

Studying the Influence of the Number Vanishing Moments of Daubechies Wavelets for the Analysis of Microstrip Lines

Mohamed Bayjja*, Mohamed Boussouis, and Naima Amar Touhami

Abstract—Using Daubechies wavelet with one, two, three, and four vanishing moments, basis functions for the efficient solution of electromagnetic integral equations are studied. Due to the vanishing moments, the moment matrices resulting in these problems are sparsified by wavelet, and consequently, the solution can be obtained rapidly. The microstrip line is examined in order to demonstrate the advantages of this suggested wavelet-moments method over the traditional moment method. To demonstrate the effectiveness and accuracy of the proposed technique, numerical results for error relative for different vanishing moments of Daubechies wavelets are presented. It is found that Daubechies wavelets with larger number of vanishing moments generally give higher accuracy.

NOMENCLATURE

MoM	Method of Moments
TEM	Transverse electromagnetic
dbN	Daubechies wavelet with N vanishing moments
nfb	Number of basic functions
nfe	Number of testing functions
Thr	Threshold
IM	Impedance Matrix
CPU	Central Processing Unit

1. INTRODUCTION

Wavelet theory is a relevant continuously emerging area in mathematical research. It has been applied to a wide range of engineering disciplines and received considerable attention in computational electromagnetics, particularly in solving integral equations [2, 10]. Researchers are now faced with an ever increasing variety of wavelet bases to choose from. While the choice of the “best” wavelet is obviously application-dependent, it can be useful to isolate a number of properties and features that are of general interest to the user. Since wavelets may have some nice properties, including symmetry, vanishing moments, compact support and orthogonality, the construction of a wavelet has become a very popular scientific research subject, especially in mathematics and engineering [23].

Planar transmission structures are widely used in microwave, millimeter-wave circuits and high speed digital circuits. These are stripline, microstrip and coplanar waveguides [1–12]. In analyzing planar microstrip structures, the method of moments (MoM) can be applied either in the spectral

Received 21 May 2016, Accepted 26 August 2016, Scheduled 12 September 2016

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domain [19–21] or in the spatial domain [9–21]. However, in the conventional form of the spatial domain approach, the Green's functions for the microstrip structures involve the evaluation of the Sommerfeld integrals, whose integrands are highly oscillatory and slowly decaying functions; hence their computation is very time consuming. However, it has been demonstrated in [21] that this problem can be obviated by using the newly-developed closed-form spatial domain Green's functions [22]. Using traditional basis functions in the MoM, the arising matrices become very large and densely populated, such that the solution of the underlying system of equations requires long computation times and huge memory [21]. It takes $O(N^2)$ units of storage and $O(N^3)$ multiplications/divisions to solve the matrix equations [2].

Wavelets found their application in solving integral equations, resulting in sparse impedance matrices. This is due to features of vanishing moments, orthogonality and multiresolution analysis in wavelets [5]. The conversion of dense matrices into a sparse form requires $O(N^2)$ operations. The algorithm for solving the resulting sparse system requires only $O(N \log 2N)$ operations, as shown in [4]. This technique will be shown to be versatile and efficient.

In this paper, we propose to study the merit of Daubechies wavelets for N number of vanishing moments in order to model the microstrip line by the MoM. The paper is organized as follows. Section 2 is devoted to microstrip line formulation of the integral equation, using Green's function. The MoM is used to approximate the integral equation. A brief review of the theory of wavelets, Daubechies wavelets, and wavelet expansion is presented. In Section 3, we report our numerical finding and demonstrate the accuracy of the proposed numerical scheme by considering numerical examples.

2. THEORETICAL FORMULATION

2.1. Method of Moments

In the analysis, we consider a shielded microstrip line with the cross-section shown in Figure 1 [7, 8].

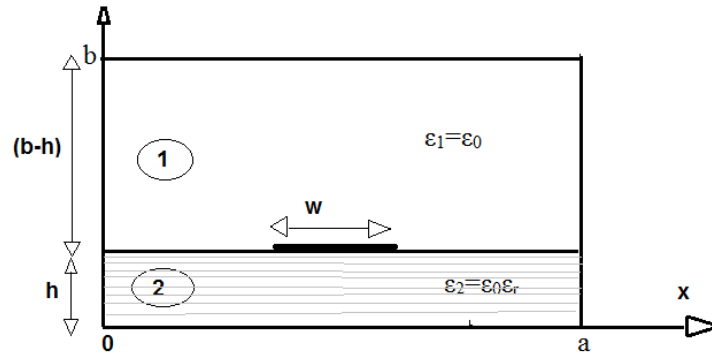


Figure 1. Shielded microstrip line geometry.

We assume that the metal and dielectric losses in this line are negligible, and the propagation mode in direction Oz is almost TEM. Green function determined by the Fourier method is written [12, 19]:

$$G(x, x_0) = \sum_{n=1}^{\infty} \frac{2}{a} \frac{1}{\beta_n} \frac{1}{Kn} \sin(\beta_n x) \sin(\beta_n x_0) \quad (1)$$

Function G satisfies the boundary and the interface conditions of the microstrip equation. The following form can be formulated and solved to determine the charge distribution [12]:

$$V(x, y_0) = V(x) = \int_{\frac{a-w}{2}}^{\frac{a+w}{2}} G(x, x_0) \rho(x_0) dx_0 \quad (2)$$

In the method of Galerkin, the weighting functions are identical to the trial functions. We make the scalar product of the relation giving $V(x)$ with $\varphi_q(x)$ by integrating on the domain of definition of

impulsion function $\varphi_q(x)$, that is:

$$\langle V(x) | \varphi_q(x) \rangle = \sum_{p=1}^{Nfe} \sum_{n=1}^{\infty} An\alpha_p\gamma_{pn} \langle \sin(\beta_n x) | \varphi_q(x) \rangle \tag{3}$$

For bases functions, choose $\varphi_q(x - xq) = \delta(x - xq)$, with $xq = (q - 0.5) * \delta$, and take the potential $V(x) = 1$ when x is located in the metal. We have:

$$\langle V(x) | \varphi_q(x) \rangle = \int_{xq-\delta/2}^{xq+\delta/2} \frac{1}{\sqrt{\delta}} dx = \begin{bmatrix} \sqrt{\delta} \\ \vdots \\ \sqrt{\delta} \end{bmatrix} \tag{4}$$

This set of equations can be written in matrix from as

$$[D] [\alpha] = [V] \tag{5}$$

2.2. Daubechies Wavelets

Wavelets ψ such as dilated and translated family is an orthonormal basis of $L^2(R)$ [10, 11].

$$\left\{ \psi_{j,n}(t) = \frac{1}{\sqrt{2^j}} \psi \left(\frac{t - 2^j n}{2^j} \right) \right\}_{(j,n) \in Z^2} \tag{6}$$

Behind this simple statement lie very different points of view that open a fruitful exchange between harmonic analysis and discrete signal processing. Orthogonal wavelets dilated by 2^j carry signal variations at the resolution 2^j .

The Daubechies is the first one to make the handle orthogonal wavelets with compact support and arbitrary regularity. We will call N the order, or the vanishing moments number of the db N wavelet. This family contains the Haar wavelet, db1, which is the simplest and certainly the oldest of wavelets. It is discontinuous, resembling a square form. The coefficients for Compactly Supported Daubechies Wavelets in [5–13] will be employed to construct the Daubechies scalets and wavelets of different orders. In general, for Daubechies scalets $h_n = 0$ for $n < 0$ and $n > 2N + 1$, the support $\phi = [0, 2N - 1]$, and the support $\psi = [1 - N, N]$, where N is the order, or the number of vanishing moments.

2.3. Wavelet Expansion

It is easier to expand a given function in a wavelet basis than to expand an unknown function in wavelets while solving the corresponding integral equation by the MoM. In this paper, we will apply the wavelet based MoM [11–17].

Therefore, $\{\psi_{m,n}\}_{m,n \in Z}$ is an orthonormal basis of $L^2(R)$. For all $f(x) \in L^2(R)$, we have

$$f(x) = \sum_{m,n} \langle f(x), \psi_{m,n}(x) \rangle \psi_{m,n}(x) \tag{7}$$

The wavelets are applied directly upon the integral equation. The density of charge will be represented as a linear combination of the set wavelet functions and scaling functions, and we obtain:

$$\rho(x0) = \sum_n a_n \emptyset_{j,n}(x0) + \sum_{m=j}^{2^j-1} \sum_n C_{m,n} \psi_{m,n}(x0) \tag{8}$$

The main mathematical properties which enable sparse matrix generation are the orthogonal and vanishing moment. A function $\psi(x)$ is said to have vanished moment of N order if:

$$\int_{-\infty}^{+\infty} x^n \psi(x) dx = 0 \quad \forall n = 0, 1 \dots (N - 1) \tag{9}$$

Applying Equation (8) into Equation (2), we obtain the set of matrix equation given by [14–16]:

$$\begin{bmatrix} [Z_{\emptyset,\emptyset}] & [Z_{\emptyset,\psi}] \\ [Z_{\psi,\emptyset}] & [Z_{\psi,\psi}] \end{bmatrix} \begin{bmatrix} a_n \\ c_{m,n} \end{bmatrix} = \begin{bmatrix} \langle V, \emptyset_{j,n} \rangle \\ \langle V, \psi_{m,n} \rangle \end{bmatrix} \tag{10}$$

where:

$$\begin{aligned}
 [Z_{\emptyset, \emptyset}] &= \left\langle \emptyset_{j,n}, \int_{\frac{a-w}{2}}^{\frac{a+w}{2}} \emptyset_{j,n} G(x, x_0) dx \right\rangle \\
 [Z_{\emptyset, \psi}] &= \left\langle \emptyset_{j,n}, \int_{\frac{a-w}{2}}^{\frac{a+w}{2}} \psi_{m,n} G(x, x_0) dx \right\rangle \\
 [Z_{\psi, \emptyset}] &= \left\langle \psi_{m,n}, \int_{\frac{a-w}{2}}^{\frac{a+w}{2}} \emptyset_{j,n} G(x, x_0) dx \right\rangle \\
 [Z_{\psi, \psi}] &= \left\langle \psi_{m,n}, \int_{\frac{a-w}{2}}^{\frac{a+w}{2}} \psi_{m,n} G(x, x_0) dx \right\rangle
 \end{aligned} \tag{11}$$

3. NUMERICAL RESULTS AND DISCUSSIONS

The geometrical and physical parameters of the microstrip line are summarized in Table 1.

Table 1. Parameters of structure.

Parameters	ε_{r1}	h (mm)	w/h	b (mm)	nfe	nfb	v	Frequency
Values	9.6	0.635	0.1 to 5	$25 + h$	10	250	3×10^8	2 GHz to 18 GHz

3.1. Moment Method

In this section, we study the convergence of moment method for various nfb values. Figure 2 shows the characteristic impedance in function of W/h , for several values of nfb.

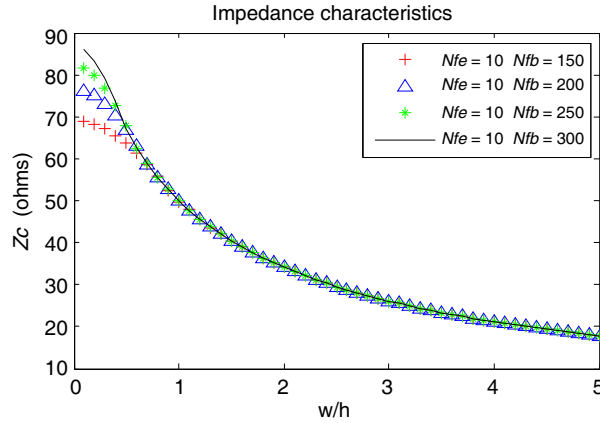


Figure 2. Characteristic impedance of microstrip line.

We observe that the characteristic impedance decreases when the ratio W/h increases. This variation is due to the microstrip line width, w , and when w increases, the capacity of the line increases. In Figure 2, we see that we need $Nfb = 250$ pulses per ribbon to obtain a stable solution. This result is in a good agreement with the ones published in [12], [19], and [15].

3.2. Moments Method and Daubechies Wavelet

Most applications of wavelet bases exploit their ability to efficiently approximate particular classes of functions with few nonzero wavelet coefficients. In this section, we study the merit of number of

vanishing moments in order to modeling the microstrip line bay the MoM.

It has been previously proved that a basis pulse functions of MoM is in a good agreement with the experimental results [21]. For this reason, the MoM was chosen as a reference for the accuracy evaluation. Figure 3 shows the characteristic impedance of the shielded microstrip line using MoM and Daubechies wavelet with one, two, three, and four vanishing moments. We see a good agreement between the two methods, especially for number of vanishing moments superior to two for Daubechies wavelet. These results agree with the previous work published in [12, 15, 19].

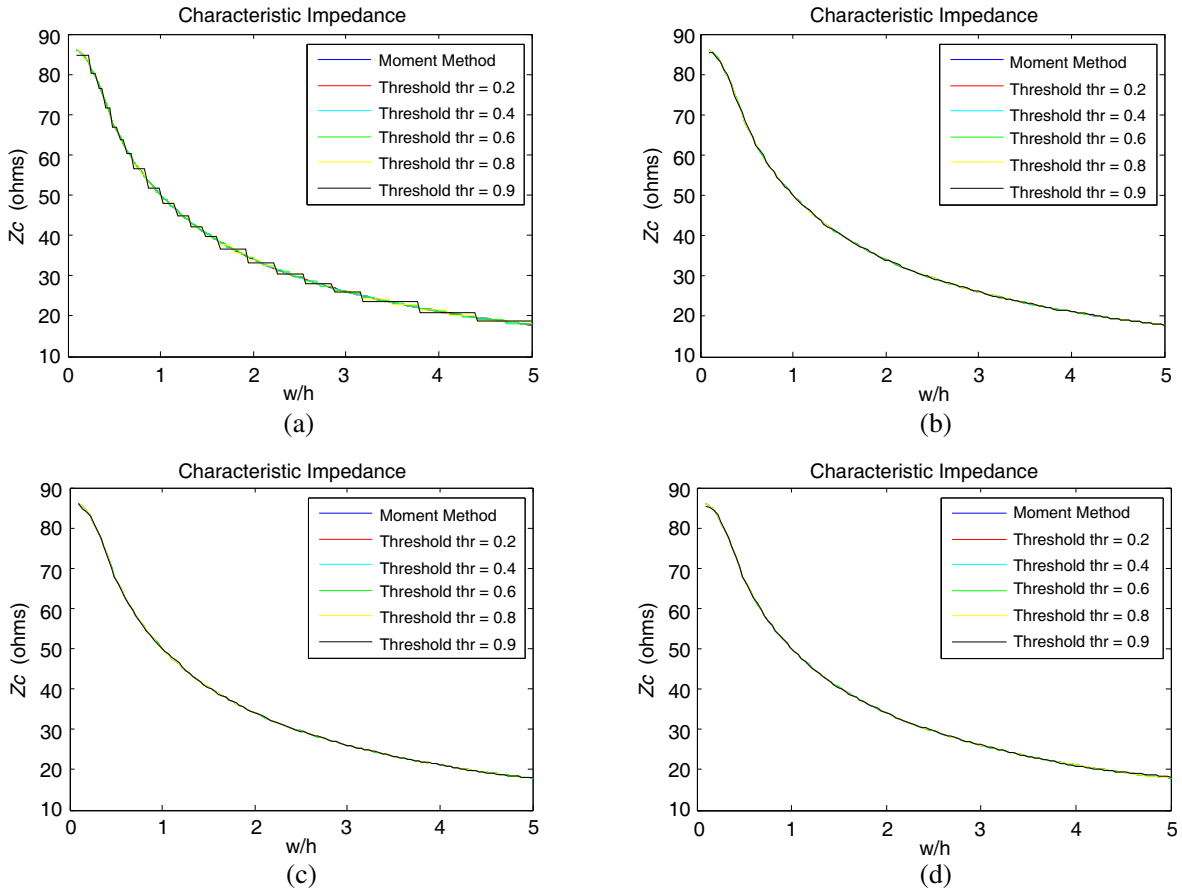


Figure 3. Characteristic impedance of the microstrip line versus w/h with different thresholds: (a) db1, (b) db2, (c) db3, and (d) db4.

Table 2. Comparison of the computation time, and sparsity the IM for conventional MoM, db1, db2, db3 and db4 wavelet bases in characterizing microstrip line.

	MoM	db1	db2	db3	db4
Sparsity %		46.14	66.16	70.80	73.60
CPU Time to reverse IM (ms)	7.931	7.554	5.252	4.559	3.648
CPU Time reduction %		4.75	33.77	42.51	54

The calculations are made on a PC computer with AMD Dual-Core 1.30 GHz CPU, 2 Gb memory and Windows 7 Professional system. Table 2 compares the performance of Daubechies wavelet bases with one, two, three, and four vanishing moments in terms of the computational time and percentage sparsity achieved in the IM. As seen in Table 2, the Daubechies wavelet expansion extracts the variation

of characteristic impedance more rapidly than the conventional MoM. On the other hand, it is clear that with increasing the vanishing moment's number of the Daubechies wavelets, the CPU Time decreases, and the sparsity of IM increases. In particular, db4 wavelet appears to be the most appropriate choice for solving the microstrip line integral equation. Fig. 4 shows the sparsity pattern of the most ill-conditioned IM for different Daubechies wavelets. By means of Fig. 4, priori locations of non-significant matrix elements have been estimated, and evaluation of small elements at impedance matrix has been avoided.

The relative error in function of the threshold for various Daubechies wavelets is shown in Fig. 5. We

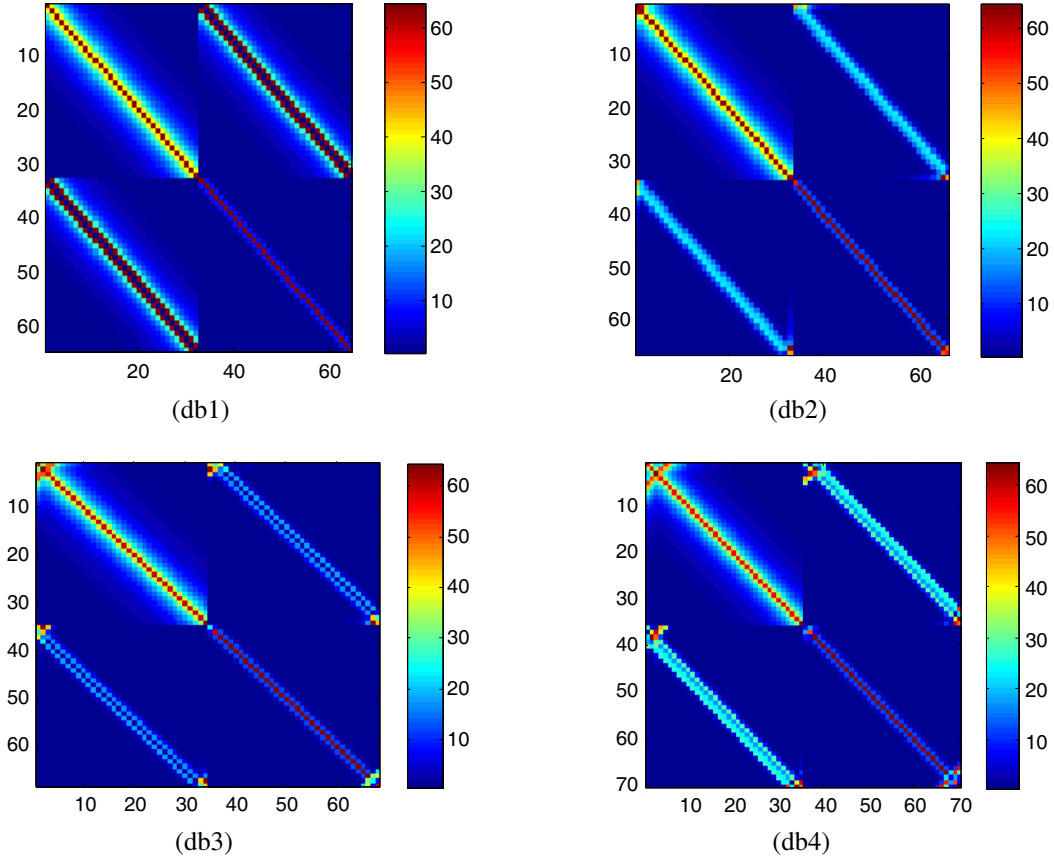


Figure 4. Gray-image of a typical coefficient matrix.

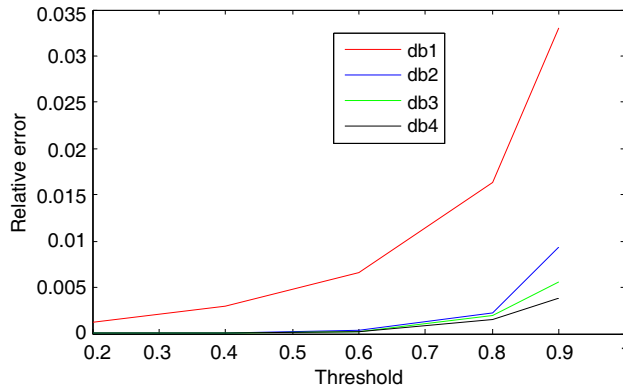


Figure 5. Relative error of the characteristic impedance versus threshold for different Daubechies wavelet.

observe that the relative error increases exponentially when increasing the threshold, due to canceling of the impedance matrix elements. On the other hand, it is clear that with increasing the vanishing moment's number of the Daubechies wavelets, the relative error decreases. These results agree with previous work published in [21].

4. CONCLUSIONS

In this paper, we propose a numerical method for evaluating the characteristic impedance of microstrip line based on Daubechies wavelet with one, two, three, and four vanishing moments. A rigorous integral equation formulation for the charge distribution on the microstrip line surface with finite thickness of an isotropic stratified medium is derived. The comparison between MoM and Daubechies wavelet provides a good agreement. A sparse matrix equation is attained from the microstrip line integral equation by using this technique (The sparsity of 73.60% is obtained using db4 wavelets). These examples demonstrate that the Daubechies wavelet with increasing number of vanishing moments works effectively in terms of computational time and numerical accuracy. The error decreases when the number of vanishing moments of Daubechies wavelet increases (Fig. 5). The propose method is suitable for the analysis of wideband metal-dielectric composite problems with high accuracy.

REFERENCES

1. Poza, D. M., *Microwave Engineering*, 4th edition, Wiley, Dec. 2011.
2. Francomanoa, E., A. Tortorici, et al., "Wavelet-like bases for thin-wire integral equations in electromagnetics," *Journal of Computational and Applied Mathematics*, Vol. 175, 77–86, Feb. 2004.
3. Gibson, W. C., *The Method of Moments in Electromagnetics*, 2nd edition, Taylor & Francis, 2015.
4. Rickard, Y., "An efficient wavelet-based solution of electromagnetic field problems," *Applied Numerical Mathematics*, Vol. 58, No. 4, 472–485, Apr. 2008.
5. Mallat, S., *A Wavelet Tour of Signal Processing: The Sparse Way*, Elsevier Inc., 2009.
6. Zhu, X., G. Lei, and G. Pan, "On application of fast and adaptive periodic Battle-Lemarie wavelets to modeling of multiple lossy transmission lines," *Journal of Computational Physics*, Vol. 132, 299–311, 1997.
7. Kusananl, S. R. and C. Nguyen, "Efficient space-domain analysis of microstrip lines using wavelet transform," *Microwave and Optical Technology Letters*, Vol. 9, No. 1, May 1995.
8. Oueslat, N. and T. Aguilì, "New implementation of the moment method based on the impedance operator to study the dispersion characteristics of microstrip lines," *International Journal of Computer Science*, Vol. 11, No. 5, Sep. 2014.
9. Harrington, R. F., *Field Computation by Moment Methods*, William Perkins, 1992.
10. Goswami, J. C. and A. K. Chan, *Fundamentals Wavelets Theory, Algorithms, and Applications*, John Wiley & Sons, 2011.
11. Pan, G., *Wavelets in Electromagnetics and Device Modeling*, John Wiley & Sons, Inc., 2003.
12. Gupta, K. C., et al., *Microstrip Lines and Slotlines*, 2nd edition, Artech House, 1996.
13. Daubechies, I., *Ten Lectures on Wavelets*, The Society for Industrial and Applied Mathematics, 1992.
14. Maurya, V. N. and A. K. Maurya, "Application of Haar wavelets and method of moments for computing performance characteristics of electromagnetic materials," *American Journal of Applied Mathematics and Statistics*, Vol. 2, No. 3, 96–105, 2014.
15. Metlevskis, E. and V. Urbanavicius, "Analysis of charge distribution on rectangular microstrip structures," *Physical Aspects of Microwave and Radar Applications*, Vol. 119, 2011.
16. Goswami, J. C., et al., "On solving first-kind integral equations using wavelets on a bounded interval," *IEEE Transactions on Antennas and Propagation*, Vol. 43, No. 6, Jun. 1995.

17. Lashab, M., F. Benabdelaziz, and C. Zebiri, "Analysis of electromagnetics scattering from reflector and cylindrical antennas using wavelet-based moment method," *Progress In Electromagnetics Research*, Vol. 76, 357–368, 2007.
18. Danesfahani, R., S. Hatamzadeh-Varmazyar, E. Babolian, and Z. Masouri, "Applying Shannon wavelet basis functions to the Method of Moments for evaluating the radar cross section of the conducting and resistive surfaces," *Progress In Electromagnetics Research B*, Vol. 8, 257–292, 2008.
19. Davidson, D. B. and J. T. Aberle, "An introduction to spectral domain Method-of-Moments formulations," *IEEE Antennas and Propagation Magazine*, Vol. 46, No. 3, Jun. 2004.
20. Oguzer, T., F. Kuyucuoglu, and I. Avcin, "Electromagnetic scattering from layered strip geometries: The Method of Moments study with the sinc basis," *Turk. J. Elec. Eng. & Comp. Sci.*, Vol. 19, No. 3, 2011.
21. Oberschmid, G. and A. F. Jacob, "Battele-Lemarié wavelets for the analysis of microstrip lines," *Institut fur Hochfrequenztechnik, TU Braunschwei*, Annual Report, 1995.
22. Park, I., R. Mittra, and M. I. Aksun, "Numerically efficient analysis of planar microstrip configurations using closed-form Green's functions," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 43, No. 2, Feb. 1995.
23. Li, Y. and S. Yangb, "Construction of symmetric or anti-symmetric B-spline wavelets and their dual wavelets," *International Journal of Computer Mathematics*, Vol. 88, No. 5, Mar. 2011.