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Design of Multi-Stage Power Divider Based on the Theory of Small Reflections

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Abstract—This paper presents a novel multi-way multi-stage power divider design method based on the theory of small reflections. Firstly, the application of the theory of small reflections is extended from transmission line to microwave network. Secondly, an explicit closed-form analytical formula of the input reflection coefficient, which consists of the scattering parameters of power divider elements and the lengths of interconnection lines between each element, is derived. Thirdly, the proposed formula is applied to determine the lengths of interconnection lines. A prototype of a 16-way 4-stage power divider working at 4 GHz is designed and fabricated. Both the simulation and measurement results demonstrate the validity of the proposed method.

1. INTRODUCTION

Power dividers are widely used in microwave and millimeter-wave systems and devices such as antenna feeders, power amplifiers [1, 2]. In recent applications, the required channels in these multi-channel systems increase persistently. In many occasions, number of ways is so large that a single-stage power divider does not satisfy the requirement. Then a multi-stage structure, such as tree-style 2^{n}-way power dividers consisting of n-stage two-way dividers, is applied [3–7]. For a multi-stage power divider, interconnection lines are used to connect power divider elements [3]. Therefore, the design of a multi-way multi-stage power divider have to consider not only the parameters of two-way power divider elements, but also the lengths of the interconnection lines.

One type of the most popular two-way power dividers is Wilkinson power divider. After it was first proposed by Wilkinson in 1960, many improved Wilkinson power dividers have been proposed. Oraizi et al. improved its bandwidth [8, 9]. Li and Wang designed a divider with arbitrary power division ratio [10]. Wu et al. studied dual-band power divider [11, 12] and miniaturization of Wilkinson power divider [13, 14]. Most of the researches focus on single-stage power divider.

In this paper, the effect of different lengths of interconnection lines within multi-stage power divider is studied. The performance of the multi-stage power divider depends on the lengths of the interconnection lines; the reflection coefficient of the multi-stage power divider at center frequency will increase with improper lengths. Currently, the most common way to determine the lengths of interconnection lines is brute-force parameter search using commercial simulation software; many iterations of calculation must be performed to calculate the lengths of interconnection lines of the multi-stage power divider. When the power divider has many stages, or the electrical size of the power divider is too large, each iteration of calculation will take a long time, or even unrealistic in practice. Therefore, a direct method or an analytical formula is required to determine the lengths of the interconnection lines within multi-stage power divider.

This paper presents a novel design method of multi-way multi-stage power divider based on the theory of small reflections. In Section 2, the theory of small reflections extended from transmission line

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to microwave network is introduced. Based on the theory, the total reflection wave in the multi-stage power divider can be obtained by the sum of all partial waves reflected from power divider elements. It is a novel analytical method for a multi-stage power divider, which is discussed in the first part of Section 3. The design of interconnection lines for a multi-stage power divider are described in the second part of Section 3. In Section 4, comparison between reflection coefficient calculated by the proposed analytical method and reflection coefficient simulated by ADS are provided to verify the novel analytical method. In Section 5, the proposed design method is verified by the measurement results of the prototype of a 16-way 4-stage power divider.

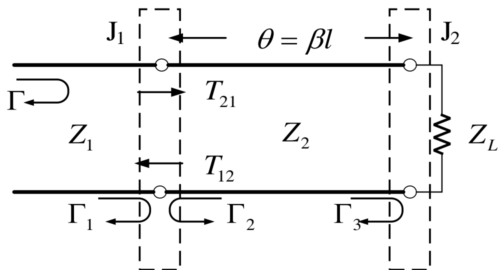


Figure 1. The theory of small reflections.

2. THE THEORY OF SMALL REFLECTIONS FOR MICROWAVE NETWORK

According to the theory of small reflections [15], the total reflection wave in the discontinuous transmission line can be obtained by the sum of all partial waves passed junction J_1 to the left, as shown in Fig. 1.

$$\Gamma = \Gamma_1 + T_{12}T_{21}\Gamma_3 e^{-2j\theta}\sum_{n=0}^{\infty}\Gamma_2^n\Gamma_3^n e^{-2jn\theta} = \Gamma_1 + \frac{T_{12}T_{21}\Gamma_3 e^{-2j\theta}}{1-\Gamma_2\Gamma_3 e^{-2j\theta}} \tag{1}$$

where $\Gamma_1 = (Z_2 - Z_1)/(Z_2 + Z_1)$, $\Gamma_2 = -\Gamma_1$, $\Gamma_3 = (Z_L - Z_2)/(Z_L + Z_2)$ are partial reflection coefficients for junction J_1 and J_2, $T_{21} = 1 + \Gamma_1$, $T_{12} = 1 + \Gamma_2$ are partial transmission coefficients.

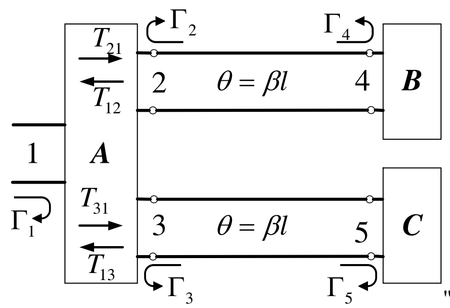


Figure 2. The theory of small reflections for microwave network.

Replacing junction J_1 in Fig. 1 with a 3-port microwave network **A** and junction J_2 with two single-port microwave networks **B** and **C** is depicted in Fig. 2. Under the assumption that output ports 2 and 3 of **A** are isolated, the total reflection is given by

$$\begin{aligned}\Gamma &= \Gamma_1 + T_{21}T_{12}\Gamma_4 e^{-2j\theta}\sum_{n=0}^{\infty}\Gamma_2^n\Gamma_4^n e^{-2jn\theta} + T_{31}T_{13}\Gamma_5 e^{-2j\theta}\sum_{n=0}^{\infty}\Gamma_3^n\Gamma_5^n e^{-2jn\theta}\\ &= \Gamma_1 + \frac{T_{21}T_{12}\Gamma_4 e^{-2j\theta}}{1-\Gamma_2\Gamma_4 e^{-2j\theta}} + \frac{T_{31}T_{13}\Gamma_5 e^{-2j\theta}}{1-\Gamma_3\Gamma_5 e^{-2j\theta}}\end{aligned} \tag{2}$$

According to transmission line theory, formula (2) can be written as:

$$\Gamma \approx S_{11}^{A} + S_{21}^{A} S_{12}^{A} S_{11}^{B} e^{-2j\theta} + S_{31}^{A} S_{13}^{A} S_{11}^{C} e^{-2j\theta} \tag{3}$$

This result shows a intuitive idea that the total reflection is dominated by the reflection from the microwave network **A**, and the reflections from the microwave networks **B** and **C**. The S_{21}^{A} and S_{12}^{A} terms account for the transmission coefficients when the incident wave travels forward and backward on the upper branch. Meanwhile, S_{31}^{A} and S_{13}^{A} terms account for the transmission coefficients when the incident wave travels forward and backward on the lower branch. The $e^{-2j\theta}$ term accounts for the phase delay.

3. DESIGN OF MULTI-STAGE POWER DIVIDER

3.1. Analysis of Input Reflection Coefficient for Multi-Stage Power Divider

The power divider elements closest to the output ports are defined as 1-stage power divider. Two adjacent 1-stage power dividers are connected to an extra power divider element by two interconnection lines which are defined as a 2-stage power divider. 3-stage to N-stage power dividers are defined in the same way as shown in Fig. 3. To ensure that signals of output ports are in phase, all the interconnection lines that connect two i-stage power dividers to an extra power divider element to construct a $(i+1)$-stage power divider are of the same length L_i.

The input reflection coefficient of an N-stage power divider can be derived by the scattering parameters **S** of power divider elements and lengths of interconnection lines.

The input reflection coefficient is given

$$\Gamma_N = S_{11} + S_{12} \cdot S_{21} \cdot \Gamma_{N-1} \cdot e^{-j2\beta L_{N-1}} + S_{13} \cdot S_{31} \cdot \Gamma_{N-1} \cdot e^{-j2\beta L_{N-1}} \tag{4}$$

where Γ_N is the input reflection coefficient of N-stage power divider, and Γ_{N-1} is that of $(N-1)$-stage power divider.

Formula (4) is simplified as:

$$\Gamma_N = S_{11} \left\{ 1 + \sum_{m=1}^{N-1} \left\{ \left[2 \cdot |S_{21}|^2 \right]^m \cdot e^{j\alpha_m} \right\} \right\} \tag{5}$$

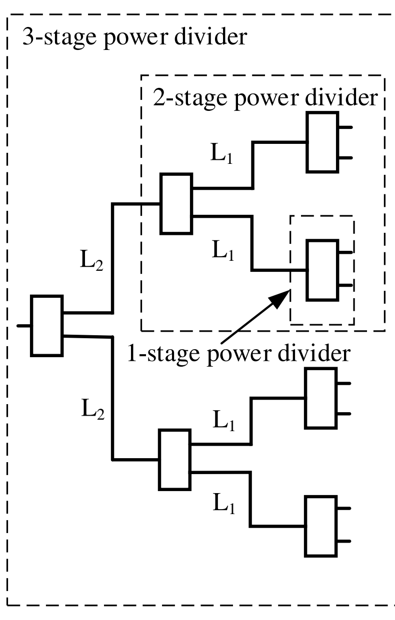


Figure 3. Structure of a multi-stage power divider including interconnection lines.

where $\alpha_m = \sum_{n=N-m}^{N-1} 2(\phi_0 - \beta L_n)$ is the total phase of partial reflections from m-stage power divider to the input port of N-stage power divider. In practice, the insert loss of power divider element is small, therefore $2 \cdot |S_{21}|^2 \approx 1$, and produces:

$$\Gamma_N \approx S_{11} \left\{ 1 + \sum_{m=1}^{N-1} e^{j\alpha_m} \right\} \tag{6}$$

If the length of interconnection lines is not appropriate, the worst case scenario is that the partial reflections are in phase at the input port of the multi-stage power divider, i.e., $\alpha_m = 0$ and produces:

$$\Gamma_N \approx N \cdot S_{11} \tag{7}$$

The input reflection coefficient of the N-stage power divider is approximately N times of that of individual power divider element. However, Γ_N can be reduced through the following design method.

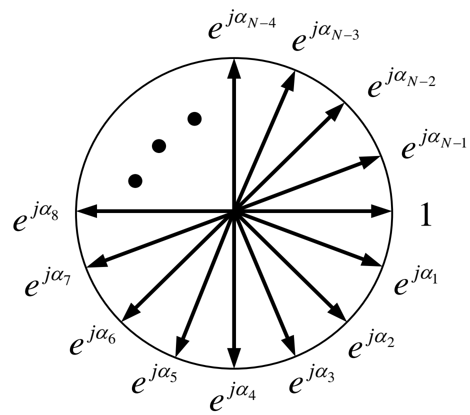


Figure 4. Unit vectors of different phases.

3.2. Design of the Lengths for Interconnection Lines

In order to obtain perfect match at center frequency, Γ_N should be equal to zero at center frequency. From Equation (6), it can be written as:

$$\Gamma_N \approx S_{11} \left\{ 1 + \sum_{m=1}^{N-1} e^{j\alpha_m} \right\} = 0 \tag{8}$$

The lengths L_m of interconnection line can be obtained by solving Equation (8). It is obvious that there are many solutions for Equation (8). A specific solution is given as follows.

Because $S_{11} \neq 0$, Equation (8) can be simplified as:

$$1 + \sum_{m=1}^{N-1} e^{j\alpha_m} = 0 \tag{9}$$

The left part of Equation (9) is the sum of unit vectors with different phases, as shown in Fig. 4.

If $\alpha_1, \alpha_2, \ldots, \alpha_N$ have the same phase interval $2\pi/N$ in the unit circle and equally distributed, one specific solution is:

$$\alpha_m - \alpha_{m-1} = 2(\phi_0 - \beta L_{N-m}) = -\frac{2\pi}{N} + 2k\pi \tag{10}$$

where k is any integer. Hence,

$$L_i = \frac{\phi_0 + \pi/N - k\pi}{\beta} \tag{11}$$

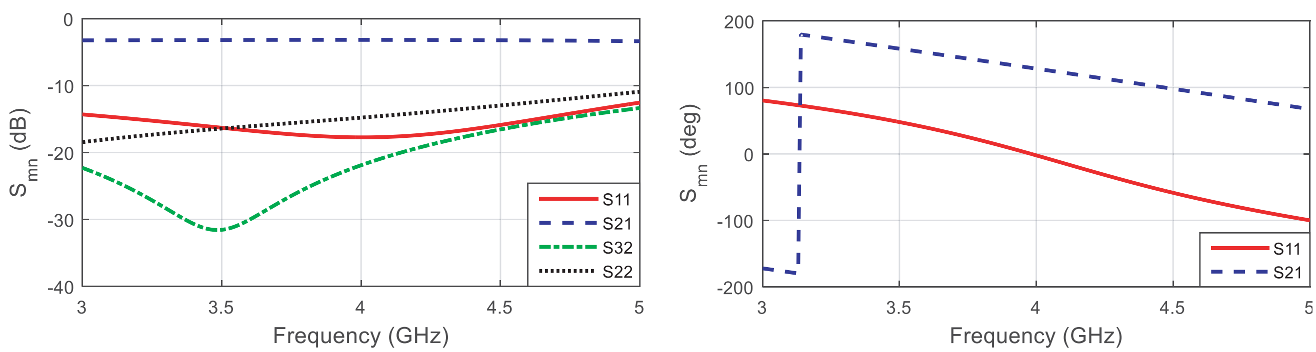
where $i = 1, 2, \ldots, N-1$. In order to make power divider physically realizable L_i should satisfy:

$$L_i \geq \left(2^i - 1\right)/2 \cdot D \tag{12}$$

where D is the distance between two adjacent output ports. In order to get wider impedance bandwidth, L_i should be as short as possible.

4. SIMULATION

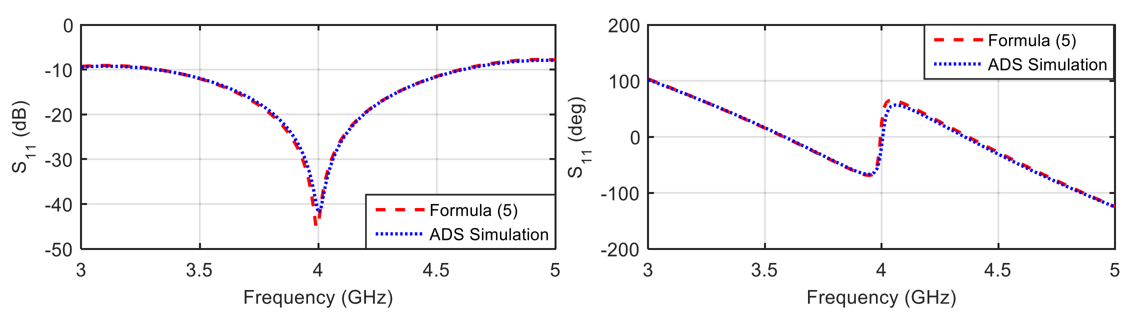
A microstrip Wilkinson power divider element is designed on a 0.508 mm thick dielectric substrate with $\varepsilon_r = 3.66$, and center frequency is at 4 GHz. The characteristic impedances of all the three ports are 50 ohms, and the corresponding widths are 1.09 mm. The width and length of the quarter-wavelength lines are 0.58 mm and 11.41 mm, respectively. The shunt resistor is 100 ohms. The magnitude and phase of the scattering parameters for the power divider element are shown in Fig. 5. The input reflection coefficient is far smaller than 1 and the isolation larger than 20 dB at center frequency, thus satisfy the foundation of the theory of small reflections discussed in Section 2.



(a) Magnitude of scattering parameters (b) Phase of scattering parameters

Figure 5. Scattering parameters of the power divider element.

A 4-way 2-stage power divider consisting of three Wilkinson power dividers is designed. The lengths of interconnection lines are set to 27.44 mm, which make the 2-stage power divider perfect match at center frequency. The input reflection coefficients are calculated by formula (5) and simulated by Agilent Design System (ADS) 2011. The calculated and simulated results are shown in Fig. 6. It can be observed that the magnitude and phase of input reflection coefficients agree very well. The deviations are less than 0.012 in magnitude and 5 degrees in phase over the frequency band from 3 to 5 GHz. In Fig. 6(a), it can be seen that the valley points of the two curves are below -40 dB at center frequency of 4 GHz.



(a) The magnitude of S11 (b) The phase of S11

Figure 6. Comparison of S_{11} calculated by formula (5) and ADS simulation for a 4-way two-stage power divider.

5. FABRICATION AND MEASUREMENTS

A 16-way 4-stage power divider is designed with perfect match at center frequency based on the new design method. The power divider elements are the same as that in Section 4. The interconnection lines between different divider elements are carefully calculated. The lengths of interconnection lines corresponding to different stages are: $L_1 = 27.44$ mm, $L_2 = 38.76$ mm, $L_3 = 95.34$ mm. The interconnection lines are curved to decrease the length of the 4-stage power divider. A prototype is fabricated and shown in Fig. 7.

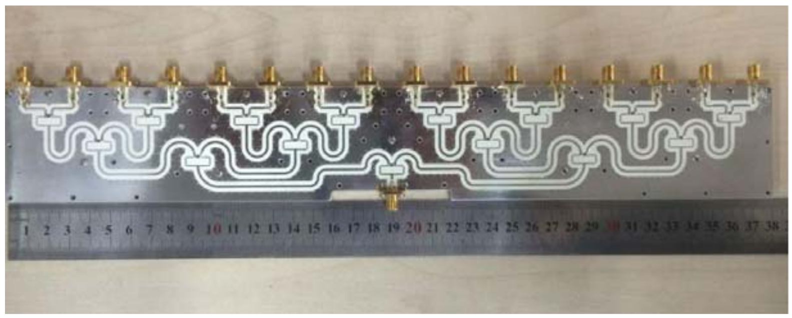


Figure 7. 16-way 4-stage power divider.

The proposed multi-stage power divider is measured by Rohde-Schwarz ZVA24 vector network analyzer. The reflection coefficient of input port obtained by the proposed theory and the measurement of the fabricated prototype are shown in Fig. 8. It can be seen that the two reflection coefficients agree very well, and their minimums occur at center frequency 4 GHz. The minimum value of the measured input reflection coefficient is -30.3 dB. The disagreement of measured S_{11} magnitude is mainly caused by measurement error and manufacturing imperfection. The 4-stage power divider with worst return loss at 4 GHz is also designed, and the lengths of interconnection lines corresponding different stages are: $L_1 = 27.44$ mm, $L_2 = 38.76$ mm, $L_3 = 95.34$ mm. Its return loss is shown as the green dotted line in Fig. 8. Over the frequency band 3.8–4.2 GHz, its return loss is worse than 10 dB, which is not acceptable. Other important parameters for the fabricated 16-way 4-stage power divider are shown Figs. 9–11, which satisfy our design requirements. The validity of the proposed design method is confirmed.

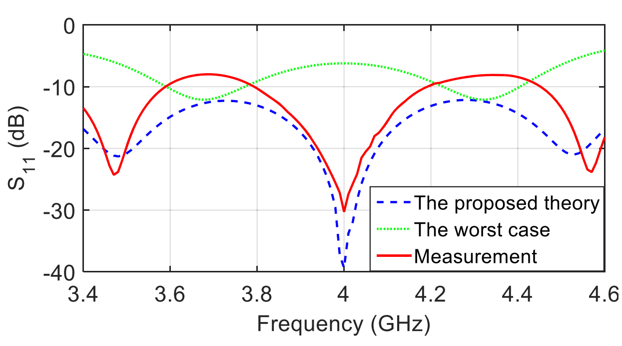


Figure 8. Calculated and measured reflection coefficient of 4-stage power divider designed with perfect match at 4 GHz, and calculated reflection coefficient of 4-stage power with worst return loss at 4 GHz.

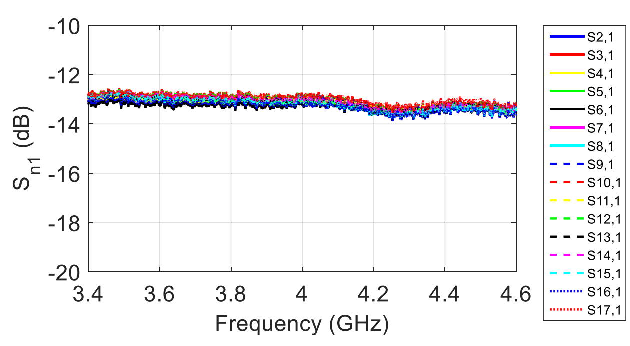


Figure 9. Magnitude of measured transmission coefficients.

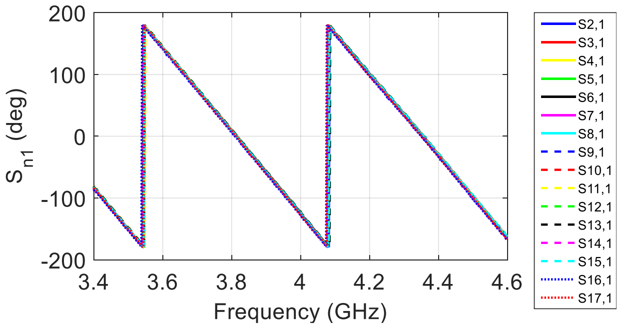


Figure 10. Phase of measured transmission coefficients.

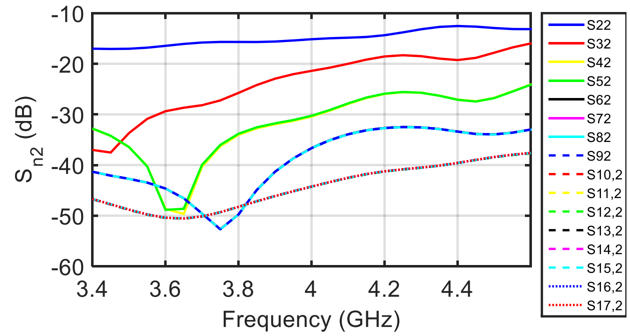


Figure 11. Measured values of reflection coefficient and mutual coupling for output ports.

6. CONCLUSION

This paper presents a method based on the theory of small reflections to design electrically large multi-stage power divider. From the theory of small reflections, formula (5) can be derived, which describes the relationship between the lengths of the interconnection lines and the input reflection coefficient of N-stage power divider. With formula (11) which is derived from formula (5), the lengths of interconnection lines can be determined by design requirements after the two-way power divider element is designed. In the worst case scenario, the input reflection coefficient for N-stage power divider is approximately N times of that of each power divider element. However, by carefully choosing the lengths of interconnection lines using the proposed method, partial reflections are cancelled out, achieving minimum reflection at specified frequency. The design efficiency is highly improved compared with the optimization method using simulation software such as ADS and HFSS. The measured result shows that the fabricated 16-way 4-stage power divider indeed has least refection at center frequency, thus validates the propose method.

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