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# Study on Attitude Control Method for Zero-Doppler Steering in Space Borne SAR System

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**Abstract**—For the spaceborne synthetic aperture radar (SAR) system, in order to alleviate the complexity of the imaging algorithm and to improve the accuracy of the applications of SAR images, attitude steering is required to reduce the Doppler centroid to 0 Hz. In published literature, two-dimensional attitude steering, including yaw and pitch steering, is employed for elliptic orbiting SAR systems. This paper proposes a new steering approach involving only yaw steering to suppress the Doppler centroid of the mid-range to theoretically 0 Hz with a low residual Doppler centroid at the edge of the range extent. This may reduce the complexity of the attitude control system. The comparison of the performances of the current applied methods and the proposed approach is carried out with a simulation, and the effectiveness of the new approach is validated by the results.

## 1. INTRODUCTION

In spaceborne synthetic aperture radar (SAR) systems, the Doppler centroid is not zero in the conventional broadside mode with zero beam attitudes due to the earth rotation and eccentricity of the orbit [1, 2]. The large Doppler centroid may result in serious coupling of range and azimuth variables and increasing the difficulty of focusing. This may also result in degradation in image registration accuracy, interferometry accuracy, scalloping correction performance for ScanSAR processing and other factors [2, 3]. In order to suppress the Doppler centroid to 0 Hz, attitude steering method is applied to point the antenna beam centerline in the direction of Doppler zero-line. The effectiveness of the attitude steering methods has been validated in advanced space borne SAR systems including TerraSAR-X [4, 5], to name a few.

Several attitude steering policies can be found in current literature. Ref. [1] proposes a yaw steering method which works perfectly for circular orbiting space borne SAR systems, but this method does not work well in elliptic orbit cases, which will generate a Doppler frequency at the scale of hundreds of Hz. Refs. [6, 7] and [8] proposes a Total Zero Doppler Steering (TZDS) method by exploiting an additional pitch steering. This method is applied in the TerraSAR-X system and suppresses the Doppler centroid to tens of Hz all over the range swath [9, 10]. However, this method suppresses the Doppler centroid at the cost of increasing the complexity of the SAR system since it requires attitude steering on two dimensions. Besides, in some SAR systems, there are always some constraints on the attitude of the satellite for specific purpose [11, 12], which also sets a limit on the applications of the 2-dimensional attitude steering method.

By analyzing the aforementioned attitude steering methods for elliptic orbiting SAR systems, it can be found that the pitch steering angle is usually very small in nearly circular orbit cases. This implies that an attitude steering method with only yaw steering may suffice to suppress the Doppler centroid to 0 Hz for the mid-range at the cost of causing a low residual Doppler centroid at the edge of the range extent. It can reduce the complexity of the attitude control system while not affecting

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the accuracy of the SAR products. This paper studies the attitude steering problem and proposes a new method to calculate the yaw steering angle with zero pitch steering angle or with a given pitch steering angle. The former formula can be used to perform 1-D yaw steering in the SAR systems with one coordinate axis pointing toward the earth center [6, 7]; the latter can be used to perform 1-D yaw steering in the SAR systems with one coordinate axis pointing perpendicular to the earth surface [5] or to refine the performance of TZDS.

This paper is organized as follows. Section 2 derives the formula for calculating the yaw steering angle of the new method. Section 3 gives the simulation results to compare the performances of the new method and the currently applied methods. Section 4 gives a discussion and draws the conclusion.

## 2. THE GENERAL METHOD FOR ZERO DOPPLER STEERING

### 2.1. Geometry for SAR Attitude Steering

The Doppler frequency $f_{\text{dop}}$ of a point target is given by [13]

$$f_{\text{dop}} = -\frac{2}{\lambda}\frac{(\vec{R}_S - \vec{R}_T)\cdot(\dot{\vec{R}}_S - \dot{\vec{R}}_T)}{R} \tag{1}$$

where $\lambda$ is the wavelength; $\vec{R}_S$ and $\vec{R}_T$ are the position vectors of satellite and target, respectively; $\dot{\vec{R}}_S$ and $\dot{\vec{R}}_T$ are the corresponding first order derivatives; $R$ is the range distance from the satellite to the target; $\cdot$ is the inner product operator.

Let $f_{\text{dop}}$ equal 0. Expanding the terms in the brackets, substituting the relation $\vec{R}_T\cdot\dot{\vec{R}}_T = 0$ for earth rotation, and shifting the term irrelevant to the target to the other side yield

$$\vec{R}_S\cdot\dot{\vec{R}}_T + \dot{\vec{R}}_S\cdot\vec{R}_T = \vec{R}_S\cdot\dot{\vec{R}}_S. \tag{2}$$

From the derivation process, it may be contended that Eq. (2) is the function for zero-Doppler plane. To get a concrete point of this contention, we proceed to derive an algebraic form of Eq. (2) in terms of coordinates.

First, three coordinate systems depicted in Figure 1 are defined as follows.

(1) Earth centered coordinate system $E - x_e y_e z_e$: The origin coincides with the Earth center, and the $z_e$-axis is along the angular momentum direction of the Earth; the $x_e$-axis is pointing to the ascending node of the orbit, while the $y_e$-axis is directed to complete a right-handed Cartesian system.

(2) Orbit plane coordinate system $E - x_o y_o z_o$: The origin coincides with the Earth center, while its $z_o$-axis is along the angular momentum direction of the satellite; the $x_o$-axis is pointing to the perigee, while the $y_o$-axis is directed to complete a right-handed Cartesian system.

(3) Satellite local coordinate system $S - x_s y_s z_s$: The origin coincides with the satellite mass center, while its $z_s$-axis is along the angular momentum direction of the satellite; the $x_s$-axis is pointing to the satellite away from the earth center, while the $y_s$-axis is directed to complete a right-handed Cartesian system.

Then the translations between these three coordinate systems are as following:

(1) From $E - x_o y_o z_o$ to $E - x_e y_e z_e$

$E - x_o y_o z_o$ needs to be rotated about $z_o$-axis by $\omega$, where $\omega$ is the argument of perigee, and then be rotated about the $x$-axis by $i$, where $i$ is the inclination, so the rotation matrix from the orbit plane coordinate system $E - x_o y_o z_o$ to the earth centered coordinate system $E - x_e y_e z_e$ is

$$A_{eo} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix}\begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

(2) From $S - x_s y_s z_s$ to $E - x_o y_o z_o$

$S - x_s y_s z_s$ needs to be translated by $\overrightarrow{SE}$ and then be rotated about $z_o$-axis by $f$, where $f$ is the true anomaly. The rotation matrix is

$$A_{os} = \begin{bmatrix} \cos f & -\sin f & 0 \\ \sin f & \cos f & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

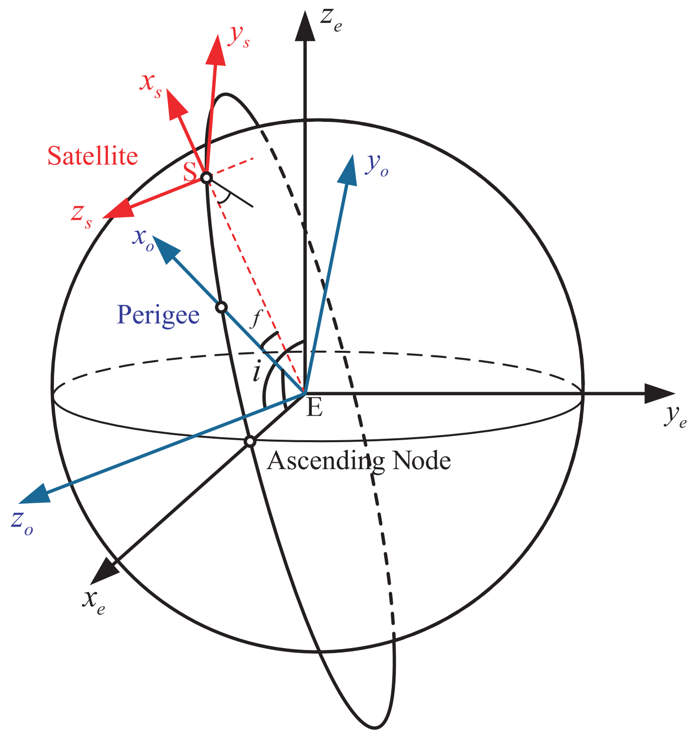


**Figure 1.** Geometry of space borne SAR.

### 2.2. Derivation of Steering Angles for SAR Attitude Control

In the orbit plane coordinate system $E - x_o y_o z_o$, we have the satellite position vector $\vec{R}'_S$, and its derivative $\dot{\vec{R}}'_S$ is

$$\begin{aligned} \vec{R}'_S &= \frac{a(1-e^2)}{1+e\cos f}\left[\cos f,\ \sin f,\ 0\right]^{\mathrm{T}} \\ \dot{\vec{R}}'_S &= \sqrt{\frac{\mu}{a(1-e^2)}}\left[-\sin f,\ \cos f + e,\ 0\right]^{\mathrm{T}} \end{aligned}. \tag{5}$$

where $a$ is the semi-major axis length, $e$ the eccentricity, and $\mu$ the earth gravitational constant

In the earth centered coordinate system $E - x_e y_e z_e$, the position vector of the target $\vec{R}_T$ and its first order derivative $\dot{\vec{R}}_T$ can be expressed as

$$\vec{R}_T = \begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix}, \quad \dot{\vec{R}}_T = \omega_e \begin{bmatrix} -y_T \\ x_T \\ 0 \end{bmatrix} = \omega_e \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix}. \tag{6}$$

where $\omega_e$ is the angular velocity of the Earth. Denoting that

$$A_c = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{7}$$

using the relations that $\vec{R}_S = A_{eo}\vec{R}'_S$ and $\dot{\vec{R}}_S = A_{eo}\dot{\vec{R}}'_S$, replacing the inner product of two vectors by the corresponding matrix multiplication, and substituting Eqs. (5) and (6) into Eq. (2), a plane equation with respect to $\vec{R}_T$ drops out as

$$\vec{R}_S \cdot \dot{\vec{R}}_S = \left[\omega_e \vec{R}'^{T}_S A^{\mathrm{T}}_{eo} A_c + \dot{\vec{R}}'^{T}_S A^{\mathrm{T}}_{eo}\right] \vec{R}_T. \tag{8}$$

of which the normal vector is

$$\vec{n}=\left[\omega_e\vec{R}_S^{\prime T}A_{eo}^{\mathrm{T}}A_c+\dot{\vec{R}}_S^{\prime T}A_{eo}^{\mathrm{T}}\right]^T. \tag{9}$$

In $S-x_sy_sz_s$, without attitude steering, the unit vector of the beam center line can be expressed as

$$\vec{l}=[-\cos\gamma,\ 0,\ \varepsilon\sin\gamma]^{\mathrm{T}}. \tag{10}$$

where $\varepsilon$ is an indicator variable that describes radar's looking to the right ($-1$) or left ($+1$) of the orbital velocity vector.

Assume that the angles for yaw and pitch steering are $\theta_p$ and $\theta_y$, respectively, then the coordinate of the beam center line vector in $S-x_sy_sz_s$ becomes $\vec{l}'=A_{sa}\vec{l}$, where the rotation matrix $A_{sa}$ is

$$A_{sa}=\begin{bmatrix}1&0&0\\0&\cos\theta_y&-\sin\theta_y\\0&\sin\theta_y&\cos\theta_y\end{bmatrix}\begin{bmatrix}\cos\theta_p&-\sin\theta_p&0\\\sin\theta_p&\cos\theta_p&0\\0&0&1\end{bmatrix}. \tag{11}$$

Converted into $E-x_ey_ez_e$, the coordinate of $\vec{l}'$ is $\vec{l}'=A_{eo}A_{os}A_{sa}\vec{l}$.

For zero Doppler steering, the beam center line must be in the zero Doppler plane, hence $\vec{l}'$ should be perpendicular to $\vec{n}$, which can be written as

$$\vec{n}\cdot\vec{l}'=\left[\omega_e\vec{R}_S^{\prime T}A_{eo}^{\mathrm{T}}A_c+\dot{\vec{R}}_S^{\prime T}A_{eo}^{\mathrm{T}}\right]A_{eo}A_{os}A_{sa}\vec{l}=\left[\omega_e\vec{R}_S^{\prime T}A_{eo}^{\mathrm{T}}A_cA_{eo}+\dot{\vec{R}}_S^{\prime T}\right]A_{os}A_{sa}\vec{l}=0. \tag{12}$$

The last simplification employs the relation that $A_{eo}^{-1}=A_{eo}^{\mathrm{T}}$.

(1) 2-D steering

Observe that the second dimension of $\vec{l}$ is 0, so $A_{os}A_{sa}\vec{l}$ can be viewed as a linear combination of the first and third column vectors of $A_{os}A_{sa}$. In order to satisfy Eq. (12), it is sufficient to have

$$\begin{array}{l}\left[\omega_e\vec{R}_S^{\prime T}A_{eo}^{\mathrm{T}}A_cA_{eo}+\dot{\vec{R}}_S^{\prime T}\right]\mathrm{col}_1\left(A_{os}A_{sa}\right)=0\\\left[\omega_e\vec{R}_S^{\prime T}A_{eo}^{\mathrm{T}}A_cA_{eo}+\dot{\vec{R}}_S^{\prime T}\right]\mathrm{col}_3\left(A_{os}A_{sa}\right)=0\end{array}. \tag{13}$$

where $\mathrm{col}_i(\cdot)$ denotes the $i$th column of the matrix in the brackets. Since $\mathrm{col}_3(A_{os}A_{sa})$ relates only to the yaw steering angle $\theta_y$, the second equation of Equation (13) can be solved for $\theta_y$, then $\theta_p$ can be determined from the first equation by using $\theta_y$. For convenience of reference, the steering angles are listed here as $\theta_{yo}$ and $\theta_{po}$.

$$\begin{array}{l}\theta_{yo}=\tan^{-1}\left(\dfrac{k_1\sin i\cos(\omega+f)}{k_2(1+e\cos f)-k_1\cos i}\right)\\\theta_{po}=k\tan^{-1}\left(\dfrac{ek_2\sin f}{\sqrt{[k_1\sin i\cos(\omega+f)]^2+[k_2(1+e\cos f)-k_1\cos i]^2}}\right)\end{array}. \tag{14}$$

where

$$k_1=\omega_e\frac{a(1-e^2)}{1+e\cos f},\quad k_2=\sqrt{\frac{\mu}{a(1-e^2)}},\quad k=\begin{cases}-1 & k_2(1+e\cos f)-k_1\cos i\geq 0\\1 & k_2(1+e\cos f)-k_1\cos i<0\end{cases}. \tag{15}$$

(2) 1-D yaw steering with zero pitch steering angle

Let $\theta_p=0$ in Eq. (12), by collecting together the terms related to $\sin\theta_y$ and $\cos\theta_y$, respectively, we have

$$[k_1\cos i-k_2(e+\cos f)]\sin\theta_y+[k_1\sin i\cos(\omega+f)]\cos\theta_y=\varepsilon ek_2\sin f\cot\gamma. \tag{16}$$

Employing the triangular identity $\sin\theta_y\cos\theta+\cos\theta_y\sin\theta=\sin(\theta_y+\theta)$ yields

$$\theta_y=\sin^{-1}\left(\varepsilon\tan\theta_{po}\cot\gamma\right)+\theta_{yo}. \tag{17}$$

where $\theta_{yo}$ and $\theta_{po}$ are written as in Eq. (14).

(3) 1-D yaw steering with a given pitch steering angle

Given a pitch steering angle $\theta_p$, the yaw steering angle can be solved as

$$\theta_y = \sin^{-1}\left[\frac{\varepsilon \tan\theta_{po} \cot\gamma \cos\theta_p}{\sqrt{1+\sin^2\theta_p \cot^2\gamma}}\right] + \theta_{yo} - \theta. \tag{18}$$

where

$$\theta = \tan^{-1}\left(\varepsilon \sin\theta_p \cot\gamma\right). \tag{19}$$

## 3. SIMULATION RESULTS

For the convenience of reference, the analytical form of the steering law employed by TerraSAR-X is listed here:

$$\theta_y = \tan^{-1}\left(\frac{\sin i \cos(\omega+f)}{N-\cos i}\right), \quad \theta_p = k' \cos^{-1}\left(\frac{1+e\cos f}{\sqrt{1+e^2+2e\cos f}}\right), \quad k' = \begin{cases} 1 & 0 \le f < \pi \\ -1 & \pi \le f < 2\pi \end{cases}. \tag{20}$$

where $N$ is the number of revolutions per day. There is a little improvement on the accuracy of this steering law by replacing $N$ with $\omega_s/\omega_e$, where $\omega_s$ is the instantaneous angular velocity of satellite. We label the steering law that TerraSAR-X employs and its improvement edition as TZDS and TZDM, respectively, label the steering law in Eq. (17) as OLY and the steering law in Eq. (18) with a given pitch angle as OLYT. The parameters used in the simulation are listed in Table 1. The orbit is a kind of low sun synchronous orbits which most current SAR systems adopt. The beam width angle offers a range swath of 30 kilometers.

**Table 1.** Orbit elements and SAR system parameters for simulation.

| Denotation | Description | Value | Value Units |
|---|---|---|---|
| $a$ | Semi Major | 6892.137 | km |
| $e$ | Eccentricity | 0.0011 | |
| $i$ | Inclination | 97.42 | deg |
| $\omega$ | Argument of Perigee | 90 | deg |
| $f_0$ | Carrier Frequency | 9.6 | GHz |
| $\gamma$ | Off-Nadir Angle | 30 | deg |
| $\theta_b$ | Beam Width Angle | 2 | deg |
| $T$ | Orbit cycle | 4.2 | $h$ |

The yaw and pitch steering angles that the above methods employ and the residual Doppler centroid at the mid-range and at the edge of the range extent are demonstrated in Figure 2. The scales of the residual Doppler frequency are tabulated in Table 2. Based on the simulation results, it can be concluded that:

(1) For the mid-range, the Doppler centroid is reduced to 0 Hz by employing OLY and OLYT, while there is a small residue by employing TZDS and TZDM.

(2) Compared with the residual Doppler frequency over the range extent by employing TZDS and TZDM, there is a variation of ±20 Hz of the residual Doppler centroid. This low variation will not add the difficulty of focusing due to the azimuth over sampling.

(3) The residual Doppler centroid can be reduced from ±20 Hz by employing TZDS to ±5 Hz by employing TZDM. The residual Doppler centroid can be reduced further to almost 0 Hz by employing the yaw steering angle of OLYT.

According to the analysis and simulation shown in the manuscript, the proposed altitude steering method has the following advantages. Firstly, the Doppler centroid of mid-range can always be 0 Hz,

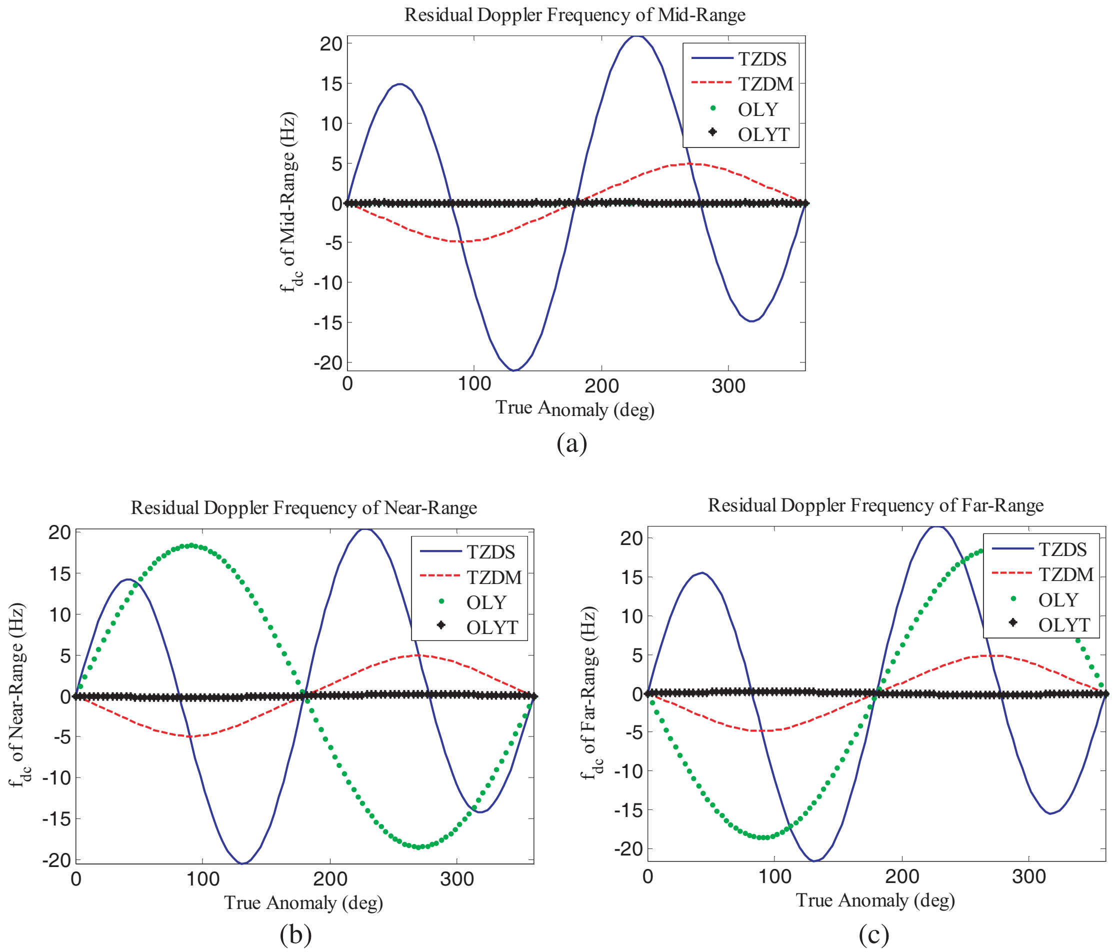


**Figure 2.** Residual Doppler centroid over the range by employing different steering laws. (a) Residual doppler at mid-range. (b) Residual doppler at near-range. (c) Residual doppler at far-range.

**Table 2.** The scale of the residual doppler frequency.

| | TZD | TZDM | OLY | OLYT |
|---|---|---|---|---|
| Mid-Range | $\pm 21.0$ Hz | $\sim 0$ Hz | $\sim 0$ Hz | $\sim 0$ Hz |
| Near-Range | $\pm 20.5$ Hz | $\pm 4.9$ Hz | $\pm 18.4$ Hz | $\pm 0.2$ Hz |
| Far-Range | $\pm 21.5$ Hz | $\pm 4.9$ Hz | $\pm 18.4$ Hz | $\pm 0.2$ Hz |

and the residual Doopler centroid at the edge of range extent is low and will not cause defocus. Secondly, the system complexity is lower with only yaw steering than 2D altitude steering. Thirdly, the proposed algorithm can be applied in circular and elliptic orbit cases.

## 4. CONCLUSION

This paper discusses the problem of attitude control in SAR system, and proposes a new method to suppress the residual Doppler centroid by employing 1-D yaw steering when the pitch angle is set to 0 or a constant value. Compared with 2-D steering law in current literature, the proposed method can reduce the complexity of the attitude control system and can be used to refine the performance of current steering laws.

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