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A Set of Simple Numerical Pattern Synthesis Algorithms for Anti-Jamming with Superdirective Receiving Array

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Abstract—Although a superdirective array can acquire maximum directive gain with electrically small array, in some practical applications, low sidelobe and deep nulls are also important, which can effectively inhibit directional interference. In this work, a set of simple superdirective pattern synthesis methods are proposed. By introducing diagonal loading factor and adding virtual jamming constraints, they can keep suitable tradeoff among directive gain, efficiency and anti-jamming performance. Besides, easy realization is another good feature of the proposed methods.

1. INTRODUCTION

Aiming to maximize directive gain and overcome external interference with electrically small arrays, the concept of superdirective pattern synthesis has been proposed [1–4]. It claims the theoretical possibility of arbitrarily high directivity from an array of given aperture or overall length, even if element spacing is sufficiently compact. Relative algorithms are termed super-gain arrays techniques [5]. Based on the theoretical assumption, sensor array with ultra-small aperture can also acquire the same directive gain or signal-to-noise ratio (SNR) as conventional array with half-wavelength aperture. This is quite attractive in engineering application. Especially in HF band, to achieve high angular resolution and suppress external noise, electrical size of conventional receiving array is tremendous as its working wavelength is 10–100 m. Huge array brings a lot of inconvenience, such as high cost, poor mobility, being vulnerable to attack, etc. Consequently, miniaturization of huge arrays generates considerable interest.

However, classical superdirective arrays have some inherent shortcomings, such as low radiation efficiency and poor robustness. For example, consider a 9-element linear broadside array of copper half-wave dipoles with a overall length of $1/4\lambda$. If the specified directive gain reaches 8.5 times greater than a single dipole, corresponding array efficiency is only 10^{-14}, and corresponding jitter amplitude of excitation currents must be controlled within 10^{-11} [5], which is technically unrealistic at present. Therefore, certain constraint conditions must be imposed to make super-gain array more feasible in practical situations. Newman et al., Zhou et al., etc. have introduced sensitivity constraint in optimization of maximum directive gain. Zhang et al. apply steering vector mismatch to model the uncertainty of a super-gain array. Most of these approaches can be classified as diagonal-loading method [6–10]. Based on these theoretical contributions, superdirective arrays gradually enter engineering application stage. Meanwhile, people find that constrained optimal directivity (CODG) methods still need to be improved. Under given array distribution, such as uniform circular array, their first sidelobe levels remain almost unchanged, which is irrelevant with elements number. This provides array with rather limited anti-jamming effect. After all, ultimate design objective is to reserve signal of interest (SOI) while suppressing or even eliminating external noise and disturbance. Therefore, it will be meaningful to study some optimal algorithms which can realize sidelobe or null control.

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Refs. [11, 12] adopt the second-order cone programming (SOCP) in reduction of sidelobe levels. But its parameter selection criteria are rather complex, and the method requires array to be accurately calibrated, which is not practical in engineering. Traditional linear constrained optimal directive gain (L-CODG) method can easily form deep nulls at prescribed direction [13–16], but it can hardly satisfy requirements of robustness and efficiency indicators. Here, a set of new pattern synthesis techniques are proposed. The first method forms deep and broaden nulls to suppress high-power interference from centralized direction. The second method uses low sidelobe level to inhibit dispersed interference. By introducing diagonal loading factor, superdirective arrays obtain effective compromise among efficiency, robustness and anti-jamming performance. Besides, the proposed methods are simpler in engineering implementation.

The rest of this paper is arranged as follows. In Section 2, problems and existing solutions are reviewed. Section 3.1 introduces the nulling based method. Section 3.2 analyzes performance of the proposed method by numerical examples. Similarly, Section 4.1 gives the low-sidelobe based method. Relative numerical analysis is shown in Section 4.2. Section 5 draws a final conclusion.

2. PROBLEM FORMULATION

In order to facilitate analysis, we assume that an antenna array consists of M isotropic elements which uniformly distribute at known locations. Applying a set of complex excitation weights $\mathbf{w}$, radiation pattern of the array can be steered towards a predetermined direction (θ_0, ϕ_0). For HF superdirective array, directivity $G(\theta_0, \phi_0)$ and efficiency η are key indexes, and they should meet following relationship:

$$G(\theta_0, \phi_0) = \frac{\mathbf{w}^{\mathrm{H}}\mathbf{N}\mathbf{w}}{\mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w}}, \ \eta = \frac{\mathbf{w}^{\mathrm{H}}\mathbf{N}\mathbf{w}}{M\mathbf{w}^{\mathrm{H}}\mathbf{w}} \tag{1}$$

where $\mathbf{N} = \mathbf{a}(\theta_0, \phi_0)\mathbf{a}^{\mathrm{H}}(\theta_0, \phi_0)$ and $\mathbf{R} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} \sin\theta \mathbf{a}(\theta, \phi)\mathbf{a}^{\mathrm{H}}(\theta, \phi)d\theta d\phi$. $\mathbf{a}(\theta, \phi)$ is steering vector of the array, and $(\cdot)^{\mathrm{H}}$ denotes Hermitian transpose. Using distortionless constraint $\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_0, \phi_0) = 1$ over signal of interest (SOI) direction, the above formula can be simplified as:

$$G(\theta_0, \phi_0) = \frac{1}{\mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w}}, \ \eta = \frac{1}{M\mathbf{w}^{\mathrm{H}}\mathbf{w}} = \frac{1}{MK} \tag{2}$$

where K represents sensitivity indicator. The smaller K is, the more robust the array will be.

For a certain superdirective array, classical optimal directive gain (ODG) method only seeks the maximization of $G(\theta_0, \phi_0)$. It brings about low efficiency and high sensitivity to array uncertainty, both of which are unacceptable. Omitting derivation, its final computation formula is:

$$\mathbf{w}_{opt} = \mathbf{R}^{-1}\mathbf{a}(\theta_0, \phi_0), \qquad G_{opt}(\theta_0, \phi_0) = \mathbf{a}^{\mathrm{H}}(\theta_0, \phi_0)\mathbf{R}^{-1}\mathbf{a}(\theta_0, \phi_0) \tag{3}$$

In order to improve its engineering practicality, sensitivity-constrained optimal directive gain (S-CODG) method is proposed, which is written as:

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{R}\mathbf{w}, \qquad \textit{subject to} \quad \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_0, \phi_0) = 1, \qquad \|\mathbf{w}\|^2 = K. \tag{4}$$

The corresponding excitation weight is:

$$\hat{\mathbf{w}}_{opt} = \frac{(\mathbf{R} + \lambda\mathbf{I})^{-1}\mathbf{a}(\theta_0, \phi_0)}{\mathbf{a}^{\mathrm{H}}(\theta_0, \phi_0)(\mathbf{R} + \lambda\mathbf{I})^{-1}\mathbf{a}(\theta_0, \phi_0)} \tag{5}$$

where λ is a scalar multiplier associated with K. Although smaller K value can bring better robustness and higher efficiency, it will also make directive gain worse. A practical super-gain array just needs to guarantee the dominance of system background noise, i.e., array efficiency η always has prescribed minimum (maximum for K) in different frequency bands. For example, at 10 MHz, external receiver noise is typically -55 dB larger than internal receiver noise. Assume that each dipole element is connected to a high-impedance preamplifier with a noise figure of 10 dB, and a 10 dB essential "cushion" is needed to ensure that external noise dominates. Then, the specified array efficiency η_0 should be no less than -35 dB. Further, according to [8], if minimum array efficiency η_0 is given, selection interval of sensitivity K is subjected to:

$$\frac{1}{M} \leq K \leq \min\left(\frac{1}{M\eta_0}, \frac{\mathbf{a}^{\mathrm{H}}(\theta_0, \phi_0)\mathbf{R}^{-2}\mathbf{a}(\theta_0, \phi_0)}{[\mathbf{a}^{\mathrm{H}}(\theta_0, \phi_0)\mathbf{R}^{-1}\mathbf{a}(\theta_0, \phi_0)]^2}\right) \tag{6}$$

Still revolving around robustness problem, Ref. [10] proposes a uncertainty-constrained optimal directive gain (U-CODG) method. Considering steering vector mismatch always exists in real array system, we can only get its estimate. Assume $\mathbf{a}(\theta_0, \phi_0)$ as desired steering vector, and $\hat{\mathbf{a}}$ represents mismatched steering vector. The relationship between $\mathbf{a}$ and $\hat{\mathbf{a}}$ will meet following uncertainty constraint:

$$[\hat{\mathbf{a}} - \mathbf{a}(\theta_0, \phi_0)]^{\mathrm{H}} \mathbf{C}^{-1} [\hat{\mathbf{a}} - \mathbf{a}(\theta_0, \phi_0)] \leq 1 \tag{7}$$

where $\mathbf{C}$ represents constraint matrix. Apparently, it is a Euclid ellipsoidal constraint problem. Without losing generality, we make $\mathbf{C} = \mu \mathbf{I}$, where $\mathbf{I}$ is identity matrix, and μ takes the maximum axle length of ellipsoidal. Thus, the above inequality can degenerate into sphere constraint problem:

$$\|\hat{\mathbf{a}} - \mathbf{a}(\theta_0, \phi_0)\|^2 \leq \mu \tag{8}$$

On the other hand, in Formula (3), replacing $\mathbf{a}(\theta_0, \phi_0)$ with $\hat{\mathbf{a}}$ will yield estimation value of directive gain $\hat{G}_{opt}$. Considering these two aspects, the U-CODG method can be represented as:

$$\max_{\hat{\mathbf{a}}} \hat{G}_{opt} \qquad subject \ \ to \quad \|\hat{\mathbf{a}} - \mathbf{a}(\theta_0, \phi_0)\|^2 \leq \mu \tag{9}$$

Omitting derivation, the corresponding excitation weight is represented as:

$$\hat{\mathbf{w}} = \frac{(\mathbf{R} + \frac{1}{\lambda}\mathbf{I})^{-1} \mathbf{a}(\theta_0, \phi_0)}{\mathbf{a}^H(\theta_0, \phi_0)(\mathbf{R} + \frac{1}{\lambda}\mathbf{I})^{-1} \mathbf{R} (\mathbf{R} + \frac{1}{\lambda}\mathbf{I})^{-1} \mathbf{a}(\theta_0, \phi_0)} \tag{10}$$

where λ denotes scalar multiplier associated with μ. As seen, the solution is very similar to S-CODG method. In Formula (10), matrix $\mathbf{R}$ can further be decomposed as $\mathbf{R} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^{\mathrm{H}}$, in which $\mathbf{U}$ represents the eigenvector matrix, and $\mathbf{\Lambda}$ is a diagonal matrix composed by the eigenvalues of $\mathbf{R}$. Make $\mathbf{z} = \mathbf{U}^{\mathrm{H}} \mathbf{a}(\theta_0, \phi_0)$. Based on the analysis of [10], by choosing a proper mismatch value of $\mu / \|\mathbf{z}\|^2$, the same robustness and efficiency can be acquired as S-CODG method.

Although the above methods effectively improve the engineering value of a super-gain array, they still have some deficiency. On one hand, S/U-CODG can make super-gain array have sufficiently high radiation efficiency and robustness against random variations of array. On the other hand, their constant sidelobe level depth provides limited anti-jamming effect under strong interference environment. Therefore, the ability of forming deep nulls and low sidelobe levels at prescribed direction will be more beneficial in some situations.

3. NUMERICAL METHOD 1

3.1. Theory and Implementation

Assuming that there are P external signals not of interest (SNOI) which are from (θ_i, ϕ_i), $i = 1, 2, \cdots, P$, the constraints providing level control over sidelobe and null directions can be written as:

$$\mathbf{w}^{\mathrm{H}} \mathbf{a}(\theta_0, \phi_0) = 1, \ \mathbf{w}^{\mathrm{H}} \mathbf{a}(\theta_i, \phi_i) = \epsilon_i, \ i = 1, 2, \cdots, P \tag{11}$$

Making $\mathbf{A} = [\mathbf{a}(\theta_0, \phi_0), \mathbf{a}(\theta_1, \phi_1), \cdots, \mathbf{a}(\theta_P, \phi_P)]$ and $\mathbf{g} = [1, \epsilon_1, \cdots, \epsilon_P]^{\mathrm{H}}$, the above equation can be further simplified as $\mathbf{A}^{\mathrm{H}} \mathbf{w} = \mathbf{g}$.

In order to maximize directive gain, we must minimize denominator of $G(\theta_0, \phi_0)$. Consequently, we get:

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}} \mathbf{R} \mathbf{w}, \quad subject \ to \ \ \mathbf{A}^{\mathrm{H}} \mathbf{w} = \mathbf{g} \tag{12}$$

Lagrange method can be applied to solve it for $\mathbf{w}$. Ignoring derivation, the result can be denoted as:

$$\mathbf{w} = \mathbf{R}^{-1} \mathbf{A} (\mathbf{A}^{\mathrm{H}} \mathbf{R}^{-1} \mathbf{A})^{-1} \mathbf{g} \tag{13}$$

Further, array efficiency η can be rewritten as:

$$\eta = \frac{1}{M \mathbf{g}^{\mathrm{H}} \left[\mathbf{A}^{\mathrm{H}} \mathbf{R}^{-1} \mathbf{A}\right]^{-1} \left[\mathbf{A}^{\mathrm{H}} \mathbf{R}^{-2} \mathbf{A}\right] \left[\mathbf{A}^{\mathrm{H}} \mathbf{R}^{-1} \mathbf{A}\right]^{-1} \mathbf{g}} \tag{14}$$

Aiming to improve array efficiency and robustness, diagonal loading factor Δd is introduced. Therefore, Equation (14) can be corrected as:

$$\eta' = \frac{1}{M\mathbf{g}^{\mathrm{H}}[\mathbf{A}^{\mathrm{H}}(\mathbf{R}+\Delta d\mathbf{I})^{-1}\mathbf{A}]^{-1}[\mathbf{A}^{\mathrm{H}}(\mathbf{R}+\Delta d\mathbf{I})^{-2}\mathbf{A}][\mathbf{A}^{\mathrm{H}}(\mathbf{R}+\Delta d\mathbf{I})^{-1}\mathbf{A}]^{-1}\mathbf{g}} = \frac{1}{Mf(\Delta d)} \quad (15)$$

The function $\eta' = 1/[Mf(\Delta d)]$ is monotone increasing as Δd in interval $[0, +\infty]$, which can be verified by numerical method. Therefore, for a given array efficiency η_0, the following inequality should be satisfied:

$$\lim_{\Delta_d \to 0} \frac{1}{Mf(\Delta d)} \le \eta_0 \le \lim_{\Delta_d \to +\infty} \frac{1}{Mf(\Delta d)} \quad (16)$$

By numerical simulation tests, when $\Delta d \ge 1$, η_0 will approach the maximum value.

As a conclusion, we implement the proposed method in the following steps:

Step 1) According to practical SNOI direction, set nulling constraints by formula $\mathbf{A}^{\mathrm{H}}\mathbf{w} = \mathbf{g}$.

Step 2) Based on the inequality in Eq. (16), set suitable efficiency η_0, initial diagonal loading value Δd and incremental step δ.

Step 3) Make $\Delta d = \Delta d + \delta$; use function in Eq. (15) to compute practical η'.

Step 4) If $\eta' \le \eta_0$, return to Step 3); otherwise, iteration ends and go to Step 5).

Step 5) Use Δd to compute final weight vector $\mathbf{w}$:

$$\mathbf{w} = (\mathbf{R}+\Delta d\mathbf{I})^{-1}\mathbf{A}(\mathbf{A}^{\mathrm{H}}(\mathbf{R}+\Delta d\mathbf{I})^{-1}\mathbf{A})^{-1}\mathbf{g} \quad (17)$$

3.2. Numerical Examples and Analysis

In order to analyze practical performance of the proposed method, assume a compact circular array model which consists of 11 idealized short vertical dipole elements. The array radius is 4 m, and working frequency is 12 MHz. SOI is from 180°, and SNOI is from 300°. Three virtual interference constraints are added to form a broaden null with −30 dB depth. Disturbance spacing is 10°. Desired array efficiency should be above 24%. Make initial loading value $\Delta d = 5 \times 10^{-3}$ and incremental step $\delta = 10^{-3}$. By computation, when the total iteration number is 95 and final $\Delta d = 0.1$, specified efficiency is satisfied. With reference to Formula (2), the corresponding array sensitivity factor is 0.378, and directive gain is 17.7 dB. Fig. 1(a) shows corresponding beampatterns. Other patterns by S-CODG and U-CODG methods are drawn together to form contrast.

In order to present the comparative results better, Table 1 lists specific performance indicators. As can be seen, S/U-CODG methods satisfy the same array efficiency as the proposed method does. However, their sidelobe levels located at SNOI direction are much higher (−14.61 dB, −14.09 dB

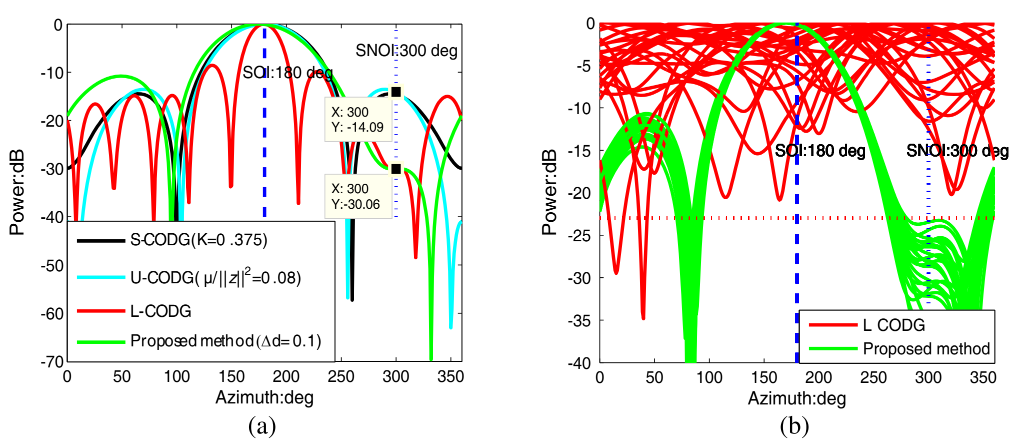

Figure 1. Numerical examples. (a) Pattern Contrast. (b) Robustness Contrast.

Table 1. Performance indicators ($\Delta d = 0.1$).

Method	Directive Gain(dB)	Sensitivity	HPBW(deg)	Efficiency(%)	Null Depth(dB)
S-CODG	18.6	0.375	69	24.25	−14.61
U-CODG	19.0	0.698	66	24.20	−14.09
L-CODG	27.5	2.1×10^4	29	$4.3{\times}10^{-4}$	−30.0
DL-CODG	17.7	0.378	72	24.03	−30.07

(HPBW: half-power beamwidth.)

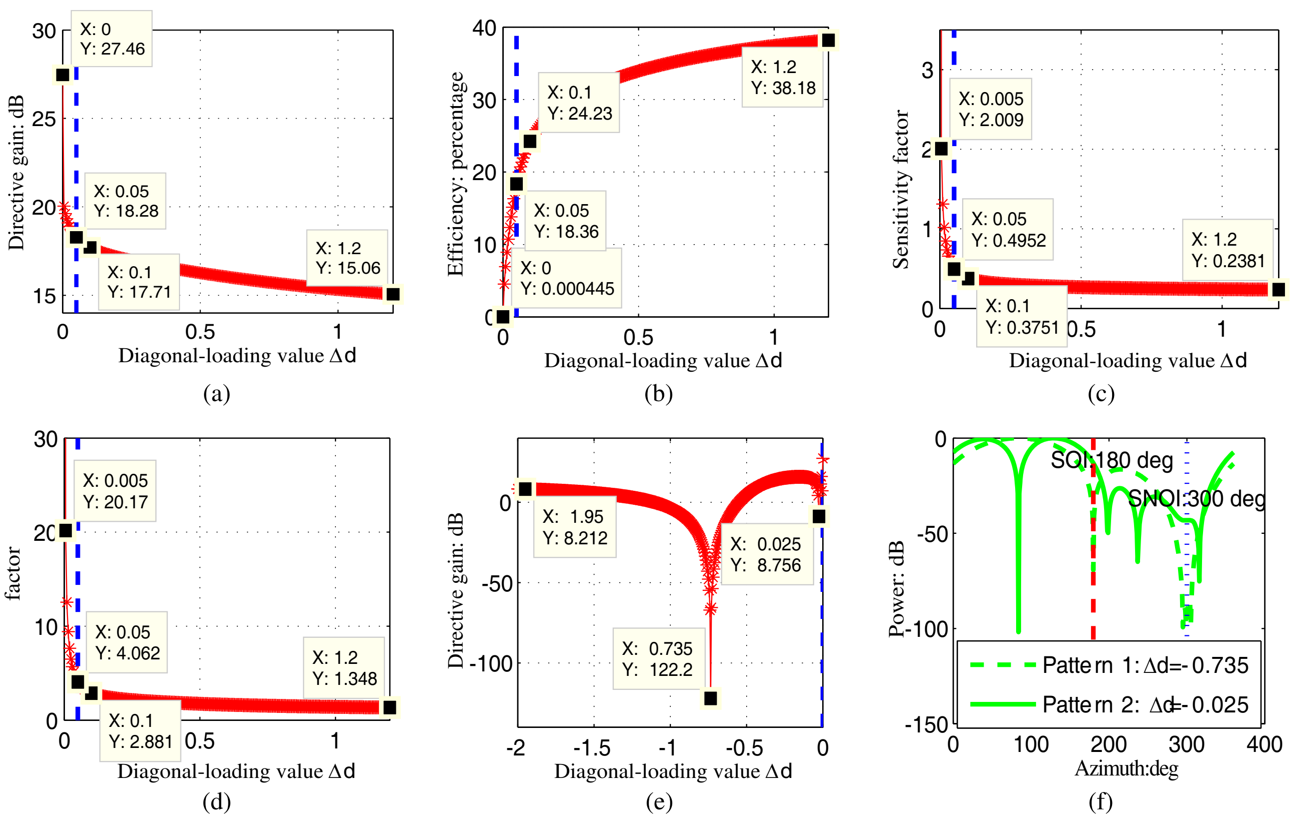

Figure 2. Performance analysis. (a) Directivity corresponding to a given Δd. (d) η' corresponding to a given Δd. (c) Sensitivity corresponding to a given Δd. (d) Q factor corresponding to a given Δd. (e) Directivity corresponding to a negative Δd. (f) Patterns under negative Δd

respectively), which verifies their limited anti-jamming performance. Although the pattern by L-CODG method has deep enough null level (−30.0 dB) at interference direction, its array efficiency (only $4.3{\times}10^{-4}$) is far below specified value. Besides, its considerably high sensitivity indicator (2.1×10^4) also reflects that the pattern will be very unstable when confronting array uncertainty. For example, we add −35 dB (Standard Deviation 0.0178) random amplitude errors and 3° (Standard Deviation 0.0524) random phase errors into array model. Both meet independent Gaussian distribution. Fig. 1(b) shows corresponding patterns under 20 times Monte-Carlo tests. Obviously, the pattern by L-CODG has serious distortion while that of DL-CODG still keeps robust. Therefore, the proposed method makes a better tradeoff among efficiency, robustness and anti-jamming ability.

Further, to evaluate the performance of the proposed method more comprehensively, some relevant indicator curves such as directive gain, array efficiency, sensitivity, Q factor are plotted in Fig. 2. As seen, when $\Delta d = 1.2$, efficiency index is 38.18%, and sensitivity is 0.2381. When $\Delta d = 0$, the proposed method will convert into L-CODG method. On this condition, although directivity is up to 27.46 dB, radiation efficiency is only $4.45 \times 10^{-4}\%$, which cannot be realized in practical engineering. When $\Delta d < 0$, as shown in Fig. 2(e), array directivity is generally smaller than that of positive loading. In interval $\Delta d \in [-1.1, -0.5]$, directive gain is negative, which means that mainlobe distortion occurs. Especially when $\Delta d = -0.735$, directive gain has minimum -122.2 dB, which even forms a deep null at desired SOI position. Corresponding beampattern is plotted in Fig. 2(f). Consequently, in order to keep high directive gain and avoid pattern distortion, positive loading value should be selected.

4. NUMERICAL METHOD 2

4.1. Theory and Implementation

As can be seen from above, virtual nulling techniques are suitable for high power interference, especially when SNOI comes from a concentrated direction. If interferences are more, and they present dispersion distribution, low sidelobe will produce more significant suppression effect. Corresponding mathematical expression can be written as:

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}(\mathbf{R} + \Delta d\mathbf{I})\mathbf{w}, \quad \textit{subject to} \;\; \mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_0, \phi_0) = 1, \;\; |\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_i, \phi_i)| \leq SLL \tag{18}$$

where (θ_0, ϕ_0) denotes desired mainlobe position, and (θ_i, ϕ_i) belongs to specified sidelobe region. SLL is specified maximum sidelobe level.

In order to solve the above problem, the following numerical iteration steps are adopted:

Step 1) Assume that P virtual interferences ($P \geq 6$) are located in specified sidelobe region. Set initial nulling constraints:

$$\min_{\mathbf{w}_0} {\mathbf{w}_0}^{\mathrm{H}}(\mathbf{R} + \Delta d\mathbf{I})\mathbf{w}_0, \quad \textit{subject to} \;\; {\mathbf{w}_0}^{\mathrm{H}}\mathbf{a}(\theta_0, \phi_0) = 1, \;\; |{\mathbf{w}_0}^{\mathrm{H}}\mathbf{a}(\theta_i, \phi_i)| = l_0 < SLL \tag{19}$$

where l_0 denotes initial null depth which should be smaller than SLL. According to Formula (17), get initial weight vector $\mathbf{w}_0$.

Step 2) Use weight vector $\mathbf{w}_0$ to compute amplitude response and find local maximum l_j located at (θ_j, ϕ_j) in each sidelobe (Boundary points cannot be ignored). Compute corresponding difference level between l_j and SLL:

$$\Delta l_j = \frac{l_j}{|l_j|}(SLL - |l_j|) \tag{20}$$

Step 3) Use the following constraints to compute step weight vector $\Delta\mathbf{w}$:

$$\min_{\Delta\mathbf{w}} \Delta\mathbf{w}^{\mathrm{H}}(\mathbf{R} + \Delta d\mathbf{I})\Delta\mathbf{w}, \quad \textit{subject to} \;\; \Delta\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_0, \phi_0) = 0, \;\; |\Delta\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_j, \phi_j)| = \Delta l_j \tag{21}$$

Step 4) Update weight vector: $\mathbf{w} = \mathbf{w}_0 + \Delta\mathbf{w}$. Compute amplitude response in sidelobe region. If response level is less than specified SLL, iteration ends. Otherwise, return to Step 2).

4.2. Numerical Examples and Analysis

Still use the array model in Section 3: 11-element circular array, 4 m array radius, 12 MHz working frequency. Assume mainlobe direction as 180°. Specified sidelobe region is located in interval $[0°, 100°] \bigcup [260°, 360°]$, and desired maximum sidelobe level is $SLL \leq$-30 dB. In order to satisfy the constraints, we set 6 virtual interferences $(20°, 60°, 100°, 260°, 300°, 340°)$ which are evenly distributed in sidelobe region. Diagonal loading value $\Delta d = 0.1$ and initial null depth is -55 dB. After several iterations, array pattern will become stable, which is shown in Fig. 3(a). As can be seen, L-CODG and the proposed method are both favorable on sidelobe level index. However, on array efficiency, L-CODG is only 0.287%, and the proposed method is 4.8%. On sensitivity, L-CODG is 31.66, and the proposed method is 1.89. Apparently, the proposed method has higher array efficiency and better robustness.

In order to further verify the above conclusion, Table 2 lists specific indicators of both methods under different SLL levels. We can find that as SLL decreases, array efficiency and robustness by the proposed method correspondingly increase (η' increases by 10.3%, and K falls by 8.89). Meanwhile, the array still keeps a relatively high directive gain (only falls 3.55 dB). On the other hand, L-CODG method has no obvious performance improvement. When $SLL = -30$ dB, it reaches the maximum

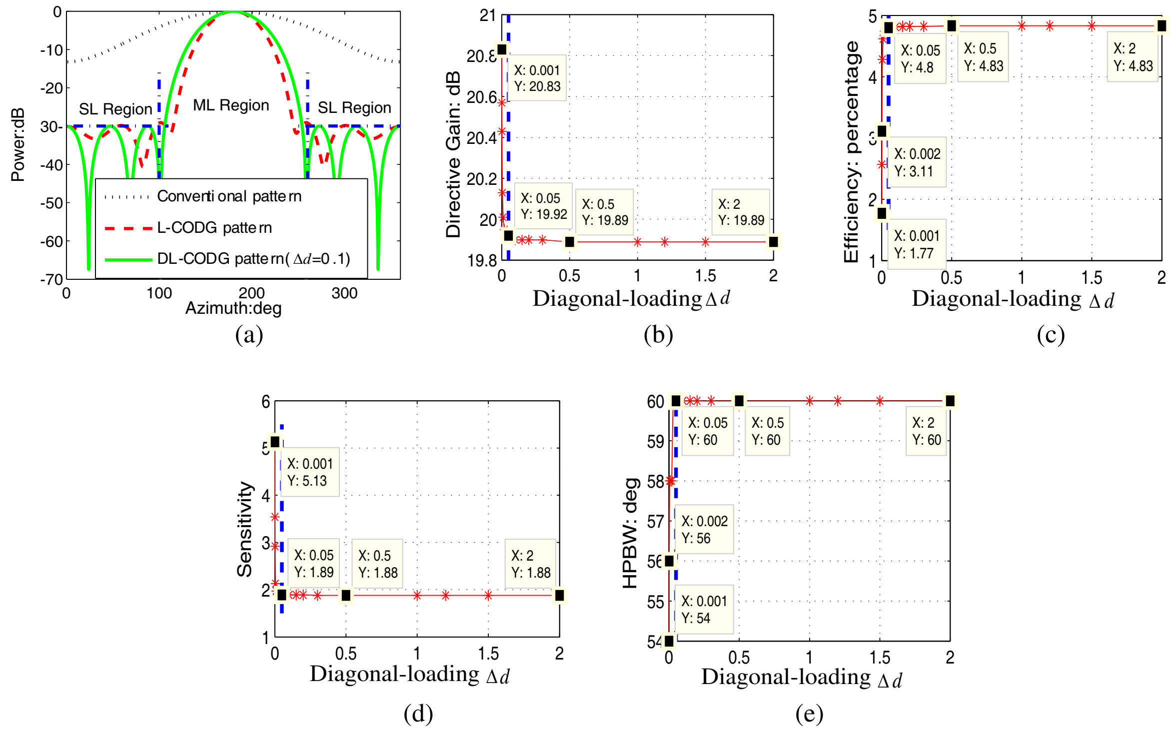

Figure 3. Performance analysis($SLL = -30$ dB). (a) Pattern contrast. (b) Directivity corresponding to a given Δd. (c) η' corresponding to a given Δd. (d) Sensitivity corresponding to a given μ. (e) HPBW corresponding to a given Δd.

Table 2. Performance indicators ($\Delta d = 0.1$).

Specified SLL (dB)	Method	Directive Gain (dB)	Sensitivity	HPBW (deg)	Efficiency(%)
-15	L-CODG	20.37	2499	54	0.004
	DL-CODG	22.17	9.7	48	0.935
-25	L-CODG	21.47	104.3	50	0.087
	DL-CODG	19.82	1.53	60	5.95
-35	L-CODG	21.7	40.54	50	0.22
	DL-CODG	19.37	1.32	62	6.862
-40	L-CODG	21.74	78.68	49	0.12
	DL-CODG	18.95	1	64	9.06
-45	L-CODG	21.76	104	48	0.087
	DL-CODG	18.62	0.81	66	11.24

(HPBW: half-power beamwidth.)

efficiency 0.287% and minimum sensitivity 31.66, which is still worse than the proposed method on condition of $SLL = -15$ dB.

In addition to reducing SLL level, choosing suitable loading value Δd is also helpful in improving array efficiency and robustness, which can be verified in Figs. 3(b)–3(e). As can be seen, when $\Delta d \in [0, 0.005]$, increasing loading value will have significant effect on array performance. Efficiency increases from 1.77% to 4.8%, and sensitivity decreases from 5.13 to 1.89. When $\Delta d \geq 0.05$, the change trend of each indicator will be flat. When $\Delta d \geq 0.5$, array performance will be no longer improved as Δd increases.

Therefore, aiming to overcome low efficiency and poor robustness of a superdirective array, setting lower sidelobe and larger diagonal loading value is recommended.

5. CONCLUSION

In this work, a set of simple superdirective pattern synthesis methods are proposed. By adding virtual interferences and diagonal loading value as constraints, an array can make a better tradeoff among directive gain, efficiency and anti-jamming performance. The proposed methods adopt numerical iteration solution and have simple parameter selection criteria, which is convenient for engineering realization.

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