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The Research of Reciprocal Relations for Nonlinear Quadripole in the Magnetic Field

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Abstract—This paper deals with experimental verification of reciprocity relations for nonlinear quadripole, such as Hall transducer. It is shown that the matrix of quadripole resistances can be decomposed into the sum of matrices. The classical reciprocity relations will be valid for linear part of matrix of resistances.

1. INTRODUCTION

Symmetry relations of kinetic coefficients for thermodynamics processes were found by Onsager 1931 [1]. He used linear approaching, the condition of invariance of macroscopic motion in relation to the reversion of time and the supposition that middle relaxation of spontaneous fluctuations in the system takes place in accordance with macroscopic laws. Onsager reciprocal relations are fundamental, and they are valid not only in thermodynamics, but also in mechanics [2], theories of antennas [3], and acoustics [4].

Theoretical analysis has shown that for any homogeneous, stationary and isotropic medium which is in the external magnetic field of $\bar{B}_e$, classic reciprocal relations must be valid even at the presence of non-linearity [5] and in inhomogeneous field [6]. Both theoretical reasonings and partial results require experimental verification in more general conditions.

2. RECIPROCAL RELATIONS

Experimental verification of reciprocal relations is offered for the matrix $\hat{R}$ of resistances of nonlinear multipole, determined as

$$\varphi_k = \sum_{m=1}^{M} R_{km}\left(\bar{I}, \bar{B}_e\right) I_m. \tag{1}$$

Here k, m are numbers of contacts of multipole, M — their amount, φ_k — electric potential of kth contact of multipole I_m — current going through m contact, $\bar{I}$ — the vector of currents, made from the components of I_m.

The verified relations are based on the supposition that

$$R_{km}\left(\bar{I}, \bar{B}_e\right) = R_{mk}\left(-\bar{I}, -\bar{B}_e\right), \tag{2}$$

and the components of resistances matrix $\hat{R}$ can be decomposed into the sum:

$$R_{km}\left(\bar{I}, \bar{B}_e\right) = R^{A}_{km} + R^{H}_{km}\left(\bar{B}_e\right) + R^{NL}_{km}\left(\bar{I}\right), \tag{3}$$

where R^{A}_{km} are components of linear resistances matrix $\hat{R}^{A}$ which does not depend on currents and external magnetic field and which are responsible for resistance of asymmetry. $R^{H}_{km}(\bar{B}_e)$ are the

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components of matrix $\hat{R}^H$ which does not depend on currents. Matrix $\hat{R}^H$ is responsible for Hall effect. $R_{km}^{NL}(\bar{I})$ are components of matrix $\hat{R}^{NL}$ which does not depend on any external magnetic field and which are responsible for non-linear phenomena.

From Eqs. (2) and (3), one can get experimentally checked reciprocal relations:

$$\begin{aligned} R_{km}^A &= R_{mk}^A, \\ R_{km}^H\left(\bar{B}_e(\bar{r})\right) &= R_{mk}^H\left(-\bar{B}_e(\bar{r})\right) = -R_{mk}^H\left(\bar{B}_e(\bar{r})\right). \end{aligned} \tag{4}$$

3. THE METHODS OF VERIFICATION

To experimentally verify the reciprocal relations of Eqs. (2) and (4), we have applied Hall transducer (Fig. 1) in the quadripole mode. So with its help, it is possible to distinctly register the influence of external magnetic field of B_e. Then for measuring of non-diagonal elements of resistances matrix, we should measure voltages in two modes with a few values of current of i_k:

(i) Contacts 1 and 3 are connected to the current source of i_k, and the voltage $u_1 = \varphi_2 - \varphi_4$ between contacts 2 and 4 is measured, and here $I_1 = i_k$, $I_3 = -i_k$, $I_2 = I_4 = 0$.

(ii) Contacts 2 and 4 are connected to the source of current of i_k, and the voltage $u_2 = \varphi_1 - \varphi_3$ between contacts 1 and 3 is measured, and here $I_2 = i_k$, $I_4 = -i_k$, $I_1 = I_3 = 0$.

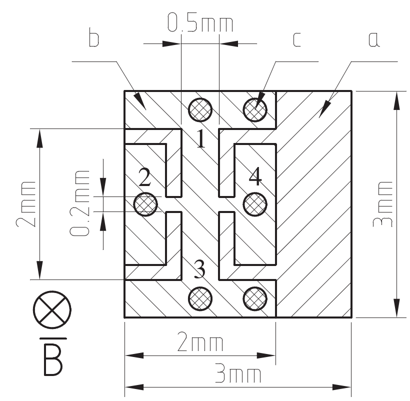

Figure 1. Geometry of galvanomagnetic transducer under research.

From the measured voltages of u_1 and u_2, we exclude thermal e.m.f of transducer contacts and the systematic errors made by the measuring instruments. For this purpose, the values of voltage measured in zero external magnetic-field are approximated by polynomials

$$\tilde{u}_1(i) = \tilde{a}_0 + \tilde{a}_1 i + \tilde{a}_2 i^2 + \ldots, \tilde{u}_2(i) = \tilde{b}_0 + \tilde{b}_1 i + \tilde{b}_2 i^2 + \ldots \tag{5}$$

Here and further the approximated polynomials and their coefficients are marked with tilda at the top. In these expressions, coefficients $\tilde{a}_0$ and $\tilde{b}_0$ are the systematic error of measuring of voltage. Through the obtained dependences, we determine the nonlinear functions $\tilde{R}_1(i)$ and $\tilde{R}_2(i)$ according to formulas

$$\begin{aligned} \tilde{R}_1(i) &= \frac{\tilde{u}_1(i) - \tilde{a}_0}{i} = \tilde{a}_1 + \tilde{a}_2 i + \tilde{a}_3 i^2 + \ldots = \tilde{a}_1 + \tilde{R}_1^{NL}(i), \\ \tilde{R}_2(i) &= \frac{\tilde{u}_2(i) - \tilde{b}_0}{i} = \tilde{b}_1 + \tilde{b}_2 i + \tilde{b}_3 i^2 + \ldots = \tilde{b}_1 + \tilde{R}_2^{NL}(i). \end{aligned} \tag{6}$$

These functions according to Eqs. (1)–(4) are related to the components of non-diagonal elements of the resistance matrix $\hat{R}$ of Eq. (2) in the following way:

$$\begin{aligned} R_1 &= \frac{u_1}{i_k} = \frac{\varphi_2 - \varphi_4}{i_k} = R_{21} - R_{23} + R_{43} - R_{41}, \\ R_2 &= \frac{u_2}{i_k} = \frac{\varphi_1 - \varphi_3}{i_k} = R_{12} - R_{14} + R_{34} - R_{32}. \end{aligned} \tag{7}$$

The extraction of intercommunicating resistance (linearization) which is independent of current was carried out in the following way:

$$a_1\left(\bar{B}_e\right) = \frac{u_1(i) - \widetilde{a}_0}{i} - \widetilde{R}_1^{NL}(i),$$
$$b_1\left(\bar{B}_e\right) = \frac{u_2(i) - \widetilde{b}_0}{i} - \widetilde{R}_2^{NL}(i). \quad (8)$$

For estimating linearization efficiency, we calculated standard deviations (SD) of values of functions $a_1(\bar{B}_e)$ and $b_1(\bar{B}_e)$ at different values of controlling current and fixed external field of $\bar{B}_e$. Linearization is considered as effective, if the calculated SD does not exceed SD of measuring noise. If this condition is fulfilled at different values of the external magnetic-field induction, then it confirms the idea that nonlinear part of matrix of resistances in Eq. (3) does not depend on induction of external magnetic-field of $\bar{B}_e$.

In functions $a_1(\bar{B}_e)$ and $b_1(\bar{B}_e)$, formulas (8) according to relations (1), (3) and (7), it is possible to distinguish components $R_1^H(\bar{B}_e)$ and $R_2^H(\bar{B}_e)$ which depend on external magnetic-field of B_e, and components R_1^A and R_2^A which do not depend on external magnetic-field B_e, accordingly

$$a_1\left(\bar{B}_e\right) = R_1^A + R_1^H\left(\bar{B}_e\right), \quad b_1\left(\bar{B}_e\right) = R_2^A + R_2^H\left(\bar{B}_e\right),$$

for which the next reciprocal relations follow from formulas (4) and (7):

$$R_1^A = R_2^A = R_A, \quad R_1^H\left(\bar{B}_e\right) = -R_2^H\left(\bar{B}_e\right) = R_H\left(\bar{B}_e\right). \quad (9)$$

Then asymmetry resistance R_A and Hall resistance R_H can be calculated with elements not depending on current [2]

$$R_A = \frac{a_1\left(\bar{B}_e\right) + b_1\left(\bar{B}_e\right)}{2}, \quad R_H\left(\bar{B}_e\right) = \frac{a_1\left(\bar{B}_e\right) - b_1\left(\bar{B}_e\right)}{2}. \quad (10)$$

4. EXPERIMENTAL TECHNIQUE

Experimental verification of reciprocal relations of formulas (8), (9) and corresponding to them formulas (2) and (4) were done for Hall sensors of the type PHE602117, PHE606117, which are a film (a) of 10 µm thick, made from the strongly alloyed InSb. The film is placed on a substrate (b) made of intrinsic GaAs (Fig. 1). Pads to copper conductors are soldered by indium (c).

Measurements were done with the help of a special hardware-software complex. The complex consists of adjustable current source, Helmholtz coils, nanovoltmeter [4] and commutating systems of contacts of the researched quadripole. Hall transducer and the Helmholtz coils were placed into the center of permalloy screen to reduce the impact of the geomagnetical field on the results of measurements.

5. RESULTS AND DISCUSSION

At the first stage, we checked the hypothesis that non-linearity of volt-ampere characteristics of Hall transducer does not depend on induction of external magnetic-field. For this purpose, we measured functions $R_1(i)$ and $R_2(i)$ of transducer PHE602117 when the Helmholtz coils are switched off (Fig. 2). The values of functions were averaged for every value of current in 10 measurings. The obtained dependences were approximated by fourth-order polynomials according to expression (5), and the linear functions (8) were built. We find that increasing polynomials degree has no influence on the accuracy of function estimation for our experimental data. SD of measuring noise here is approximately 12 µΩ for resistance of R_1, and it is 4 µΩ for resistance of R_2.

The SD of measured resistances from mean resistance are shown in the first column of Table 1. As evident from the table, SD of the linear resistances $\delta[a_1]$ and $\delta[b_1]$ are two orders less than SD of initial resistances $\delta[R_1]$ and $\delta[R_2]$. The dependences of Hall resistance R_H and asymmetry R_A were built according to the obtained linear resistances from the control current according to formula (10). These dependences are shown in the first column of Table 2. SD of the obtained resistances do not exceed SD of measured noise in the 50 ... 100 mA currents range. The non-zero Hall resistance of R_H shows the value of Earth magnetic field uncompensated with the magnetic shield.

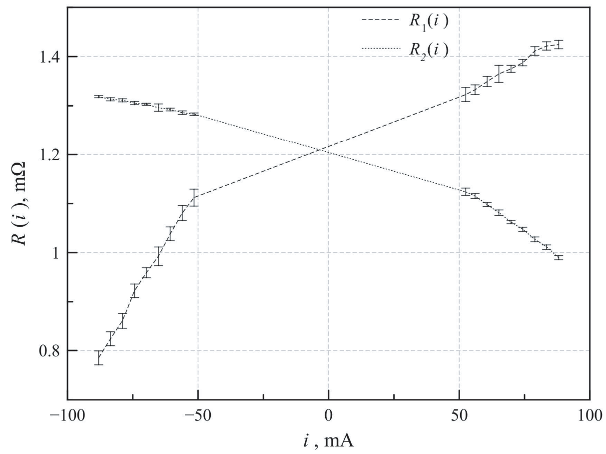

Figure 2. Experimental dependences of resistances $R_1(i)$ and $R_2(i)$ are measured in the residual magnetic field.

Table 1. SD of measured resistances.

B_e, μT	0	300	−300	100	−100	50	−50	10	−10
$\delta[R_1]$, μΩ	236	240	244	245	246	249	249	248	253
$\delta[R_2]$, μΩ	109	101	98	96	98	93	92	91	90
$\delta[a_1]$, μΩ	4	8	16	15	13	18	18	17	22
$\delta[b_1]$, μΩ	1	10	14	16	14	19	21	22	23

Table 2. Estimations of asymmetry (R_A) and Hall (R_H) resistances and its standard deviations.

B_e, μT	0	300	−300	100	−100	50	−50	10	−10
R_A, μΩ	1511	1511	1509	1510	1515	1515	1517	1517	1517
$\delta[R_A]$, μΩ	2	8	14	15	13	18	19	19	23
R_H, μΩ	7	3492	−3476	1198	−1184	626	−611	168	−153
$\delta[R_H]$, μΩ	2	3	4	4	3	3	4	5	3

Further, we made measurements under the external magnetic field, produced by Helmholtz coils. The results of processed data are shown in Table 1 and Table 2. When they were analyzed, we used polynomials obtained at the stage of calibration, when $B_e = 0$. The increasing $\delta[a_1]$, $\delta[b_1]$ and $\delta[R_A]$ concerned with fluctuations of environment temperature about 1°C during experiment. Temperature coefficient of resistance does not exceed 50 μΩ/ °C for applied Hall transducer. Also, we have found that if taking into account the SD of the noise of measurement $(R_H(\bar{B}_e) + R_H(-\bar{B}_e))/2 \approx R_H(\bar{B}_e = 0)$, the resistance of asymmetry does not change when the sign of the field is changed.

We also perform this experiment for Hall transducers PHE606117A with sensitivity 6 Ω/T. The distinguishing feature of volt-ampere characteristics of this transducer is high nonlinearity. SD of resistances R_1 and R_2 exceed 1300 μΩ. For linear resistances a_1 and b_1, we obtain SD about 70 μΩ. At the same time, SD of Hall resistance do not exceed 20 μΩ. Thus, the obtained reciprocal relations allow us to use Hall transducer with significant nonlinear volt-ampere characteristics for precision measuring of magnetic field.

Thus, the experiments confirmed the validity of decomposition in Eq. (3), reciprocal relations (9) and consequently, reciprocal relations (4) and (2).

6. CONCLUSION

We find a technique for extraction of linear response for nonlinear quadripole. We did experiments for measuring resistance matrix components of Hall transducers. We experimentally verify that the classical reciprocal relations for elements of resistance matrix are valid for linear response on the external magnetic field. The accuracy of the provided measurements is about $5\,\mu\Omega$. These relations can be applied to improving accuracy of Hall magnetometers. We demonstrate decreasing measurement error of Hall transducer into 50 times.

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