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Wander and Spreading of Gaussian-Schell Model Beams Propagating through Anisotropic Marine-Atmospheric Turbulence

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Abstract—The effects of anisotropic turbulence on the wander and spreading of Gaussian-Schell model beams propagating in non-Kolmogorov marine-atmospheric channel are investigated. Expressions for beam wander and long-term beam spreading are derived in all conditions of marine-atmospheric turbulence. Our results indicate that the beam wander and spreading of Gaussian-Schell model beams are lower in the anisotropic turbulence than the beam in isotropic turbulence. This model can be evaluated ship-to-ship/shore optical laser communication system performance.

1. INTRODUCTION

The propagation of laser beams through marine atmospheric turbulence is a subject of considerable importance in connection with free-space optical (FSO) communication between ship and ship/shore [1]. As an optical beam propagates through the maritime atmosphere, it will experience random deflections due to refractive turbulence. As a result, the beam will wander randomly in the plane transverse to the propagation direction at some distance from its source [2].

Recently, the effects of beam wander caused by terrestrial atmospheric turbulence on the scintillation and irradiance distribution [3–5], Strehl ratio [6], and bit-error rate [7] of the ground-to-satellite laser communication systems [8] have attracted lots of attention. In the turbulence of maritime atmosphere, some authors discussed the angle of arrival fluctuation (imaging wander) effects on infrared imaging system [9], beam wander and long term beam spread for Gaussian beams propagating through Kolmogorov turbulence [10, 11]. However, these studies mentioned above were presented without considering the anisotropy of turbulence. Published theoretical papers were dedicated to the interpretation of experiments on the anisotropy [12, 13] and non-Kolmogorov [14, 15] characters of turbulence scatterers. The effects of anisotropic turbulence on propagating stochastic electromagnetic beams are discussed in [16]. These papers indicate that the anisotropy and non-Kolmogorov power spectrum are in accordance with experimental data. Understanding the effects of anisotropy and non-Kolmogorov turbulence on the wander and spreading of Gaussian-Schell model (GSM) beams propagation through marine-atmosphere is necessary for the design of nautical laser radar and optical communication system [17]. The propagation of sinh-Gaussian pulse and cosh-Gaussian pulse beam has been studied in [18, 19]. The beam wander of Gaussian wave propagating through non-Kolmogorov turbulence in atmospheric channel was studied in [20]. The quantization Gaussian laser beam propagating in a turbulent of atmosphere channel [21] and the degree of beam coherence on the multi-Schell quantization beam distortion were discussed in [22]. However, to the best of our knowledge, the effects of anisotropy and non-Kolmogorov turbulence in marine-atmospheric channel on wander and spreading of GSM beams have not been studied.

In this paper, we put forward a model of wander and spreading for GSM beams propagating in anisotropy, clean (regardless of the aerosol scattering) and non-Kolmogorov maritime turbulence, and

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analyze the influence of anisotropy and non-Kolmogorov turbulence on the wander and spreading of GSM beams.

2. NON-KOLMOGOROV POWER SPECTRUM OF ANISOTROPIC MARINE-ATMOSPHERIC TURBULENCE

In this paper, we assume, as in [17, 23], that the anisotropy of marine-atmospheric turbulence exists only along the z direction of propagation, and we consider that circular symmetry is maintained on the orthogonal plane to the propagation direction, thus we can neglect the vector properties of laser beams [23]. The non-Kolmogorov power spectrum of anisotropic marine-atmospheric turbulence can be given by the expression

$$\varphi(\kappa_\zeta,\zeta)=A(\alpha)\tilde{C}_n^2\zeta^2\exp(-\kappa_\zeta^2/\kappa_H^2)\left[1-0.061\,(\kappa_\zeta/\kappa_H)+2.836(\kappa_\zeta/\kappa_H)^{3-\alpha/2}\right](\kappa_\zeta^2+\kappa_0^2)^{-\alpha/2}, \quad (1)$$

where $\tilde{C}_n^2$ denotes the generalized refractive index structure parameter with units $m^{3-\alpha}$; ζ is an effective anisotropic factor; α denotes the general spectral power law value; $\kappa_H=c_0(\alpha)/l_0$; l_0 denotes the inner scale of turbulence; $\kappa_0=2\pi/L_0$; L_0 denotes the outer scale of turbulence, $\kappa_\zeta=(\kappa_z^2+\zeta^2\kappa^2)^{1/2}$, $\kappa=(\kappa_x^2+\kappa_y^2)^{1/2}$, κ is the spatial frequency of turbulent fluctuations, and $A(\alpha)$ is defined by

$$A(\alpha)=\frac{\Gamma(\alpha-1)}{4\pi^2}\cos\left(\frac{\alpha\pi}{2}\right),\quad 3<\alpha<5, \quad (2)$$

where $\Gamma(\cdot)$ denotes the gamma function, $c_0(\alpha)$ is given by

$$c_0(\alpha)=\left\{\pi A(\alpha)\left[\Gamma\left(\frac{3-\alpha}{2}\right)\frac{3-\alpha}{3}-0.061\Gamma\left(2-\frac{\alpha}{2}\right)\frac{4-\alpha}{3}+2.836\Gamma\left(3-\frac{3\alpha}{4}\right)\frac{4-\alpha}{2}\right]\right\}^{\frac{1}{\alpha-5}}. \quad (3)$$

3. LONG-TERM AVERAGE BEAM SPREADING IN ANISOTROPIC MARINE-ATMOSPHERIC TURBULENCE

Laser beam spreading induced by the turbulence has been mainly concerned with the determination of the long-term average beam spreading, i.e., the effects of both beam spreading and wander are included in [2]. The long-term beam spreading of GSM beams [24] through atmospheric turbulence is given by

$$w_{LT}^2(z)=\omega_0^2+\frac{4z^2}{k^2}\left(\frac{1}{\omega_0^2}+\frac{1}{\rho_{s0}^2}\right)+T(z), \quad (4)$$

where ω_0 represents the source's transverse size, $k=2\pi/\lambda$ the wave number of light, with λ being the wavelength, z the propagation path length, ρ_{s0} the transverse coherent width of the source, and $T(z)=\frac{4}{3}\pi^2z^3\int_0^\infty\kappa^3\varphi(\kappa_\zeta,\zeta)d\kappa$ the turbulent spreading factor.

Using Eq. (1), we have

$$T(z)=\frac{4}{3}\pi^2A(\alpha)\tilde{C}_n^2\zeta^2z^3\int_0^\infty\kappa^3\exp(-\kappa_\zeta^2/\kappa_H^2)\left[1-0.061(\kappa_\zeta/\kappa_H)+2.836(\kappa_\zeta/\kappa_H)^{3-\alpha/2}\right](\kappa_\zeta^2+\kappa_0^2)^{-\alpha/2}d\kappa. \quad (5)$$

The turbulent spreading factor of the beam can be reduced to (see Appendix A)

$$\begin{aligned}T(z)=&\frac{2}{3}\pi^2A(\alpha)\tilde{C}_n^2\zeta^{-2}z^3\left\{\kappa_0^{4-\alpha}\frac{\Gamma(\alpha/2-2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(2;3-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right.\\&+\kappa_0^{4-\alpha}\Gamma(2-\alpha/2){}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha}{2}-1;\frac{\kappa_0^2}{\kappa_H^2}\right)-\frac{0.061}{\kappa_H}\kappa_0^{5-\alpha}\Gamma(5/2)\left[\frac{\Gamma(\alpha/2-5/2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(\frac{5}{2};\frac{7-\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right.\\&\left.+\frac{\Gamma(5/2-\alpha/2)}{\Gamma(5/2)}\left(\frac{\kappa_0}{\kappa_H}\right)^2{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha-3}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right]+\frac{2.836}{\kappa_H^{3-\alpha/2}}\kappa_0^{4-5\alpha/4}\Gamma(7/2-\alpha/4)\left[\frac{\Gamma(3\alpha/4-7/2)}{\Gamma(\alpha/2)}\right.\\&\left.\left.{}_1\mathrm{F}_1\left(\frac{7}{2}-\frac{\alpha}{4};\frac{9}{2}-\frac{3\alpha}{4};\frac{\kappa_0^2}{\kappa_H^2}\right)+\frac{\Gamma(7/2-3\alpha/4)}{\Gamma(7/2-\alpha/4)}\left(\frac{\kappa_0}{\kappa_H}\right)^{3\alpha/2-7}{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{3\alpha}{4}-\frac{5}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right]\right\}. \quad (6)\end{aligned}$$

In Eq. (6), the former two terms in the right-hand part of Eq. (6) characterize the beam spreading caused by the component of the von Karman spectrum of turbulence, and the latter four terms in the right-hand part of Eq. (6) denote contributions of the component of the high wave number spectral bump to turbulent spreading of the beam [2].

Substituting Eq. (6) into Eq. (4), we obtain the long-term beam spreading of GSM beams in the anisotropic marine-atmospheric non-Kolmogorov turbulence

$$\begin{aligned} w_{LT}^2(z) = \omega_0^2 + \frac{4z^2}{k^2}\left(\frac{1}{\omega_0^2}+\frac{1}{\rho_{s0}^2}\right) + \frac{2}{3}\pi^2 A(\alpha)\tilde{C}_n^2\zeta^{-2}z^3\left\{\kappa_0^{4-\alpha}\frac{\Gamma(\alpha/2-2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(2;3-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right. \\ +\kappa_0^{4-\alpha}\Gamma(2-\alpha/2){}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha}{2}-1;\frac{\kappa_0^2}{\kappa_H^2}\right)\frac{0.061}{\kappa_H}\kappa_0^{5-\alpha}\Gamma(5/2)\left[\frac{\Gamma(\alpha/2-5/2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(\frac{5}{2};\frac{7-\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right. \\ \left.+\frac{\Gamma(5/2-\alpha/2)}{\Gamma(5/2)}\left(\frac{\kappa_0}{\kappa_H}\right)^2{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha-3}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right]+\frac{2.836}{\kappa_H^{3-\alpha/2}}\kappa_0^{4-5\alpha/4}\Gamma(7/2-\alpha/4)\left[\frac{\Gamma(3\alpha/4-7/2)}{\Gamma(\alpha/2)}\right. \\ \left.\left.{}_1\mathrm{F}_1\left(\frac{7}{2}-\frac{\alpha}{4};\frac{9}{2}-\frac{3\alpha}{4};\frac{\kappa_0^2}{\kappa_H^2}\right)+\frac{\Gamma(7/2-3\alpha/4)}{\Gamma(7/2-\alpha/4)}\left(\frac{\kappa_0}{\kappa_H}\right)^{3\alpha/2-7}{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{3\alpha}{4}-\frac{5}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right]\right\}. \end{aligned} \quad (7)$$

4. WANDER OF GSM BEAMS IN ANISOTROPIC MARINE-ATMOSPHERIC TURBULENCE

Beam wander is characterized by the random displacement r_c of the instantaneous center of the beam as it propagates through anisotropic marine-atmospheric turbulence, and it can be statistically described by the variance of this displacement, i.e., $\langle r_c^2\rangle$. The general model of beam wander that is valid under all conditions of turbulence is given by Andrews and Phillips [2]:

$$\langle r_c^2\rangle = 4\pi^2k^2w^2(z)\int_0^z\int_0^\infty \kappa\varphi(\kappa_\zeta,\zeta)\exp[-\kappa^2w_{LT}^2(\xi)]\left\{1-\exp\left[-\frac{2z^2\kappa^2(1-\xi/z)^2}{k^2w^2(z)}\right]\right\}d\kappa d\xi, \quad (8)$$

where $w^2(z) = \omega_0^2 + 4z^2(1/\omega_0^2 + 1/\rho_{s0}^2)/k^2$ is the free-space beam radius at distance z.

It is known that beam wander is mostly caused by the refraction of large-scale turbulence near the transmitter, in the geometrical optics approximation, and the last term in Eq. (8) becomes [2]

$$1-\exp\Big[-\frac{2z^2\kappa^2(1-\xi/z)^2}{k^2w^2(z)}\Big] \approx \frac{2z^2\kappa^2(1-\xi/z)^2}{k^2w^2(z)}, \qquad z\kappa^2/k \ll 1. \quad (9)$$

On substituting Eq. (9) into Eq. (8), we have

$$\langle r_c^2\rangle = 8\pi^2\int_0^z(1-\xi/z)^2\int_0^\infty \kappa^3\varphi(\kappa_\zeta,\zeta)\exp[-\kappa^2w_{LT}^2(\xi)]d\kappa d\xi. \quad (10)$$

After some lengthy mathematical manipulations (see Appendix B), the integral for κ in Eq. (10) is given as

$$\begin{aligned} N(\xi) = \frac{1}{2}A(\alpha)\zeta^{-2}\tilde{C}_n^2\left\{\kappa_0^{4-\alpha}\frac{\Gamma(\alpha/2-2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(2;3-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\right. \\ +\kappa_0^{4-\alpha}\Gamma(2-\alpha/2){}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha}{2}-1;\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\frac{0.061}{\kappa_{HL}}\kappa_0^{5-\alpha}\Gamma(5/2)\left[\frac{\Gamma(\alpha/2-5/2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(\frac{5}{2};\frac{7-\alpha}{2};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\right. \\ \left.+\frac{\Gamma(5/2-\alpha/2)}{\Gamma(5/2)}\left(\frac{\kappa_0}{\kappa_{HL}}\right)^2{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha-3}{2};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\right]+\frac{2.836}{\kappa_{HL}^{3-\alpha/2}}\kappa_0^{4-5\alpha/4}\Gamma(7/2-\alpha/4)\left[\frac{\Gamma(3\alpha/4-7/2)}{\Gamma(\alpha/2)}\right. \\ \left.\left.{}_1\mathrm{F}_1\left(\frac{7}{2}-\frac{\alpha}{4};\frac{9}{2}-\frac{3\alpha}{4};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)+\frac{\Gamma(7/2-3\alpha/4)}{\Gamma(7/2-\alpha/4)}\left(\frac{\kappa_0}{\kappa_{HL}}\right)^{3\alpha/2-7}{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{3\alpha}{4}-\frac{5}{2};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\right]\right\}, \end{aligned} \quad (11)$$

where $1/\kappa_{HL}^2 = 1/\kappa_H^2 + \zeta^{-2} w_{LT}^2(\xi)$, substituting Eq. (11) into Eq. (10), we finally obtain the variance of the instantaneous center of the beam in the marine-atmospheric turbulence

$$\langle r_c^2 \rangle = 8\pi z^2 \int_0^z N(\xi)(1-\xi/z)^2 d\xi. \tag{12}$$

5. NUMERICAL RESULTS AND ANALYSIS

In this section, we show the numerical discussions and analysis for the spreading and wander of GSM beams as the functions of the effective anisotropic factor ζ, general spectral power law α, inner scale l_0 of turbulence, outer scale L_0 of turbulence, transverse coherent width of the source ρ_{s0}, source's transverse size ω_0, and generalized refractive index structure parameter $\tilde{C}_n^2$ in a anisotropic marine-atmospheric turbulence channel.

We plot Figure 1 to explore the influence of the effective anisotropic factor ζ on the normalized beam width $w_{LT}(z)/w(0)$ of GSM beams. We change z from 1000 m to 2000 m, for the effective anisotropic factor $\zeta = 1$, $\zeta = 2$, $\zeta = 5$, and $\zeta = 10$, in the case $\alpha = 3.27$, $\alpha = 3.47$, $\alpha = 3.67$ and $\alpha = 3.87$, with the other parameters $\lambda = 1550\,\text{nm}$, $L_0 = 5\,\text{m}$, $l_0 = 0.001\,\text{m}$, $\tilde{C}_n^2 = 5 \times 10^{-13}\,\text{m}^{3-\alpha}$, $\omega_0 = 0.05\,\text{m}$ and $\rho_{s0} = 0.03\,\text{m}$. From Figure 1 we can see that the spreading of GSM beams decreases as the effective anisotropic factor ζ increasing ($\zeta = 1$ being the isotropic turbulence). The result is in accordance with Figure 2(a) and Figure 2(b) in [16]. Additionally, it also indicates that the normalized beam width decreases with increasing general spectral power law α, and α has slight influence on the beam spreading when the value of ζ is large enough. As a consequence, the beam spreading is lower in the anisotropic and larger general spectral power law turbulence than in the marine-atmospheric channel without considering anisotropy.

Figure 2 plots the normalized beam width $w_{LT}(z)/w(0)$ of GSM beams as a function of the source's transverse size ω_0 and the source's transverse coherent width ρ_{s0} in isotropic (a) and anisotropic (b) marine-atmospheric turbulence, with the system parameters $\lambda = 1550\,\text{nm}$, $\alpha = 11/3$, $L_0 = 5\,\text{m}$, $l_0 = 0.001\,\text{m}$, $z = 2\,\text{km}$, $\tilde{C}_n^2 = 5 \times 10^{-13}\,\text{m}^{-2/3}$, $\zeta = 1$ in (a) and $\zeta = 2$ in (b). Figure 2 shows that ρ_{s0} slightly affects the beam spreading which is instead by decreasing with the increasing ω_0. This

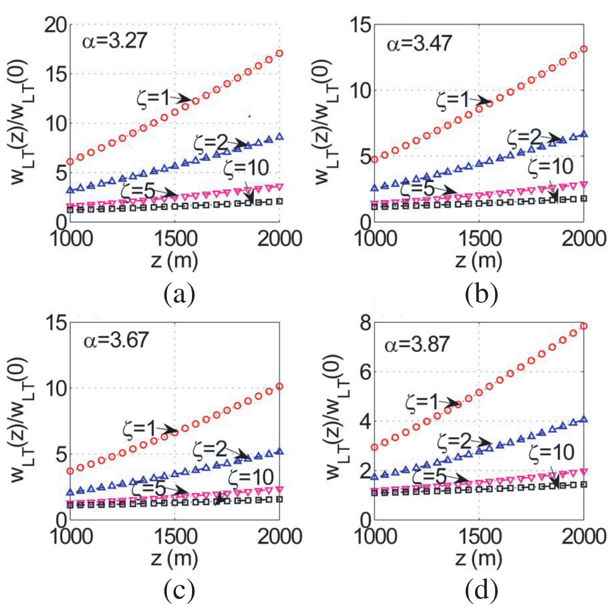


Figure 1. Spreading of GSM beam versus the effective anisotropic factor ζ and the transmission distance z in anisotropic marine-atmospheric turbulence in the case (a) $\alpha = 3.27$, (b) $\alpha = 3.47$, (c) $\alpha = 3.67$ and (d) $\alpha = 3.87$.

result agrees with Figure 1 in [25] when the transmission distance is less than 2 km. It is known that beam spreading is associated with the effect of diffraction. Larger ω_0 will lead to weaker diffraction. Thus, the beam spreading decreases with increasing ω_0.

Figure 3 demonstrates the normalized beam width $w_{LT}(z)/w(0)$ of GSM beams as a function of the inner scale l_0 of turbulence, outer scale L_0 of turbulence in isotropic (a) and anisotropic (b) marine-atmospheric channel, with the system parameters $\lambda = 1550\,\text{nm}$, $\alpha = 11/3$, $\omega_0 = 0.05\,\text{m}$, $\rho_{s0} = 0.03\,\text{m}$, $z = 2\,\text{km}$, $\tilde{C}_n^2 = 5 \times 10^{-13}\,\text{m}^{-2/3}$, $\zeta = 1$ in (a) and $\zeta = 2$ in (b). As indicated by Figure 3(a) and Figure 3(b), the normalized beam width of GSM beam increases with decreasing the inner scale l_0 when the propagation distance z is fixed. The inner scale l_0, which forms the lower limit of the inertial range, has a smaller value for strong turbulence and a larger value for weak turbulence. As a result, a smaller l_0 leads to a wider spreading of GSM beams. However, the outer scale L_0 of turbulence has little influence on the beam spreading.

Figure 4 plots the influence of the effective anisotropic factor ζ on the root-mean-square (rms) $\langle r_c^2\rangle$

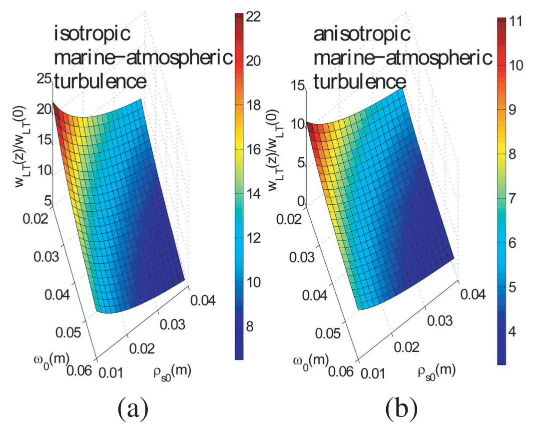


Figure 2. Spreading of GSM beam versus the source's transverse size ω_0 and the source's transverse coherent width ρ_{s0} in (a) isotropic and (b) anisotropic marine-atmospheric turbulence.

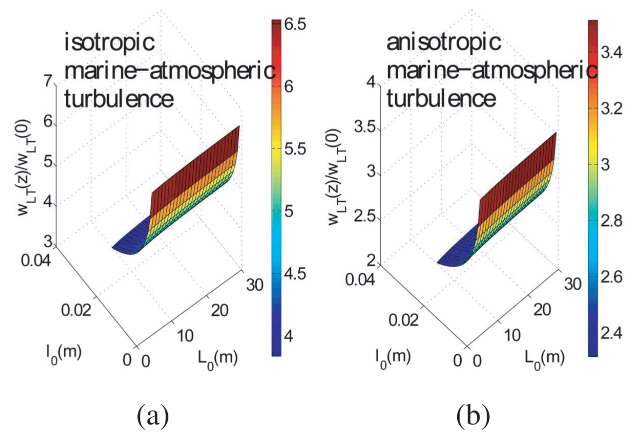


Figure 3. Spreading of GSM beam versus the inner scale l_0 of turbulence, the outer scale L_0 of turbulence in (a) isotropic and (b) anisotropic marine-atmospheric turbulence.

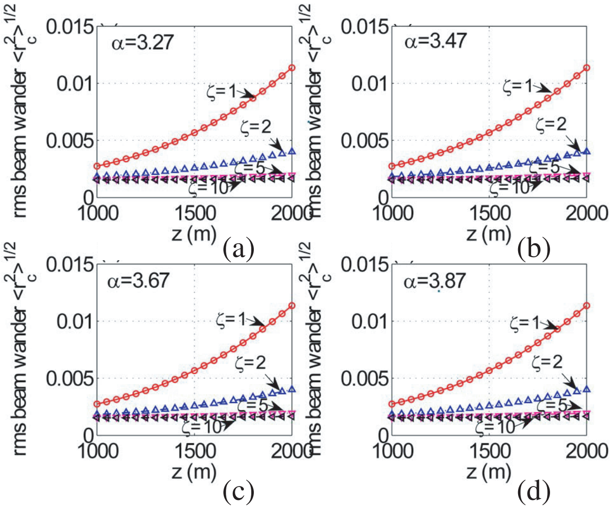
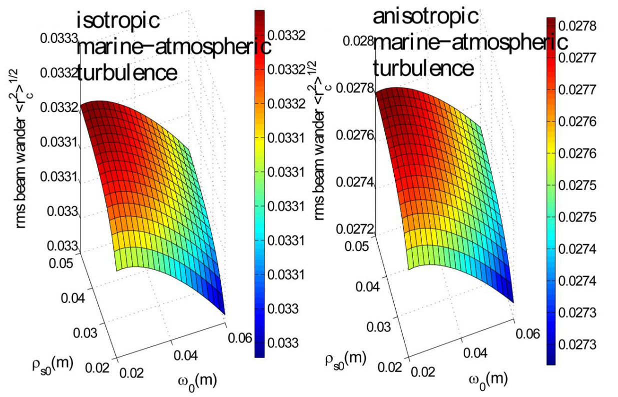
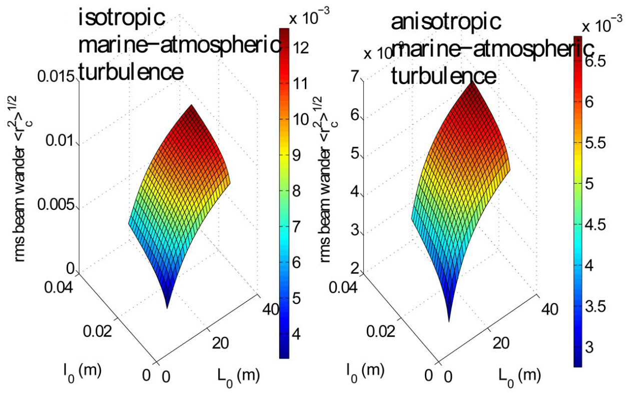


Figure 4. Wander of GSM beam versus the effective anisotropic factor ζ and the transmission distance z in anisotropic marine-atmospheric turbulence in the case (a) $\alpha = 3.27$, (b) $\alpha = 3.47$, (c) $\alpha = 3.67$ and (d) $\alpha = 3.87$.



(a) (b)

Figure 5. Wander of GSM beam versus the source's transverse size ω_0 and the source's transverse coherent width ρ_{s0} in (a) isotropic and (b) anisotropic marine-atmospheric turbulence.



(a) (b)

Figure 6. Wander of GSM beam versus the inner scale l_0 of turbulence, the outer scale L_0 of turbulence in (a) isotropic and (b) anisotropic marine-atmospheric turbulence.

of GSM beams by changing z from 1 km to 2 km, for the effective anisotropic factor $\zeta = 1$, $\zeta = 2$, $\zeta = 5$, and $\zeta = 10$, in the case $\alpha = 3.27$, $\alpha = 3.47$, $\alpha = 3.67$ and $\alpha = 3.87$, with the other parameters $\lambda = 1550\,\text{nm}$, $L_0 = 5\,\text{m}$, $l_0 = 0.001\,\text{m}$, $\tilde{C}_n^2 = 5 \times 10^{-13}\,\text{m}^{3-\alpha}$, $\omega_0 = 0.05\,\text{m}$ and $\rho_{s0} = 0.02\,\text{m}$. As indicated in Figure 4, the general spectral power law α has little influence on the beam wander. On the other hand, the wader of GSM beams decreases as the effective anisotropic factor ζ increasing. This is caused by the large scale L_0 of turbulence leading to larger beam wander and the anisotropy can suppress the divergence directionality of the beam. As a consequence, channels with anisotropic turbulence will have lower beam wander.

Figure 5 demonstrates the rms $\langle r_c^2 \rangle$ of GSM beams as a function of the source's transverse size ω_0 and the source's transverse coherent width ρ_{s0} in isotropic (a) and anisotropic (b) marine-atmospheric turbulence, with the system parameters $\lambda = 1550\,\text{nm}$, $\alpha = 11/3$, $L_0 = 5\,\text{m}$, $l_0 = 0.001\,\text{m}$, $z = 1.5\,\text{km}$, $\tilde{C}_n^2 = 5 \times 10^{-13}\,\text{m}^{-2/3}$, $\zeta = 1$ in (a) and $\zeta = 2$ in (b). Figure 5 indicates that the beam wander decreases with the increasing ρ_{s0} and ω_0 in both the isotropic and anisotropic marine-atmospheric turbulence. Larger transverse coherent width will lead to a smaller beam wander than a fully coherent beam. The partially coherent beam is more suitable for obtaining smaller beam wander. It is known that the beam wander is affected mainly by large turbulence cells size, and an increase of the beam spot size is physically equivalent to filtering out turbulence cell size which contributes to the beam wander [10].

Figure 6 shows the rms $\langle r_c^2 \rangle$ of GSM beams as a function of the inner scale l_0 of turbulence, the outer scale L_0 of turbulence in isotropic (a) and anisotropic (b) marine-atmospheric turbulence, with the system parameters $\lambda = 1550\,\text{nm}$, $\alpha = 11/3$, $\omega_0 = 0.05\,\text{m}$, $\rho_{s0} = 0.03\,\text{m}$, $z = 1.5\,\text{km}$, $\tilde{C}_n^2 = 5 \times 10^{-13}\,\text{m}^{-2/3}$, $\zeta = 1$ in (a) and $\zeta = 2$ in (b). From Figure 6(a) and Figure 6(b), we deduce that the inner scale l_0 has little influence on the beam wander which is instead remarkably affected by the outer scale L_0. This is a consequence that the beam wander is affected mainly by large turbulence cells size. Larger L_0 means larger turbulence cells size which leads to stronger beam wander. The result is also in accordance with the case in terrestrial turbulence.

6. CONCLUSIONS

In summary, we developed the expressions of long-term beam spreading and wander for GSM beams propagating through anisotropic and non-Kolmogorov turbulence of marine-atmosphere. The influence of turbulent parameters: anisotropy, non-Kolmogorov feature, the inner and outer scale of turbulence, and the system parameters: the transverse coherent width and transverse size of source on spreading and wander for GSM beams in the marine-atmospheric turbulence environment are investigated by numerical integration and simulation. We find that larger effective anisotropic factor and larger source's transverse size will generate lower beam spreading and beam wander. The beam spreading decreases with increasing general spectral power law for given effective anisotropic factor. The beam spreading increases with increasing the inner scale of turbulence; however, the inner scale has little influence on the beam wander. On the other hand, the beam wander increases with increasing the outer scale of turbulence. Nevertheless, the outer scale has little influence on the beam spreading. The results show that larger transverse coherent width will lead to a smaller beam wander. Compared to the fully coherent beam, the partially coherent beam is more suitable to obtain smaller beam wander. Our conclusions can be helpful for ship-to-ship/shore FSO system design and mission planning to control the beam wander and spreading in the marine-atmospheric turbulence environment.

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APPENDIX A.

In this appendix, we derive the turbulent spreading factor $T(z)$ of the beam for non-Kolmogorov anisotropic turbulence in the marine-atmospheric channel

$$T(z)=\frac{4}{3}\pi^2 z^3\int_0^\infty \kappa^3\varphi(\kappa_\zeta,\zeta)d\kappa, \tag{A1}$$

where $\varphi(\kappa_\zeta,\zeta)$ is given by the expression

$$\varphi(\kappa_\zeta,\zeta)=A(\alpha)\tilde{C}_n^2\zeta^2\exp(-\kappa_\zeta^2/\kappa_H^2)\left[1-0.061\,(\kappa_\zeta/\kappa_H)+2.836(\kappa_\zeta/\kappa_H)^{3-\alpha/2}\right](\kappa_\zeta^2+\kappa_0^2)^{-\alpha/2}. \tag{A2}$$

Substituting Eq. (A2) to Eq. (A1), we have

$$T(z)=\frac{4}{3}\pi^2 z^3A(\alpha)\zeta^2\tilde{C}_n^2\int_0^\infty\frac{\kappa^3}{(\kappa_\zeta^2+\kappa_0^2)^{\alpha/2}}\exp(-\kappa_\zeta^2/\kappa_H^2)\left[1-0.061(\kappa_\zeta/\kappa_H)+2.836(\kappa_\zeta/\kappa_H)^{3-\alpha/2}\right]d\kappa. \tag{A3}$$

Let $\kappa'=\zeta\kappa$, and then we obtain

$$\begin{aligned}T(z)&=\frac{4}{3}\pi^2 z^3A(\alpha)\zeta^2\tilde{C}_n^2\int_0^\infty\frac{(\zeta^{-1}\kappa')^3}{(\kappa'^2+\kappa_0^2)^{\alpha/2}}\exp(-\kappa'^2/\kappa_H^2)\left[1-0.061(\kappa'/\kappa_H)+2.836(\kappa'/\kappa_H)^{3-\alpha/2}\right]d\kappa\\&=\frac{4}{3}\pi^2 z^3A(\alpha)\zeta^{-2}\tilde{C}_n^2\int_0^\infty\frac{\kappa'^3}{(\kappa'^2+\kappa_0^2)^{\alpha/2}}\exp(-\kappa'^2/\kappa_H^2)\left[1-0.061(\kappa'/\kappa_H)+2.836(\kappa'/\kappa_H)^{3-\alpha/2}\right]d\kappa,\end{aligned} \tag{A4}$$

and then Eq. (A4) can be reduced to

$$\begin{aligned}T(z)\;=\;&\frac{4}{3}\pi^2 z^3A(\alpha)\zeta^{-2}\tilde{C}_n^2\int_0^\infty\frac{\kappa'^3}{(\kappa'^2+\kappa_0^2)^{\alpha/2}}\exp(-\kappa'^2/\kappa_H^2)d\kappa'\\&-\frac{0.061\kappa'^4}{\kappa_H(\kappa'^2+\kappa_0^2)^{\alpha/2}}\exp\left(-\frac{\kappa'^2}{\kappa_H^2}\right)d\kappa'+\frac{2.836\kappa'^{6-\alpha/2}}{\kappa_H^{3-\alpha/2}(\kappa'^2+\kappa_0^2)^{\alpha/2}}\exp\left(-\frac{\kappa'^2}{\kappa_H^2}\right)d\kappa',\end{aligned} \tag{A5}$$

By the integral formula [2]

$$\int_0^\infty\kappa^{2\mu}\frac{\exp(-\kappa^2/\kappa_H^2)}{(\kappa^2+\kappa_0^2)^{\alpha/2}}d\kappa=\frac{1}{2}\kappa_0^{2\mu+1-\alpha}\Gamma\left(\mu+\frac{1}{2}\right)\mathrm{U}\left(\mu+\frac{1}{2};\mu+\frac{3}{2}-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right), \tag{A6}$$

where $U(a;c;z)$ is the confluent hypergeometric function of the second kind and is a linear combination of functions of the first kind that can be expressed as

$$\begin{aligned}\mathrm{U}\left(\mu+\frac{1}{2};\mu+\frac{3}{2}-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\;=\;&\frac{\Gamma(\alpha/2-\mu-1/2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(\mu+\frac{1}{2};\mu+\frac{3}{2}-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\\&+\frac{\Gamma(\mu+1/2-\alpha/2)}{\Gamma(\mu+1/2)}\left(\frac{\kappa_0}{\kappa_H}\right)^{\alpha-2\mu-1}{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{1}{2}-\mu+\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right),\end{aligned} \tag{A7}$$

we obtain the expression of $T(z)$

$$T(z) = \frac{2}{3}\pi^2 A(\alpha)\tilde{C}_n^2\zeta^{-2}z^3\left\{\kappa_0^{4-\alpha}\frac{\Gamma(\alpha/2-2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(2;3-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right.$$
$$+\kappa_0^{4-\alpha}\Gamma(2-\alpha/2){}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha}{2}-1;\frac{\kappa_0^2}{\kappa_H^2}\right)-\frac{0.061}{\kappa_H}\kappa_0^{5-\alpha}\Gamma(5/2)\left[\frac{\Gamma(\alpha/2-5/2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(\frac{5}{2};\frac{7-\alpha}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right.$$
$$\left.+\frac{\Gamma(5/2-\alpha/2)}{\Gamma(5/2)}\left(\frac{\kappa_0}{\kappa_H}\right)^2{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha-3}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right]+\frac{2.836}{\kappa_H^{3-\alpha/2}}\kappa_0^{4-5\alpha/4}\Gamma(7/2-\alpha/4)\left[\frac{\Gamma(3\alpha/4-7/2)}{\Gamma(\alpha/2)}\right.$$
$$\left.\left.{}_1\mathrm{F}_1\left(\frac{7}{2}-\frac{\alpha}{4};\frac{9}{2}-\frac{3\alpha}{4};\frac{\kappa_0^2}{\kappa_H^2}\right)+\frac{\Gamma(7/2-3\alpha/4)}{\Gamma(7/2-\alpha/4)}\left(\frac{\kappa_0}{\kappa_H}\right)^{3\alpha/2-7}{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{3\alpha}{4}-\frac{5}{2};\frac{\kappa_0^2}{\kappa_H^2}\right)\right]\right\}. \quad \text{(A8)}$$

APPENDIX B.

The expression of $N(\xi)$ is given by

$$N(\xi) = \int_0^\infty \kappa^3\varphi(\kappa_\zeta,\zeta)\exp[-\kappa^2 w_{LT}^2(\xi)]d\kappa$$
$$= A(\alpha)\tilde{C}_n^2\zeta^{-2}\int_0^\infty \kappa'^3\left[1-0.061\frac{\kappa'}{\kappa_H}+2.836\left(\frac{\kappa'}{\kappa_H}\right)^{3-\alpha/2}\right]\exp\left[-\kappa'^2\left(\frac{1}{\kappa_H^2}+\frac{w_{LT}^2(\xi)}{\zeta^2}\right)\right](\kappa'^2+\kappa_0^2)^{-\alpha/2}d\kappa'$$
$$= A(\alpha)\tilde{C}_n^2\zeta^{-2}\int_0^\infty \kappa'^3\left[1-0.061\frac{\kappa'}{\kappa_H}+2.836\left(\frac{\kappa'}{\kappa_H}\right)^{3-\alpha/2}\right]\frac{\exp[-\kappa'^2/\kappa_{HL}^2]}{(\kappa'^2+\kappa_0^2)^{\alpha/2}}d\kappa', \quad \text{(B1)}$$

where $1/\kappa_{HL}^2 = 1/\kappa_H^2 + \zeta^{-2}w_{LT}^2(\xi)$, by the integral formula Eq. (A6), we obtain the expression of $N(\xi)$

$$N(\xi) = \frac{1}{2}A(\alpha)\zeta^{-2}\tilde{C}_n^2\left\{\kappa_0^{4-\alpha}\frac{\Gamma(\alpha/2-2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(2;3-\frac{\alpha}{2};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\right.$$
$$+\kappa_0^{4-\alpha}\Gamma(2-\alpha/2){}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha}{2}-1;\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\frac{0.061}{\kappa_{HL}}\kappa_0^{5-\alpha}\Gamma(5/2)\left[\frac{\Gamma(\alpha/2-5/2)}{\Gamma(\alpha/2)}{}_1\mathrm{F}_1\left(\frac{5}{2};\frac{7-\alpha}{2};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\right.$$
$$\left.+\frac{\Gamma(5/2-\alpha/2)}{\Gamma(5/2)}\left(\frac{\kappa_0}{\kappa_{HL}}\right)^2{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{\alpha-3}{2};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\right]+\frac{2.836}{\kappa_{HL}^{3-\alpha/2}}\kappa_0^{4-5\alpha/4}\Gamma(7/2-\alpha/4)\left[\frac{\Gamma(3\alpha/4-7/2)}{\Gamma(\alpha/2)}\right.$$
$$\left.\left.{}_1\mathrm{F}_1\left(\frac{7}{2}-\frac{\alpha}{4};\frac{9}{2}-\frac{3\alpha}{4};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)+\frac{\Gamma(7/2-3\alpha/4)}{\Gamma(7/2-\alpha/4)}(\frac{\kappa_0}{\kappa_{HL}})^{3\alpha/2-7}{}_1\mathrm{F}_1\left(\frac{\alpha}{2};\frac{3\alpha}{4}-\frac{5}{2};\frac{\kappa_0^2}{\kappa_{HL}^2}\right)\right]\right\}. \quad \text{(B2)}$$

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