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Range Distance Requirements for Large Antenna Measurements for Linear Aperture with Uniform Field Distribution

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Abstract—Gain reduction on measurement of distance and on sizes of test and probe antennas are discussed. We consider only the case of linear antenna with uniform field distribution. The analysis is based on time-domain (TD) physical optics (PO) method of field calculation [1]. We show that in determining the level of the side lobe there are two competing effects: (i) the decrease in the amplitude of the signal in the direction of the side lobe and (ii) reducing the maximum signal level in the direction of zero-angle. We show the optimal measurement distance with respect to the acceptable small errors of antenna gain. It is shown that optimal relation $\beta = b/a$ is about ~ 0.4 (a and b are the sizes of the antenna under test and the probe antenna). For this optimal relation, the well-known far-field distance criterion $R_0 = 2D^2/\lambda$ can be reduced by 2 times (D is the diameters of the antenna under test and λ is wavelength). Note that when b is optimal, the errors in determining of the sidelobe levels are also small and do not exceed 0.5 dB.

1. INTRODUCTION

The far-field distance criterion $R_0 = 2D^2/\lambda$ is well known. This criterion assumes small dimensions of the probe antenna, and it is valid only for uniform field distribution over the aperture. Therefore, for small probes, it was recognized that antennas with a low sidelobes levels may require a longer measurement distance [2, 3]. For example, longer distance is required for Taylor or Bayliss field distribution over the aperture [4–6]. The measurement distance requirements for some symmetrical and antisymmetrical field distributions for circular aperture antennas were investigated in [7, 8]. Universal curves giving the main lobe reduction and first sidelobe level change versus the measurement distance were furnished for various aperture distributions [8]. An empirical formula for correction of the sidelobe level of the radiation pattern of high gain antenna has been derived in [9]. This formula was obtained after a thorough analysis of a large number of different cases. From reciprocity theorem, it makes no difference whether the antenna under test is a transmitter or a receiver.

In practical measurements, to increase signal level, it is reasonable to use a probe antenna having comparable aperture with the measured antenna. In this case, by analogy with the case of a small probe, the criterion $R_0 = 2(D_a + D_b)^2/\lambda$ is generally used [10]. Here D_a and D_b are the diameters of the antenna under test and the probe antenna, respectively. However, it was shown in [11] that the range distance criterion $R_0 = 2D^2/\lambda$ may apply for big probes. Later, it was recognized that a large probe antenna yields a result closer to the far-field pattern at any range distance in the near zone compared with the results obtained by the small probe antenna [12]. However, all these results were obtained for a specific aperture field distribution, specific antenna sizes, and specific range distances.

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More general kinds of aperture distributions when the measured and probe antennas have the same order of large dimensions were investigated in [13]. These investigations were based on the near zone power transmission formula derived in [14]. It was shown that error of first sidelobe level obtained from large antenna is always smaller not only for any range distance but also for any aperture distribution than the error obtained from the small antenna. The near-zone gains versus the range distances were also computed as a reduction from the far-zone gains. It was indicated that the gain error obtained from the large antenna is greater at any range distance than the error obtained from the small antenna. For the same gain error and the same test and probe antennas size, the range distance is about $2.2 \sim 2.83$ times that for the small probe antenna depending on aperture distribution.

The physical explanation of measurement errors with respect to the radiation pattern and the gain was described as follows. The near zone radiated field from the antenna under test can be expressed as the sum of angular spectrum of plane waves [15]. At a given range distance in the near zone, the small probe antenna receives equally all the angular spectra and leads to error for the far-field, even as the large antenna receives selectively the angular spectrum in the boresight direction of the receiving antenna, which mainly contributes to the far-field pattern. This is not the case, however, for the antenna gain, which is related to the actual received power. The selective receiving of the angular spectrum may cause the measurement error for the gain.

Let us note that a method has been developed to obtain the far-field radiation pattern of an antenna at a reduced distance where the far-field distance condition is not satisfied [16]. It uses an asymptotic antenna transmission formula and angular mode deconvolution technique to determine the far-field radiation.

Another physical explanation of the effect of sidelobe reduction when the big probe was used is given in [17, 18]. Cases when the test antenna is a circular plane aperture and the probe antenna is a circular plane aperture or a small (negligible size) antenna were considered for uniform field distribution. TD PO method was used for TF calculation at the output of small and big probe antennas. This method allows obtaining analytic form results for TF for all observation points in the half-space in front of the aperture, which are presented in inverse trigonometric functions [1] or as an integral over a circle [19–21]. TF or the so-called pulse radiating characteristics (PRC) in the physical sense represents antenna field if each element of the aperture radiates a δ-pulse at the time moment $t = 0$. Frequency domain field was determined as the result of the Fourier transform of the TF. It was shown that aperture probe at nonzero angles significantly smoothes the rising PRC edge when compared with a small probe. In the boresight of the test antenna, the rising PRC edge remains short and unchangeable. This effect is also shown below for linear antenna.

Therefore, the problem of range distance requirements for large antenna measurements has been discussed for more than 60 years up to the most recent times. Despite the long historical background, a choice of criterion for the minimum distance taking into account the size of the probe antenna has not been finally elucidated.

The main aim of this article is to find the optimal size of the aperture probe. We also show the optimal measurement distance with respect to the acceptable small errors of the antenna gain.

2. METHOD AND RESULTS

Instead of the distance between both antennas R, it is convenient to use dimensionless parameter m, which represents the number of reductions of far-zone distance R_0:

$$m = \frac{R_0}{R} = \frac{2D_a^2}{R\lambda} \tag{1}$$

The field of linear aperture antenna on the frequency f at the observation point (x, y) (see Fig. 1) can be written in the following form:

$$F_z = A(f) \int_{-a}^{a} g(x') \frac{e^{ikr}}{r} dx', \quad r = \sqrt{z^2 + (x' - x)^2}. \tag{2}$$

Here, $k = 2\pi f/c$, $a = D_a/2$, D_a is the aperture length, and $g(x')$ is the amplitude distribution over the aperture. It is independent of the frequency and is symmetric: $g(x') = g(-x')$. In this case, the field amplitude F_z also will be symmetric in the coordinate x, and it is possible to put $x > 0$. Expression (2) precisely enough describes a field, for example, a linear antenna array on the basic polarization in the front half space in the area close to the axis z. Transformation of (2) to the time domain gives:

$$E_z = \int_{-\infty}^{\infty} F_z e^{-2\pi i f t} df = s(t) \otimes E_{z,0}(t, x, z). \tag{3}$$

Where $s(t)$ is the Fourier transform of $A(f)$, $\otimes$ is the convolution in the time domain, and $E_{z,0}$ is the Fourier transform of integral (2). This transform considered as the PRC of linear aperture results in replacing of the function $\exp(ikr)$ in the integral (2) by $\delta(t - r/c)$, where $\delta(t)$ is the Dirac's δ-function.

$$E_{z,0} = \int_{-a}^{a} g(x') \frac{\delta(t - r/c)}{r} dx' \tag{4}$$

The PRC can be represented in close form using the integration technique with δ-function depends on complex argument $\delta[\varphi(x')]$, where $\varphi(x')$ is any function [22]. The main contribution to the integral is determined by zeros of the equation $\varphi(x') = 0$. The equation $r - ct = 0$ has two zeros $x'_{1,2} = x \mp \sqrt{(ct)^2 - z^2}$ when $ct > z$, and there are three cases: (1) $x'_1, x'_2 \in [-a, a]$, (2) x'_1 or $x'_2 \in [-a, a]$, (3) x'_1, $x'_2 \notin [-a, a]$. Correspondingly, the PRC is written as

$$E_{z,0} = \frac{c\gamma}{\sqrt{(ct)^2 - z^2}}; \quad \gamma = \begin{cases} g(x'_1) + g(x'_2), & (z < ct < r_1) \cap (|x| < a) \\ g(x'_{12}), & r_1 < ct < r_2 \\ 0, & (ct < z) \cup (ct > r_2) \end{cases}, \tag{5}$$

where $x'_{12} = x'_1$ when $x > 0$, and $x'_{12} = x'_2$ when $x < 0$.

Here, the symbols $\cap$ and $\cup$ denote logical "and" and "or", respectively.

$$r_{1,2} = \sqrt{z^2 + (x \mp a)^2}. \tag{6}$$

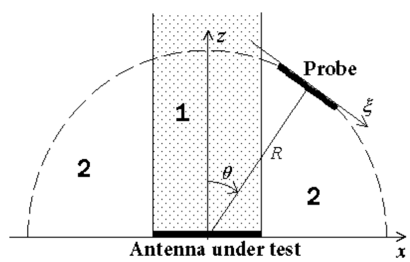


Figure 1. The scheme of measurements using the aperture probe, **1** is the projector area; **2** is the area of side lobes.

For uniform field distribution $g(x') = 1$ and according to (4) function γ can be one of three values: 2, 1 or 0. In contrast to (2) and PRC (4), (5) often can be presented in analytical form. Values γ are defined by the number of decisions of the equation $ct = r$ in an interval $|x'| < a$. Roots of this equation can be found geometrically as points of intersection of a circle with the center at the observation point (x, z) and the radius ct with the axis $z = 0$. Typical PRC for uniform distributions are shown in Fig. 2.

We will further assume that the signal from the antenna is taken by the probe representing a linear antenna of length $2b$, which moving around the measured antenna is tangential to the circle with radius R (see Fig. 1). The coordinates of the observation point in this case are expressed using the rotation angle θ and the coordinate in the probe aperture ξ, $|\xi| < b$, is as follows:

$$x = R \sin\theta + \xi \cos\theta, \quad z = R \cos\theta - \xi \sin\theta \tag{7}$$

The boundaries $r_{1,2}$ that define the point of discontinuity are transformed to

$$r_{1,2} = \sqrt{R_{1,2}^2 + (\xi \mp a \cos\theta)^2}, \tag{8}$$

where $R_{1,2} = R \mp a \sin\theta$. The influence of the size of the probe aperture on the time dependence of the received signal can be taken into account by integrating (4) over the probe aperture. Herewith $E_{z,0}$ is transformed into $E_{z,0b}$:

$$E_{z,0b} = \frac{1}{2b} \int_{-b}^{b} E_{z,0}(ct, \theta, \xi) d\xi \tag{9}$$

This integration in many cases can be done analytically, in particular, for the uniform distribution on the aperture $g(x') = 1$. There are several areas in the plane (ξ, ct), bounded by three function (7),(8): $ct = z(\xi)$, $ct = r_1(\xi)$, $ct = r_2(\xi)$, as it is shown in Fig. 2. Inside each area γ in (5) is equal to 2, 1 or 0. For a fixed $ct = \text{const}$ the boundaries of the intervals ξ_m with a constant values of γ are determined analytically from $ct = z(\xi_m)$, etc. Note, that the full interval of ct where (9) isn't equal to 0 also consists from several intervals with the same structure of the boundaries ξ_m. Although all boundaries and the integral (8) can be found analytically for arbitrary β and θ the PRC (9) is presented by cumbersome expressions. More observable expression (9) can be given as an example for the probe orientation in the boresight of the antenna $\theta = 0$; at a sufficiently large distance R taken in the direction of the signal (9) is proportional to the antenna gain. In this case, $r_{1,2} = \sqrt{R^2 + (\xi \mp a)^2}$ and

$$E_{z,0b} = \frac{c\gamma_b}{\sqrt{(ct)^2 - R^2}}, \tag{10}$$

where

$$\gamma_b = \frac{1}{2b} \int_{-b}^{b} \gamma d\xi = \begin{cases} 2, & R < ct < r_{\min} \\ (b + a - \sqrt{(ct)^2 - R^2})/b, & r_{\min} < ct < r_{\max} \\ 0, & (ct < R) \cup (ct > r_{\max}) \end{cases}, \tag{11}$$

$$r_{\min} = \sqrt{R^2 + (a - b)^2}, \quad r_{\max} = \sqrt{R^2 + (a + b)^2}. \tag{12}$$

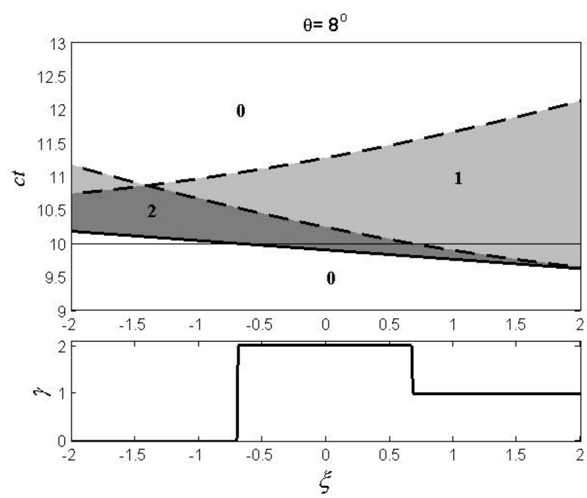


Figure 2. An example of $\gamma(\xi, ct)$ for $R = 10$, $a = 4$ and $\theta = 8^\circ$ (upper figure). The numerals 0, 1, 2 designate the areas where $\gamma = 0$, 1 or 2 correspondingly. The boundary $z(\xi)$ is shown by the solid line, the boundaries $r_{1,2}(\xi)$ are shown by dashed lines. The thin line $ct = 10$ corresponds to the lower figure showing the function $\gamma(\xi, 10)$.

To obtain the amplitude of the signal in the boresight direction as a function of the number of reductions of the far-zone distance, let us go to the frequency domain where we will find the inverse Fourier transform of (12) with the normalization multiplier $R/2a$.

$$F_R = \frac{R}{2a} \int E_{z,0b} e^{2\pi i f t} dt. \tag{13}$$

The introduction of the normalization factor is convenient because when $R \to \infty$, F_R is transformed to the radiation pattern, the main maximum of which is normalized to 1.

It is easy to show that the spectrum of the signal (13) represents simultaneously the dependence of the amplitude of the signal in the direction of the boresight as function of the number of reductions of the far-zone distance m, because $m \sim f$. The PRC, as seen in Fig. 3, is a real positive finite function, placed in the interval $ct \in [R, r_{\max}]$. The spectrum of the function will have a main lobe with a maximum at $f = 0$ and a characteristic width $(r_{\max} - R)^{-1}$. According to (12), with the increase of the probe aperture size, the time interval, in which the PRC is not equal to zero, is extended. Accordingly, the width of the main lobe is decreased, and the rate of reduction of the signal amplitude with increasing the parameter m grows.

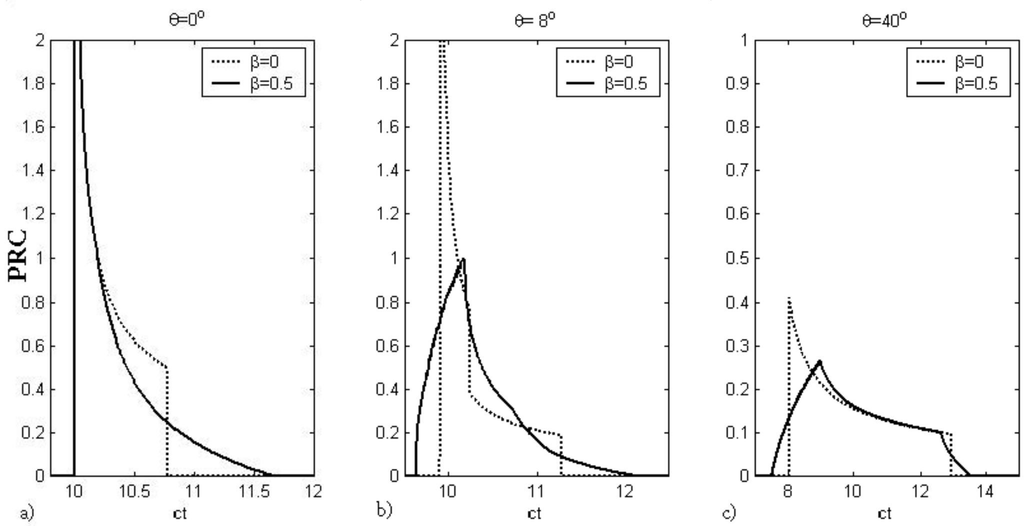
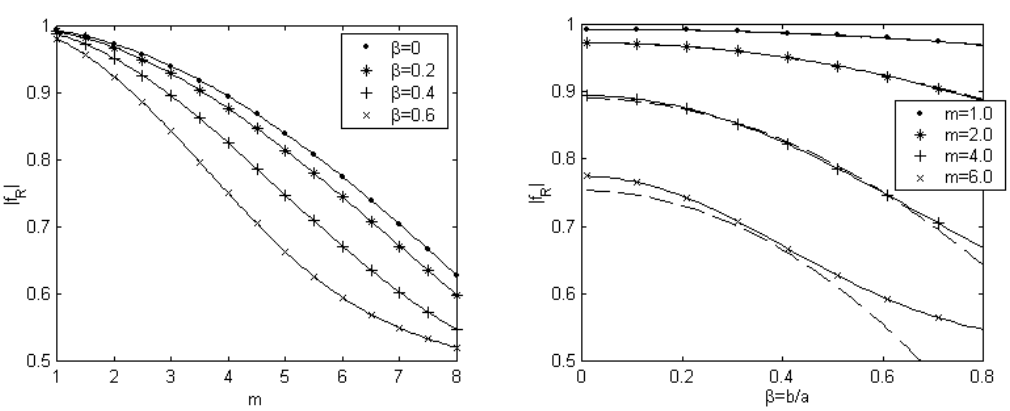


Figure 3. The PRC of linear aperture in the case of the point probe antenna ($\beta = 0$, the dotted line) and of the aperture probe antenna ($\beta = 0.5$, the solid line) at three different angles of rotation of the antenna relative to the direction of the probe antenna; $R = 10$; $a = 4$; $b = 2$.



(a) (b)

Figure 4. (a) The dependences of the amplitude of the signal in the boresight direction as function of the parameter m, with the different relations $\beta = b/a$; (b) the dependences of the amplitude of the signal in the boresight direction as function of the relation β with different parameters m. The dashed lines on the right figure correspond to the approximate formula (13).

As can be seen from the Fig. 4, the decrease in the level of the signal, that is, the gain error, varies approximately with the square-law when $m < 4 \ldots 5$. The decomposition in a Taylor series gives

$$|F_R| \approx 1 - \frac{1}{90}\left(\frac{\pi m}{4}\right)^2 \left(1 + C(m)\beta^2\right), \tag{14}$$

where $C(m) \approx 6,3 - 0,73m$ if you change m in the range $m = 1 \ldots 5$. From expression (14), the error of the gain determination depending on m and β can be expressed in the form:

$$\delta\,|F_R| = -\frac{1}{90}\left(\frac{\pi m}{4}\right)^2 \left(1 + C(m)\beta^2\right) = -\frac{1}{90}\left(\frac{\pi m_e}{4}\right)^2, \tag{15}$$

where m_e is the effective number of reductions of the far-zone distance, and it is determined by the size of the probe

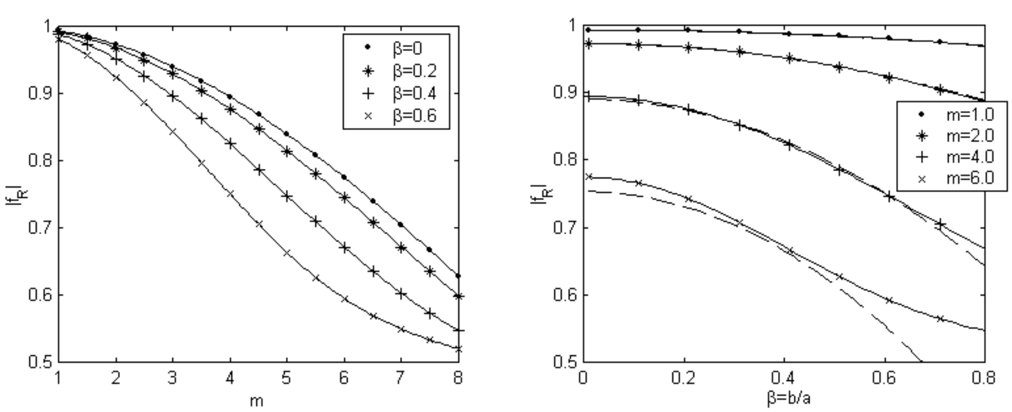
$$m_e = m\sqrt{1 + C(m)\beta^2}. \tag{16}$$

Thus the error in determining the antenna gain has a quadratic dependence on the size of the probe antenna and the parameter m.

For angles $\theta > 0$, PRC can also be represented in analytical form, similar to (10); however, the corresponding expression is rather cumbersome and not presented here. As can be seen from the graphs, the use of aperture probe in all cases leads to the PRC stretching and, consequently, to a decrease in the amplitude of the PRC. Fig. 3 shows the PRC graphs of linear aperture in the case of a point probe antenna and the aperture probe antenna at three different angles of rotation of the antenna relative to the direction of the probe antenna. The angular directions θ are chosen to correspond to the surrounding area of the main lobe and side lobes of the antenna pattern. Fig. 5 shows relative error in determining the width of the main lobe $\Delta u = \delta\{\Delta\theta_{\text{main}}\}$ as function of m and β.

In determining the level of side lobes, there are two competing effects: the decrease in the amplitude of the signal in the direction of the side lobe and reducing the maximum signal level in the direction $\theta = 0$. The result is a rather interesting effect: the error of the determination of the levels of side lobes is minimal for some value of $\beta = b/a$. Fig. 6 shows the dependence of the error of side lobe levels on m and β. As can be seen from the graphs, the optimal value β is weakly dependent on the number of side lobe and is $\sim 0,4$.

In principle, the same results for a linear aperture can be obtained in the frequency domain in the framework of the Fresnel approximation. The angular dependence of the field measured by aperture probe can be represented in analytical form through the Fresnel integrals, but the use of the PRC enables to immediately predict the result, at least on a qualitative level.



(a) (b)

Figure 5. The relative error in determining the width of the main lobe $\Delta u = \delta\{\Delta\theta_{\text{main}}\}$ as function of m and β.

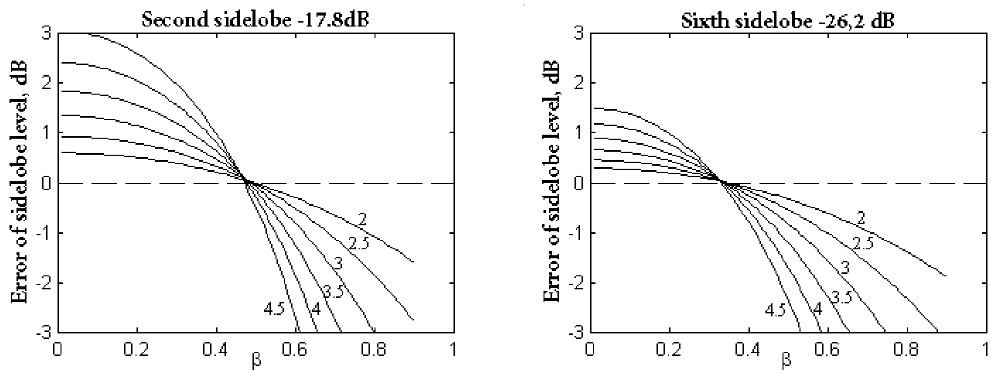


Figure 6. The errors on the peak sidelobe level of the 2nd and the 6th side lobes of linear antenna with a uniform amplitude distribution as function of the size of the probe antenna $\beta = b/a$; the numbers in the graphs indicate number of reductions of far-zone distance m.

3. CONCLUSIONS

In this article, we showed the optimal size of the aperture probe for the case of linear antenna with uniform field distribution. It is shown that the optimal length relation $\beta = b/a$ is about ~ 0.4 (a and b are the lengths of the antenna under test and the probe antenna). Moreover, the well-known far-field distance $R_0 = 2D^2/\lambda$ can be reduced by 2 times ($m = 2$) when errors of the antenna gain are ($\Delta_{|F_R|} < 5$–7%). Note that when b is optimal, the errors in determining of the sidelobe levels are also small and do not exceed 0.5 dB.

It is shown that the gain error increases uneventfully as both the function of the size of the aperture probe and the reduction of the distance between antennas. Note also that in practical measurements, we can consider the reduction of the amplitude of the signal in the direction of the boresight versus β and m (see Fig. 4) not as error, but as the known multiplier for measured gain correction. In this case, it is interesting to optimize relation β with respect to minimization of errors of antenna pattern. Moreover, it is interesting to consider range distance requirements for different plane aperture antennas with different field distributions over the aperture.

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