Z_B = 50.00\,\Omega$. Then one gets four roots of $T$ ($T_1 = -1.558 \times 10^{21}$, $T_2 = 2.744 \times 10^{20}$, $T_3 = 4.499 \times 10^{20}$, $T_4 = 8.335 \times 10^{20}$) from Eq. (46). Only $T = 4.499 \times 10^{20}$ satisfies the condition in Eq. (34). L-C elements are calculated from Eqs. (38)–(45), and (47) as $L_1 = 1.534\,\mathrm{nH}$, $C_1 = 3.604\,\mathrm{pF}$, $C_2 = 7.428\,\mathrm{pF}$, $L_2 = 0.7426\,\mathrm{nH}$, $C_3 = 1.227\,\mathrm{pF}$, $L_3 = 4.505\,\mathrm{nH}$, $L_4 = 9.284\,\mathrm{nH}$, and $C_4 = 0.5941\,\mathrm{pF}$. The cutoff frequencies $f_{C5} \sim f_{C8}$ are given as $f_{C5} = f_{C6} = 1.522\,\mathrm{GHz}$, and $f_{C7} = f_{C8} = 3.013\,\mathrm{GHz}$ from Eq. (45). Fig. 10 shows the $S$-parameters of the designed E-CRLH TL connected 10 cells in cascade. From the calculated results, the proposed approach gives us the results comparable with those in Fig. 6 of [5]. It confirms the good quality of our method for designing $S$-parameters of E-CRLH TL circuit.

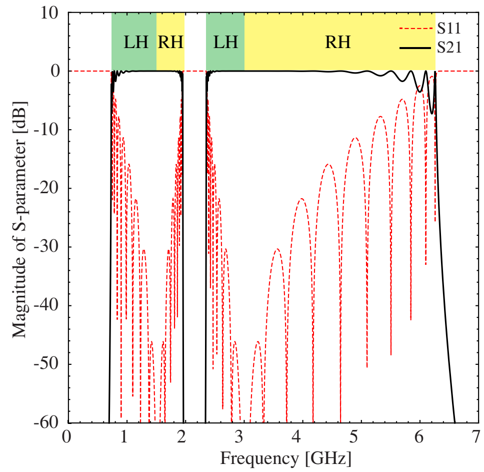


**Figure 10.** Computed $S$-parameters of E-CRLH TL for 10 cells network with L-C elements: $L_1 = 1.534\,\text{nH}$, $C_1 = 3.604\,\text{pF}$, $C_2 = 7.428\,\text{pF}$, $L_2 = 0.7426\,\text{nH}$, $C_3 = 1.227\,\text{pF}$, $L_3 = 4.505\,\text{nH}$, $L_4 = 9.284\,\text{nH}$, and $C_4 = 0.5941\,\text{pF}$.

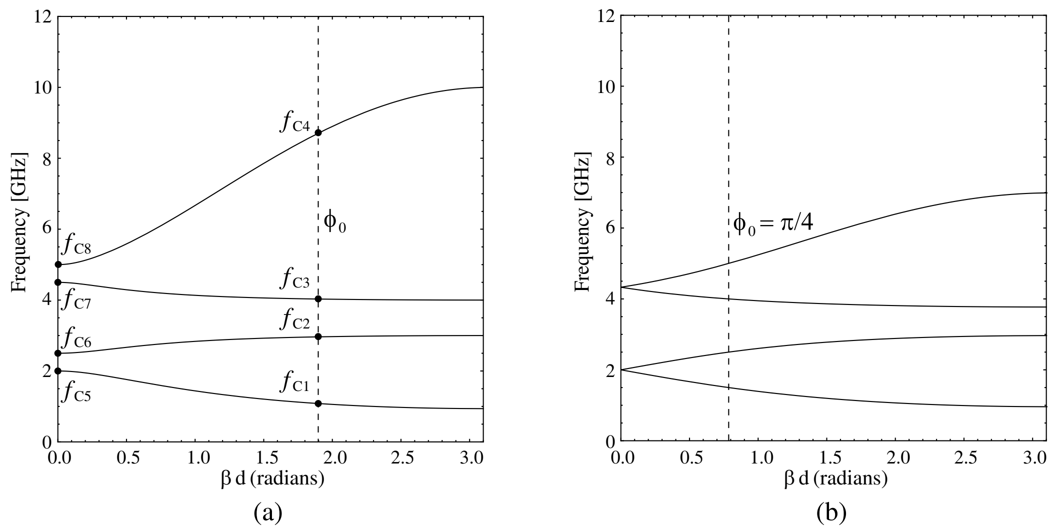


**Figure 11.** Dispersion diagrams. (a) Dispersion diagram in a general unbalanced case for a desired phase characteristic ($\phi_0 = \beta d$). (b) Dispersion diagram in a special case with the L-C elements in Table 5.

## 4. DESIRED PHASE CHARACTERISTIC DESIGN

This section presents the design of E-CRLH TL to achieve a desired phase characteristic $\phi_0 = \beta d$ ($-\phi_0$ in two LH bands, $+\phi_0$ in two RH bands) at four design frequencies $f_{C1} \sim f_{C4}$ as shown in Fig. 11(a). It is useful for phase shift devices such as directional couplers and power dividers. For this case, the design frequencies $f_{C1} \sim f_{C4}$ are determined from the equation:

$$Z_h Y_v + 1 - \cos(\phi_0) = 0. \tag{48}$$

Similarly, for Eq. (21), design equations for $C_1$, $C_2$, $L_2$, $L_3$, $L_4$, and $C_4$ are retained as shown in Eq. (27), except that $C_3$ is given by

$$C_3 = \frac{1 - \cos(\phi_0)}{\left[(x_1 + x_2 + x_3 + x_4) - (x_5 + x_6 + x_7 + x_8)\right] L_1}. \tag{49}$$

While the solutions for Eqs. (22)–(26) may be obtained easily from two solutions of Eq. (21) as the transformation given in Appendix A. For a constant Bloch impedance of the special case, design equations of $C_1$, $C_2$, $L_2$, $L_3$, $L_4$, and $C_4$ are the same as Eqs. (38)–(40), (42)–(44), and $C_3$ is calculated from Eq. (49). The relation between $L_1$ and $Z_B$ becomes

$$L_1 = Z_B \sqrt{\frac{1-\cos(\phi_0)}{2\left[(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)\right]}}. \tag{50}$$

For designing phase shift devices, it is important to achieve a desired phase characteristic at four design frequencies $f_{C1} \sim f_{C4}$ and to have good impedance matching. A numerical example for designing $\pi/4$ phase characteristic in a special case is shown in Table 5. The design procedures in the previous sections are used for obtaining these L-C elements. Fig. 11(b) shows the corresponding dispersion diagram calculated from the L-C elements in Table 5. The dispersion diagram correctly presents four designed frequencies, $f_{C1} = 1.500$ GHz, $f_{C2} = 2.500$ GHz, $f_{C3} = 4.000$ GHz, and $f_{C4} = 5.000$ GHz at the desired phase characteristics $\phi_0 = \pi/4$. Our calculated result covers the result in Fig. 2 of [9], in which the balanced case with some specified conditions is a special case in this paper. Therefore, our design equations are in a general form and can be used for many cases, while the design equations of [9] may only be used for a special case.

**Table 5.** An example of $\pi/4$ phase characteristic.

| $\phi_0 = \pi/4$ at $f_{C1} = 1.500$ GHz, $f_{C2} = 2.500$ GHz, $f_{C3} = 4.000$ GHz, $f_{C4} = 5.000$ GHz, $Z_B = 50.00\ \Omega$. $T = 8.981 \times 10^{20}$. $f_{C5} = f_{C6} = 2.000$ GHz, $f_{C7} = f_{C8} = 4.330$ GHz. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Eqs. | Sol. | $L_1$ [nH] | $C_1$ [pF] | $C_2$ [pF] | $L_2$ [nH] | $C_3$ [pF] | $L_3$ [nH] | $L_4$ [nH] | $C_4$ [pF] |
| (21), (23)<br>(23), (26) | 1, 2 | 1.52 | 2.63 | 3.65 | 0.584 | 1.22 | 3.29 | 4.56 | 0.468 |

## 5. CONCLUSION

This paper presents a new way for designing the E-CRLH TLs. All possible values of L-C elements to achieve the desired characteristics of the E-CRLH TLs are shown by using new closed-form solutions. Unlikely the previous methods, our method is helpful for various cases. The numerical results demonstrate the usefulness of the method. Thus this study contributes to the theory and applications of the E-CRLH TLs, and its results can be applied to design dual-, tri- and quad-band microwave devices.

## ACKNOWLEDGMENT

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## APPENDIX A. POSSIBLE SOLUTIONS FOR OTHER CASES

The solutions in the cases of Eqs. (22)–(26) are obtained easily from two solutions in the case of Eq. (21) by the following transformation: $x_6 \to x_7$, $x_7 \to x_6$ in the case of Eq. (22), $x_7 \to x_8$, $x_8 \to x_7$ in the case of Eq. (23), $x_5 \to x_6$, $x_6 \to x_5$, $x_7 \to x_8$, $x_8 \to x_7$ in the case of Eq. (24), $x_5 \to x_7$, $x_6 \to x_5$, $x_7 \to x_8$, $x_8 \to x_6$ in the case of Eq. (25), $x_5 \to x_6$, $x_6 \to x_5$ in the case of Eq. (26).

For example, two solutions in the case of Eq. (22) are obtained by interchanging $x_6$ and $x_7$ in Eqs. (27) $\sim$ (29) as

Solution 1 :

$$\begin{aligned}
C_1 &= \frac{B_0+\sqrt{B_0^2-4A_0}}{2x_5x_6L_1},\\
C_2 &= \left[\left(x_5+x_6-\frac{2x_5x_6}{B_0+\sqrt{B_0^2-4A_0}}-\frac{B_0+\sqrt{B_0^2-4A_0}}{2}\right)L_1\right]^{-1},\\
L_2 &= \frac{2}{(B_0+\sqrt{B_0^2-4A_0})C_2},\\
C_3 &= \frac{2}{[(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)]L_1},\\
L_3 &= \frac{B_0-\sqrt{B_0^2-4A_0}}{2x_7x_8C_3},\\
L_4 &= \left[\left(x_7+x_8-\frac{2x_7x_8}{B_0-\sqrt{B_0^2-4A_0}}-\frac{B_0-\sqrt{B_0^2-4A_0}}{2}\right)C_3\right]^{-1},\\
C_4 &= \frac{2}{(B_0-\sqrt{B_0^2-4A_0})L_4},
\end{aligned}$$

Solution 2 :

$$\begin{aligned}
C_1 &= \frac{B_0-\sqrt{B_0^2-4A_0}}{2x_5x_6L_1},\\
C_2 &= \left[\left(x_5+x_6-\frac{2x_5x_6}{B_0-\sqrt{B_0^2-4A}}-\frac{B_0-\sqrt{B_0^2-4A_0}}{2}\right)L_1\right]^{-1},\\
L_2 &= \frac{2}{(B_0-\sqrt{B_0^2-4A_0})C_2},\\
C_3 &= \frac{2}{[(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)]L_1},\\
L_3 &= \frac{B+\sqrt{B_0^2-4A_0}}{2x_7x_8C_3},\\
L_4 &= \left[\left(x_7+x_8-\frac{2x_7x_8}{B_0+\sqrt{B_0^2-4A_0}}-\frac{B_0+\sqrt{B_0^2-4A_0}}{2}\right)C_3\right]^{-1},\\
C_4 &= \frac{2}{(B_0+\sqrt{B_0^2-4A_0})L_4},
\end{aligned} \tag{A1}$$

where $A_0$ and $B_0$ are calculated as follows:

$$A_0 = \frac{D-[x_5x_6(x_7+x_8)+x_7x_8(x_5+x_6)]}{(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)}, \tag{A2}$$

$$B_0 = \frac{E-[x_5x_6+x_7x_8+(x_5+x_6)(x_7+x_8)]}{(x_1+x_2+x_3+x_4)-(x_5+x_6+x_7+x_8)}, \tag{A3}$$

and other parameters $D$, $E$, $x_i$ are the same as in Eqs. (30) $\sim$ (32).

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