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Microwave Characterization of Electrical Conductivity of Composite Conductors by Half-Wavelength Coplanar Resonator

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Abstract—The aim of this work is to characterize the electrical conductivity of composite conductors deposited on an alumina substrate. Several half-wavelength coplanar resonators are realized using several pure conductors, silver (Ag), copper (Cu), gold (Au) and tin (Sn), to compare their quality factors (Q_0), related to losses, with those from analytical methods. In the literature, losses in coplanar components have been estimated by different analytical methods. We have put in evidence the relationship between electrical conductivity of the conductor and the resonator quality factor. An overall good agreement among quality factor values obtained by the analytical formulas, by numerical simulations and by microwave measurements is observed. The surface roughness is taken into account to better estimate real conductor losses. Therefore, these analytical formulas are used to extract the electrical conductivity values of the composite conductors (Ag-aC, AgSnIn and AgSn), from measured quality factors.

1. INTRODUCTION

This work is part of a research project (Plug In Nano) whose objective is to offer a new composite conductor, instead of gold used as a coating conductor in microwave and automotive connectors. The new composite conductors developed during this project have unknown electrical conductivities. In the literature, little work has been done on the characterization of the effective electrical conductivity of materials [1, 2]. Our characterization method makes use of the relationship between the resonator quality factor (Q_0) and associated losses (α_T), which depends, among several parameters, on the electrical conductivity of the metal which forms the resonator.

Several analytical formulas for the estimation of these losses, in coplanar components (CPW), have been developed [3, 4]. First, we have verified experimentally and numerically these formulas by microwave measurements of losses and quality factor Q_0 of half-wavelength resonators realized by some known conductors such as silver (Ag), copper (Cu), gold (Au) and tin (Sn). Then, we have extracted the electrical conductivities values of the composite conductors from measured quality factors of resonators realized with these metals. Three new composite conductors are measured here, which are: silver-amorphous carbon (Ag-aC), silver-tin-indium (AgSnIn) and silver-tin (AgSn).

2. ANALYTICAL FORMULAS OF LOSSES

Three loss sources are considered in coplanar half-wavelength ($\lambda/2$) resonator, as shown in Figure 1:

- Conductor loss (α_c); because of the finite conductivity of the conductor.

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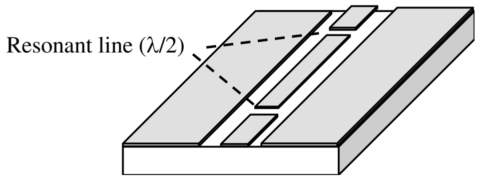

Figure 1. $\lambda/2$ CPW resonator.

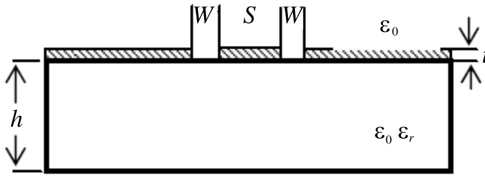

Figure 2. CPW line dimensions.

- Dielectric loss (α_d); due to the heating effects in the dielectric.
- Radiation loss (α_r); which can be split into surface wave and space wave components.

2.1. Dielectric Loss

For CPW line, the attenuation due to dielectric losses in the substrate is [5]:

$$\alpha_d = \frac{\pi}{\lambda_0} \frac{\varepsilon_r}{\sqrt{\varepsilon_{eff}}} q \tan\delta \,[\text{Np/m}] \tag{1}$$

$\tan\delta$ is the dielectric loss tangent and ε_{eff} the effective permittivity of the CPW,

$$\varepsilon_{eff} = 1 + q(\varepsilon_r - 1) \tag{2}$$

q is the filling factor,

$$q = \frac{1}{2} \frac{K(k_1)}{K(k_1')} \frac{K(k_0')}{K(k_0)} \tag{3}$$

The terms K (k_1) and K (k_0) are the complete elliptic integrals of the first kind with moduli k_0 and k_1, which are given by:

$$k_1 = \frac{\sinh(\pi S/4h)}{\sinh\left[\left(\frac{\pi(S+2W)}{4h}\right)\right]} \tag{4}$$

$$k_0 = \frac{S}{S+2W} \tag{5}$$

The terms $k'_{0,1}$ are the complementary moduli given by

$$k'_{0,1} = \sqrt{1 - k_{0,1}^2} \tag{6}$$

S, W and h are the geometric parameters of the structure (Figure 2).

2.2. Conductor Loss

The attenuation constant α_c due to conductor loss in the center strip conductor and the ground planes of the CPW is investigated by several methods.
- In the Conformal Mapping method [5], the expression of attenuation constant α_c is given below. The assumption made in deriving this expression is that thickness t of the CPW conductors is far greater than the skin depth δ in the metal. Typically, t is greater than 5δ.

$$\alpha_c = \frac{R_c + R_g}{2Z_0} \,[\text{Np/m}] \tag{7}$$

where R_c is the series resistance in ohms per unit length of the center strip conductor and is given by:

$$R_c = \frac{R_s}{4S\left(1 - k_0^2\right) K^2(k_0)} \left[\pi + \ln\left(\frac{4\pi S}{t}\right) - k_0 \ln\left(\frac{1+k_0}{1-k_0}\right)\right] \tag{8}$$

R_g is the distributed series resistance in ohms per unit length of the ground planes and is given by:

$$R_g = \frac{k_0 R_s}{4S\left(1-k_0^2\right)K^2(k_0)}\left[\pi + \ln\left(\frac{4\pi(S+2W)}{t}\right) - \frac{1}{k_0}\ln\left(\frac{1+k_0}{1-k_0}\right)\right] \tag{9}$$

and Z_0 is the coplanar waveguide characteristic impedance. The term R_s is the skin effect surface resistance given by:

$$R_s = \frac{1}{\sigma\delta}\,[\text{Ohms}] \tag{10}$$

where σ is the conductivity of the conductor in Siemens/meter and δ the skin depth.

- Ghione and Naldi [6] obtained the expression for the conductor loss through an extension of Owyang an Wu's conformal mapping approach [7]:

$$\alpha_c = \frac{R_s\sqrt{\varepsilon_{eff}}}{480\pi K(k_0)K'(k_0)\left(1-k_0^2\right)}\left\{\frac{1}{a}\left[\pi + \log\left(\frac{8\pi a(1-k_0)}{t(1+k_0)}\right)\right] - \frac{1}{b}\left[\pi + \log\left(\frac{8\pi b(1-k_0)}{t(1+k_0)}\right)\right]\right\} \tag{11}$$

- In Matched Asymptotic technique [8], both the finite thickness and non-ideal conductivity of the CPW conductors are taken into account to calculate the conductor loss. In addition, the shape of the conductor cross-section need not be rectangular, but instead can be trapezoidal.

Thus, the sides of the conductor which are normally assumed to be vertical can now be considered to be inclined at an angle θ as shown in Figure 3. In the same figure, the average center strip width and ground plane separation are indicated as $2a$ and $2b$, respectively.

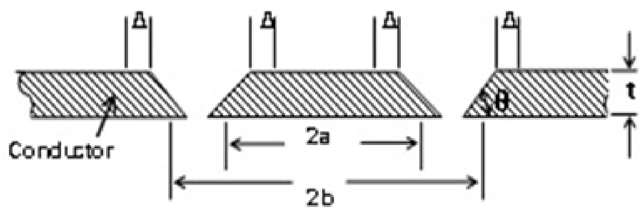

Figure 3. Coplanar waveguide conductor geometry showing the stopping distance Δ and edge profile θ.

The attenuation constant α_c due to conductor loss is given by

$$\alpha_c = \frac{R_{sm}b^2}{16Z_0\left[K^2(k)\right](b^2-a^2)}\left\{\frac{1}{a}\ln\left(\frac{2a}{\Delta}\frac{(b-a)}{(b+a)}\right) + \frac{1}{b}\ln\left(\frac{2b}{\Delta}\frac{(b-a)}{(b+a)}\right)\right\} \tag{12}$$

where

$$k = \frac{a}{b} \tag{13}$$

$$R_{sm} = \omega\mu_0 t \,\mathrm{Im}\left(\frac{\cot(k_c t) + \csc(k_c t)}{k_c t}\right) \tag{14}$$

$$k_c = \frac{2\pi}{\lambda_0}\sqrt{-j\frac{\sigma}{\omega\varepsilon_0}} \tag{15}$$

The stopping distance Δ for a given conductor thickness t, skin depth δ and edge profile θ is given in Table 1 in [8].

2.3. Radiation Loss

In addition to the conductor and dielectric losses, the loss of the energy radiated in the coplanar line contributes significantly to the total losses. It strongly depends on the frequency and geometry of the structure. For thick substrates, radiation losses are dominant because the CPW radiates much in the dielectric. The attenuation constant α_{rd} due to radiation loss is given by [9]:

$$\alpha_{rd} = \left(\frac{\pi}{2}\right)^5 \frac{1}{\sqrt{2}} \frac{(1-1/\varepsilon_r)^2}{\sqrt{1+1/\varepsilon_r}} \frac{(S+2W)^2\varepsilon_r^{3/2}}{c^3 K'(k)K(k)} f^3 \tag{16}$$

$k = k_0$ given in Equation (5), and f is the frequency. Equation (16) is supposed to be valid under the conditions of geometry (Figure 2) limited by $0.1 < S/W < 10$, thickness of the substrate $h > 3W$ and wavelength $\lambda > S + 2W$.

Total losses (α_T) which are the sum of all these components are related to the resonator unloaded quality factor Q_0 by the following formula [10]:

$$Q_0 = \frac{\pi\sqrt{\varepsilon_{eff}}}{\lambda_0 \alpha_T} \tag{17}$$

λ_0, ε_{eff} are the free space wavelength and effective permittivity, respectively, which are related to the length of the cavity resonator (l) by [5]:

$$l = \frac{\lambda_0}{2\sqrt{\varepsilon_{eff}}} \tag{18}$$

The Q_0 factor is calculated from the load quality factor Q_L at resonance by:

$$Q_0 = \frac{Q_L}{1 - 10^{(-L_A/20)}} \tag{19}$$

where

$$Q_L = \frac{f_0}{f_{2\text{–}3\,\mathrm{dB}} - f_{1\text{–}3\,\mathrm{dB}}} \tag{20}$$

L_A is the insertion loss at resonance. f_0 and $f_{1\text{–}3\,\mathrm{dB}}$, $f_{2\text{–}3\,\mathrm{dB}}$ are, respectively, the resonant frequency and the frequencies at $-3\,\mathrm{dB}$.

3. REALIZATION AND MEASUREMENT

The components are manufactured by the following steps. The 4 µm conductor layer is obtained by electro-deposition and chemical formulation of metal on an alumina substrate (Al_2O_3) with a relative dielectric permittivity of 9.8 and dielectric loss tangent of 10^{-4}. The samples are then passed to the photolithography step and etching by an etchant which depends on the metal: ferric chloride for copper, nitric acid for silver and nickel, and cyanuric acid for gold (Figure 4). Composite conductors are etched by femtosecond laser technique (Figure 5). Three $\lambda/2$ resonators have been designed and produced following the steps mentioned above, with dimensions and resonant frequencies given in Table 1.

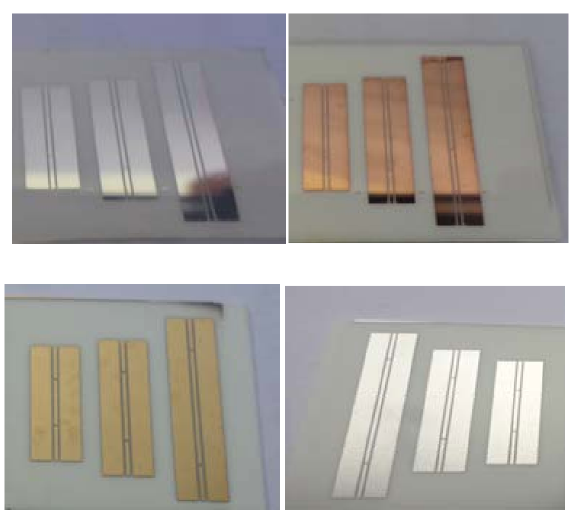

Figure 4. Realized CPW resonators on alumina substrate with pure conductors: Ag, Cu, Au and Sn (4 µm).

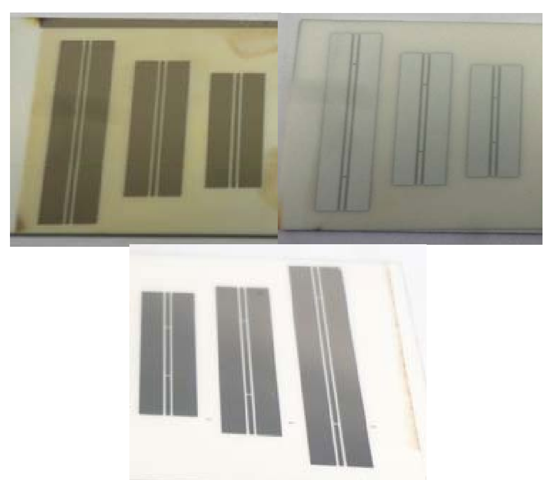

Figure 5. Realized CPW resonators on alumina substrate with composite conductors, to be characterized: Ag-aC, AgSnIn and AgSn (4 µm).

Table 1. Dimensions of fabricated $\lambda/2$ resonators.

$\lambda/2$ resonator	$S = 330\,\mu m$	$W = 146\,\mu m$	
Length ($l = \lambda/2$)	10.6 mm	6.3 mm	4.2 mm
Frequency (f_0)	6 GHz	10 GHz	15 GHz

The loaded Q factor (Q_L) is calculated by Equation (24) where the frequencies are determined from S_{12} and S_{21} transmission coefficients measured by a vector network analyzer (VNA) through coplanar probes after calibration with an adapted kit in the frequency band [10 MHz–25 GHz].

In the HFSS software, resonators were excited through two wave ports, and an absorbing boundary condition was applied in the air box to simulate radiation losses. We used the same dielectric substrate and different values of finite conductivity that corresponds to the pure conductors.

4. RESULTS AND DISCUSSION

Tables 2, 3 and 4 show the evolution of the unloaded quality factor of resonators realized with pure conductors: silver (Ag), copper (Cu), gold (Au) and tin (Sn), as a function of their electrical conductivities, with respectively the following values: 61, 58, 41 and 9 MS/m.

According to losses formulas, dielectric and radiation losses are supposed to be constant for the same design structure, and only the conductor loss changes when the metal is altered.

The results show the variation of the resonator quality factor (Q_0) as a function of the electrical conductivity of the conductor which put in evidence the relationship between these parameters.

We observe an overall good correlation between the Q_0 factors obtained by the analytical formulas, numerical simulation of HFSS and by microwave measurement of the three resonators with a small difference in some points where the Q_0 factor of theory and numerical simulation is a bit larger than

Table 2. Experimental, numerical and analytical Q_0 factors values of CPW $\lambda/2$ resonators realized by pure conductors, Ag, Cu, Au and Sn, with $f_0 = 6$ GHz.

	Quality factors values (Q_0) of CPW $\lambda/2$ resonator with $f_0 = 6$ GHz					Q_0 Mean value and uncertainty
Purs conductors	Mapping	Asymptotic	Mapping (G. Ghione)	HFSS	Measured	
Ag	142.99	152.03	144.13	142.58	139.95	144.33 ± 2.7
Cu	140.33	149.58	141.46	132.6	128.36	138.46 ± 4.7
Au	87.23	93.17	88.18	84.69	85.55	87.76 ± 1.9
Sn	39.53	21.8	40.06	28.45	28.01	31.57 ± 4.1

Table 3. Experimental, numerical and analytical Q_0 factors values of CPW $\lambda/2$ resonators realized by pure conductors, Ag, Cu, Au and Sn, with $f_0 = 10$ GHz.

	Quality factors values (Q_0) of CPW $\lambda/2$ resonator with $f_0 = 10$ GHz					Q_0 Mean value and uncertainty
Purs conductors	Mapping	Asymptotic	Mapping (G. Ghione)	HFSS	Measured	
Ag	132.28	130.81	132.63	128.3	123.15	129.34 ± 2.1
Cu	130.71	129.27	131.08	126.09	118.01	127.03 ± 2.9
Au	92.84	94.66	93.45	88.06	93.55	92.51 ± 1.47
Sn	46.95	50.38	47.47	45.03	39.98	45.96 ± 2.3

Table 4. Experimental, numerical and analytical Q_0 factors values of CPW $\lambda/2$ resonators realized by pure conductors, Ag, Cu, Au and Sn, with $f_0 = 15\,\text{GHz}$.

Purs conductors	Quality factors values (Q_0) of CPW $\lambda/2$ resonator with $f_0 = 15\,\text{GHz}$					Q_0 Mean value and uncertainty
	Mapping	Asymptotic	Mapping (G. Ghione)	HFSS	Measured	
Ag	116.56	115.92	115.86	125.3	122.07	119.42 ± 2.1
Cu	115.37	114.71	114.71	115.09	113.44	114.66 ± 0.4
Au	82.68	83.06	82.69	79.56	76.41	80.88 ± 1.5
Sn	31.12	24.53	31.42	28.03	21.66	27.35 ± 2.2

Table 5. Measured Q_0 factors of $\lambda/2$ resonators with $f_0 = 6$, 10 and 15 GHz, realized by composite conductors.

Composite conductors	Ag-aC	AgSnIn	AgSn
Q_0 factors ($f_0 = 6\,\text{GHz}$)	100.39	57.94	-
Q_0 factors ($f_0 = 10\,\text{GHz}$)	105.6	67.92	49.7
Q_0 factors ($f_0 = 15\,\text{GHz}$)	89.3	64.95	56.43

measured one. The surface roughness of conductors increases the conductor losses (α_c). We have used Hammerstad and Bekkadal formula [11], to take into account the effects of surface roughness measured with a profilometer, given by:

$$\alpha_{c,\text{rough}} = \alpha_c \left[1 + \frac{2}{\pi}\tan^{-1}\left(1.4\left(\frac{Rq}{\delta}\right)^2\right)\right] \tag{21}$$

where $\alpha_{c,\text{rough}}$ corresponds to conductor losses of a rough conductor; Rq is RMS value of the surface roughness; δ is the skin depth of the electromagnetic wave. The values of Rq measured are in the range [400, 500 nm].

The error between different extracted Q_0 factors values is indicated in the last column of Tables 2, 3 and 4. This difference can be explained by the fact that there are some loss measured in realized structures that are not taken into account in the HFSS simulation and analytical formulas as measurement errors.

The Q_0 factors of resonators realized with composite conductors to be characterized, Ag-aC, AgSnIn and AgSn, are also measured, and its values are shown in Table 2. The resonator realized with AgSn, with $f_0 = 6\,\text{GHz}$, is poorly engraved, hence unmeasurable.

We have developed a script which relates the quality factor of a resonator with the electrical conductivity of a metal that forms the resonator by different analytical formulas of losses cited in Section 2. We then extract the values of effective electrical conductivities of composite conductors from measured values of quality factor shown in Table 5. Tables 6, 7 and 8 show the values of effective electrical conductivities obtained by several analytical models of conductor loss for resonators with resonance frequencies 6, 10 and 15 GHz.

The results obtained by different analytical models are similar to each other. We observe that Ag-aC has a better electrical conductivity than AgSnIn and AgSn. The mean values of effective electrical conductivity of all performed measurements are given in Table 9.

According to these results, we can classify electrical conductivity of composite conductors characterized in this work, compared to those of pure conductors, in an ascending order like this: Sn, AgSn, AgSnIn, Au, Ag-aC, Cu, Ag.

Table 6. Effective electrical conductivity of composite conductors, Ag-aC, AgSnIn and AgSn, extracted by matched asymptotic analytical model for $\lambda/2$ resonators with resonance frequencies 6, 10 and 15 GHz.

	Extracted effective electrical conductivity (MS/m)		
Composite conductors	$f_0 = 6\,\text{GHz}$	$f_0 = 10\,\text{GHz}$	$f_0 = 15\,\text{GHz}$
Ag-aC	43.51	43.65	47.74
AgSnIn	21.38	27.34	24.58
AgSn	-	9.46	9.16

Table 7. Effective electrical conductivity of composite conductors, Ag-aC, AgSnIn and AgSn, extracted by conformal mapping analytical model for $\lambda/2$ resonators with resonance frequencies 6, 10 and 15 GHz.

	Extracted effective electrical conductivity (MS/m)		
Composite conductors	$f_0 = 6\,\text{GHz}$	$f_0 = 10\,\text{GHz}$	$f_0 = 15\,\text{GHz}$
Ag-aC	48.42	46.54	49.73
AgSnIn	28.12	25.52	27.21
AgSn	-	10.93	10.64

Table 8. Effective electrical conductivity of composite conductors, Ag-aC, AgSnIn and AgSn, extracted by conformal mapping (G. Ghione) analytical model for $\lambda/2$ resonators with resonance frequencies 6, 10 and 15 GHz.

	Extracted effective electrical conductivity (MS/m)		
Composite conductors	$f_0 = 6\,\text{GHz}$	$f_0 = 10\,\text{GHz}$	$f_0 = 15\,\text{GHz}$
Ag-aC	46.78	45.60	49.13
AgSnIn	28.53	24.47	28.55
AgSn	-	10.36	10.5

Table 9. Mean values of effective electrical conductivity of composite conductors.

Composite conductors	Ag-aC	AgSnIn	AgSn
Electrical conductivity (MS/m)	46.7 ± 1	25.5 ± 1.2	9.9 ± 0.36

5. CONCLUSION

A microwave characterization method of electrical conductivity of composite conductors is performed using coplanar half-wavelength resonators. Several analytical models are collected from literature to calculate losses in such structures and establish an analytical relation between conductivity and quality factor.

Q_0 factors values of CPW half-wavelength resonators, obtained by microwave measurement for pure conductors, are in good agreement with those obtained by analytical formulas of loss and by numerical simulations. Analytical formulas of losses are used to determine the effective electrical conductivity of composite conductors that form the resonators, from their measured quality factors. The results show that Ag-aC has a better electrical conductivity than Gold (Au). AgSnIn and AgSn have a better electrical conductivity than Tin (Sn).

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