# Length Estimation of Ballistic Targets Based on Full-Polarization Range Profiles

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**Abstract**—This study focuses on the length estimation of ballistic targets based on the full-polarization range profiles measured by the wideband full-polarization radar system. Firstly, the mathematical model of full-polarization range profiles is introduced, and the full-polarization range profiles characteristics of typical ballistic targets are analyzed by using the microwave anechoic chamber measurement data. Secondly, three methods are proposed for target length estimation based on single-channel detection synthesis, SPAN power synthesis and target characteristic polarization, respectively. Then, comparison experiments among the proposed methods are carried out. The results demonstrate that the extraction accuracy and the anti-noise performance of the method based on target characteristic polarization are better than the others. Furthermore, the influence of the signal-to-noise ratio (SNR) on the length estimation is also discussed.

## 1. INTRODUCTION

Feature extraction and recognition of ballistic targets are very important in spatial safety. Micro-motion and physical features are recognized as the most effective features. The micro-motion effect and micro-Doppler phenomenon of ballistic target have been studied in deep, and many methods for micro-motion feature extraction have been proposed [1, 2]. The physical features include physical size and scattering structure which can only be obtained by wideband and polarization radar. The physical size can be extracted from high resolution range profile (HRRP) [3], inverse synthetic aperture radar (ISAR) image [4] and three-dimensional image [5], while the HRRP is the easiest one.

HRRP reflects the distribution of the projection of scattering centers onto the line of sight (LOS), which includes the number of scattering centers and geometric structural features such as spatial distribution rule and radial length. The recognition technique based on HRRP is one of the research hotspots in target recognition area [6–9]. With the development of polarization measurement technology, wideband full-polarization radar has become an important direction of advanced radar technology. Recently, feature extraction and target recognition based on full-polarization range profiles has become one of the most important and difficult issues.

Radar target recognition methods based on range profiles mainly include: 1) the methods based on template matching, which consider the ensemble of range profile as a feature vector, and use the feature vector for target matching recognition [9]; 2) the methods based on feature transformation, such as Mellin transformation, wavelet transformation and K-L transformation [10]; 3) the methods based on physical characteristics, such as the number of scattering centers (which reflects the complication of target structure) and length of range profile. Length feature is one of the intrinsic attributes of a target and one of the most immediate foundations for discriminating the real and false targets [11]. Rough classification of targets can be achieved by utilizing the length information. The key point to obtaining

the radial length of target is to confirm the number of range cells that target includes. In practice, the existence of noise and other factors makes the length estimation difficult. Moreover, the extension of a target usually occupies only a part of a range profile. Therefore, how to choose a suitable threshold value to separate the target from the noise is essentially important for the length feature extraction. Although some researches on the length feature extraction of radar targets have been published, there is little work based on full-polarization range profiles offered by the wideband full-polarization radar systems.

The remainder of this paper is organized as follows. Section 2 introduces the mathematic model of full-polarization range profiles and analyzes the full-polarization range profiles characteristics of typical radar targets by using the data which is measured in the microwave anechoic chamber. Section 3 proposes three methods for target length estimation which are respectively based on single-channel detection synthesis, SPAN power synthesis and characteristic polarization. Section 4 analyzes the performances of the proposed methods through dynamic simulation. Section 5 concludes this paper.

## 2. ANALYSIS OF FULL-POLARIZATION RANGE PROFILES CHARACTERISTICS OF RADAR TARGETS

In general, radar target has a polarization varied effect when it is excited by electromagnetic wave. The transformation relationship can be described by a coherent polarization Sinclair Matrix which could reflect the shape, structure, dielectric property and space orientation of a target. Wideband high resolution radar usually works at optical frequencies, and the electromagnetic characteristic of a target can be approximately described by the scattering center model. In other words, the scattering echo from a radar target at high frequencies can be assumed as the sum of the echoes of a number of scattering centers. Therefore, it is equivalent to a distributed target that includes several continuous resolution cells rather than a point target, which is the foundation for the length feature extraction of radar targets.

For simplicity, we assume that radar target consists of $N$ scattering centers which distribute along the radial direction of the radar light-of-sight, and the coherent scattering matrices of the scattering centers are $s_i = \begin{bmatrix} S_{HHi} & S_{HVi} \\ S_{VHi} & S_{VVi} \end{bmatrix}$, $i = 0, \ldots, N-1$. For reciprocity target, $S_{HVi} = S_{VHi}$; the distances from other scattering centers to the first one are $r_i = \frac{c(\tau_i - \tau_0)}{2}$, $c = 3 \times 10^8$ m/s, $i = 1, \ldots, N-1$, $\tau_0 = \frac{2R_0}{c}$, and $R_0$ is the distance between the first scattering center and radar. Therefore, the system response function of target is

$$h(t) = \sum_{i=0}^{N-1} s_i \delta\left(t - \tau_i\right) \tag{1}$$

If the incident wideband electromagnetic signal is $e_i(t)$, the backscattered wave is

$$e_o(t) = h(t) * e_i(t) = \sum_{k=0}^{N-1} s_k \delta\left(t - \tau_k\right) * e_i(t) = \sum_{k=0}^{N-1} s_k e_i\left(t - \tau_k\right) \tag{2}$$

where $(*)$ represents the signal convolution.

Then, the spectrum of scattering echo is

$$E_o(\omega) = FFT\left[h(t) * e_i(t)\right] = \left(\sum_{k=0}^{N-1} s_k e^{-j\omega\tau_k}\right) E_i(\omega)$$

$$= \left(\sum_{k=0}^{N-1} s_k e^{-j\omega(\tau_k - \tau_0)}\right) E_i(\omega) e^{j\omega\tau_0} = S(\omega) E_i(\omega) e^{j\omega\tau_0} \tag{3}$$

where $E_i(\omega)$ is the spectrum of incident wideband signal $e_i(t)$, and the wideband polarization scattering formula is

$$S(\omega) = \sum_{k=0}^{N-1} s_k e^{-j\omega(\tau_k - \tau_0)} \tag{4}$$

So the corresponding signal in time filed is

$$s(t) = \sum_{k=0}^{N-1} s_k \delta \left(t - \tau'_k\right) \tag{5}$$

where $\tau'_k = \tau_k - \tau_0$.

Thereby, the full-polarization range profiles of reciprocity target can be expressed as

$$h(l) = \begin{bmatrix} H_{HH}(l) \\ \sqrt{2}H_{HV}(l) \\ H_{VV}(l) \end{bmatrix} = \sum_{k=0}^{N-1} \begin{bmatrix} |S_{HH}(k)| \\ |\sqrt{2}S_{HV}(k)| \\ |S_{VV}(k)| \end{bmatrix} \delta(l - r_k) \tag{6}$$

As shown in Equations (4)~(6), the polarization scattering characteristic of a target depends on the number, location and structure of the scattering centers which are determined by the target itself. Range profile relates to these mentioned factors and is sensitive to the polarization. In other words, range profile is quite different at different polarization channels. Fig. 1 shows that the full-polarization range profiles of some space-target model in different poses which are measured in a microwave anechoic chamber. The frequency is $8.75 \sim 10.75$ GHz. Frequency interval is $20$ MHz. Azimuth in the direction of the nose cone is $0°$. The step interval is $0.2°$, and the measured angle range is from $0°$ to $180°$.

According to Fig. 1, it is difficult to describe the scattering characteristic of target completely via the range profile of single polarization channel. Utilizing full-polarization operation mode can obtain more overall target characteristics.
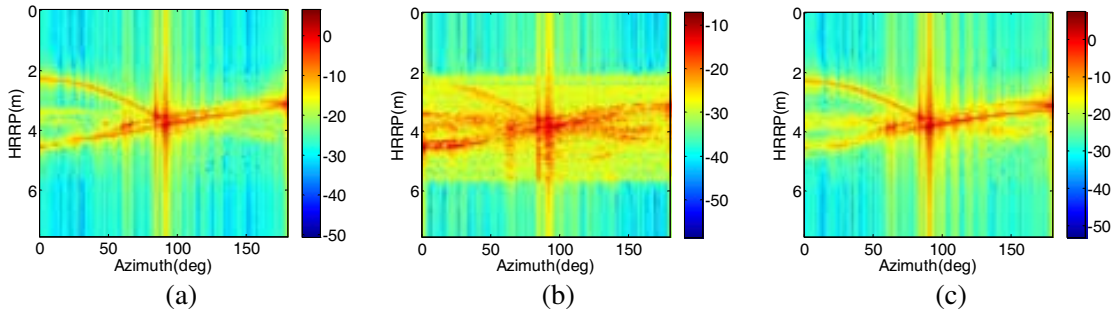


**Figure 1.** Range profile of space-target model from different polarization channels. (a) $HH$ channel, (b) $HV$ channel, (c) $VV$ channel.

## 3. LENGTH EXTRACTION METHODS BASED ON FULL-POLARIZATION RANGE PROFILES

The length of a target is intrinsic information for the artifacts discrimination. For ballistic targets, the differences between the real and false ones are as follows: (1) the length of warhead is $1 \sim 3$ m, while the length of the mother cabin is longer; (2) Simple active repeater decoy forms only a single pulse of which the length is very short, and the length of fragment is always shorter than the length of real target. Using the length information could be an effective method for target recognition.

### 3.1. Length Extraction Algorithm Based on Single Polarization Range Profile

In fact, it is difficult to obtain the target length due to the effects of noise and other factors. The target is always only a part of range profile, and it is difficult for radar to fix on the dividing point of target and noise. In addition, the target length relates closely to the target position, and estimating its position is a tough thing. The estimation formulation of the observed target length $L_s$ proposed by Hussain [13] is

$$L_s = (\max\{l|H(l) > q\} - \min\{l|H(l) > q\})\Delta r \quad l = 1, 2, \cdots N \tag{7}$$

where $H(l)$ is the amplitude of range profile, $q$ the threshold value, and $\Delta r$ the size of the range resolution cell. Then, the real target length is

$$L_T = \frac{L_s}{\cos(A_z)\cos(E_l)} \tag{8}$$

where $A_z$ and $E_l$ are the target azimuth and elevation angles with respect to the observed radar position, respectively.

The length estimation accuracy strongly depends on the threshold selection. Therefore, a suitable threshold value should be chosen carefully. In practice, there are several methods such as ones based on noise voltage, normalized threshold value, and adaptive threshold [12, 13]. As pointed out in [12], for ballistic missile target, based on normalized threshold value, the length extracted from range profile is more accurate than the others. The method is operated as follows: firstly, the power of range profile is normalized, then the normalized threshold $q$ is confirmed by the experience value. Fig. 2 shows the relationship curve between the threshold value and the radial length of conical target. From Fig. 2 we can see that the threshold value is more appropriate when it is $0.1 \sim 0.2$. For the sake of analyzing conveniently, this paper adopts the threshold value estimation method based on normalized threshold value in the following discuss.

## 3.2. Length Extraction Algorithm Based on Full-Polarization Range Profiles

From Equations (5), (6) and Fig. 2, full-polarization range profiles are closely related to the coherent polarization scattering matrix of each scatterer. Several length extraction methods based on full-polarization range profiles are proposed as follows.
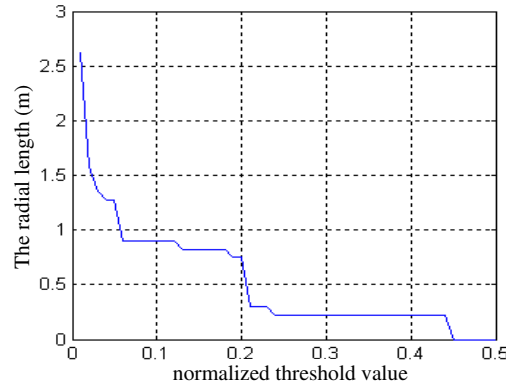


**Figure 2.** The relationship between the normalized threshold value and the target radial length (the real length is 0.9 m).

### 3.2.1. Length Extraction Method Based on Single-channel Detection Synthesis

According to Equation (6), full-polarization range profiles can be regarded as range profiles of three polarization channels. So the radial length of each channel is estimated using Equation (7), then the radial length of target is obtained through an integrated judgment as follows:

$$L_{sPQ} = (\max\{l|H_{PQ}(l) > q\} - \min\{l|H_{PQ}(l) > q\})\Delta r \quad l = 1, 2, \ldots, N \tag{9}$$

and

$$L_s = \max\{L_{sPQ}\} \tag{10}$$

where $\{P, Q\} = \{H, V\}$.

*3.2.2. Length Extraction Method Based on SPAN Power Synthesis*

The detection method based on SPAN power synthesis which uses the sum of full-polarization echo power is a common method in full polarimetric radar target detection and tracking. The extraction of the radial length feature of a target based on SPAN power is

$$H(l) = \sum_{k=0}^{N-1} \left[ |S_{HH}(k)|^2 + \left| \sqrt{2} S_{HV}(k) \right|^2 + |S_{VV}(k)|^2 \right] \delta(l - r_k) \tag{11}$$

and

$$L_s = \left( \max\{l|H(l) > q\} - \min\{l|H(l) > q\} \right) \Delta r \tag{12}$$

*3.2.3. Length Extraction Method Based on Target Characteristic Polarization*

As aforementioned, the substance of length estimation based on range profile is to determine the dividing point of target and noise in the range axis. Especially for the weaker scattering component of a target, since the response from a target is always mixed with noise and the SNR is low, using the methods based on single-channel detection synthesis or SPAN power synthesis may cause misjudgment of the dividing point. In order to improve the accuracy of the radial length extraction by using the polarization information adequately, we should first work out the characteristic polarization of each scattering center by utilizing the coherent polarization matrix of target, then choose the maximal eigenvalue as the characteristic polarization range profile of target, at last obtain the estimation of the radial length. As shown from Equation (5),

$$s(t) = \sum_{k=0}^{N-1} U_k^T \left[ \begin{array}{cc} \lambda_1(k) & 0 \\ 0 & \lambda_2(k) \end{array} \right] U_k \delta(t - \tau_k') \tag{13}$$

where $U_k$ is the eigenvector of the polarization scattering matrix of the $k$-th scattering center, and $\lambda_1(k)$ and $\lambda_2(k)$ are the eigenvalues of the polarization scattering matrix, $\lambda_1(k) \geq \lambda_2(k)$.

Therefore, the polarization range profile of target is

$$H_\lambda(l) = \sum_{k=0}^{N-1} \lambda_1(k) \delta(l - r_k) \tag{14}$$

And the radial length is

$$L_s = \left( \max\{l|H_\lambda(l) > q\} - \min\{l|H_\lambda(l) > q\} \right) \Delta r \tag{15}$$

## 4. SIMULATION AND ANALYSIS

### 4.1. Simulation and Analysis Based on Scattering Mechanism Model

The radar target model consists of five scatterers A, B, C, and D, E (The corresponding scattering mechanism includes sphere, 45-dihedral, cylinder, left helical line and right helical line). All of these points are located in the same line, and the distance between two nearest points is 1.0 m, shown in Fig. 3. A1, A2, A3, and A4 represent the distances of sphere-dihedral, dihedral-cylinder, cylinder-left helical line, and left helical line-right helical line, respectively. In order to simulate the wideband radar observation of the simulated scene, we generate the electromagnetic data using the MLFMM method offered by FEKO. The simulation parameters are: the range of frequency is $9.75 \sim 10.25$ GHz, the step frequency 10 MHz, and the radar observes these scattering points in the direction of the radial distribution of scattering points. Then the full-polarization wideband radar echo is obtained under the stated conditions. Furthermore, a different complex Gaussian noise component is superimposed until the generation of scattering echo is completed.

Figure 4 shows the radar high resolution range profiles of each polarization channels when the SNR is 20 dB. From Fig. 4, the range profiles of the target are quite different at different polarization modes. At $HH$ and $VV$ channels, the scattering intensity of sphere, cylinder, left helical line and right helical
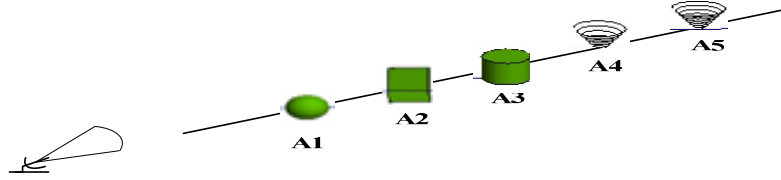
**Figure 3.** The location of canonical scatterers.
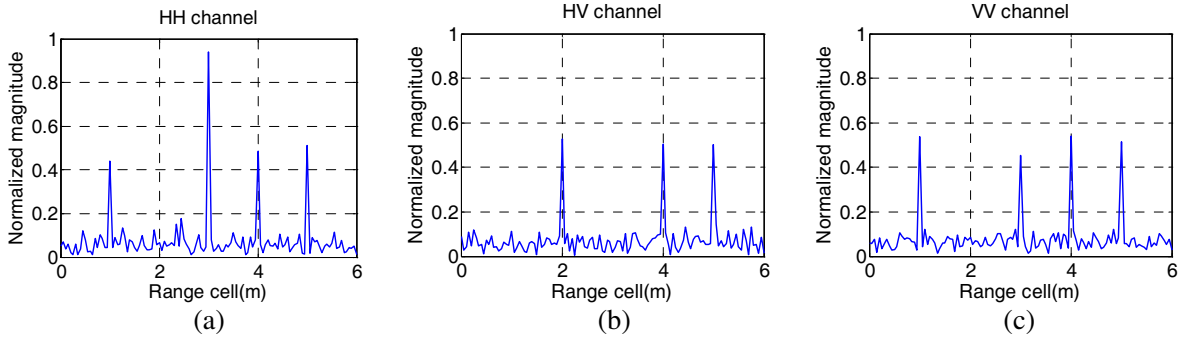


**Figure 4.** The amplitude of range profile at each polarization channel when the SNR is 20 dB. (a) $HH$ channel, (b) $HV$ channel, (c) $VV$ channel.

line is strong, and the scattering centers can be extracted effectively, while the scattering intensity of 45-dihedral is so week that the echo is drowned in the noise and the scattering center can't be isolated. In comparison, in the range profiles at $HV$ and $VH$ channel, the scattering intensity of dihedral, left helical line and right helical line is stronger, while there are not visible scattering centers corresponding to sphere and cylinder. In short, using the range profile of the target in single channel can't extract the scattering centers completely, no matter which polarization mode is chosen. And then accurate length or distance information of the target cannot be acquired. That is why the data sets from different polarization channels need to be fused. Then the performances of three extraction algorithms based on full-polarization range profiles present in Section 3.2 are compared and analyzed.

Normalized threshold value $q$ is chosen as 0.12, and then the distances of target groups are estimated by three different full-polarization length extraction methods. Table 1 shows the experiment results when the SNR is 20 dB, where method 1, 2 and 3 correspond to the length feature extraction method based on single-channel detection synthesis, SPAN power synthesis, and target characteristic polarization, respectively. From Table 1, it is clear that the misjudgment rate of method 1 is significantly greater than the other two methods, and A1, A2, A4, which are quite different from the real distances, are invalid estimations. According to the result of method 1, under the condition that some scatterers show weak scattering mechanism in some polarization channels, the method 1 does not fuse various polarization channel data effectively, so it hardly reduces the misjudgment rate. In comparison, the accurate rates of method 2 and method 3 are higher, and the errors between the estimated results and the real ones are within 10 percent. According to the results of method 2 and method 3, these two methods fuse the data of different polarization channels effectively and improve the accurate rate of length extraction.

Furthermore, the performances of these two effective methods (method 2 and method 3) in terms of different SNR values are compared. Figs. 5(a) and (b) show the extraction results of A1 and A3 respectively. As shown in Fig. 5, the results of method 3 are still accurate under the condition of lower SNR, and have a better performance, while the accuracy of method 2 reduces with the decreasing of the SNR. The results demonstrate that the method based on target characteristic polarization has a better anti-noise capacity.

**Table 1.** Estimated lengths of three methods based on full-polarization range profiles (SNR = 20 dB).

| Method / Interval | Method 1 | Method 2 | Method 3 | True value |
|---|---|---|---|---|
| A1 | 1.97 | 0.97 | 0.98 | 1.00 |
| A2 | 1.93 | 1.07 | 1.00 | 1.00 |
| A3 | 1.03 | 0.97 | 0.99 | 1.00 |
| A4 | 0.61 | 1.03 | 1.02 | 1.00 |



(a)                                                    (b)
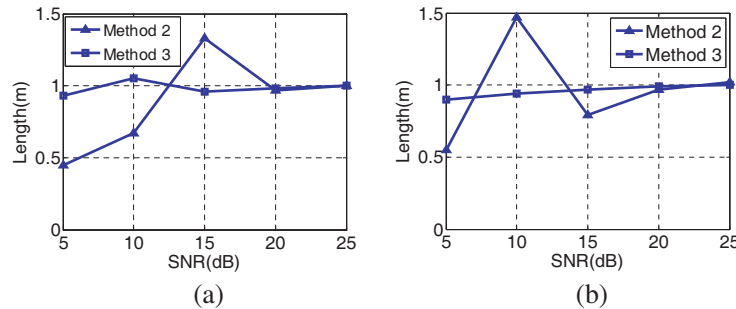
**Figure 5.** The extraction results in terms of the SNR. (a) The extraction results of A1, (b) the extraction results of A3.

## 4.2. Analysis Based on the Simulation Platform of Space Target Recognition

In order to validate the effectiveness of the proposed radial length extraction methods of target further, we also carry out the simulation experiment through our spatial target recognition system. The full-polarization scattering data of the space target model measured in the microwave anechoic chamber are shown in Fig. 6(a). The real length is 2.2 m, and the simulation parameters are set as follows: the frequency is $8.75 \sim 10.75$ GHz; the frequency interval is 20 MHz; the number of frequency samples is 101; the azimuth angle steps from $0°$ to $180°$; the step interval is $0.2°$. Fig. 6(b) shows the range profile when the azimuth is $90°$. As shown from Fig. 6(b), the scattering centers of target mainly include the nose cone, central ring and tail skirt. The distance between the nose cone and the tail skirt is the real length of the target when the wave radiates vertically.

The echo data from a moving target using the simulation platform of space target recognition are generated via the following steps [14, 15]. Firstly, the ballistic of target is simulated according to the set ballistic parameters (including the space target launch position and the parameters of space target



(a)                                                    (b)
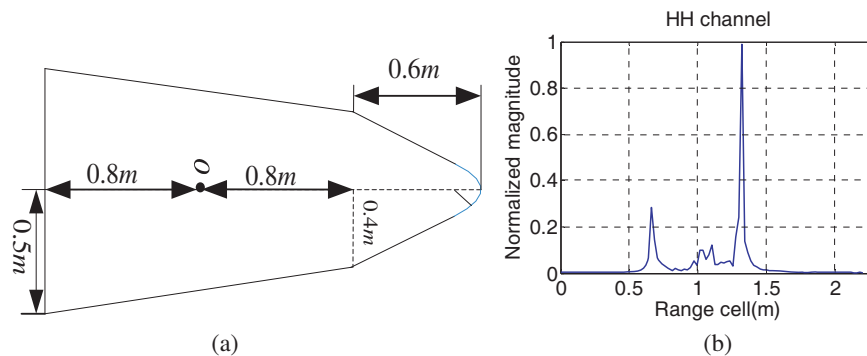
**Figure 6.** Target shape and range profile of static state. (a) The target model measured in microwave anechoic chamber, (b) range profile when the azimuth is $90°$ ($HH$ channel).
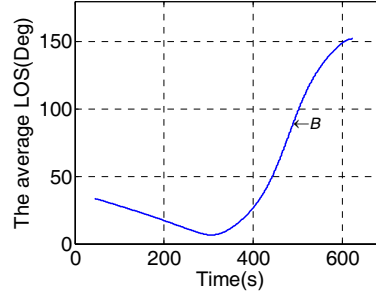
**Figure 7.** The average azimuth with the varied time.

shutdown point). Secondly, with the combination of the ballistic data, locations of radar netting and micro-motion parameters, the dynamic sequence of target azimuth angle is obtained. Finally, according to the measured data in the microwave anechoic chamber, the full-polarization echo of target is simulated under the condition of full-pose and full-polarization. The scenario parameters are set as follows: the space target launch position is 62.45°E and 14.61°N, and the height is 0 km; the velocity of space target shutdown point is 4 km/s, the height is 60 km, and the tilt angle is 30°; the radar position is 79.02°E and 15.05°N, and the height is 0 km. Fig. 7 shows the curve of the average line-of-sight angle of the space target change with time. From Fig. 7, the angle between radar and the direction of radial axis of target experiences a process that the angle decreases from large to small (about $50° \sim 15°$) and then increases from small to large ($15° \sim 150°$) quickly.

The chosen line-of-sight angle is 90°. According to the geometric relationship between radar and target, at this moment the projection of target onto the direction of the radar line-of-sight has the maximal radial length. Under the conditions of different SNRs, each of these three length extraction methods is executed 100 times through Monte-Carlo method, and the estimated results of target length based on the normalized threshold are shown as Table 2, in which the single channel ($HH$), method 2 and method 3 represent the length extraction method of range profile based on $HH$ polarization mode, SPAN power synthesis and target characteristic polarization, respectively.

**Table 2.** The extraction results under the conditions of different SNRs (unit: m).

| Extraction method           SNR | 5 dB | 10 dB | 15 dB | 20 dB | 25 dB | true value |
|----------------------------------|------|-------|-------|-------|-------|------------|
| Single channel ($HH$)            | 1.71 | 2.20  | 1.82  | 2.10  | 2.20  | 2.20       |
| Method 2                         | 1.96 | 1.24  | 1.98  | 2.27  | 2.20  | 2.20       |
| Method 3                         | 2.22 | 2.17  | 2.12  | 2.20  | 2.20  | 2.20       |

As shown in Table 2, when SNR is low, the extraction length information of the extraction method using single channel feature or method 2 is distorted, so the length feature is invalid. For these two methods, if more accurately estimated length is needed, the SNR of range profile should be above 20 dB. When SNR is up to 25 dB, the noise has no effect on the length extraction. The anti-noise capacity of method 3 is better than the other two methods, and the estimated error is still within 5% even SNR is reduced to 5 dB.

## 5. CONCLUSION

Range profile could reflect physical properties of the scatterers. How to extract features with clear physical meaning and reflecting the essential attributes of radar targets from range profile is worthy for further study. From this viewpoint, this paper focuses on the length feature extraction based on full-polarization range profiles. Three methods are proposed, based on single-channel detection synthesis, SPAN power synthesis and target characteristic polarization, respectively. In addition, the anti-noise

performance of each method is analyzed. Experiment results verify that the method based on target characteristic polarization shows a better performance than the others. Length feature is one of the most effective features for discriminating targets and artifacts. For real application, there are still many issues which need to be concerned, and further advancements are required: how to determine the relative position between the radar and the target; how to improve the estimation accuracy of target length using polarization information; how to estimate the 2-D length feature using polarization information. These problems will be addressed in near future.

## ACKNOWLEDGMENT

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