

Accurate Extraction of High Quality Factor of Dielectric Resonators from Measurements

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Abstract—We present a revised Cauchy method to accurately extract the high quality factor of dielectric resonators from measurements. Since the losses displace all the zeros and poles of the transfer function horizontally to the left in the complex plane, the accurate evaluation of the unloaded quality factor of microwave resonators can be achieved based on the complex frequency transformation. The results show that if the three-point method is employed, the accuracy of the quality factor values deteriorates when the input/output coupling is strong. Nevertheless, a nearly constant factor value can be obtained by our proposed technique whether the input/output couplings are weak or strong. This algorithm provides an alternative method to measure the unloaded quality factor when the signal-to-noise ratio is high.

1. INTRODUCTION

Microwave resonators are widely used in a variety of applications, including filters, multiplexers, oscillators, frequency meters, and tuned amplifiers [1]. Among the numerous parameters of resonator, the unloaded quality factor (Q_0) representing the inherent losses of the resonator is one of the most critical parameters and presents a fundamental upper limit on the resonator performance [2]. Its value varies notably among various microwave resonators. Hence, accurate and rapid measurements of the Q_0 factor are essential in the performance assessments of microwave circuit and characterization of material properties.

Dielectric resonators (DRs) are massively used in electronics and telecommunications applications as high quality factor components for narrow-band filters. The higher the Q_0 factor value is, the better is the selectivity performance and the lower are the losses. At the same time, it brings difficulty for accurate measurement of the value. An accurate calculation of the unloaded Q_0 factor requires not only the scope of the conductivity of the metal walls but also the dielectric loss tangent of the materials. Moreover, there is a considerable uncertainty regarding the loss tangent value of dielectric materials from batch to batch. Based on these considerations, measurement is the only approach that can provide an accurate evaluation of the unloaded Q_0 factor [3].

In general, the measurement of loaded quality factor (Q_L) is routinely performed through one- or two-port measurement [3]. There are various methods discussed in the literature for measuring Q_L factor, e.g., the graphical method, transmission technique, reflection technique, et al. [4]. However, some of these methods require sophisticated mathematical treatment or complicated procedures [5]. In addition, the 3-dB “three-point” method is used to determine the Q_L factor [6]. Usually, the Q_L factor is referred as the measured loaded Q_L factor of this resonator together with external circuitry and coupling coefficients [6]. For an accurate measurement of the Q_0 factor, we need to use very loose

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coupling between the probe and the resonator to minimize the loading on the resonator. The unloaded Q_0 factor thereupon can be approximated by the loaded Q_L factor. However, measurements with loose coupling usually result in low SNR that may affect the accuracy of the final results [6], especially for the case of high quality factor.

Recently, the Cauchy method has been employed to extract the unloaded Q_0 factor from measurements on filters [7–12]. In the initial stage, it cannot be utilized to deal with the lossy filters. Lampérez et al. [8] improved the method by solving three polynomials in one step, and as a consequence, it is not restricted to lossless networks. However, the degree of accuracy is not high enough. In this paper, we propose a revised Cauchy method to accurately extract the high quality factor of dielectric resonators from measurements. Based on the proposed technique, a nearly constant unloaded Q_0 factor can be obtained whether the input/output couplings are weak or strong.

2. CAUCHY METHOD

As a rule of thumb, a two-port reciprocal filter can be characterized by the reflection and transmission functions, S_{11} and S_{21} . As usual, the two functions can be expressed as a ratio of two n th-degree polynomials

$$S_{11}(s) = \frac{F(s)}{E(s)} = \frac{\sum_{k=0}^n a_{1k}s^k}{\sum_{k=0}^n b_k s^k}, \quad S_{21}(s) = \frac{P(s)}{E(s)} = \frac{\sum_{k=0}^{n_z} a_{2k}s^k}{\sum_{k=0}^n b_k s^k}, \quad (1)$$

where $s = j\omega$ is the complex frequency variable, n is the filter order and n_z is the number of finite transmission zeros. $a_{1k} = [a_{10}, \dots, a_{1n}]$, $a_{2k} = [a_{20}, \dots, a_{2n_z}]$, $b_k = [b_0, \dots, b_n]$ is the polynomial coefficients of $F(s)$, $P(s)$ and $E(s)$, respectively. $S_{11}(s)$ and $S_{21}(s)$ share a common denominator $E(s)$. This low-pass prototype filter can be easily obtained from a band-pass filter using the band-pass to low-pass prototype transformation.

We get the values of the transmission and reflection parameters simultaneously at a set of N frequency points $S_i = j\omega_i$ ($i = 1, 2, \dots, N$) by measurements. Then, the generated rational polynomials S_{11} and S_{21} should equal the measured S_{m11} and S_{m21} , respectively. Therefore, in a linear time invariant system, the polynomial coefficients can be constructed as

$$\sum_{k=0}^n a_{1k}s_i^k - S_{m11}(s_i) \sum_{k=0}^n b_k s_i^k = 0, \quad i = 1, 2, 3, \dots, N, \quad (2)$$

$$\sum_{k=0}^{n_z} a_{2k}s_i^k - S_{m21}(s_i) \sum_{k=0}^n b_k s_i^k = 0, \quad i = 1, 2, 3, \dots, N. \quad (3)$$

The complex coefficients a_{1k} , a_{2k} can be evaluated by solving the following equation;

$$\frac{S_{11}(s_i)}{S_{21}(s_i)} = \frac{F(s_i)}{P(s_i)} = \frac{\sum_{k=0}^n a_{1k}s_i^k}{\sum_{k=0}^{n_z} a_{2k}s_i^k} = \frac{S_{m11}(s_i)}{S_{m21}(s_i)} \quad (4)$$

The Equation (4) can be rewritten in matrix notation as:

$$[S_{m21}V_n - S_{m11}V_{n_z}] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = [0] \quad (5)$$

where $S_{m11} = \text{diag}\{S_{m11}(s_i)\}$, $S_{m21} = \text{diag}\{S_{m21}(s_i)\}$, a_1 and a_2 are the vectors containing the polynomial coefficients $a_1 = [a_{10}, \dots, a_{1n}]^T$, $a_2 = [a_{20}, \dots, a_{2n_z}]^T$, and V_m ($m = n$ or n_z) is a $(N \times m + 1)$ Vandermonde matrix. The complex coefficients are obtained by solving (5) through the singular value decomposition and total least squares method.

Once $P(s)$ and $F(s)$ are extracted, the roots of $E(s)$ can be evaluated by the Feldkeller's equation

$$E(s)E*(-s) = F(s)F*(-s) + P(s)P*(-s) \quad (6)$$

The roots of $E(s)$ and $E^*(s)$ are in pairs with opposite real part; by selecting those with negative real part, the poles of the filter are obtained, as well as the polynomial $E(s)$.

We proposed a modified Cauchy method to extract the high quality factor of dielectric resonators from measurements. First, we utilize the Cauchy method to generate the rational polynomials based on lossless. Then, we can obtain the rough range of loss factor σ according to the roots of $E(s)$ polynomial, since $E(s)$ is a strict Hurwitz polynomial, as all its roots lie in the left half of the s plane, and the losses displace all the zeros and poles of the transfer function horizontally to the left [3].

In the second stage, the losses are considered through a complex frequency transformation

$$s' = \frac{f_0}{BW} \frac{1}{Q_0} + j \frac{f_0}{BW} \left(\frac{\omega_m}{\omega_0} - \frac{\omega_0}{\omega_m} \right) = (\sigma + \Delta\sigma) + j\omega \quad (7)$$

where BW and f_0 are bandwidth and center frequency of the filter, σ is the dissipation factor and $\Delta\sigma$ represents the displacement between the extracted and experimental results. In this way, all the zeros and poles of the transfer function are displaced horizontally to the left by an amount $(\sigma + \Delta\sigma)$. The value of $\Delta\sigma$ can be then estimated by imposing that

$$|S_{11}(s')|^2 + |S_{21}(s')|^2 - |S_{m11}(\omega_i)|^2 - |S_{m21}(\omega_i)|^2 = 0 \quad (8)$$

Finally, the unloaded Q_0 factor is determined by

$$Q_0 = \frac{f_0}{BW} \cdot \frac{1}{\sigma + \Delta\sigma}. \quad (9)$$

During the process, the amounts of sample data are very little and the computing time is extremely short, allowing the method in real time applications.

3. APPLICATION

To show the potentialities offered by the proposed technique, the extraction of the Q_0 factor from the simulated S -parameters of a DR is first presented.

In this paper, the employed ring DR has a dielectric constant of 45. The single DR cavity structure is shown in Fig. 1, where the inner, outer diameter and height is 22.26 mm, 62.30 mm and 24.70 mm respectively. The length and width of the metal cavity are 92.10 mm and 68.66 mm, and the height is 86.20 mm. The distance from the vertex along the y axis to the opposite side is 94.12 mm. The filter has been simulated using a EM simulator. The loss factors (conductor loss and dielectric loss) are included in the simulated responses.

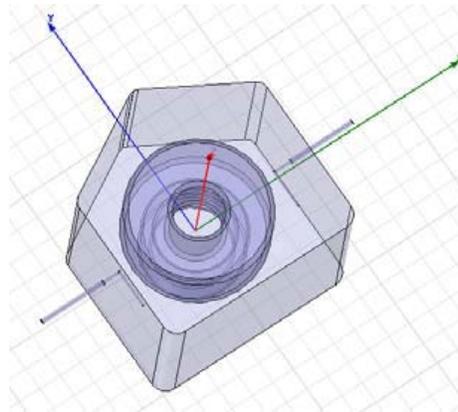


Figure 1. Diagram of the simulated single DR cavity.

At first, we extract the unloaded Q_0 factor when the input/output couplings are strong. The proposed algorithm is applied with $n = 1$, $n_z = 1$ and $N = 84$. The magnitude and phase response of the dielectric resonators (S_{11} and S_{21}) is shown in Fig. 2. The black dot lines are the simulated results

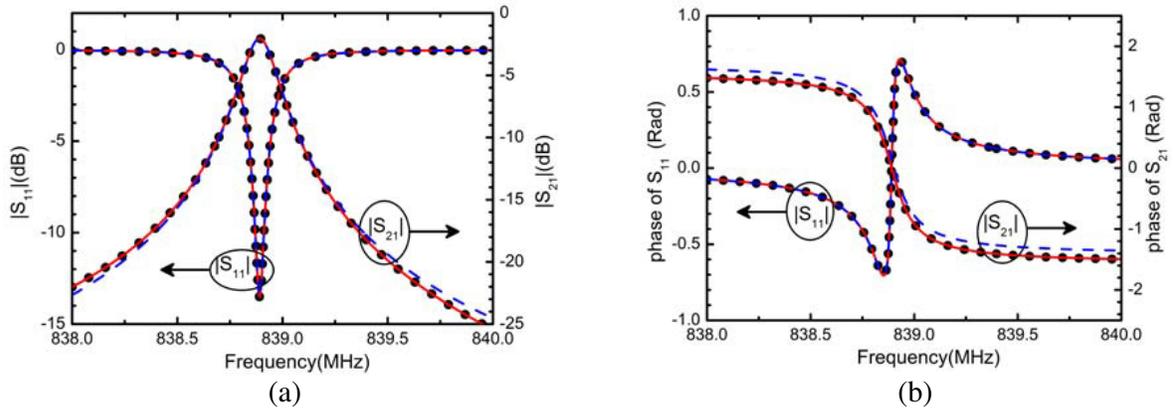


Figure 2. Simulated (black dot) and extracted responses of the dielectric resonators by our method (red solid lines) and [10] (blue dashed line); (a) magnitude response, (b) phase response.

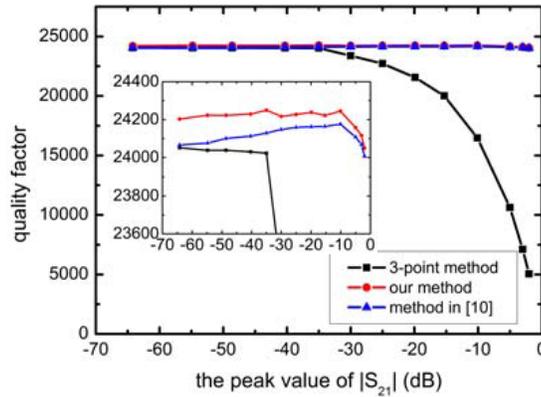


Figure 3. The Q_0 factor obtained from the simulated data by different methods. The inset shows a zoomed-in region of the quality factor.

while the red solid and blue dashed lines are the extracted responses by two different methods, our proposed and that in [10], respectively.

Figure 2 shows an excellent agreement between the extracted polynomial by our method and the simulated response. By inspecting the response of Fig. 2, it can be identified that the central frequency is 838.8910 MHz; the 3 dB bandwidth is 166 kHz. The peak value of $|S_{21}| = -2.0$ dB suggests that the input/output couplings are strong. The response of S_{21} in high stop band declines more quickly than that of the low stop band owing to the transmission zero locating around 843.5 MHz. The transmission zero in a single-cavity is caused by the coupling between source and load [13]. By virtue of the relation in (9), the value of the Q_0 factor can then be obtained. Result shows that the effective Q_0 factor is equal to 24050.8 (as shown in Fig. 3). According to the three-point method, the Q_0 factor is 5053.6. As expected, there is a tremendous difference between the values obtained from the two different methods when the coupling is strong enough. At the same time, the extracted polynomial by the method in [10] is also presented in Fig. 2. Even a good fit with the measured S_{11} can be observed, and the accuracy of S_{21} fitting is poor when the axis is far away from the central frequency. Thus, the accuracy of the Q_0 factor decreases which is shown in the inset of Fig. 3.

Then the input/output coupling strength is weakened. The Q_0 factor values obtained from the three different methods are shown in Fig. 3. From the figure, we can clearly see that, as the input/output coupling strength weakens; the Q_0 factor deriving from the three-point method is varied from 5053.6 to 24051.1. However, the values which are obtained by our proposed algorithm nearly remain unchanged. It is worthy to note that when the peak value of $|S_{21}|$ is equal to -30.0 dB, the difference between the

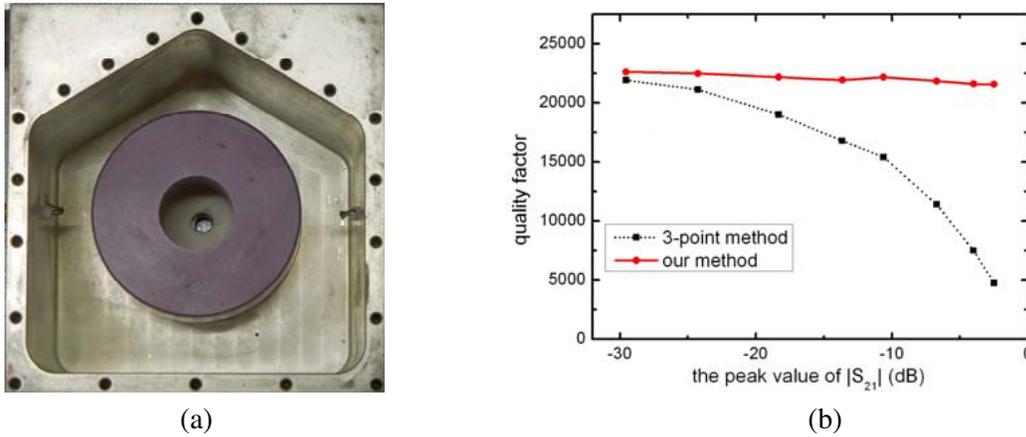


Figure 4. (a) Diagram of the manufactured single DR cavity; (b) the Q_0 factor obtained from experimental data.

values is about 3.1%. If the coupling strength is further weakened, for example, the $|S_{21}|$ peak value equal to -40.0 dB, this difference is reduced to 0.8%. The extracted Q_0 factor by the method in [10] is also presented. From the figure, we will find that this value decreases as the input/output coupling strength decreases. Needless to say, the accuracy is seriously impaired when the input/output coupling is loose.

In order to demonstrate the validity of our proposed method, we also extract the Q_0 factor from experimental data, which is obtained from the vector network analyzers. The employed ring DR is made by low-loss ceramic materials and the cavity structure is shown in Fig. 4(a). The structure of the single DR cavity is the same as that in Fig. 1. The results are shown in Fig. 4(b). It also shows that the Q_0 factor derived from the three-point method is varied from 4749.2 to 21929.4, while the values obtained by our proposed algorithm nearly remain unchanged. It can be seen that the results provided from the experimental data are identical to those of the simulated.

Generally speaking, the measurement is completed under a very weak coupling strength in order to reduce the effect of coupling. However, low coupling usually result in low SNR that may affect the accuracy of the measurements. From Fig. 3, it is clear that the accuracy of the two-port analysis deteriorates when the input/output coupling is strong by the three-point method. But our results show that the coupling strength plays little role in the extraction of the Q_0 factor. It must be stressed here that although the measurement parameter is the loaded Q_L factor, the unloaded Q_0 factor can be approximated by the loaded Q_L factor by our proposed method. This algorithm is very promising for it is independent of coupling strength.

4. CONCLUSION

We present a revised Cauchy method for extracting unloaded Q_0 factor of dielectric resonators. Using the complex frequency transformation, the accurate evaluation of the unloaded Q_0 factor of dielectric resonators can be realized. The results indicate that the accuracy of the obtained unloaded Q_0 factor by the three-point method deteriorates when the input/output coupling is strong. However, the values obtained by our proposed algorithm are independent of different coupling strengths. It provides us an alternative method to accurately and reliably extract unloaded Q_0 factor of dielectric resonators from S -parameters without the limit of loose coupling.

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