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Research on Minimum Energy Excited to Plasma Coating for Reducing Radar Cross Section of Target

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Abstract—To reduce the radar cross section (RCS) of a target, plasma coating on perfectly electric conducting plate is studied in this paper. Nonuniform helium plasma produced by a minitype solid rocket engine is with collisional and unmagnetized. Energy excited for generating helium plasma is investigated. Based on the collisional, unmagnetized, and cold plasma model, backscattering RCS is computed by using finite-difference time-domain method. Principle of RCS reduction is explained. To find minimum input energy while RCS reduced, relationship between input power and RCS reduction is discussed, and numerical optimization is also implemented. We can identify optimal parameters and choose the best electron density profile under condition of given input power level.

1. INTRODUCTION

Stealth aircraft design tends to make use of special surface shape or microwave-absorbing coating, sometimes, of both to reduce the radar cross section (RCS). However, these techniques may not be effective in some situations where the use is for a wide-band radar wave frequency [1]. Since 1990s, plasma stealth technology has been paid attention to [2–9]. It is found that inhomogeneous plasma behaves as frequency selective medium and can be used as a broadband absorbing layer in general shielding and military stealth applications [2]. For complex target surfaces on ship and aircraft, the geometry model can be divided into some simple parts approximately to analyze the RCS at the beginning. These parts, such as flat plate, cone frusta, spheroid and edge, etc., have been modeled isolately. The target response to the radar wave is dependent upon a limited number of individual parts. We can focus on these responses of our attention instead of the composite target. Here, we choose a perfectly electric conducting (PEC) plate coated with plasma as an investigated target. Helium plasma is produced by a minitype solid rocket engine. Electron density profile affects the reflection, absorption and transmission of the electromagnetic energy in plasma for such application. In literature, different electron density functions are considered such as exponential, parabolic, steepened, hyperbolic, tangent, sinusoidal and exponential with time variation [4–6, 8]. In laboratory, we use the spatial distribution of plasma electron density which is measured for illustration [10]. So it is very important to find a theoretical model to suit for the electron density profile. Another important issue is that the energy required to sustain the plasma must be concerned [2, 7]. It is significant to use minimum energy to produce plasma coating for reducing the RCS of target. To calculate the electromagnetic wave propagation in plasma, the finite-difference time-domain (FDTD) method is adopted [3, 8, 9].

In this paper, the electromagnetic wave interaction with nonuniform plasma is studied. The FDTD method used in collisional, unmagnetized, cold plasma is introduced in Section 2. Numerical experiment is verified. To validate the proposed model, bistatic RCS of a PEC plate coated with plasma is calculated in Section 3. The principle of RCS reduction is explained. In the case of minimum RCS, Section 4

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involves numerical study to determine the minimum input power. The best-suited electron density profile is discussed. Finally, important conclusions are obtained.

2. METHODOLOGY

2.1. FDTD Method in Collisional, Unmagnetized, Cold Plasma

The complex relative dielectric constant for plasma is [11]

$$\varepsilon_r(\omega) = \varepsilon_r'(\omega) - j\varepsilon_r''(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + \upsilon^2} - j\frac{\upsilon}{\omega}\frac{\omega_p^2}{\omega^2 + \upsilon^2} \tag{1}$$

where $\omega_p = \sqrt{N_e e^2/\varepsilon_0 m_e}$ is the plasma frequency, N_e the plasma electron density, e the electron charge, m_e the electron mass, ε_0 the free-space permittivity, and ω the angular frequency of the incident electromagnetic wave. Here we use the continuous wave, sinusoidal function as incident source, and υ is the electron momentum-transfer collision rate.

When the frequency of incident wave is a constant, the plasma can be regarded as a lossy dielectric. The FDTD calculation needs to know the dielectric constant ε and electrical conductivity σ of every cell. The equations are given in [12], and we have

$$\varepsilon = \varepsilon_0 - \frac{e^2 N_e}{m_e\left(\omega^2 + \upsilon^2\right)} \tag{2}$$

$$\sigma = \frac{N_e e^2 \upsilon}{m_e\left(\omega^2 + \upsilon^2\right)} \tag{3}$$

where the collision rate υ is also constant, and ε and σ only depend on the variation of N_e. If N_e is a variable, the plasma is nonuniform. Furthermore, the plasma that we discuss is unmagnetized, so the equivalent magnetic loss $\sigma_m = 0$. Thus we can use the conventional FDTD method to calculate the electromagnetic fields in collisional, unmagnetized, cold, nonuniform plasma.

2.2. Numerical Experiment

The plasma can be taken as a lossy dielectric when the incident frequency is a constant. To validate the method used in plasma, we calculate the bistatic RCS of a lossy dielectric cube. The edge length of the cube is 0.5 m, and the relative permittivity is $\varepsilon_r = 2.0 - j0.5$. A 750 MHz x-direction polarized plane electromagnetic wave is incident from the $-z$-direction, and θ is used to denote the angle between incident direction and z-axis. Figure 1 shows the bistatic RCS of the dielectric cube for E plane and H plane. We can see that most of the calculated results agreed well with the volume integral equation (VIE) method solution [13] except for some weak scattering region. The validity of the FDTD method that we used is verified.

3. PEC PLATE COATED WITH NONUNIFORM PLASMA

3.1. Numerical Results

A square PEC plate with side length of 40 mm and thickness of 4 mm, located in x-y plane, is coated with a 60 mm thick plasma shroud. A 10 GHz x-direction polarized plane electromagnetic wave is incident from the $-z$-direction. The coordinate system can be seen in Figure 2.

We assume that the plasma is produced by a minitype solid rocket engine in helium gas. Different nonuniform plasma flow fields can be generated by choosing different engine parameters and propellants. The electron density distributions measured with electrostatic probe are given in [10]. The measured results show that the electron density is approximately Gaussian distributed as shown in Equation (4), and we have

$$N_e(z) = N_{e0}\exp\left(-z^2/L^2\right) \tag{4}$$

where N_{e0} is the peak electron density and L the gradient scale length. Here, we set $N_{e0} = 1.1\times10^{17}/\text{m}^3$, $L = 0.03$ m, and $\upsilon = 1.85\times10^{11}$ Hz [14]. Figure 3 shows the bistatic RCS of the plate with and without

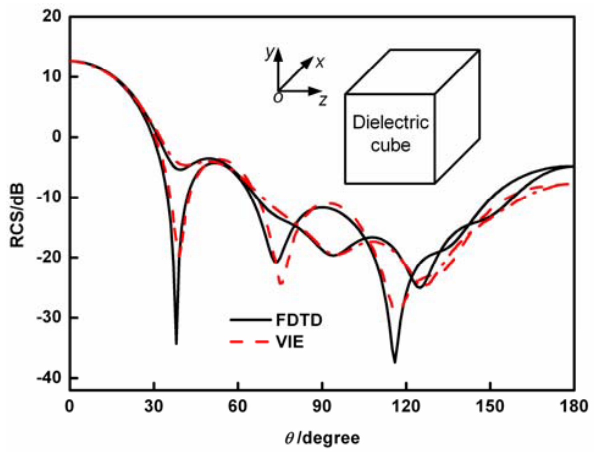


Figure 1. Bistatic RCS of the dielectric cube.

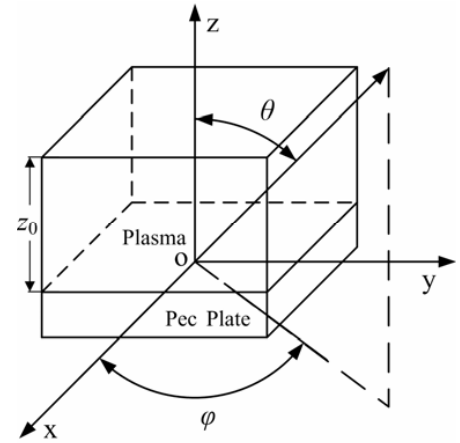


Figure 2. The coordinate system of PEC plate covered with nonuniform plasma. The electron density has a Gaussian variation in the z-direction and uniform distribution in the x-y plane.

a plasma shroud. We can see that there is an obvious RCS reduction from $\theta = 0^\circ$ to 90°, where the plate is covered with plasma. The largest reduction of 8.21 dB is obtained in the backscatter direction, and forward scattering is a little increased compared with that of the unshielded plate.

3.2. Attenuation of Electromagnetic Waves in Plasma

From Figure 3, we can see that significant backscatter RCS reduction is achieved. In this part, we will analyze the principle of the electromagnetic wave attenuation in plasma.

According to Equation (1), we can find the propagation constant k for an electromagnetic wave in collisional plasma

$$k = k_0\sqrt{\varepsilon_r(\omega)} = k_r + jk_i \tag{5}$$

where k_0 is propagation constant in free space, and k_r and k_i are the real and imaginary parts of the propagation constant, respectively. The electron-ion and electron-neutral collisions cause the existence of the imaginary part of the propagation constant, and the imaginary part describes the attenuation rate when electromagnetic wave propagates in the plasma. For an electromagnetic wave which enters the plasma at $z = z_0$, propagates to the PEC plate at $z = 0$, and then returns through the plasma, the net reflected power in the plasma is estimated by

$$P_n = P_0 \exp\left[-4\mathrm{Im}\left(\int_{z_0}^{0} k(z)dz\right)\right] \tag{6}$$

where P_0 is the power incident on the vacuum-plasma interface.

The double-pass attenuation of the electromagnetic wave is

$$Att\,(\mathrm{dB}) = \left|10\lg\frac{P\left(z_0\right)}{P_0}\right| = \left|17.36\mathrm{Im}\left(\int_{z_0}^{0} k(z)dz\right)\right| \tag{7}$$

Consequently, the reduction of the received power is mainly due to the absorption of electromagnetic wave in plasma, and the RCS of target is also reduced.

4. THE RELATIONSHIP BETWEEN INPUT POWER AND RCS

The plasma needs to be sustained to compensate for atomic processes by an input energy in any practical system. It is significant to find the minimum input power while RCS is reduced. In this part, the relationship between the input power and RCS reduction will be discussed, and numerical optimization will be implemented.

An approximation for the power per unit volume required to sustain plasma is [2]

$$P/V = \frac{N_e E_i}{\tau} \tag{8}$$

where P is the continuous-wave power deposited in plasma, V the plasma volume, E_i the energy to generate an electron-ion pair, and τ the plasma lifetime. The power per unit volume for noble gas plasma reduces to

$$P/V = K_r N_e^2 E_i \tag{9}$$

where K_r is the two-body recombination rate constant. Vidmar [2] has shown that plasma lifetimes in a noble gas are much longer than in air, which reduces the power requirement significantly. As an example, a plasma with $N_e = 10^{17}/\text{m}^3$ at 760 torr in air, the plasma lifetime is 10 ns; E_i is 33.7 eV; P/V is 54 MW/m^3 for impact ionization. However, P/V in helium gas under the same condition as that in air is only 672 W/m^3. So the power optimization study is implemented assuming that the plasma is produced in helium gas by a minitype solid rocket engine. The electron density in $+z$ direction is approximately Gaussian distribution as shown in Equation (4). We adjust N_{e0} and L to discuss the relationship between the input power and the reduction in RCS.

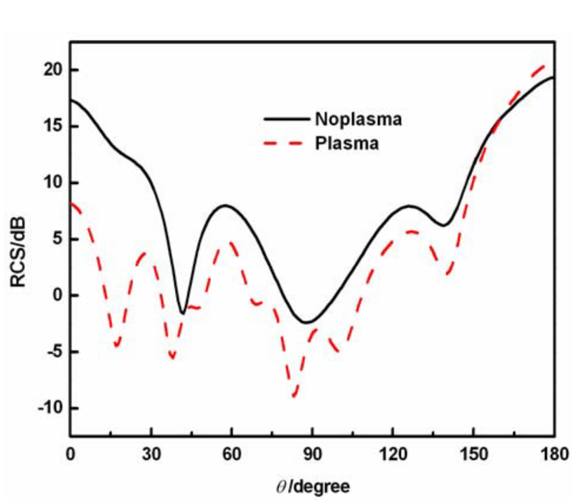


Figure 3. Bistatic RCS versus angle θ for the PEC plate with and without a plasma shroud.

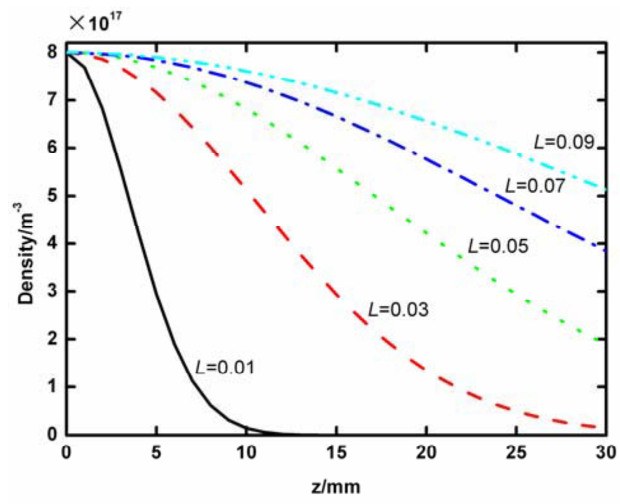


Figure 4. Electron density variation in $+z$-direction while $L = 0.01$, 0.03, 0.05, 0.07, 0.09 m, respectively.

4.1. Plasma-Coated PEC Plate

Here, we consider the same plasma-coated PEC plate as depicted in Section 3.1. The geometric model is also shown in Figure 2. The incident wave frequency is equal to 10 GHz, and the effective collision rate is 1.85×10^{11} Hz. In order to make electromagnetic wave of different frequencies be absorbed or refracted in plasma, N_{e0} is varied from $2 \times 10^{17}/\text{m}^3$ to $2.9 \times 10^{18}/\text{m}^3$, with an increment of $3 \times 10^{17}/\text{m}^3$. Five discussed L values are 0.01, 0.03, 0.05, 0.07, and 0.09 m, respectively. So there are 50 parameter sets needing to be considered.

Figure 4 shows the electron density distribution corresponding to five L values in $+z$-direction. We can see that the smaller the L is, the steeper the variation of electron density is. For helium plasma, K_r is 1.0×10^{-14} m^3/s, and the corresponding E_i is 42 eV. We can calculate the power required in a volume V_c using Equation (9), and V_c is the volume of every FDTD cell. The total power required to sustain the plasma can be obtained by adding the power over all cells together.

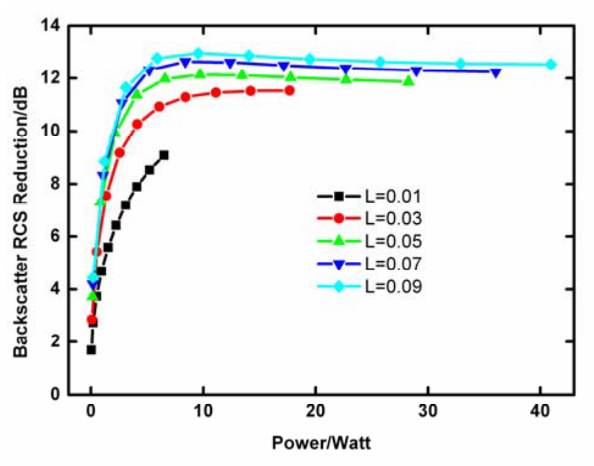


Figure 5. Backscatter RCS reduction versus input power while scale length $L = 0.01$, 0.03, 0.05, 0.07, 0.09 m, respectively for plasma-coated PEC plate.

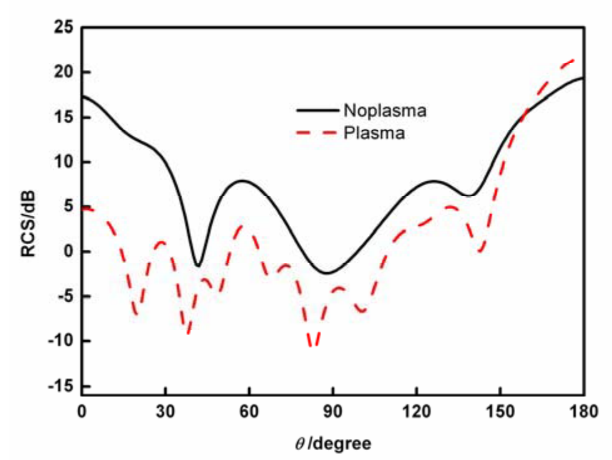


Figure 6. Bistatic RCS of the PEC plate without plasma and the PEC plate with plasma shroud after optimization.

Table 1. Corresponding parameters for different electron density profiles.

Scale length L (m)	0.01	0.03	0.05	0.07	0.09
Input power (W)	6.49	17.69	9.71	8.40	9.54
Maximum backscatter RCS reduction (dB)	9.09	11.53	12.14	12.64	12.96
Electron density ($\times 10^{18}/\mathrm{m}^3$)	2.9	2.9	1.7	1.4	1.4

The bistatic RCSs for all 50 sets have been calculated. The difference between the backscatter RCSs for the PEC plate with and without plasma is defined as RCS reduction. Figure 5 shows the RCS reduction versus input power required, corresponding to five L values. We can see that the backscatter RCS reductions are not simply monotone increasing as the input power increasing, but increase with input power up to optimal points, and then show some tendency to saturate, except for the conditions $L = 0.01$ m and $L = 0.03$ m. That is because of the limitation of the peak density. Moreover, as L increases, backscatter RCS decreases. In real-life system, the electron density distribution is nonuniform in order to make plasma absorbing or refracting the electromagnetic wave of different frequencies. Therefore, the bigger size the L is, the smaller RCS is obtained under the condition that the requirement of absorption frequency range is satisfied.

The corresponding parameters of different electron density profiles are shown in Table 1. The results at $L = 0.07$ m and $L = 0.09$ m are better than others, while the peak electron densities are both $1.4 \times 10^{18}/\mathrm{m}^3$. Comparing two cases, 8.4 W input power can get RCS reduction of 12.64 dB while $L = 0.07$ m, and 9.54 W input power can get RCS reduction of 12.96 dB while $L = 0.09$ m. Although the obtained RCS reduction corresponding to $L = 0.09$ m is greater than that corresponding to $L = 0.07$ m, the required input power per unit RCS reduction while $L = 0.07$ m is less than that while $L = 0.09$ m. Thus, the optimal values are $N_{e0} = 1.4 \times 10^{18}/\mathrm{m}^3$ and $L = 0.07$ m. The corresponding bistatic RCS is shown in Figure 6. A reduction of 12.64 dB in the backscattering direction is obtained.

4.2. Plasma-Coated PEC Sphere

We take the plasma-coated PEC sphere for another example. The PEC sphere with diameter of 30 mm is coated with a 30 mm thick plasma shroud. An x-direction polarized plane electromagnetic wave is incident from the $+z$-direction. The coordinate system can be seen in Figure 7. The plasma parameters are same as these in Section 4.1. We calculate the RCS reduction versus input power

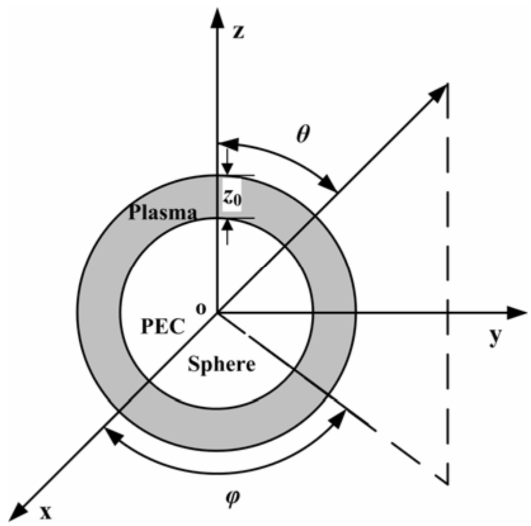
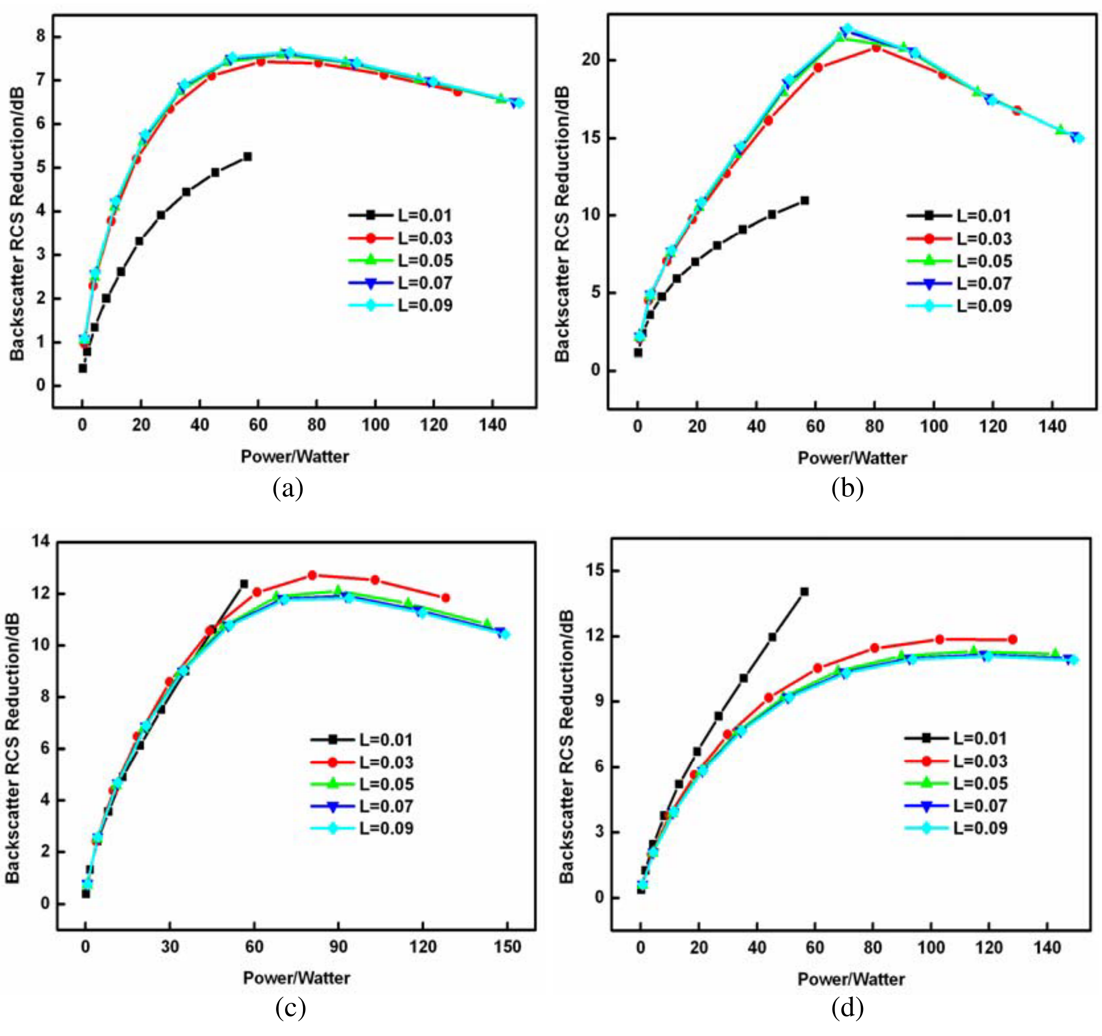
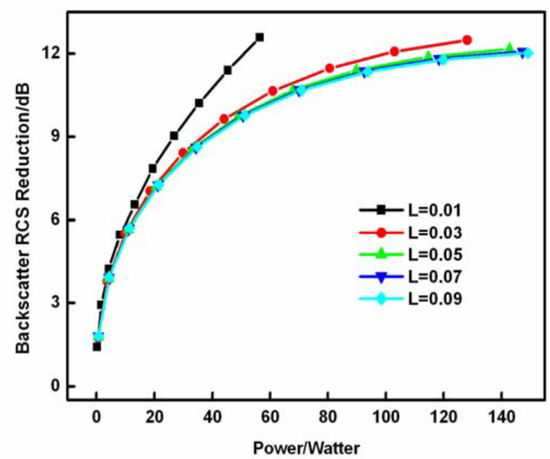


Figure 7. The coordinate system of PEC sphere covered with nonuniform plasma. The electron density has a Gaussian variation along radial axis.





(e)

Figure 8. Backscatter RCS reduction versus input power required corresponding to five L values while the frequencies of incident wave are 5, 7, 12 and 15 GHz, respectively for plasma-coated PEC sphere. (a) $f = 5$ GHz, (b) $f = 7$ GHz, (c) $f = 10$ GHz, (d) $f = 12$ GHz, (e) $f = 15$ GHz.

Table 2. The optimal values for corresponding incident wave frequencies.

Incident wave frequency (GHz)	5	7	10	12	15
Optimal scale length L (m)	0.03	0.05	0.03	0.01	0.01
Optimal input power (W)	60.96	67.88	80.63	56.39	56.39
Maximum backscatter RCS reduction (dB)	7.43	21.44	12.71	14.03	12.58
Optimal electron density ($\times 10^{18}/m^3$)	2	2	2.3	2.9	2.9

required corresponding to five L values while the frequencies of incident wave are 5, 7, 10, 12 and 15 GHz, respectively. The calculated results can be seen in Figure 8. From it, we can see that the tendencies of the backscatter RCS reduction versus input power required are almost the same while the incident wave frequencies are different. The backscatter RCS reductions are not simply monotone increasing as the input power increasing, but increase with input power up to optimal points, and then show some tendency to decrease, or show some tendency to saturate, except the condition $L = 0.01$. That is also because of the limitation of the peak density.

From Figure 8 we can also see that the optimal values are different while the frequencies are different. According to the analysis in Section 4.1, we can get optimal values for corresponding incident wave frequencies which can be seen in Table 2. It can be found that the optimal electron density increases as the incident wave frequency increases. The maximum backscatter RCS reduction while $f = 7$ GHz is much greater than others. That is because the attenuation of incident wave in different frequency is different in plasma.

5. CONCLUSION

We have discussed the FDTD method in collisional, unmagnetized, and cold plasma computation. The numerical results of an example agree well with the corresponding VIE method solution. The backscattering RCS of plasma-coated PEC plate and plasma-coated PEC sphere have been simulated. The relationship between the input power required to sustain plasma and the RCS reduction has been discussed. Generally speaking, the backscattering RCS reduces monotonically with input power to an optimal value and then shows some tendency to saturate, or to decrease. It is very significant to use FDTD method to identify the optimal parameters.

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REFERENCES

1. Knott, E. F., J. F. Shaeffer, and M. T. Tuley, *Radar Cross Section*, Scitech Publ. Inc., Raleigh, NC, 2004.
2. Vidmar, R. J., "On the use of atmospheric pressure plasma as electromagnetic reflectors and absorbers," *IEEE Trans. on Plasma Science*, Vol. 18, No. 4, 733–741, 1990.
3. Yang, L. X., Y. T. Xie, and P. P. Yu, "Study of bandgap characteristics of 2D magnetoplasma photonic crystal by using M-FDTD method," *Microwave and Optical Technology Letters*, Vol. 53, No. 8, 1778–1784, 2011.
4. Tang, D. L., A. P. Sun, X. M. Qiu, and K. Chu, "Interaction of electromagnetic waves with a magnetized nonuniform plasma slab," *IEEE Trans. on Plasma Science*, Vol. 31, No. 3, 405–410, 2003.
5. Liu, M., X. Hu, Z. Jiang, S. Zhang, C. Lan, and Y. Pan, "Reflection of a wave from a thin plasma layer attached to a metal plate by finite-difference time-domain analysis," *Plasma Sources Sci. Technol.*, Vol. 16, 614–618, 2007.
6. Zobdeh, P., R. Sadighi-Bonabi, H. Afarideh, E. Yazdani, and R. Rezaei Nasirabad, "Using the steepened plasma profile and wave breaking threshold in laser-plasma interaction," *Contributions to Plasma Phys.*, Vol. 48, 555–560, 2008.
7. Gurel, C. S. and E. Oncu, "Frequency selective characteristics of a plasma layer with sinusoidally varying electron density profile," *Int. J. Infrared Millimeter Waves*, Vol. 30, 589–597, 2009.
8. Chaudhury, B. and S. Chaturvedi, "Study and optimization of plasma-based radar cross section reduction using three-dimensional computations," *IEEE Trans. on Plasma Science*, Vol. 37, No. 11, 2116–2127, 2009.
9. Liu, J. F., X. L. Xi, G. B. Wan, and L. L. Wang, "A high efficient SO-FDTD method for magnetized collisional plasma," *Journal of Electromagnetic Waves and Applications*, Vol. 26, Nos. 14–15, 1911–1921, 2012.
10. Zeng, X. J., P. Ma, Z. F. Yu, Z. J. Wang, X. Y. Ma, and J. Li, "Experimental investigation and analysis on jet-plasma stealth in air surroundings," *Journal of Experiments in Fluid Mechanics*, Vol. 22, No. 1, 49–54, 2008 (in Chinese).
11. Heald, M. A. and C. B. Wharton, *Plasma Diagnositics with Microwaves*, Krieger, New York, 1978.
12. Howatson, A. M., *An Introduction to Gas Discharges*, Pergamon, New York, 1976.
13. Bakir, O., "Domain decomposition based hybrid methods for solving real-life electromagnetic scattering and radiation problems," Ph. D. Thesis, University of Michigan, 2012.
14. Itikawa, Y., "Effective collision frequency of electrons in gases," *Phys. Fluids*, Vol. 16, No. 6, 831–835, 1973.