Application of Quasi-TEM Surface Impedance Approach to Calculate Inductance, Resistance and Conductor Losses of Multiconductor Microstrip Line System

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Abstract—In this paper a recent new quasi-TEM surface impedance approach has been applied to fast characterization of multiconductor microstrip lines in terms of inductance and resistance matrices and conductor losses. Application examples for frequency-dependent parameters of interconnect circuits with up to five conductors (three-, four-, and five-strips) have been reported. The propagation characteristics and attenuation of multimode symmetrical multiconductor system are obtained. The effectiveness of the applied approach is confirmed by comparison of the computed numerical results with those obtained by full-wave simulators. They are found to be in good agreement.

1. INTRODUCTION

In previous works a depth treatment has been given to the simulation and modeling of single and two coupled microstrip transmission lines using quasi-TEM (Transverse ElectroMagnetic) surface impedance approach [1–3]. The host medium has been considered multilayer and all the layers are homogeneous. For the inhomogeneous case shown electromagnetic fields in the material are not TEM mode. There are bound to have longitudinal components of electric field, E, and magnetic field, H, to account for the boundary conditions between dielectric and the finite conductivity of the conductors. However for low enough frequencies, the longitudinal electromagnetic (EM) field is very small in magnitude as compared to the transverse field. Therefore the system is said to approach quasi-TEM modes and the generalized telegraphists equation can be used to describe the instantaneous voltage and current relationship in the system. For structures such as microstrip lines and striplines in printed circuit board (PCB), a valid frequency range would be frequency $<= 10\,\text{GHz}$ for quasi-TEM to be valid. This is often beyond the highest harmonics of many digital applications used today [4, 5]. Therefore, the model developed will be extended in this paper to the case of transmission line system with more than two conductor strips. Multiconductor Transmission lines are of a great importance in high frequency system design and are often the basic components in the front-end of an antenna system. Moreover, the very fast advances in integrated circuit technology have caused renewed interest in the computation of the parameters of multiconductor microstrip lines such as the propagation constant, attenuation constant or characteristic impedance. Previous attempts to solve the problem can be resumed in two main methods [4, 5]. The first one is the use of full-wave simulations. The full wave analysis allows for the determination, in addition to the TEM mode of propagation, of other higher order modes of propagation that may exist. These higher order modes generally have cut-off frequencies bellow which they are highly attenuated and for all practical proposes do not propagate. Obviously, this analysis involves a greater degree of complexity than the TEM analysis. In most cases, the full-wave simulations

are based on Finite Difference (FD) methods [4, 6], or the Integral Equation (IE) method [7–9]. These methods are generally very time-consuming and most of theme require expensive licenses. Secondly, one can resort to quasi-static formulations [5, 10–12] that make use of assumptions and approximations on the field distributions in the transmission line. These formulations make use of the hypothesis that it is possible, under certain conditions, to neglect the axial components of the fields compared to the longitudinal ones, thus obtaining an approximated characterization of the fundamental modes of the conductor microstrip line. This make the analysis significantly easier. This alternative is the most desirable if one wants to develop quick computing tools. In this case, the characteristics of the configuration are extracted from the electrostatic capacitance per-unit-length (p.u.l.) and inductance p.u.l. of the structure. In this work, a recent improved and accurate quasi-TEM technique based on the surface impedance approach for characterizing printed multiconductor transmission lines at high frequencies has been used [3]. The authors have presented a new strategy based on the surface impedance concept for the calculation of the inductance matrix p.u.l. The technique is a quasi-analytical, because it is based on a model that performs numerical integration and derivations. The quasi-analytical model makes use of the well known transmission line formalism [13–15]. In [3], this formalism made use of the spectral Green's functions that represent the solution to the integral equation of a magnetic or electric line current in the presence of multilayer dielectric media. Therefore this formalism can then be used for a wide range of printed multiconductor transmission lines having different number of conductors and dielectric stratifications such as the multiconductor microstrip line system studied in this paper. The model assumes that the conductor strips have rectangular cross section with finite thickness, are infinitely long, and a homogeneity in the transverse plane. The model accuracy has been verified for a single and two coupled microstrip lines. In this paper the accuracy and the effectiveness of the previous developed quasi-TEM surface impedance technique has been carried out through the calculation of conductor loss for three, four and five coupled conductor-strips. It is shown that the obtained numerical results compared with those obtained with full wave commercial simulators are in good agreement.

2. BRIEF REVIEW

2.1. Multiconductor Transmission Lines under Quasi-TEM Approach

The quasi-TEM approach makes use of the fact that the longitudinal component of the EM fields can be considered negligible. This involves that the transversal curl of the electric field is zero and, therefore, we can express this field in terms of the gradient of a scalar function with transversal variables. This scalar function (actually, the electric potential) satisfies Laplace's equation in the transversal plan of the structure. Therefore, we can characterize the fundamental mode that propagates through the transmission line from the study of a two-dimensional electrostatic problem. The fact of neglecting the longitudinal components of electric and magnetic fields compared to the transverse components allows for the definition, in a unique way, of the concepts of voltage and current for each conductor of the transmission line. For a multiconductor system it is possible to construct N_c dimension column vectors whose components are the values of the voltage, $\mathbf{V}$, and the current, $\mathbf{I}$, supported by each conductor. Combining the approximation made for fields with Maxwell's equations, we can relate the two vectors by the well known telegrapher's equations as done in [4]. For a multiconductor transmission line system with N_c conductor strips embedded on a lossless dielectric substrate, the telegrapher's equations(after some mathematical processing) can be written as follow:

$$\mathbf{V} \left(\frac{k_z}{\omega} \right)^2 = [\hat{\mathbf{L}}][\mathbf{C}] \, \mathbf{V} \tag{1}$$

$$\mathbf{I} \left(\frac{k_z}{\omega} \right)^2 = [\mathbf{C}][\hat{\mathbf{L}}] \, \mathbf{I} \tag{2}$$

where $[\hat{\mathbf{L}}]$ is the complex inductance p.u.l. defined as

$$[\hat{\mathbf{L}}] = [\mathbf{L}] - \frac{\mathrm{j}}{\omega}[\mathbf{R}] \tag{3}$$

$[\mathbf{C}]$, $[\mathbf{L}]$, and $[\mathbf{R}]$ are the p.u.l. capacitance, inductance, and series resistance, respectively.

A closer inspection would reveal that the results are a system of eigenvalue problems given by equations (Eqs. (1) and (2)). The resolution will lead to a $N_c \times N_c$ diagonal matrix whose elements are the eigenvalues of the propagating modes:

$$[\mathbf{k_z}] = \begin{bmatrix} k_{z1}^2 & 0 & \dots & 0 \\ 0 & k_{z2}^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & k_{zN_c}^2 \end{bmatrix} \tag{4}$$

where $k_{zi} = \beta_i - \mathrm{j}\alpha_i$, $i = 1, 2, \dots, N_c$, is the complex wave number of the i-th mode from which the imaginary part (α_i) represents the conductor loss and the real part (β_i) represents the propagation constant.

2.2. Quasi-TEM Surface Impedance

For a real conductor in an EM field, fields extend into the conductor, but decrease rapidly with distance from the surface due to the well-known skin effect. To avoid the complication of solving Maxwell's equations inside conductors, it is usual to make use of the concept of surface impedance. The surface impedance provides the boundary condition for fields outside the conductor, and accounts for the dissipation and energy stored inside the conductor. The main idea of the surface impedance approach in the study of microstrip lines with rectangular cross section, as it has been presented in [1] and [3], is to replace the thick conducting strips with a zero-thickness strips that carry an equivalent surface current density. The main advantage of this technique is to avoid the solution of a wave equation for the potential vector inside the thick strip. There are many different ways to choose and compute the surface impedance of a rectangular microstrip lines. One of those methods is the technique proposed by Marqués et al. [1] and that has been improved recently in [3]. The main advantage of this technique is that it can be combined with a quasi-TEM approach to form the so-called quasi-TEM surface impedance approach. Following the calculation steps reported by [3], the surface impedance of a single rectangular microstrip line can be obtained as:

$$Z_s = \frac{\Omega}{\sigma} \left[\frac{1+\Gamma}{1-\Gamma} \coth(\Omega t) - \frac{\Gamma}{1-\Gamma} \coth(\Omega t/2) \right]. \tag{5}$$

where $\Gamma = H_x^+ / H_x^-$ is the ratio between the tangential magnetic fields, H_x, on the upper ($^+$) and lower ($^-$) interfaces of the conducting strip, $\Omega = (1 + \mathrm{j})/\delta$, δ is the skin depth in the metal, σ is the metal conductivity, and t is the metalization thickness.

3. NUMERICAL RESULTS AND DISCUSSION

Therefore, in this work, we focus on the accurate computation of the capacitance p.u.l. and the complex inductance p.u.l. matrices of multiconductor transmission lines from which we extract the propagation and the attenuation constants. We should note that for all the studied multiconductor configurations in this work, as in [3], the accuracy and the stability of our numerical codes have been carried out, in first step, via the comparison with the results provided only by the method reported in [1]. This is due to the fact that the available commercial tools do not provide, in straightforward way, the inductance p.u.l. of the structures under study in this work. However, for the extracted propagation and attenuation constants we do have available commercial tools that allow for such comparison.

3.1. Three Conductor-Strips

Figure 1 shows the cross section for open three-strip line with the following parameters:

t: thickness of conductor-strip $= 1 \, \text{mm}$.
w: width of conductor-strip $= 3 \, \text{mm}$.

h: height of dielectric material $= 1\,\text{mm}$.

S: distance between two adjacent conductor-strips $= 2\,\text{mm}$.

ϵ_r: dielectric constant $= 2$.

σ: conductivity $= 4e7$.

As we have shown in the previous sections, to obtain the surface impedance on a conductor strip we should first calculate Γ (Eq. (5)), that is, the ratio between the values of the transversal magnetic field (H_x) up and bellow the strip. In Figure 2 we plot the variation of the transversal magnetic field ($H_x(x = 0, y)$) in terms of normalized y/h for two different microstrip line configurations. These results are used to compute the surface impedance of the different microstrip line configurations studied in this paper. Figure 3 shows the variation of the real and imaginary parts of the surface impedance, Z_s, versus the conductor thicknesses (t) in the case of microstrip line configuration in Figure 1. The results using the approach in [1] are also reported for comparison. The difference between these results and those obtained using the recent surface impedance approach increases for higher values of t.

The results of the complex inductance p.u.l matrix of the system are shown in Table 1. The results obtained using the approximation in [1] are reported for comparison. As expected from the computation of the surface impedance (Figure 3), the imaginary part of the inductance (resistance p.u.l.) obtained using the approach in [3] used in this work differs significantly from the results using the approximation as long as the frequency increases. Instead, the real parts (inductance p.u.l.) obtained by the two approaches differ very little. To verify the accuracy of our results, we plot in Figure 4(a) the behavior of the modal attenuation ($\alpha_i, i = 1, 2, 3$) due to conductor losses as a function of the frequency. Our
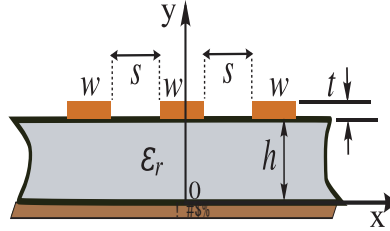

Figure 1. Cross-section of an open three-strip microstrip line.
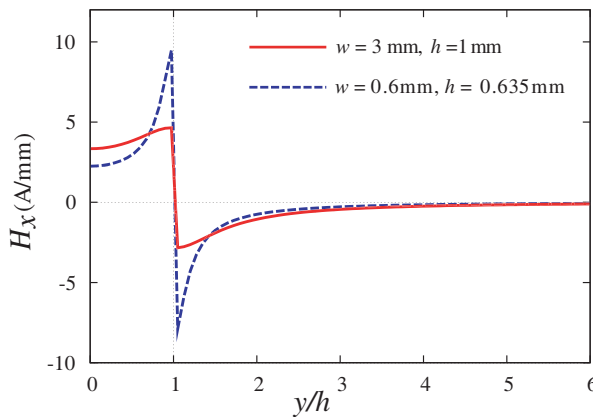

Figure 2. Transversal magnetic field, H_x, along a single conductor-strip. *Continuous line*: configuration in Figure 1 and Figure 8. *Dashed line*: configuration in Figure 5.
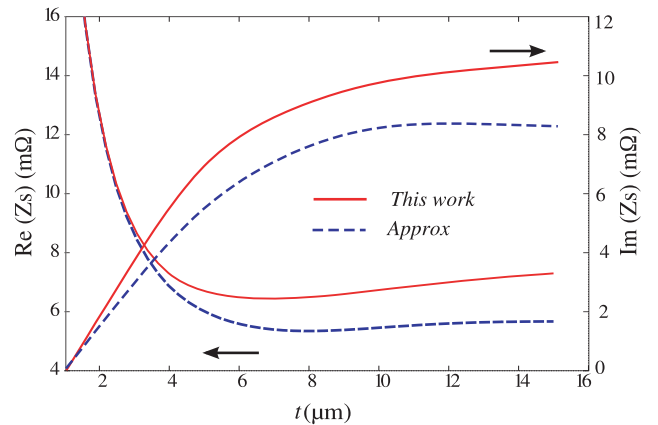
Figure 3. Real and imaginary parts of the surface impedance versus strip thickness, t, for the microstrip line configuration in Figure 1 and Figure 8. Frequency is $1\,\text{GHz}$. *This work*: using the approach developed in [3]. *Approx*: the results obtained using the approximation in [1].

Table 1. Elements of inductance and resistance p.u.l matrices of the open three-strip microstrip line system, L_{ij} (nH/m) and R_{ij} (Ω/m), respectively for weak skin effect (@ 1 GHz) and strong skin effect (@ 10 GHz).

|		Weak skin effect				Strong skin effect			
i	j	L_{ij}		R_{ij}		L_{ij}		R_{ij}	
		This work	Ref. [1]	This work	Ref. [1]	This work	Ref. [1]	This work	Ref. [1]
1	1	233.02	231.92	4.0965	3.4134	232.61	232.57	13.373	11.090
1	2	18.844	18.847	0.1187	0.1000	18.899	18.900	0.4019	0.3345
1	3	4.4690	4.4697	0.0265	0.0223	4.5249	4.5252	0.0814	0.0677
2	2	232.61	0.4202	4.1241	3.4366	232.18	232.15	13.465	11.166
2	3	18.844	18.847	0.1187	0.1000	18.899	18.900	0.4019	0.3345
3	3	233.02	231.92	4.0965	3.4134	232.61	232.57	13.373	11.090
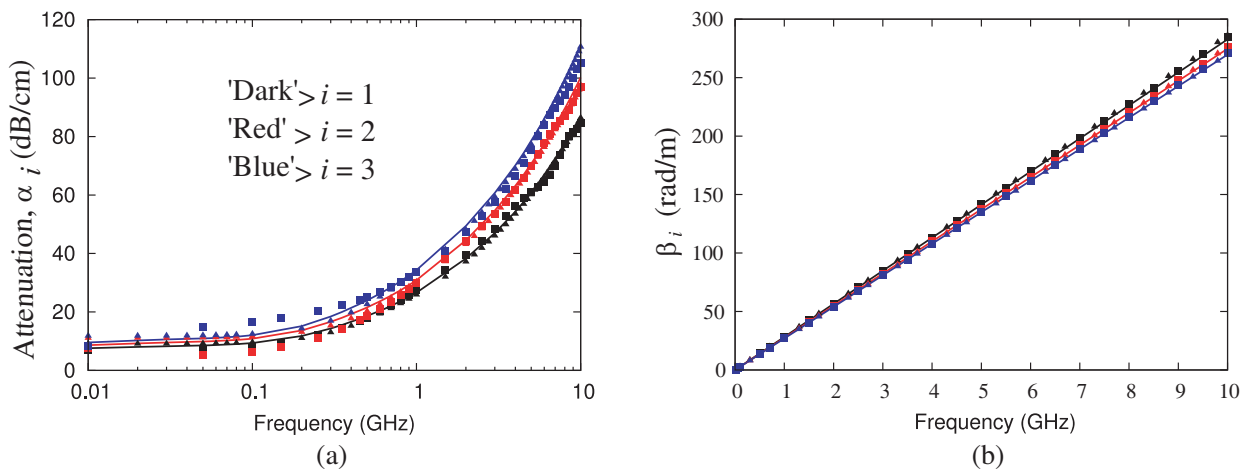
Figure 4. (a) The modal conductor losses and (b) propagation constant versus frequency for an open coupled three microstrip line. Solid: this work, Triangle: HFSS, Square: ADS.

results are compared with those provided by the finite elements solver HFSS and Agilent ADS, and both set of results are in good agreement in the considered frequency range. In Figure 4(b) we plot the modal propagation constants ($\beta_i, i = 1, 2, 3$). It is clear that our results and those obtained by HFSS and ADS are almost indistinguishable.

3.2. Four-Coupled Conductor-Strips

Figure 5 shows the cross section for open four-strip line with the following parameters:

    t: thickness of conductor-strip $= 10\,\mu$m.

    w: width of conductor-strip $= 0.6$ mm.

    h: height of dielectric material $= 0.635$ mm.

    S: distance between two adjacent conductor-strips $= 0.3$ mm.

    ϵ_r: dielectric constant $= 9.8$.

    σ: conductivity $= 5.1e7$.

This structure supports four fundamental modes. In Figure 6 we report the variation of real and imaginary parts of the surface impedance as a function of the conductor strip thickness. Here also the
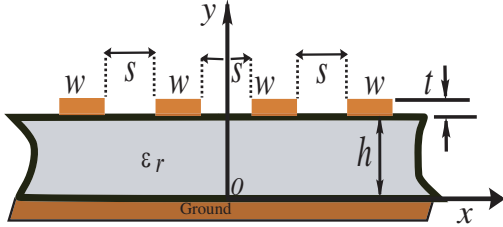
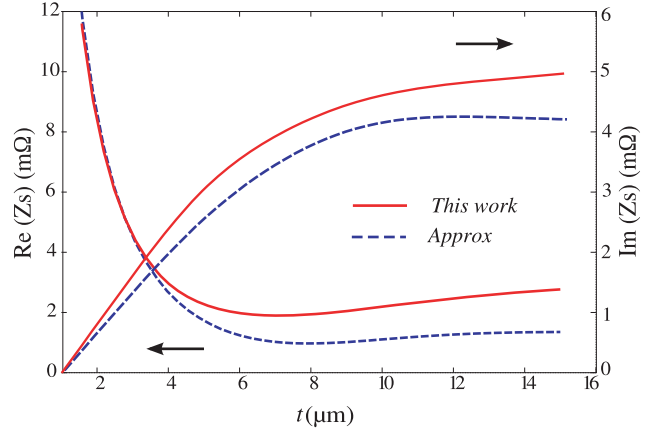**Figure 5.** Cross-section of an open four-strip microstrip line.

Figure 6. Real and imaginary parts of the surface impedance versus strip thickness, t, for the microstrip line configuration in Figure 5. Frequency is 1 GHz. *This work*: using the approach developed in [3]. *Approx*: the results obtained using the approximation in [1].
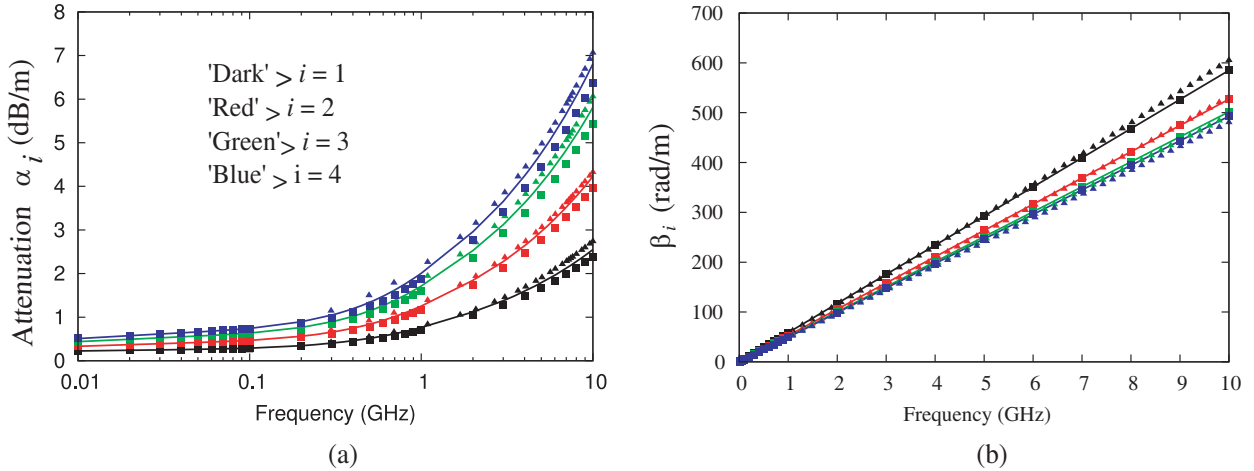

(a)   (b)

Figure 7. (a) Modal attenuation and (b) propagation constant for the open coupled four microstrip line versus frequency in Figure 5. Solid: this work, Triangle: HFSS, Square: ADS.

results obtained by the approach in [1] are reported for comparison. As in Figure 3, it can be seen that as long as the strip thickness increases more discrepancies are observed between the two techniques. The effect of this difference can be seen in the calculation of the real and imaginary parts of the complex inductance p.u.l. reported in Table 2. To show the accuracy of the approach used in this work for the case of four coupled microstrip line, we plot in Figure 7 the modal attenuation (Figure 7(a)) and propagation constant (Figure 7(b)). The good agreement of our results with those obtained by ADS and HFSS is very clear for both attenuation and propagation constant.

3.3. Five-Coupled Conductor-Strips

Figure 8 shows the cross section for open five-strip line with the following parameters:

t: thickness of conductor-strip $= 10\,\mu$m.

w: width of conductor-strip $= 3\,$mm.

Table 2. Elements of inductance and resistance p.u.l matrices of the open four-strip microstrip line system, L_{ij} (nH/m) and R_{ij} (Ω/m), respectively.

i	j	L_{ij} @ 1 GHz		R_{ij} @ 1 GHz		L_{ij} @ 10 GHz		R_{ij} @ 10 GHz	
		This work	Ref. [1]	This work	Ref. [1]	This work	Ref. [1]	This work	Ref. [1]
1	1	0.4271	0.4270	15.045	14.095	0.4253	0.4252	53.150	49.780
1	2	0.1215	0.1215	0.8592	0.8131	0.1216	0.1216	3.4055	3.2027
1	3	0.0486	0.0486	0.9573	0.9021	0.0487	0.0487	3.6319	3.4097
1	4	0.0241	0.0241	0.5741	0.5407	0.0242	0.0242	2.1812	2.0475
2	2	0.4203	0.4202	15.933	14.932	0.4184	0.4183	56.492	52.917
2	3	0.1196	0.1196	0.5854	0.5552	0.1197	0.1197	2.3750	2.2354
3	3	0.4203	0.4202	15.933	14.932	0.4184	0.4183	56.492	52.917
4	2	0.0486	0.0486	0.9573	0.9021	0.0487	0.0487	3.6319	3.4097
4	3	0.1215	0.1215	0.8592	0.8131	0.1216	0.1216	3.4055	3.2027
4	4	0.4271	0.4270	15.045	14.095	0.4253	0.4252	53.150	49.780

Table 3. Elements of inductance and resistance p.u.l matrices of the open five-strip microstrip line system, L_{ij} (nH/m) and R_{ij}(Ω/m), respectively.

i	j	L_{ij} @ 1 GHz		R_{ij} @ 1 GHz		L_{ij} @ 10 GHz		R_{ij} @ 10 GHz	
		This work	Ref. [1]	This work	Ref. [1]	This work	Ref. [1]	This work	Ref. [1]
1	1	233.02	232.94	4.0138	3.5227	232.61	232.58	13.262	11.708
1	2	18.842	18.844	0.1163	0.1028	18.896	18.897	0.3981	0.3523
1	3	4.4547	4.455	0.0253	0.0224	4.5086	4.5088	0.0774	0.0684
1	4	1.8982	1.8984	0.0071	0.0063	1.9033	1.9034	0.0304	0.0269
1	5	1.2149	1.2161	0.0504	0.0504	1.0672	1.0673	0.0252	0.0223
2	2	232.60	232.52	4.0393	3.5452	232.17	232.15	13.354	11.790
2	3	18.780	18.783	0.1126	0.0996	18.835	18.835	0.3859	0.3416
2	4	4.4441	4.4446	0.0245	0.0217	4.4959	4.4960	0.0746	0.0659
3	3	232.59	232.51	4.040	3.5489	232.17	232.14	13.356	11.792
4	3	18.780	18.783	0.1126	0.0996	18.835	18.835	0.3859	0.3416
5	3	4.4547	4.455	0.0253	0.0224	4.5086	4.5088	0.0774	0.0684

h: height of dielectric material $= 1$ mm.

S: distance between two adjacent conductor-strips $= 2$ mm.

ϵ_r: dielectric constant $= 2$.

σ: conductivity $=$ 4e7.

The five coupled conductor-strip configuration supports the propagation of five fundamental modes. The elements of the complex inductance p.u.l. are shown in Table 3 where the results obtained by the approximation in [1] are reported for comparison. Both weak skin effect (@ 1 GHz) and strong skin effect (@ 10 GHz) are reported. We should note that for all the tables giving the elements of inductance p.u.l. and resistance p.u.l. matrices we have not reported all the elements. This is due to the structure symmetry (all the conductor strips have the same properties and are equally spaced). The other matrices elements in tables Table 1, Table 2, and Table 3 can be obtained in straightforward way from the reported ones. This makes the numerical calculations fast and reduces considerably the
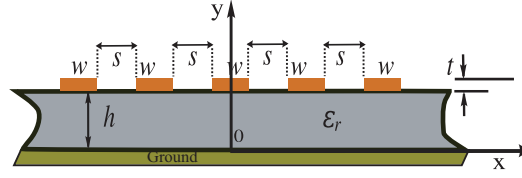
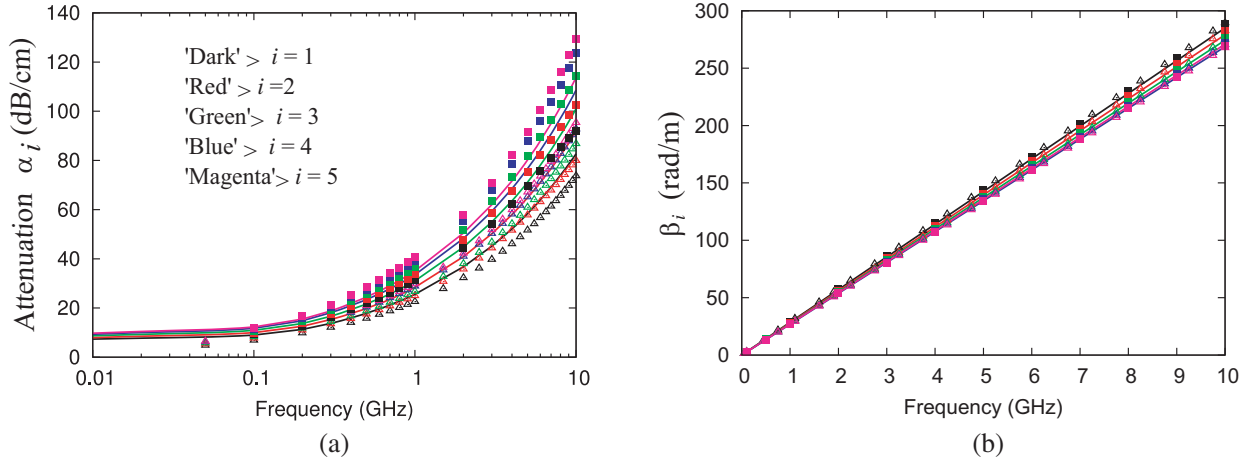**Figure 8.** Cross-section of an open five-strip microstrip line.

Figure 9. Conductor losses and propagation constant for an open coupled five microstrip line versus frequency. Solid: this work, Triangle: HFSS, Square: ADS.

computing time. Finally, we plot in Figure 9 the modal attenuation $(\alpha_i, i = 1, \ldots, 5)$ and propagation constant $(\beta_i, i = 1, \ldots, 5)$ for the five coupled multiconductor microstrip line. The figure shows that our results for the attenuation are very close to those provided by the full-wave simulator HFSS. Moreover, it can be seen that the difference between our results (and also HFSS results) compared to those provided by ADS increases for the higher frequencies as shown in Figure 9(a). However, our results for the modal propagation constant (Figure 9(b)) and those obtained using HFSS and ADS are in good agreement.

4. CONCLUSION

In this paper, we have presented the numerical modeling for three-, four-, and five-strip open microstrip lines embedded in a lossless dielectric substrate. The results obtained using the quasi-TEM surface impedance technique for the calculation of inductance per unit length, resistance per unit length and attenuation due to conductor-loss agree well with those found by the commercial simulators. This shows once more, despite its simplicity, the accuracy of the present numerical quasi-TEM surface impedance model to deal with large variety of multiconductor systems.

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