

“Double-Humped Effect” in the Turbulent Collision Magnetized Plasma

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Abstract—Statistical moments of the spatial power spectrum of multiple scattered ordinary and extraordinary waves in the turbulent collision magnetized plasma with aligned anisotropic electron density irregularities are investigated using modify smooth perturbation method taking into account diffraction effects. Correlation function and variances of the phase fluctuations are obtained for arbitrary correlation function of the electron density fluctuations. “Double-humped Effect” is investigated analytically and numerically using the anisotropic Gaussian spectral function of electron density irregularities for the polar ionospheric F-region applying the experimental data.

1. INTRODUCTION

Investigation of the anisotropy of the F layer irregularities is of great interest in ionospheric physics. It is well known that the irregularities are elongated in the direction of the geomagnetic field of lines, but especially at high latitudes they are also found to be anisotropic in the plane perpendicular to the external magnetic field vector. The anisotropy is usually investigated by correlation analysis of radio signals from satellites, observed by space receiver on the ground. The fluctuations in amplitude and phase (scintillations) of radio waves passing through the ionosphere are caused by spatial irregularities in the electron density. The spatial fluctuations in phase cause fluctuations of the angle of arrival at the observation points. Broadening of the spatial power spectrum (SPS) of scattered electromagnetic (EM) waves in the turbulent collision magnetized plasma for both power-law and anisotropic Gaussian correlation functions of electron density fluctuations was analyzed in [1] using the complex ray (-optics) approximation.

“Double-humped Effect”, the features of the SPS, spectral width and shift of its maximum of multiple scattered EM waves in the turbulent collisionless magnetized plasma were considered in [2, 3]. The influence of the collision frequency between plasma particles on the statistical characteristics of scattered EM waves in the turbulent magnetized plasma with both electron density and external magnetic field fluctuations was investigated in [4].

The present paper reports the results of an analysis of the SPS, which is related to fluctuations of the radio refractive index in the F region. Analytical expressions of the correlation function of phase fluctuations of scattered both ordinary and extraordinary waves are obtained for the arbitrary correlation functions of electron density fluctuation using modify smooth perturbation method taking into account the diffraction effects. The “Double-humped Effect” in the collision magnetized plasma with elongated electron density irregularities is considered for the first time. Formation of a gap in the SPS of scattered radiation is analyzed numerically for the polar ionospheric F -region using the anisotropic Gaussian spectral function of electron density fluctuations. This spectrum contains angle of aligned plasma irregularities to the direction of an external magnetic field and anisotropy factor. The numerical calculation is computed based on experimental data. Finally, paper concludes the obtained results and their significance.

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2. STATISTICAL CHARACTERISTICS OF THE PHASE FLUCTUATIONS IN THE COLLISION MAGNETIZED PLASMA

Electric field in the turbulent collision magnetized plasma satisfies the wave equation:

$$\left(\frac{\partial^2}{\partial x_i \partial x_j} - \Delta \delta_{ij} - k_0^2 \varepsilon_{ij}(\mathbf{r}) \right) \mathbf{E}_j(\mathbf{r}) = 0, \quad (1)$$

where components of the dielectric permittivity are [5]:

$$\begin{aligned} \varepsilon_{xx} &= 1 - \frac{vg}{g^2 - u}, & \varepsilon_{xy} &= -\varepsilon_{yx} = i \frac{v\sqrt{u} \cos \alpha}{g^2 - u}, & \varepsilon_{xz} &= -\varepsilon_{zx} = -i \frac{v\sqrt{u} \sin \alpha}{g^2 - u}, \\ \varepsilon_{yy} &= 1 - \frac{v(g^2 - u \sin^2 \alpha)}{g(g^2 - u)}, & \varepsilon_{yz} &= \varepsilon_{zy} = \frac{uv \sin \alpha \cos \alpha}{g(g^2 - u)}, & \varepsilon_{zz} &= 1 - \frac{v(g^2 - u \cos^2 \alpha)}{g(g^2 - u)}, \end{aligned} \quad (2)$$

with: $g = 1 - is$, $s = \nu_{eff}/\omega$, $\nu_{eff} = \nu_{ei} + \nu_{en}$ is the effective collision frequency of electrons with other plasma particles, α is the angle between the Z -axis (the direction of the wave propagation) and the static external magnetic field \mathbf{H}_0 in the YZ principle plane; $\omega_p(\mathbf{r}) = [4\pi N(\mathbf{r})e^2/m]^{1/2}$ is the plasma frequency, $N(\mathbf{r})$ is the electron concentration, $u(\mathbf{r}) = (eH_0(\mathbf{r})/m c \omega)^2$ and $v(\mathbf{r}) = \omega_p^2(\mathbf{r})/\omega^2$ are the magneto-ionic parameters. At high frequency the effect of ions can be neglected.

Wave field we introduce as [2] $E_j(\mathbf{r}) = E_{0j} \exp(\varphi_1 + \varphi_2 + i k_\perp y + i k_0 z)$ ($k_\perp \ll k_0$). Electron density fluctuations are random function of the spatial coordinates $v(\mathbf{r}) = v_0 [1 + n_1(\mathbf{r})]$; $\varepsilon_{ij}(\mathbf{r}) = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)}(\mathbf{r})$, $|\varepsilon_{ij}^{(1)}(\mathbf{r})| \ll 1$. First component represents zero-order approximation and fluctuations of a complex phase are of the order $\varphi_1 \sim \varepsilon_{ij}^{(1)}$, $\varphi_2 \sim \varepsilon_{ij}^{(1)2}$. Using Equation (1) in a zero approximation we obtain the set of algebraic equations for the electric field components.

Polarization coefficients can be obtained using [5]:

$$\frac{\langle E_y \rangle_{1,2}}{\langle E_x \rangle_{1,2}} = -iP''_j - sP'_j, \quad \frac{\langle E_z \rangle_{1,2}}{\langle E_x \rangle_{1,2}} = i(m_1 - s^2 m_3) + s m_2 \equiv i\Gamma''_j + s\Gamma'_j, \quad (3)$$

where: $P''_j = \frac{2\sqrt{u_L}}{u_T \mp \sqrt{a_0}} [(1-v) \mp s^2 \alpha_0]$, $P'_j = \frac{2\sqrt{u_L}}{u_T \mp \sqrt{a_0}} [1 \pm \alpha_0(1-v)]$, upper sign (index 1) corresponds to the ordinary wave; the lower sign (index 2) to the extraordinary wave; $u_T = u \sin^2 \alpha$, $u_L = u \cos^2 \alpha$,

$$\begin{aligned} \alpha_0 &= \frac{b_0}{2\sqrt{a_0}} \frac{1}{u_T \mp \sqrt{a_0}}, & a_0 &= u_T^2 + 4u_L [(1-v)^2 - s^2], & b_0 &= 8(1-v)u_L, & m_1 &= \frac{\sqrt{u_T}}{t_0} (v + \sqrt{u_L} P''_j), \\ m_2 &= \frac{\sqrt{u_T}}{t_0^2} [v(t_0 + t_3) + \sqrt{u_L}(t_3 P''_j + t_0 P'_j)], & m_3 &= \frac{\sqrt{u_T}}{t_0^2} [vt_3 + \sqrt{u_L} P''_j(3-v) + \sqrt{u_T} t_3 P'_j], \\ t_0 &= 1 - u - v + v u_L, & t_3 &= 3 + u - 2v. \end{aligned}$$

Taking into account the conditions characterizing the smooth perturbation method [6, 7]:

$$\left| \frac{\partial \varphi_1}{\partial z} \right| \ll k_0 |\varphi_1|, \quad \left| \frac{\partial^2 \varphi_1}{\partial z^2} \right| \ll k_0 \left| \frac{\partial \varphi_1}{\partial z} \right|, \quad \left| \frac{\partial \varphi_2}{\partial z} \right| \ll k_0 |\varphi_2|, \quad \left| \frac{\partial^2 \varphi_2}{\partial z^2} \right| \ll k_0 \left| \frac{\partial \varphi_2}{\partial z} \right|,$$

in the first approximation we obtain the stochastic differential equation for the phase fluctuations:

$$\left[\frac{\partial^2 \varphi_1}{\partial x_i \partial x_j} + \frac{\partial \varphi_0}{\partial x_i} \frac{\partial \varphi_1}{\partial x_j} + \frac{\partial \varphi_1}{\partial x_i} \frac{\partial \varphi_0}{\partial x_j} - \delta_{ij} \left(\Delta_\perp + 2ik_\perp \frac{\partial \varphi_1}{\partial y} + 2ik_0 \frac{\partial \varphi_1}{\partial z} \right) - k_0^2 \varepsilon_{ij}^{(0)} \right] E_{0j} = 0 \quad (4)$$

where $\Delta_\perp = (\partial^2 \varphi_1 / \partial x^2) + (\partial^2 \varphi_1 / \partial y^2)$ is the transversal Laplacian.

The Fourier transform of the Equation (4) gives differential equation for 2D spectral function:

$$\frac{\partial \psi}{\partial z} + \frac{id_1 - \tilde{d}_2}{\Gamma'' k_x + i(2k_0 - s\Gamma' k_x)} \psi(\mathbf{k}, z) = -\frac{k_0^2}{\Gamma'' k_x + i(2k_0 - s\Gamma' k_x)} [Q_1(\mathbf{k}, z) + isQ_2(\mathbf{k}, z)] \quad (5)$$

where: $\tilde{d}_1 = -P'' k_x (k_y + k_\perp) + \Gamma'' k_0 k_x$, $\tilde{d}_2 = k_y (k_y + 2k_\perp) + s [P' k_x (k_y + k_\perp) - \Gamma' k_x k_0]$, $\tilde{Q}_1(\mathbf{k}, z) = Z' n(\mathbf{k}, z)$, $\tilde{Q}_2(\mathbf{k}, z) = Z'' n(\mathbf{k}, z)$, $Z' = Z_{xx} + Z_{xy} + Z_{xz}$, $Z'' = \tilde{Z}_{xx} + \tilde{Z}_{xy} + \tilde{Z}_{xz}$, $Z_{xx} = \varepsilon_{xx}$, $Z_{xy} =$

$\varepsilon_{xy}^{(1)}\mathbf{P}'' + s^2\tilde{\varepsilon}_{xy}^{(1)}\mathbf{P}'$, $Z_{xz} = \varepsilon_{xz}^{(1)}\Gamma'' + s^2\tilde{\varepsilon}_{xz}^{(1)}\Gamma'$, $\tilde{Z}_{xx} = -\tilde{\varepsilon}_{xx}^{(1)}$, $Z_{xy} = -\varepsilon_{xy}^{(1)}\mathbf{P}' + \tilde{\varepsilon}_{xy}^{(1)}\mathbf{P}''$, $Z_{xz} = -\varepsilon_{xz}^{(1)}\Gamma' + \tilde{\varepsilon}_{xz}^{(1)}\Gamma''$; components of the permittivity tensor can be easily restored from Equation (2); $\mathbf{k} = \{k_x, k_y\}$ contains wave vector components in the directions X and Y axis, respectively. Gyrotropy of plasma is revealed in elliptic polarization of normal waves; anisotropy is appeared in the direction of propagation depending on their characteristics (polarization, refractive index and absorption).

The solution of Equation (5) satisfying the boundary condition $\psi(k_x, k_y, z = 0) = 0$ is:

$$\begin{aligned}
 \psi(\mathbf{k}, L) = & i\frac{k_0}{2}(Z' + isZ'') \int_{-\infty}^{\infty} dz' \exp\left(\frac{1}{4k_0^2} \left\{ \left[\tilde{d}_2\Gamma''k_x - \tilde{d}_1(2k_0 - s\Gamma'k_x) \right] \right. \right. \\
 & \left. \left. - i \left[\tilde{d}_1\Gamma''k_x + \tilde{d}_2(2k_0 - s\Gamma'k_x) \right] \right\} \cdot (L - z')\right) n_1(\mathbf{k}, z'), \quad (6)
 \end{aligned}$$

L is a distance travelling by the wave in the ionospheric plasma. The variance and correlation function of the phase fluctuations in the points \mathbf{r} and $\mathbf{r} + \boldsymbol{\rho}$ have the following forms:

$$\langle \varphi_1^2(\mathbf{r}) \rangle = \frac{\pi k_0^2}{2}(Z' + isZ'')^2 \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \frac{\tilde{G}_1 + i\tilde{G}_2}{\tilde{G}_1^2 + \tilde{G}_2^2} \left\{ 1 - \exp\left[(\tilde{G}_1 - i\tilde{G}_2)L\right] \right\} V_n(k_x, k_y, i\tilde{G}_3 - \tilde{G}_4), \quad (7)$$

$$\begin{aligned}
 W_\varphi(k_x, k_y, L) \equiv \langle \varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r} + \boldsymbol{\rho}) \rangle = & \frac{\pi k_0^2 L}{2}(Z'^2 + s^2 Z''^2) \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y V_n \\
 & \left[k_x, k_y, -\frac{\tilde{d}_1\Gamma''k_x + \tilde{d}_2(2k_0 + \Gamma'k_x)}{4k_0^2} \right] \cdot \exp(-ik_x\rho_x - ik_y\rho_y), \quad (8)
 \end{aligned}$$

where:

$$\begin{aligned}
 \tilde{G}_1 &= \frac{1}{k_0^2} \left[(2\Gamma''k_\perp + \mathbf{P}''k_0)k_xk_y + \frac{s}{2}(\Gamma''\mathbf{P}' - \mathbf{P}''\Gamma')k_\perp k_x^2 \right], \\
 \tilde{G}_2 &= \frac{1}{2k_0^2} \left[\Gamma''(\Gamma''k_0 - \mathbf{P}''k_\perp)k_x^2 + 2k_0k_y^2 + 2s(\mathbf{P}' - \Gamma'k_\perp)k_xk_y + s^2\Gamma'(\Gamma'k_0 + \mathbf{P}'k_\perp)k_x^2 \right], \\
 \tilde{G}_3 &= \frac{1}{4k_0^2} \left\{ [\Gamma''(2k_0^2 - k_y^2) - 2\mathbf{P}''k_0k_\perp] k_x + s(\mathbf{P}''\Gamma' - \mathbf{P}'\Gamma'')k_x^2k_y \right\}, \\
 \tilde{G}_4 &= \frac{1}{4k_0^2} \left[(4k_0k_\perp - \Gamma''\mathbf{P}''k_x^2)k_y + s(2\mathbf{P}'k_0k_\perp - 2\Gamma'k_0^2 - \Gamma'k_y^2)k_x - s^2\Gamma'\mathbf{P}'k_x^2k_y \right];
 \end{aligned}$$

ρ_y and ρ_x are distances between observation points spaced apart in the principle and perpendicular planes, respectively; the asterisk indicates the complex conjugate. From Equation (8) follows that in non-magnetized plasma $H_0 = 0$ at $\chi = 1$ (isotropic case), neglecting the diffraction effects ($\mu \equiv (k_\perp/k_0) = 0$), the power spectrum of the phase fluctuations $W_\varphi(k_x, k_y, L)$ and 3D power spectrum of the electron density fluctuations $V_n(k_x, k_y, k_z)$ are related by the well-known formula [8]

$W_\varphi(k_x, k_y, L) = 2\pi(r_e\lambda)^2 LV_n(k_x, k_y, k_z = 0)$, where r_e is the classical electron radius.

Second order approximation of the phase fluctuations yields the stochastic differential equation:

$$\begin{aligned}
 \frac{\partial^2 \varphi_2}{\partial x \partial y} (s\Gamma' + i\Gamma'') - \frac{\partial^2 \varphi_2}{\partial x \partial y} (s\mathbf{P}' + i\mathbf{P}'') + i\frac{\partial \varphi_2}{\partial x} [k_0(s\Gamma' + i\Gamma'') - k_\perp(s\mathbf{P}' + i\mathbf{P}'')] - \frac{\partial^2 \varphi_2}{\partial y^2} - 2ik_\perp \frac{\partial \varphi_2}{\partial y} \\
 - 2ik_0 \frac{\partial \varphi_2}{\partial z} = \frac{\partial \varphi_1}{\partial x} \frac{\partial \varphi_1}{\partial y} (s\mathbf{P}' + i\mathbf{P}'') + \left(\frac{\partial \varphi_1}{\partial y} \right)^2, \quad (9)
 \end{aligned}$$

and the solution is written as:

$$\begin{aligned}
 \text{Re}\langle \varphi_2(k_x, k_y, L) \rangle = & \frac{\pi}{2}k_0 \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \frac{1}{\tilde{G}_1^2 + \tilde{G}_2^2} \left\{ A \left[L\tilde{A}_1 + \frac{\tilde{A}_1\tilde{G}_1 - \tilde{B}_1\tilde{G}_2}{\tilde{G}_1^2 + \tilde{G}_2^2} - \frac{\tilde{A}_2}{\tilde{G}_1^2 + \tilde{G}_2^2} \exp(\tilde{G}_1L) \right] - \right. \\
 & \left. \cdot B \left[L\tilde{B}_1 + \frac{\tilde{B}_1\tilde{G}_1 + \tilde{A}_1\tilde{G}_2}{\tilde{G}_1^2 + \tilde{G}_2^2} - \frac{\tilde{B}_2}{\tilde{G}_1^2 + \tilde{G}_2^2} \exp(\tilde{G}_1L) \right] V_n(k_x, k_y, i\tilde{G}_3 - \tilde{G}_4) \right\}. \quad (10)
 \end{aligned}$$

where: $A = Z'^2 - s^2 Z''^2$, $B = 2sZ'Z''$, $\tilde{A}_1 = -\left[P''\tilde{G}_1 k_x k_y + \tilde{G}_2(k_y^2 + P'k_x k_y)\right]$, $\tilde{B}_1 = \tilde{G}_1(k_y^2 + P'k_x k_y) - P''\tilde{G}_2 k_x k_y$, $\tilde{A}_2 = (\tilde{A}_1\tilde{G}_1 - \tilde{B}_1\tilde{G}_2) \cos(\tilde{G}_2 L) + (\tilde{B}_1\tilde{G}_1 + \tilde{A}_1\tilde{G}_2) \sin(\tilde{G}_2 L)$, $\tilde{B}_2 = (\tilde{B}_1\tilde{G}_1 + \tilde{A}_1\tilde{G}_2) \cos(\tilde{G}_2 L) - (\tilde{A}_1\tilde{G}_1 - \tilde{B}_1\tilde{G}_2) \sin(\tilde{G}_2 L)$.

Transverse correlation function of a scattered field: $W_{EE^*}(\boldsymbol{\rho}) = \langle E(\mathbf{r})E^*(\mathbf{r} + \boldsymbol{\rho}) \rangle$ is expressed via correlation function and the variances of the phase fluctuations [9, 10]:

$$W_{EE^*}(\boldsymbol{\rho}, k_{\perp}) = E_0^2 \exp \left[\frac{1}{2} (\langle \varphi_1^2(\mathbf{r}) \rangle + \langle \varphi_1^{2*}(\mathbf{r} + \boldsymbol{\rho}) \rangle) + \langle \varphi_1(\mathbf{r})\varphi_1^*(\mathbf{r} + \boldsymbol{\rho}) \rangle + 2\text{Re}\langle \varphi_2 \rangle \right] \cdot \exp(-i\rho_y k_{\perp}) \quad (11)$$

where: E_0^2 is the intensity of an incident radiation. If a distance L traveling by the wave in a turbulent collision magnetized plasma is substantially big, $L \gg (l_n^2/\lambda)$, the diffraction effects become essential.

SPS of a scattered field in case of an incident plane wave $W(k, k_{\perp})$ is calculated by Fourier transform of the transversal correlation function of a scattered field

$$W(k, k_{\perp}) = \int_{-\infty}^{\infty} d\rho_y W_{EE^*}(\rho_y, k_{\perp}) \exp(ik\rho_y). \quad (12)$$

where k is a transverse component of the wave vector of a scattered field evaluating from the correlation function [6, 7].

On the other hand, if the angular spectrum of an incident wave has a finite width and its maximum is directed along the Z -axis, SPS of scattered radiation is given by the expression [9–11]:

$$I(k) = \int_{-\infty}^{\infty} dk_{\perp} W(k, k_{\perp}) \exp(-k_{\perp}^2 \beta^2), \quad (13)$$

where β characterizes the dispersal of an incident radiation (disorder of an incident radiation).

Let Δ_+ and Δ_- designate the displacements of the SPS maximum in the principle YZ plane of scattered ordinary and extraordinary waves in the turbulent collision magnetized plasma; Σ_{\pm} are the widths of the SPS spectrum of these waves. These statistical characteristics are obtained by differentiating correlation function of the phase fluctuations [1–3, 10]

$$\Delta_{\pm} = \frac{2}{i} \frac{\partial W_{\varphi}}{\partial \rho_y} \Big|_{\rho_x = \rho_y = 0}, \quad \Sigma_{\pm} = - \frac{\partial^2 W_{\varphi}}{\partial \rho_y^2} \Big|_{\rho_x = \rho_y = 0}. \quad (14)$$

3. NUMERICAL CALCULATIONS

In most studies the irregularities are elongated in the magnetic east-west direction [12, 13], but sometimes the orientation is found to be rather in the north-south direction [14]. In the F -region, field-aligned irregularities were observed continuously up to the upper F region with high frequency radar [15]: Since the irregularities are observed in the F region at and near the reflection level, their size is comparable or larger than the Fresnel zone, defined as $(2\lambda h)^{1/2}$ where λ is the radar wavelength and h is the reflection altitude. This size is in the range 7 km (for 3 MHz at 250 km) and 3 km (for 9 MHz at 150 km). Irregularities arise from the east-west direction both in the E and F regions [16].

Analytical and numerical calculations will be carried out for the anisotropic 3D Gaussian spectral function [1, 17]:

$$V_N(k_x, k_y, k_z) = \sigma_N^2 \frac{l_{\perp}^2 l_{\parallel}}{8\pi^{3/2}} \exp \left(-\frac{k_x^2 l_{\perp}^2}{4} - p_1 \frac{k_y^2 l_{\parallel}^2}{4} - p_2 \frac{k_z^2 l_{\parallel}^2}{4} - p_3 k_y k_z l_{\parallel}^2 \right), \quad (15)$$

where: $p_1 = (\sin^2 \gamma_0 + \chi^2 \cos^2 \gamma_0)^{-1} [1 + (1 - \chi^2)^2 \sin^2 \gamma_0 \cos^2 \gamma_0 / \chi^2]$, $p_2 = (\sin^2 \gamma_0 + \chi^2 \cos^2 \gamma_0) / \chi^2$, $p_3 = (1 - \chi^2) \sin \gamma_0 \cos \gamma_0 / 2\chi^2$, σ_n^2 is the mean-square fractional deviation of electron density. This spectral function contains parameter $\chi = l_{\parallel} / l_{\perp}$ (the ratio of longitudinal and transverse linear sizes of plasma irregularities) measuring the anisotropy of irregularities and the inclination angle γ_0 of elongated

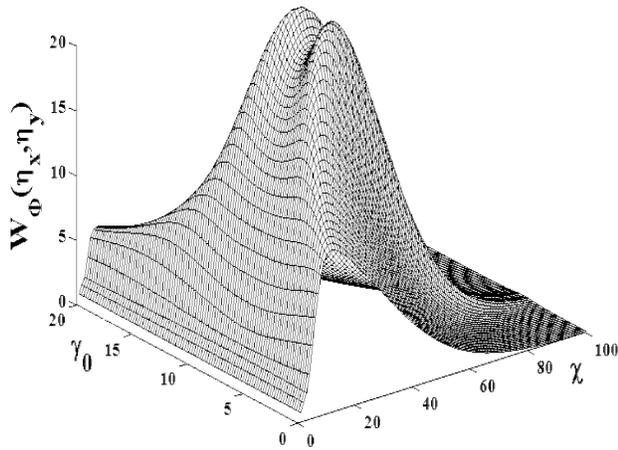


Figure 1. Dependence of the phase correlation function on the parameter of anisotropy and inclination angle.

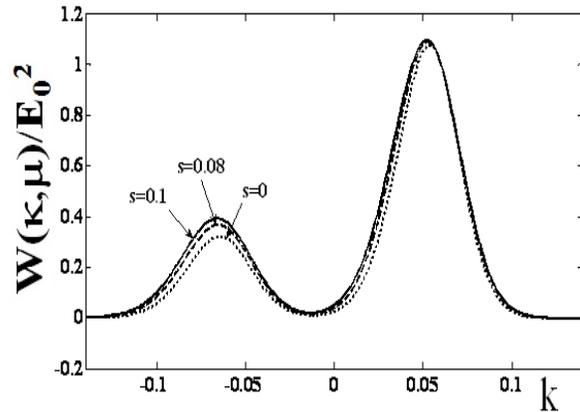


Figure 2. Dotted line corresponds to the collisionless magnetized plasma ($s = 0$), dashed line corresponds to the collision frequency $s = 0.08$, solid line to the $s = 0.1$.

irregularities with respect to the external magnetic field. Anisotropy of the shape of irregularities is connected with the difference of the diffusion coefficients in the field align and field perpendicular directions. Knowledge of the power spectrum of ionospheric refractive index fluctuations can lead to an understanding of the physical processes that characterize the region of the ionosphere under study.

Numerical calculations of the statistical parameters are carried out for the polar ionosphere ($\alpha = 0^\circ$), frequency of an incident EM wave is 3 MHz; plasma parameters are $u = 0.22$, $v = 0.28$; diffraction parameter is $\mu = 0.06$, $\sigma_n^2 \sim 10^{-5}$.

Figure 1 depicts 3D double-humped shape of the phase correlation function for plasma irregularities with characteristic linear scale $l_{\parallel} = 600$ m, varying parameters are in the interval: $\chi = 1 - 100$, $\gamma_0 = 0^\circ - 20^\circ$. Fig. 2 shows curves of the SPS calculated by Equation (12). Large scale plasma irregularities are aligned along the external magnetic field ($\gamma_0 = 0^\circ$) having linear scale $l_{\parallel} = 80$ km. The gap in the SPS arises at propagating distance $L \approx 92$ km of the ordinary EM wave in the ionospheric plasma. The “Double-humped Effect” of the ordinary wave in the turbulent collision magnetized plasma with large scale electron density irregularities ($l_{\parallel} = 64$ km) elongated with the external geomagnetic field is shown in Fig. 3.

Increasing collision frequency in the interval $s = 0 - 0.3$ SPS broadens symmetrically and its depth increases. Particularly, in the collision magnetized plasma at $s = 0.3$ minimum and maximum are located at $k = 0.01$ and $k = \pm 0.5$, respectively and the broadening of the SPS two times exceeds its width in the collisionless plasma.

Figure 4 depicts the evaluation of a double-humped shape of the SPS in the collisionless ($s = 0$) magnetized plasma at: $l_{\parallel} = 64$ km, $\chi = 200$, $\beta = 10$, $\rho_x = 0$, $L = 160$ km varying inclination angle in the interval $\gamma_0 = 0^\circ - 3^\circ$. If the direction of prolate large-scale plasma irregularities coincides with the magnetic lines of force ($\gamma_0 = 0^\circ$), intensities of both ordinary and extraordinary waves have a double-humped shape with maximums symmetrically with respect to $k = 0$; a gap arises in the same direction. By increasing angle to $\gamma_0 = 2.5^\circ$ internal slopes of the SPS oscillates; for $\gamma_0 = 3^\circ$ oscillations become smooth and the spectrum broadens.

In the collision magnetized plasma varying parameters γ_0 and s (Fig. 5) at $\gamma_0 = 0^\circ$ and $s = 0.1$ the spectrum has a symmetrical double-humped shape (curve 1); if $\gamma_0 = 3^\circ$, however increasing collision frequency in the interval $s = 0.01 - 0.1$ SPS symmetrically broadens; maximums are at: $k = 0.635$; $k = 0.731$ and $k = 0.825$; the depth of a gap increases. Particularly, at $s = 0.01$ (the curve 2) internal slopes of the SPS oscillates; at $s = 0.1$ oscillations become smooth (curve 3) and then disappear. The degree of elongation gives an appreciable effect. Formation of a gap at $\gamma_0 = 3^\circ$, $s = 0.1$, $l_{\parallel} = 64$ km,

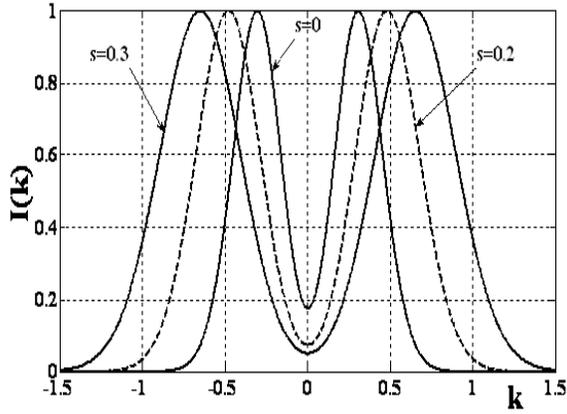


Figure 3. SPS of scattered ordinary wave for: $l_{\parallel} = 64$ km, $\chi = 200$, $\gamma_0 = 0^\circ$, $\alpha = 0^\circ$, $\beta = 10$, $L = 160$ km.

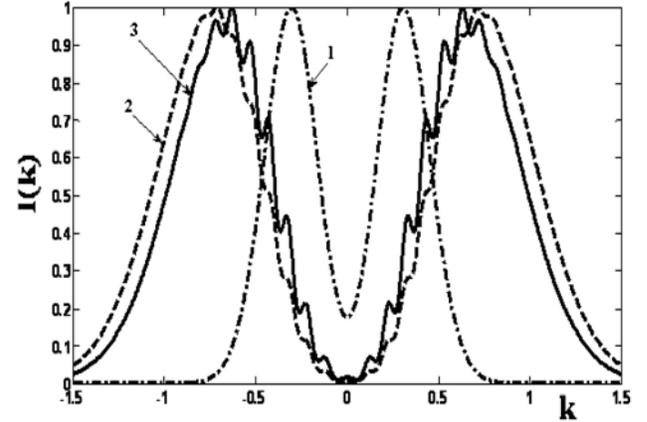


Figure 4. Dependence of the intensity of scattered ordinary wave in the collisionless magnetized plasma on the nondimensional wave parameter at different angle of inclination of prolate irregularities with respect to the external magnetic field $\gamma_0 = 0^\circ$ (curve 1), $\gamma_0 = 2.5^\circ$ (curve 2), $\gamma_0 = 3^\circ$ (curve 3).

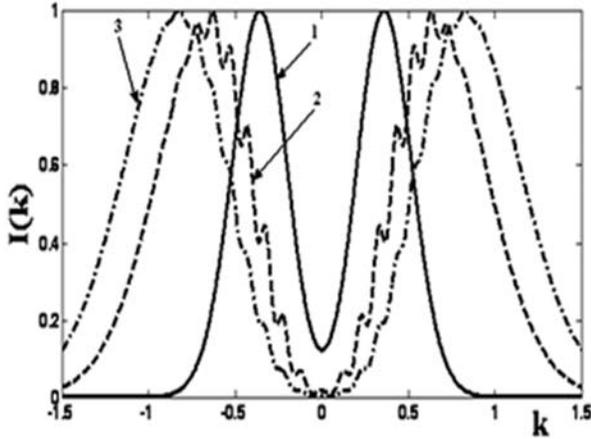


Figure 5. Evaluation of a double-humped shape of the intensity of scattered radiation in the collision magnetized plasma when $\gamma_0 = 0^\circ$, $s = 0$ (curve 1), $\gamma_0 = 3^\circ$, $s = 0.01$ (curve 2), $\gamma_0 = 3^\circ$, $s = 0.1$ (curve 3).

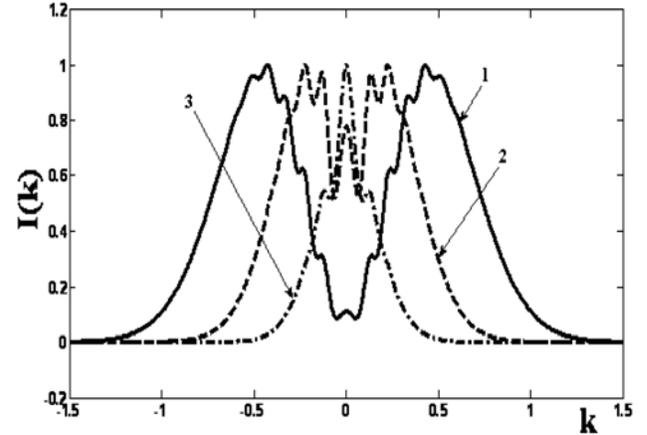


Figure 6. Formation of a gap at fixed angle of inclination $\gamma_0 = 3^\circ$ when $\chi = 100$ (curve 3), $\chi = 120$ (curve 2), $\chi = 150$ (curve 1).

$\beta = 10$, $\eta_x = 0$, $L = 160$ km is shown in Fig. 6.

Second order statistical moments: shift of maximum and broadening of the SPS of the ordinary and extraordinary waves scattered in the collision magnetized plasma are calculated at the following parameters: $l_{\parallel} = 3$ km, $L = 100$ km, $s = 10^{-3}$. Shift of maximum of the SPS of the ordinary wave always exceeds maximum displacement of the extraordinary wave. In the interval $\gamma_0 = 10^\circ - 20^\circ$ the ratio is equal to $\Delta_+/\Delta_- = 7$. At $\gamma_0 = 10^\circ$ shift of maximum for both waves is located at $\chi = 30$ (Fig. 7) and tends to the saturation increasing parameter χ . Fig. 8 shows dependence of the broadening Σ_- of the SPS for the extraordinary wave as a function of the parameter of anisotropy χ for different inclination angle γ_0 . If $\gamma_0 = 5^\circ$ and $\gamma_0 = 12^\circ$, maximums of the function Σ_- are located at $\chi = 16$ and

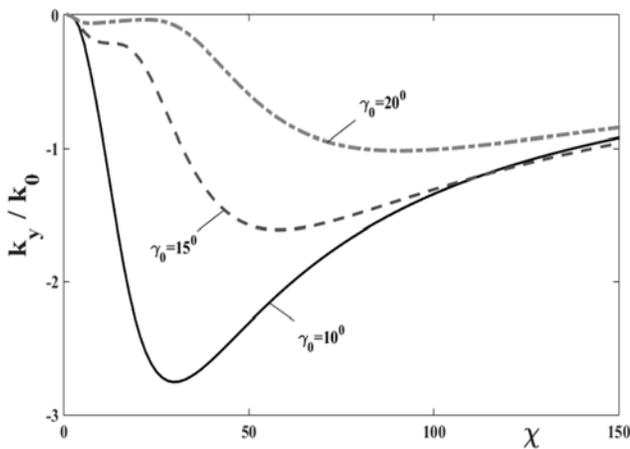


Figure 7. Plots of the maximum displacement Δ_- of the SPS of the extraordinary wave as a function of χ , at fixed γ_0 in the principle plane.

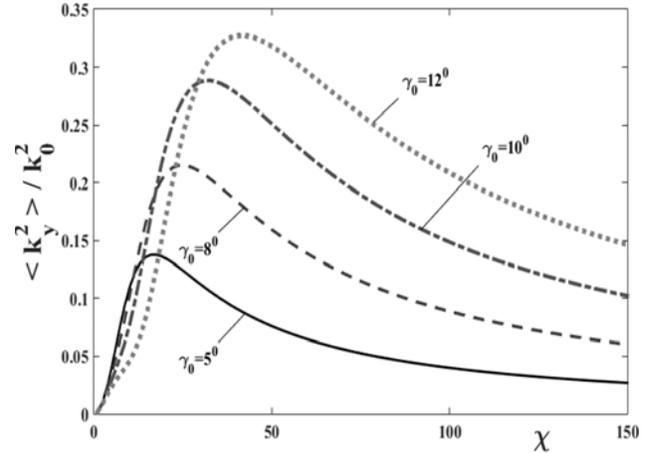


Figure 8. Broadening of the SPS of the extraordinary wave versus parameter of anisotropy χ at different inclination angle γ_0 .

$\chi = 42$, respectively. In the interval $\gamma_0 = 5^\circ - 12^\circ$ amplitude of the function Σ_- increases in 2.4 times and the spectrum broadens in two times; ratio of the widths of the ordinary and extraordinary waves does not depends on the inclination angle γ_0 : $\Sigma_+ / \Sigma_- = 7$.

4. CONCLUSION

Second order statistical moments of scattered ordinary and extraordinary EM waves in the polar ionospheric plasma: correlation function of the phase fluctuation and the SPS are investigated analytically using modify smooth perturbation method. Polarization coefficients, anisotropy factor, angle of inclination of elongated plasma irregularities with respect to the external magnetic field and the diffraction effects were taking into account. Numerical calculations were carried out for anisotropic Gaussian correlation function; frequency of an incident wave is 3 MHz. The “Double-humped Effect” in the SPS of multiple scattered ordinary and extraordinary waves in the turbulent collision magnetized plasma has been revealed for the first time. If large scale electron density irregularities are strongly aligned along the external magnetic field, SPS of both waves has a double-humped shape and a gap arises in the elongation direction. By increasing collision frequency SPS broadens, its depth increases and maximum are symmetrically displaced. Broadening of the SPS in the collision magnetized plasma two times exceeds its width in the collisionless plasma. In the collision magnetized plasma varying inclination angle in the interval ($\gamma_0 = 0^\circ - 3^\circ$) at small frequencies $s = 0.01$ internal slopes of a double-humped shape oscillates; increasing collision frequency, at $s = 0.1$ oscillations become smooth and then disappear.

“Double-humped Effect” and formation of a gap in the SPS is a result of the diffusion processes in the field aligned and perpendicular directions in the polar ionospheric F region. Evaluation of a double-humped shape in this region substantially depends on the collision frequency between plasma particles, the features of electron density irregularities (anisotropy factor, angle of inclination of prolate irregularities with respect to the geomagnetic line of forces, the mean-square fractional deviation of electron density and distance traveling by the ordinary and extraordinary waves). The absorption is due to scattering of the radio waves into plasma oscillations by the irregularities. Absorption of this type could play a significant part in heating experiments. The present results could be applied to naturally occurring or artificially created irregularities and could find practical application in propagation of short-wavelength radio waves in the Earth’s ionosphere, where random plasma irregularities are aligned with the geomagnetic field; also in communication, at observations of EM waves propagation in the upper atmosphere and remote sensing in optics and be useful in development of principles of remote sensing.

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