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First Principles Cable Braid Electromagnetic Penetration Model

Larry K. Warne*, William L. Langston, Lorena I. Basilio, and William A. Johnson

Abstract—The model for penetration of a wire braid is rigorously formulated. Integral formulas are developed from energy principles for both self and transfer immittances in terms of potentials for the fields. The detailed boundary value problem for the wire braid is also set up in a very efficient manner; the braid wires act as sources for the potentials in the form of a sequence of line multipoles with unknown coefficients that are determined by means of conditions arising from the wire surface boundary conditions. Approximations are introduced to relate the local properties of the braid wires to a simplified infinite periodic planar geometry. This is used to treat nonuniform coaxial geometries including eccentric interior coaxial arrangements and an exterior ground plane.

1. INTRODUCTION

Electromagnetic penetration of shielded cables is an important and interesting subject with a long history. Early work on solid shields can be found in Schelkunoff's paper [1]. Eddy current penetration of cable shields is discussed in Kaden's book [2] along with some models for apertures in thick screens. Measurements of braid coupling are given in [3]. A classic text on cable braids is Vance's book [4]. Lee's book [5] also has valuable information on cables and shielding. The porpoising contribution to the transfer inductance of a cable braid was introduced by Tyni [6]. Various improvements in the geometrical description were made by Sali [7] and a discussion of the low frequency diffusion is given by Zhou and Gong [8]. Kley [9] improved and assembled all these contributions into a complete semi-empirical model where some parameters were based on measurements of typical commercial cables. The book by Tesche et al. [10] also has a nice summary of these models. These models are quite useful and identify the fundamental penetration mechanisms. Nevertheless, a first principles model directly based on the braid geometry would be desirable, particularly if a cable deviates at all from typical geometries employed in commercial cables.

This paper rigorously formulates the cable braid penetration problem. The braid geometry is illustrated in Figure 1, showing a surface mesh on the wires of the braid in a planar approximation to the cylindrical geometry. We start with a coaxial topology shown in Figure 2(a) and use the electric energy and elastance to define the self capacitance and the transfer capacitance. The integral quantities for the transfer immittances, shown as sources in Figure 2(b), that describe the coupling, as well as the self immittances, are identified. The immittances depend on the external (outside the metallic conductors) potentials representing the fields. Approximations are introduced to make use of the solution of the braid in a planar geometry with periodic symmetries (to simplify the analysis of the penetrations). These approximate results make clear the dependence of the immittances on limiting potential constants arising as one moves away from the braid; this approach facilitates a simple treatment of nonuniform coaxial arrangements [11] such as an exterior ground plane and an interior eccentric coax. A line multipole representation for the wire charges and currents is used to simplify the description. Images of these charges are used to treat adjacent dielectric material surfaces. The electric coupling shown in Figure 3(a) is treated first. Magnetic coupling is then treated; both hole, shown in Figure 3(b), and

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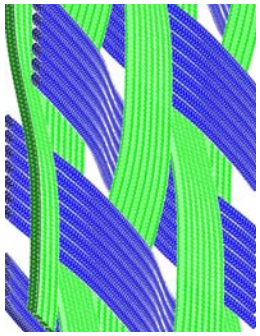


Figure 1. A planar approximation of a braid (in this case a numerical mesh representation is shown) highlighting the individual wires making up the strip carriers.

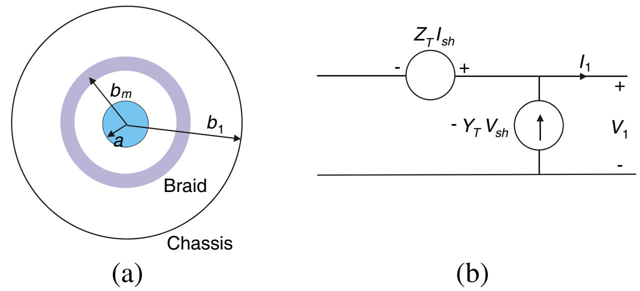


Figure 2. (a) Braided coax chassis topology. (b) Transfer immittance source model for braided cable. The upper conductor in the depicted model represents, for example, the center conductor of the inner coaxial transmission line, and the direction of propagation is taken to the right, in positive z.

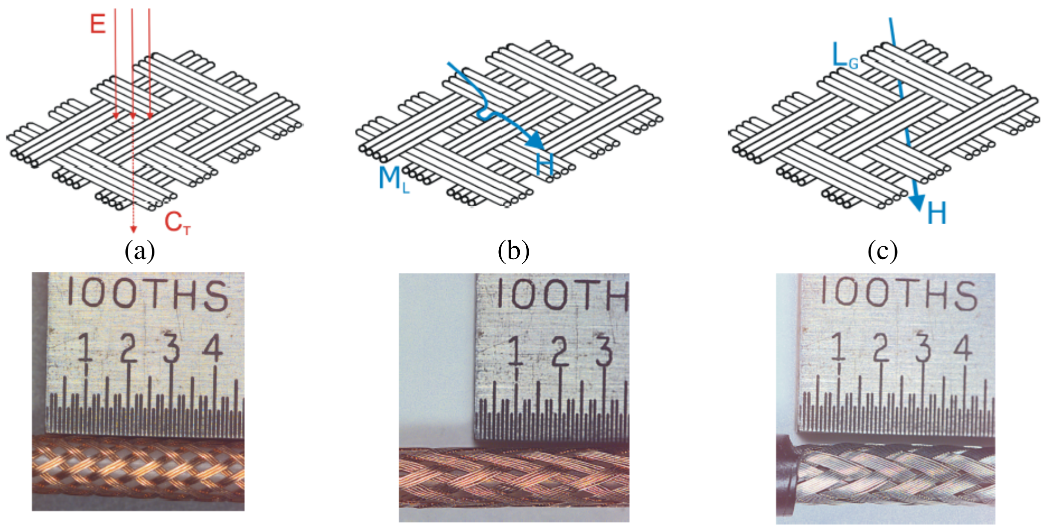


Figure 3. (a) Electric field penetration of a planar approximation to the braid, which is connected to the transfer capacitance C_T; relatively low optical coverage braid illustrating the inherent braid apertures (figure for Remee 1400 cable from [12]). (b) Magnetic penetration of braid apertures illustrated with approximate planar braid model; M_L is part of the transfer inductance L_T; medium optical coverage braid (figure for Belden 9201 cable from [12]). (c) Intrabraid magnetic porpoising penetration; L_G is also part of the transfer inductance L_T; high optical coverage braid where the intrabraid porpoising contribution is the dominant transfer impedance contribution (figure for Belden 8240 cable from [12]).

porpoising, shown in Figure 3(c), contributions are included in a self-consistent way. Magnetic energy is used to define the self and transfer inductances. Next, the magnetic diffusion into the conductor is introduced in the magnetic problem.

2. ELECTRIC COUPLING

This section formulates the transfer admittance per unit length Y_T or transfer capacitance per unit length C_T, of the braid penetration, as well as the inner coaxial admittance per unit length Y_1, or capacitance per unit length C_1. Even though the braid has a large but finite conductivity, the asymptotic form of these local quantities for large conductivities can be arrived at by treating the braid wires as perfect electric conductors. We will assume that there is enough incidental contact for the braid wires to locally be at an equal potential, which for convenience we usually take to vanish. Time dependence $e^{-i\omega t}$ is suppressed.

2.1. Energy Formulas for Elastances & Capacitances

We intend to take conductor 1 as the center conductor and conductor 2 as the chassis. It turns out to be convenient to initially use the elastances [13] (because of the open circuit or zero charge side conditions associated with these elements) defined by

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \tag{1}$$

The capacitance matrix (with the braid as a reference) is the inverse of the elastance matrix $C = S^{-1}$. In the case where $S_{11}S_{22} \gg S_{12}S_{21}$ we find $C_{jj} \approx 1/S_{jj}$, $j = 1, 2$ and in a reciprocal media

$$C_{12} = C_{21} = C_m \approx -S_m / (S_{11}S_{22}) \approx -C_{11}C_{22}S_m \tag{2}$$

where $S_m = S_{12} = S_{21}$. The continuity equation yields a connection between net current I and charge per unit length q on a conductor

$$\oint_S \underline{J} \cdot \underline{n} dS = -\frac{\partial}{\partial t} \int_V \rho_v dV \to \frac{\partial I}{\partial z} = -\frac{\partial q}{\partial t} \tag{3}$$

Hence we can determine the current changes over a short periodic section of length ℓ along the line as

$$I_j(z + \ell) - I_j(z) = -\frac{dQ_j}{dt} = -\frac{d}{dt}(\ell q_j), \quad j = 1, 2 \tag{4}$$

The power removed from a periodic section of line is minus the derivative of the stored electric energy W_e

$$V_1 [I_1(z + \ell) - I_1(z)] + V_2 [I_2(z + \ell) - I_2(z)] = -\frac{d}{dt} W_e \tag{5}$$

Taking two sources, and denoting the resulting electric fields due to these by subscripts 1 and 2 then by superposition the total electric field is $\underline{E} = \underline{E}_1 + \underline{E}_2$. Equating electric energies in a region (circuit to field quantities), where the constitutive relation between the displacement and electric field is $\underline{D} = \varepsilon \underline{E}$, gives

$$W_e = \frac{1}{2} S_{11} Q_1^2 + S_m Q_1 Q_2 + \frac{1}{2} S_{22} Q_2^2 = \frac{1}{2} \int_V \underline{D} \cdot \underline{E} dV \tag{6}$$

where ε is the electric permittivity, assumed to be piecewise constant. The 1 problem using the elastances has a charge Q_1 on the center conductor with an equal and opposite charge on the braid shield and no charge on the chassis conductor. The 2 problem has a charge Q_2 on the chassis with equal and opposite charge on the braid shield and no charge on the center conductor. We can identify the self elastances as

$$S_{11} Q_1^2 = \int_V \varepsilon E_1^2 dV = -\int_V \nabla \phi_1 \cdot \underline{D}_1 dV \tag{7}$$

where the volume V includes both the region between center conductor and braid and the region between braid and chassis. From $\nabla \times \underline{E} = 0$ we can set $\underline{E} = -\nabla \phi$, and use

$$\nabla \cdot (\phi_1 \underline{D}_2) = \nabla \phi_1 \cdot \underline{D}_2 + \phi_1 \nabla \cdot \underline{D}_2 = \nabla \phi_1 \cdot \underline{D}_2 + \phi_1 \rho_{v2} \tag{8}$$

where we also used Gauss's law $\nabla \cdot \underline{D} = \rho_v$. Then applying Eq. (8) (with the 2 replaced by 1) outside the conductors (where the volume charge is zero) and using the divergence theorem (S is the closed surface which includes the surfaces of the center conductor, braid wires, and chassis, as well as two surfaces at fixed axial positions one braid period ℓ apart along the line) gives (and similarly for S_{22}).

$$S_{11} Q_1^2 = -\oint_S \underline{n} \cdot (\phi_1 \underline{D}_1) dS = V_1 Q_1 \tag{9}$$

or $S_{11} = V_1/Q_1$ with $Q_2 = 0$.

The mutual elastance is identified as

$$Q_1 Q_2 S_m = \int_V \varepsilon \underline{E}_1 \cdot \underline{E}_2 dV = -\int_V \nabla \phi_2 \cdot \underline{D}_1 dV \tag{10}$$

Figure 4(a) shows the field $\underline{E}_1$ generated by a positive charge Q_1 on the center conductor with no net charge on the chassis, as well as the field $\underline{E}_{sh} = \underline{E}_2$ generated by a positive charge $Q_{sh} = -Q_2$ on the braided shield with no net charge on the chassis; in this case the voltage is taken to be $V_{sh} = -V_2$. We see from Figure 4(a) that the potential V_1 on the center conductor will be negative with respect to the braided shield (taken to have zero potential) when the center conductor is uncharged ($Q_1 = 0$) and the braided shield is positively charged ($Q_{sh} > 0$); hence we expect $S_m > 0$. The distributed self capacitances per unit length are $C_{11} = \ell C_1$ and $C_{22} = \ell C_2 = \ell C_{sh}$ with $Q_1 = q_1 \ell$ and the transfer capacitance per unit length is related by $C_m = -\ell C_T$ with $Q_{sh} = q_{sh}\ell$.

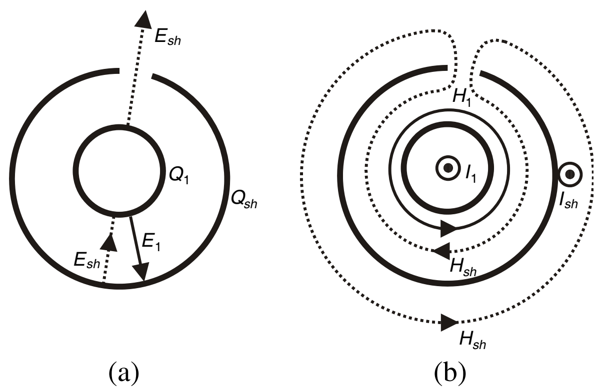


Figure 4. (a) Field E_1 generated by charge Q_1 and field E_{sh} generated by charge Q_{sh}. (b) Field H_1 generated by I_1 and field H_{sh} generated by I_{sh}.

Using (8) we can rewrite the mutual elastance as

$$Q_1 Q_2 S_m = -\oint_S \phi_2 \underline{D}_1 \cdot \underline{n} dS = -\int_{S_c} \phi_2 \underline{D}_1 \cdot \underline{n} dS = V_{12} Q_1 \tag{11}$$

where S_c is the center conductor surface, which is (V_{12} is the voltage on the open circuited conductor 1, or V_1, excited by a charge on conductor 2) $S_m = V_1/Q_2$ with $Q_1 = 0$. If we selected ϕ_1 and $\underline{D}_2$ we would end up with V_2/Q_1 with $Q_2 = 0$. Depending on which field we choose to represent with the potential we will end up with a different surface integral in the end. In the first case the closed surface integral is focused on the 1 region inside the braid because the field $\underline{D}_1$ is generated by charge on the center conductor (in the second case the closed surface integral is focused on the 2 region outside the braid because the field $\underline{D}_2$ is generated by charge on the chassis).

The transmission line equation for the interior current is

$$\frac{dI_1}{dz} = -Y_1 V_1 - Y_T V_{sh} = -Y_1 V_1 + i\omega \left(C_T / C_{sh} \right) q_{sh} \tag{12}$$

with self admittance $Y_1 = -i\omega C_1$ and transfer admittance $Y_T = -i\omega C_T$. In situations where the exterior region is not strictly a transmission line, it is often convenient to avoid using an exterior transmission line voltage as the drive and instead use the exterior charge per unit length connected to the exterior shield current by means of the continuity equation

$$\nabla \cdot \underline{J}_{sh} = i\omega \rho_{sh} \rightarrow \frac{dI_{sh}}{dz} = i\omega q_{sh} \tag{13}$$

2.1.1. Self Capacitance Approximate Evaluation

The self capacitance for nonuniform geometries is estimated for the 1 problem by breaking up the volume V into two parts $V = V_0 + \Delta V$, selecting an auxiliary volume V_0 to extend from the center conductor to a distance out near the braid

$$S_{11}Q_1^2 = \left(\int_{V_0} + \int_{\Delta V}\right) D_1 E_1 dV \approx \int_{V_0} D_1 E_0 dV + \int_{\Delta V} D_1 E_1 dV \tag{14}$$

where we approximate the electric field in the V_0 region by $\underline{E}_0$ the field for a solid shield at the boundaries of V_0. The auxiliary problem is taken to have the same charge $Q_0 = Q_1$ but the center conductor potential with a solid shield at the inner and outer boundaries of the closed surface S_0 (bounding volume V_0) is slightly different than with the braid (potential $V_0 \neq V_1$). Using scalar potentials and (8) (with the 1 replaced by 0 and the 2 replaced by 1) we obtain

$$\int_{V_0} D_1 E_0 dV \approx -\oint_{S_0} \phi_0 \underline{D}_1 \cdot \underline{n} dS = -\left(\int_{S_c} + \int_{S_0}\right) \phi_0 \underline{D}_1 \cdot \underline{n} dS \tag{15}$$

where S_0 (with a standard integral symbol) represents the outer cylinder of the closed surface S_0 stood off from the braid wires. If we note that ϕ_0 should be constructed to vanish on the solid auxiliary shield S_0 then the final term vanishes. We have used periodicity of the fields and potential to drop the surface integrals on the ends of the periodic region in z. Also with the potential ϕ_1 and using (8) (with the 2 replaced by 1) gives

$$\int_{\Delta V} D_1 E_1 dV = -\oint_{\Delta S} \phi_1 \underline{D}_1 \cdot \underline{n} dS = -\left(\int_{S_0} + \int_{S_w} + \int_{S_{\text{chassis}}}\right) \phi_1 \underline{D}_1 \cdot \underline{n} dS \tag{16}$$

Noting that ϕ_1 should be constructed to vanish on the braid wire surface S_w, the integral on S_w vanishes; the chassis is open circuited in the 1 problem and hence on this surface S_{chassis} we expect $\int_{S_{\text{chassis}}} \underline{D}_1 \cdot \underline{n} dS = 0$; with ϕ_1 equal to a constant on S_{chassis} the final integral also vanishes. Hence we finally have

$$S_{11}Q_1^2 \approx -\int_{S_c} \phi_0 \underline{D}_1 \cdot \underline{n} dS - \int_{S_0} \phi_1 \underline{D}_1 \cdot \underline{n} dS \approx V_0 Q_1 - \int_{S_0} \phi_1 \underline{D}_1 \cdot \underline{n} dS \tag{17}$$

where $\phi_0 = V_0$ (not to be confused with the auxiliary volume V_0) on S_c and the integral yields minus the center conductor charge.

Now near the braid, but still far from the individual braid wires, we have the local behavior (where $\underline{\rho} = x\underline{e}_x + y\underline{e}_y$ is the two-dimensional position vector and the nearest local braid mean position is $\underline{\rho}_m = x_m\underline{e}_x + y_m\underline{e}_y$, where $\underline{n} \times (\underline{\rho} - \underline{\rho}_m) = 0$) (in this asymptotic expression we define $\underline{n}$ as pointing in from the exterior chassis region)

$$\phi_1 \sim -\underline{E}_0 \cdot \left(\underline{\rho} - \underline{\rho}_m + \underline{n}\phi_b/E_0\right) \tag{18}$$

For a fixed standoff distance on S_0 from we can set

$$d_0 = \underline{n} \cdot \left(\underline{\rho}_0 - \underline{\rho}_m\right) \tag{19}$$

For a circular shield like in the eccentric coax with $\underline{E}_0 = E_\rho \underline{e}_\rho = E_0 \underline{e}_\rho$ we have $\phi_1 \sim E_0(b - \rho) + \phi_b$ and with $\underline{n} = -\underline{e}_\rho$ we have $d_0 = b - b_0$ where b is the mean braid radius and b_0 is the radius of S_0. Note that the electric field E_0 and the potential ϕ_b are in general varying around the circumference (for example, in the eccentric coax) but the ratio ϕ_b/E_0 is a constant dependent only on the local braid wire geometry (in the planar approximation to the braid). Then (noting that $\underline{n} = -\underline{e}_\rho$ on S_0 from the ΔV region) inserting the local potential (18)

$$S_{11}Q_1^2 \approx (C_{00}V_0)^2 / C_{11} \approx V_0 Q_1 - (d_0 + \phi_b/E_0) \int_{S_0} E_0 \underline{D}_1 \cdot \underline{n} dS \approx C_{00} V_0^2 + (d_0 + \phi_b/E_0) \int_{S_0} \varepsilon E_0^2 dS \tag{20}$$

where the charge in the auxiliary problem is taken to be the same $Q_1 = Q_0 = C_{00}V_0$ and we have used the approximation $S_{11} \approx 1/C_{11}$. Now dividing the second and final equalities of (20) by $C_{00}V_0^2$, inverting both sides of the resulting expression, and changing to capacitances per unit length (with $C_{00} = \ell C_0$) we have

$$C_1 \approx C_0 - (d_0 + \phi_b/E_0) \int_{S_0} \varepsilon \left(E_0/V_0\right)^2 dS/\ell \tag{21}$$

We can write a similar expression for the exterior capacitance per unit length of the chassis to braid C_2.

2.1.2. Transfer Capacitance Approximate Evaluation

The mutual or transfer capacitance is determined for nonuniform geometries from the cross terms. We break up the volume into two parts (where we approximate $E_1 \approx E_0$ in the auxiliary volume V_0 and $\underline{n}$ points out of V_0 and ΔV)

$$\begin{aligned} S_m Q_1 Q_2 &= \int_V \varepsilon \underline{E}_1 \cdot \underline{E}_2 dV \approx \int_{V_0} \underline{D}_2 \cdot \underline{E}_0 dV + \int_{\Delta V} \underline{E}_2 \cdot \underline{D}_1 dV = -\oint_{S_0} \phi_0 \underline{D}_2 \cdot \underline{n} dS - \oint_{\Delta S} \phi_2 \underline{D}_1 \cdot \underline{n} dS \\ &\approx -\left(\int_{S_c} + \int_{S_0}\right) \phi_0 \underline{D}_2 \cdot \underline{n} dS - \left(\int_{S_0} + \int_{S_w} + \int_{S_{\text{chassis}}}\right) \phi_2 \underline{D}_1 \cdot \underline{n} dS \end{aligned} \tag{22}$$

where in V_0 we have used the scalar potential ϕ_0 and Eq. (8) (with 1 replaced by 0), and in ΔV we have used the scalar potential ϕ_2 and Eq. (8), along with the divergence theorem. If we note that ϕ_0 should be constructed to vanish on the auxiliary shield S_0 then the second integral vanishes. Also, since ϕ_2 is constructed to vanish on the braid wire surface the integral over S_w of the preceding equation vanishes. In addition in this section, ϕ_1 is constructed with an open circuited chassis surface in the 1 problem, so there is no net charge on S_{chassis} giving $\int_{S_{\text{chassis}}} \underline{D}_1 \cdot \underline{n} dS = 0$ (where ϕ_2 is constant on S_{chassis}), and ϕ_2 is constructed with an open circuited center conductor surface in the 2 problem, so there is no net charge on S_c giving $\int_{S_c} \underline{D}_2 \cdot \underline{n} dS = 0$ (where ϕ_1 and ϕ_0 are constant on S_c). Hence the V_0 integration entirely vanishes and in the final line of the preceding ΔS integration the chassis term also vanishes. Therefore we can write the mutual elastance as

$$S_m Q_1 Q_2 = \int_V \varepsilon \underline{E}_1 \cdot \underline{E}_2 dV \approx \int_{\Delta V} \underline{E}_2 \cdot \underline{D}_1 dV = -\int_{S_0} \phi_2 \underline{D}_1 \cdot \underline{n} dS \tag{23}$$

Now we note that near S_0 we can write the potential in the 2 problem as (since there is no charge on the center conductor in the 2 problem)

$$\phi_2 \sim -\phi_c \approx (\underline{n}\phi_c/E_0) \cdot \underline{E}_0^{sh} \tag{24}$$

where here we take $\underline{n}$ again to point inward toward the center conductor region which is consistent with the sign from the ΔV integration (we usually evaluate the outward exterior field $\underline{E}_0^{sh} = -E_0^{sh}\underline{n}$ at the mean braid wire location of S_w for a solid shield rather than on S_0 since this field exists exterior to the braid shield) the ratio $-\phi_c/E_0$ is a constant for a given braid geometry (in the local planar approximation to the braid excited by E_0) and the normal field E_0^{sh} in general varies around the braid shield (which can be determined from the exterior potential ϕ_{sh} for a solid shield at the braid center line as $-E_0^{sh} = -\partial\phi_{sh}/\partial n$). Then the mutual elastance is

$$S_m \approx -(-\phi_c/E_0) \frac{1}{Q_1 Q_2} \int_{S_0} E_0^{sh} \underline{D}_1 \cdot \underline{n} dS \tag{25}$$

where the center conductor charge Q_1 normalizes the integration of the normal component of the interior displacement in the 1 problem and the chassis charge Q_2 normalizes the normal component of the exterior field level at the braid shield location. Now we approximate the field $\underline{D}_1 \approx \underline{D}_0$ on S_0 and replace $Q_1 = Q_0$ and $Q_2 = -Q_{sh}$, where $Q_{sh} = C_{00}^{sh} V_{sh}$, $C_{00}^{sh} = \ell C_{sh}$ and V_{sh} is the exterior voltage from the approximate solid shield to the chassis

$$S_m \approx (-\phi_c/E_0) \frac{1}{Q_0 C_{00}^{sh}} \int_{S_0} \left(E_0^{sh}/V_{sh}\right) \underline{D}_0 \cdot \underline{n} dS \tag{26}$$

Now setting $Q_0 = \ell q_0$, $D_0 = -\underline{D}_0 \cdot \underline{n}$, and using (2) with $C_m = -C_T \ell$, gives

$$C_T \approx (\phi_c/E_0)\, C_1 \int_{S_0} \left(E_0^{sh}/V_{sh} \right) (D_0/q_0)\, dS/\ell \tag{27}$$

where we have approximated $C_{22} \approx C_{00}^{sh}$ and $C_{11} = \ell C_1$.

2.1.3. Uniform Cylindrical Geometry

The simple case where the geometry is a uniform cylinder both inside and outside has field

$$\varepsilon E_0/\left(C_0 V_0\right) = D_0/q_0 = 1/\left(2\pi b_0\right) \tag{28}$$

$$\varepsilon E_0^{sh}/\left(C_{sh} V_{sh}\right) = 1/\left(2\pi b\right) \tag{29}$$

and capacitance per unit length $C_0 = 2\pi\varepsilon/\ln(b_0/a)$. Inserting (28) into (21) gives

$$C_1 \approx C_0 + \{(b - b_0) + \phi_b/E_0\} \frac{\partial C_0}{\partial b_0} \approx C_0\left(b_0 \to b + \phi_b/E_0\right) = \frac{2\pi\varepsilon}{\ln\left[\left(b + \phi_b/E_0\right)/a\right]} \tag{30}$$

The transfer capacitance is determined by inserting the fields from Eqs. (28) and (29) into Eq. (27)

$$C_T \approx (\phi_c/E_0)\, \frac{C_1 C_{sh}}{2\pi b\varepsilon} \tag{31}$$

A physical picture of these quantities for a circular coax is simple. The self capacitance per unit length is preserved by the replacement of radial position b by the effective position $b + \phi_b/E_0$. For the transfer capacitance, when the braid is driven from the outside with a field $E_\rho = E_0^{sh} = V_{sh} C_{sh}/(2\pi b\varepsilon)$ at $\rho = b$, giving rise to an interior potential receding from the braid toward the interior, which asymptotically is $-\phi_c$ (where ϕ_c/E_0 is the shadow-side constant determined, say, approximately from the planar problem) on the open circuited center conductor, then the resulting center conductor charge per unit length which must be supplied to bring this conductor to zero potential is $q_s = C_T V_{sh} = -C_1\phi_c$, where C_1 is the capacitance per unit length of the inner coax; the source current is minus the time derivative of this charge.

2.1.4. Other Cable Cross Sections

The preceding integral forms Eqs. (27) and (21) can be applied to nonuniform geometries. The first case of an exterior ground plane is the most common type of exterior arrangement. The second case of an eccentric coax represents a starting point for considering interior multiconductor arrangements.

Exterior Ground Plane Case When the exterior transmission line problem consists of a solid circular cable of radius b, with center height h above a ground plane the exterior capacitance per unit length is

$$C_{sh} = q_{sh}/V_{sh} = 2\pi\varepsilon/\ln\sqrt{\frac{h + h_e}{h - h_e}} = 2\pi\varepsilon/\mathrm{Arccosh}\,(h/b) \tag{32}$$

where $h_e = \sqrt{h^2 - b^2}$ is the effective line charge position. Taking a cylindrical coordinate system to be at the center of the cylinder, the radial field at the outer shield surface is

$$E_\rho\,(b, \varphi) = \frac{q_{sh}}{2\pi\varepsilon b} \frac{h_e}{h + b\sin\varphi} = E_0^{sh} \tag{33}$$

Inserting the fields from Eqs. (28) and (33) into Eq. (27) gives

$$C_T \approx (\phi_c/E_0)\, \frac{C_1 C_{sh}}{2\pi\varepsilon b} \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{h_e}{h + b\sin\varphi} d\varphi \approx (\phi_c/E_0)\, \frac{C_1 C_{sh}}{2\pi b\varepsilon} \tag{34}$$

where we have used the identity that the average of the integrand over the azimuth is unity [14].

Note that in this simplified approach we have taken only the symmetric ($m = 0$) part of the interior potential of the cylindrical coax. Although other azimuthal modes will exist, due to the asymmetric exterior excitation, the net source charge q_s should be determined by this symmetric component.

Interior Eccentric Coax In addition to the ground plane exterior let us consider the situation when the interior is an eccentric coax [13]. We place a cylindrical coordinate system at the center of the outer cylinder of radius b_0 (we will take this cylinder to be a height h above an exterior ground plane). The center of the inner cylinder of radius a is displaced a distance d from the center of the outer cylinder. Initially we orient the displacement d to be downward toward the ground plane, however later we also consider the case where it is rotated by $\pi/2$ with respect to the ground plane. The potential is taken to vanish on the outer cylinder and equal V_0 on the inner cylinder. The capacitance per unit length is [13]

$$q_0/V_0 = C_0\left(b_0\right) = 2\pi\varepsilon/\mathrm{Arccosh}\left(\frac{a^2+b_0^2-d^2}{2ab_0}\right) \tag{35}$$

where q_0 is the charge per unit length on the inner cylinder. The radial electric field at the outer cylinder boundary is

$$\frac{1}{q_0}E_{0\rho}\left(b_0,\varphi\right) = \frac{1}{2\pi\varepsilon b_0}\frac{y_c}{y_1+b_0\sin\varphi} \tag{36}$$

where (the primed quantities replace b_0 by b and will be used shortly)

$$2y_c d = \sqrt{\left[\left(b_0-a\right)^2-d^2\right]\left[\left(b_0+a\right)^2-d^2\right]};\quad 2y_c' d = \sqrt{\left[\left(b-a\right)^2-d^2\right]\left[\left(b+a\right)^2-d^2\right]} \tag{37}$$

$$y_1 = \sqrt{y_c^2+b_0^2} = \left(-a^2+b_0^2+d^2\right)/(2d);\quad y_1' = \sqrt{y_c'^2+b^2} \tag{38}$$

The self capacitance from Eq. (21), using interior field Eq. (36), is

$$\begin{aligned} C_1 &\approx C_0 - \left[(b-b_0)+\phi_b/E_0\right]\frac{1}{2\pi b_0\varepsilon}C_0^2\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{y_c}{y_1+b_0\sin\varphi}\right)^2 d\varphi \\ &\approx C_0 + \left[(b-b_0)+\phi_b/E_0\right]\frac{\partial C_0}{\partial b_0} \approx C_0\left(b+\phi_b/E_0\right) = 2\pi\varepsilon/\mathrm{Arccosh}\left[\frac{a^2+\left(b+\phi_b/E_0\right)^2-d^2}{2a\left(b+\phi_b/E_0\right)}\right] \end{aligned} \tag{39}$$

where we used the identity that the average of the integrand over the azimuth is y_1/y_c.

The transfer capacitance from Eq. (27), using exterior field Eq. (33) and interior field Eq. (36), is given by

$$C_T \approx \left(\phi_c/E_0\right)\frac{C_1 C_{sh}}{2\pi\varepsilon b}\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{h_e}{h+b\sin\varphi}\right)\left(\frac{y_c}{y_1+b_0\sin\varphi}\right)d\varphi \approx \left(\phi_c/E_0\right)\frac{C_1 C_{sh}}{2\pi\varepsilon b}\left(\frac{by_c-b_0h_e}{by_1-b_0h}\right) \tag{40}$$

where we used the identity that the average of the integrand over the azimuth is $(by_c-b_0h_e)/(by_1-b_0h)$. To avoid introducing a somewhat arbitrary $b_0\sim b$ in this result we make the replacement $b_0\to b$ using the primed quantities in Eqs. (37) and (38)

$$C_T \approx \left(\phi_c/E_0\right)\frac{C_1 C_{sh}}{2\pi\varepsilon b}\left(\frac{y_c'-h_e}{y_1'-h}\right) \tag{41}$$

Note that the final ratio in Eq. (41) is unity if $d=0$ (which makes the field of the interior problem uniform in azimuth) or if $h\to\infty$ (which makes the field of the exterior problem uniform in azimuth).

We can rotate the angle φ in the interior field formula (36) versus in the exterior field formula (33) in order to rotate the displacement d in the eccentric coax relative to the outer short circuit field; such a rotation by $\pi/2$ gives the transfer capacitance

$$C_T \approx \left(\phi_c/E_0^{sh}\right)\frac{C_1 C_{sh}}{2\pi\varepsilon b}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{h_e}{h+b\sin\varphi}\frac{y_c}{y_1+b_0\cos\varphi}d\varphi \approx \left(\phi_c/E_0^{sh}\right)\frac{C_1 C_{sh}}{2\pi\varepsilon b}\frac{b^2y_1y_c+b_0^2hh_e}{y_1^2b^2+h_e^2b_0^2} \tag{42}$$

where we have used the identity that the average of the integrand over the azimuth is $(b^2y_1y_c+b_0^2hh_e)/(y_1^2b^2+h_e^2b_0^2)$. Again to avoid having a somewhat arbitrary $b_0\sim b$ it is probably better to simplify the final expression by letting $b_0\to b$

$$C_T \approx \left(\phi_c/E_0^{sh}\right)\frac{C_1 C_{sh}}{2\pi\varepsilon b}\frac{y_1'y_c'+hh_e}{y_1'^2+h_e^2} \tag{43}$$

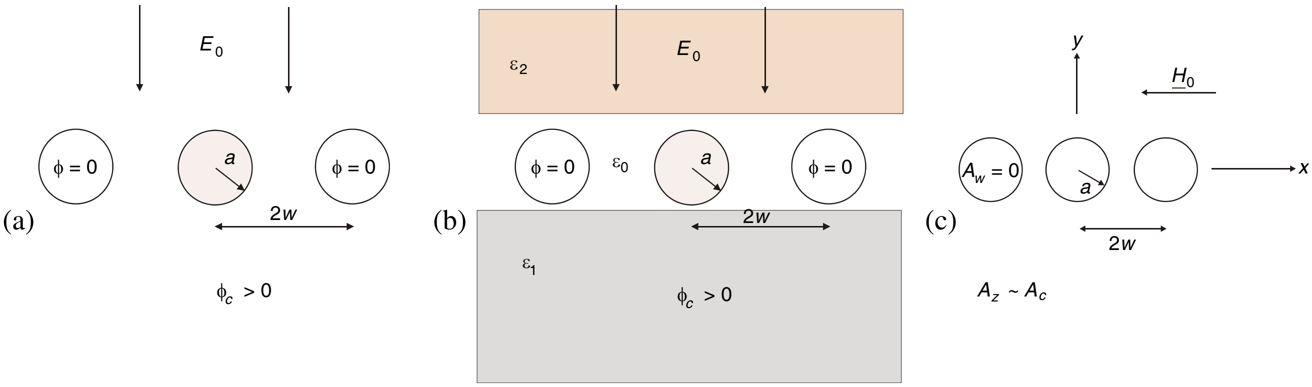


Figure 5. (a) Illustration of the braid transfer potential ϕ_c with drive field E_0. (b) Dielectric materials surround the planar braid layer. (c) Illustration of planar braid driven by uniform magnetic field to determine magnetic flux per unit length constants.

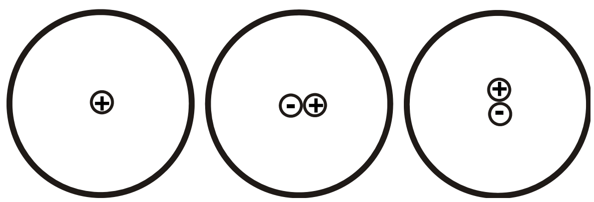


Figure 6. A sequence of line multipole charges to represent the transverse variations of the electric field.

2.2. Electric Multipole Representation

In the preceding sections, we have assumed that the values of ϕ_c/E_0 and of ϕ_b/E_0 have been determined. The way we will actually determine these quantities is by solving for the potential surrounding a periodic cell of the braid. This could be done in the actual cylindrical braid, but as an approximation, and because the planar shield is of interest in its own right, we will concentrate at present on the planar problem as depicted in Figure 5(a). The drive potential in the planar problem will be taken as $\phi_{inc} = E_0 y$ where $y = 0$ is at the braid center.

It is efficient to represent the electric scalar potential by an electric multipole summation as shown in Figure 6 to capture the transverse field behavior [15]. The potential for an axially varying line charge $q(s')$ is

$$\phi_{\text{scatt}} = \frac{1}{4\pi\varepsilon}\int \frac{q(s')}{|\underline{r}-\underline{r}'|}ds' = -\frac{q_n}{4\pi\varepsilon}\ln\left[\frac{(s-s_n/2)+\sqrt{\rho^2+(s-s_n/2)^2}}{(s+s_n/2)+\sqrt{\rho^2+(s+s_n/2)^2}}\right] \tag{44}$$

where the charge is discretized as pulses of strength q_n, we take the end positions of the N wire segments to be denoted by $\underline{r}_n^{\pm}$, $n = 1, \dots, N$, the segment length is s_n, the vector along the axis of the segment is $\underline{s}_n = \underline{r}_n^+ - \underline{r}_n^-$ with unit vector $\underline{e}_{sn} = \underline{s}_n/s_n$, the center location of the segment is $\underline{r}_n^c = (\underline{r}_n^+ + \underline{r}_n^-)/2$, the projected distance along the segment is denoted by $s = \underline{e}_{sn}\cdot(\underline{r}-\underline{r}_n^c)$, and ρ is the transverse distance from the segment axis, with the vector distance perpendicular to the segment $\underline{\rho} = -\underline{e}_{sn}\times[\underline{e}_{sn}\times(\underline{r}-\underline{r}_n^c)]$.

The lattice parameters are now used to image this potential contribution in one periodic cell over the planar braid model. The two lattice vectors associated with the two carrier directions are taken as $\underline{u}$ and $\underline{v}$. The components of the lattice vectors along and perpendicular to the direction of a particular braid segment are taken as $\underline{u} = u_{sn}\underline{e}_{sn} + \underline{u}_{\rho n}$ and $\underline{v} = v_{sn}\underline{e}_{sn} + \underline{v}_{\rho n}$. Thus we have the total potential

$$\phi_{\text{scatt}}^{\text{tot}} = \sum_{n=1}^{N}\frac{q_n}{4\pi\varepsilon}\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\ln\left[\frac{(s-s_n/2-ju_{sn}-kv_{sn})+|\underline{r}-\underline{r}_n^+-j\underline{u}-k\underline{v}|}{(s+s_n/2-ju_{sn}-kv_{sn})+|\underline{r}-\underline{r}_n^--j\underline{u}-k\underline{v}|}\right] \tag{45}$$

The constant potential condition on each n segment $\phi_{tot} + \phi_{inc} = V_{n'}$ around the braid wires uses the evaluation $s = s_{n'}$ and $\underline{\rho} = \underline{\rho}_{n'}$ or $\underline{r} = \underline{r}_{n'}$ where the observation or match points are (a is the braid wire radius) taken as $s_{n'} = \underline{e}_{sn} \cdot (\underline{r}_{n'} - \underline{r}_n^c)$, $\underline{\rho}_{n'} = -\underline{e}_{sn} \times \underline{e}_{sn} \times (\underline{r}_{n'}^c - \underline{r}_n^c) + a\underline{e}_{\rho n'}$, and $\underline{r}_{n'} = \underline{r}_{n'}^c + a\underline{e}_{\rho n'}$. To construct the unit vector perpendicular to the n' wire $\underline{e}_{\rho n'}$ we need a vector $\underline{r}_0$ linearly independent of $\underline{s}_{n'}$. Then we can take (the choice is obviously not unique)

$$\underline{e}_{\rho n'} = -\underline{e}_{sn'} \times \underline{e}_{sn'} \times (\underline{r}_0 - \underline{r}_{n'}^c) \,/\, |\underline{e}_{sn'} \times \underline{e}_{sn'} \times (\underline{r}_0 - \underline{r}_{n'}^c)| \tag{46}$$

We can rotate the vector to obtain other observation points around the wire

$$\underline{e}_{\rho n'}^{m'} = \cos\left(m'\pi/M\right)\underline{e}_{\rho n'} + \sin\left(m'\pi/M\right)\underline{e}_{sn'} \times \underline{e}_{\rho n'}, \quad m' = 0, 1, \ldots, 2M-1 \tag{47}$$

where $\underline{\rho}_{n'}^{(m')}$ and $\underline{r}_{n'}^{(m')}$ go with this generalization of the vectors for $m' = 0$.

The monopole moments are not sufficient to match the potential condition at many points around the wire so we include a series of line multipole moments in the potential, which for a given position n, is written as (where $q_n = \underline{p}^{(0)}$)

$$\phi_{\text{scatt}}^n = -\frac{1}{4\pi\varepsilon}\sum_{m=0}^{M}\underline{p}^{(0)}\underline{p}^{(1)}\ldots\underline{p}^{(m)} \cdot \nabla_t^m \ln\left[\frac{(s - s_n/2) + \sqrt{\rho^2 + (s - s_n/2)^2}}{(s + s_n/2) + \sqrt{\rho^2 + (s + s_n/2)^2}}\right] \tag{48}$$

where $\nabla_t = \underline{e}_x\partial/\partial x + \underline{e}_y\partial/\partial y$ is the "del" operator transverse to the particular wire segment and the meaning of the "dot" product notation will be made clear on the following pages. Analogous to Eq. (45) the total potential is

$$\phi_{\text{scatt}}^{tot} = \sum_{n=1}^{N}\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\phi_{\text{scatt}}^n \tag{49}$$

Now the final matching equation to determine the $2M$ multipole moments on each of N segments imposes the constant $V_{n'} = \phi = \phi_{\text{scatt}}^{tot} + \phi_{inc}$ with $s = s_{n'}$ and $\underline{r} = \underline{r}_{n'}^{(m')}$ with $n' = 1, \ldots, N$ and $m' = 0, 1, \ldots, 2M-1$. Note that the above matching positions $(\underline{\rho}_{n'}, s_{n'})$ depend on the the source point n, m as well as the observation point n', m'.

Once the potential ϕ is found, with the potential on the braid taken, say, to vanish $V_{n'} = 0$, we can then proceed to find the potential constant behaviors of interest. For $y < 0$ (the shadow side of the shield) we evaluate the total potential far from the braid to find $\phi \to \phi_c$, $y \to -\infty$. For $y > 0$ (the illuminated side of the shield) we evaluate the potential to find $\phi \to E_0 y + \phi_b$, $y \to +\infty$. Normalizing by the drive field E_0 we find the desired constants ϕ_c/E_0 and ϕ_b/E_0.

2.2.1. Multipole Evaluation

The transverse derivatives in Eq. (48) are needed to obtain the line multipole moments. Writing the derivatives of the transverse "del" operator in local cylindrical coordinates of the segment and taking $\partial/\partial\varphi = 0$ (as in the monopole term) the dipole term is found by applying

$$\underline{p}^{(1)} \cdot \nabla_t = \left(p_x^{(1)}\cos\varphi + p_y^{(1)}\sin\varphi\right)\frac{\partial}{\partial\rho} \tag{50}$$

There are two independent terms here: $\sin\varphi$ and $\cos\varphi$; there is only a single independent function of ρ. Next the quadrupole term is

$$\begin{aligned}\underline{p}^{(1)}\underline{p}^{(2)} \cdot \nabla_t\nabla_t &= \frac{1}{2}\left(p_x^{(1)}p_x^{(2)} + p_y^{(1)}p_y^{(2)}\right)\left(\frac{\partial}{\partial\rho} + \frac{1}{\rho}\right)\frac{\partial}{\partial\rho} \\ &+\frac{1}{2}\left[\left(p_x^{(1)}p_x^{(2)} - p_y^{(1)}p_y^{(2)}\right)\cos(2\varphi) + \left(p_x^{(1)}p_y^{(2)} + p_x^{(2)}p_y^{(1)}\right)\sin(2\varphi)\right]\left(\frac{\partial}{\partial\rho} - \frac{1}{\rho}\right)\frac{\partial}{\partial\rho}\end{aligned} \tag{51}$$

There are only three independent terms here: a constant 1, $\sin 2\varphi$, and $\cos 2\varphi$; there are only two independent functions of ρ. This follows from having $\partial^2/\partial x^2$, $\partial^2/\partial y^2$, and $\partial^2/\partial x\partial y$ derivatives taken. Note that the functions $\sin(m\varphi)$ and $\cos(m\varphi)$ have the same functional form of ρ for each value of m.

For long segments compared to their radius we could also introduce a two-dimensional redundancy between double x and double y derivatives by virtue of the two-dimensional Laplace equation. This would imply only two independent quantities for each value of m because $\partial^2/\partial x^2 = -\partial^2/\partial y^2$. In the preceding expression we would select $p_x^{(1)} p_x^{(2)} + p_y^{(1)} p_y^{(2)} = 0$ so that the constant term vanishes. This relation can be imposed on all coefficients so that additional higher order multipole terms do not influence lower order Fourier terms.

Radial Derivatives Because the logarithm is only a function of ρ our starting point for these terms is the radial derivative of the monopole. We can write this as (where we impose the two-dimensional Laplace relation between the coefficients used to generate the multipole terms) [16]

$$\phi_n = -\frac{1}{4\pi\varepsilon} \sum_{m=0}^{M} \left[p_e^{(m)} \cos(m\varphi) + p_o^{(m)} \sin(m\varphi) \right] L_m \left(\rho, \frac{\partial}{\partial \rho} \right) \ln \left[\frac{(s - s_n/2) + \sqrt{\rho^2 + (s - s_n/2)^2}}{(s + s_n/2) + \sqrt{\rho^2 + (s + s_n/2)^2}} \right] \quad (52)$$

2.3. Electric Multipoles with Dielectric Materials

Now we consider the electric problem when dielectric materials are present. With the approximate local planar model we take two dielectric half spaces (the exterior 2 half space could also be truncated into a finite thickness layer to model the outer jacket) about the braid as shown in Figure 5(b). Because we are representing the braid wires by line multipole moment segments, these charge multipoles can be imaged in the dielectric interfaces. We first decompose the total field into a uniform electric displacement in the y direction $D_0 = \varepsilon E_0$ (where ε is equal to ε_j with $j = 0$, 1, 2 depending on which region the observation is made) in addition to a field generated by the multipolar charges. These charges can be imaged in the dielectric interfaces to represent the potential in the various regions. Let us consider a charge q in the center region at $y = 0$ and the interfaces are at $y = h_2$ and $y = -h_1$ with respect to this charge position. The incident uniform field potential can be written as the linear functions $\phi_{inc} = yD_0/\varepsilon_0$, $-h_1 < y < h_2$ or $\phi_{inc} = (y - h_2)D_0/\varepsilon_2 + h_2 D_0/\varepsilon_0$, $y > h_2$ or $\phi_{inc} = (y + h_1)D_0/\varepsilon_1 - h_1 D_0/\varepsilon_0$, $y < -h_1$. If driven from above (transfer capacitance problem) we would have $D_0 = \varepsilon_2 E_0$ whereas when driven from below (self capacitance of inner coax problem) we would have $D_0 = \varepsilon_1 E_0$.

When viewed from the central braid region the potential due to this charge can be written as

$$\begin{aligned} \left(\frac{4\pi\varepsilon_0}{q} \right) \phi = & \frac{1}{\sqrt{x^2 + y^2 + z^2}} + \ldots + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2} \frac{1}{\sqrt{x^2 + (y - 2h_2)^2 + z^2}} \\ & + \frac{\varepsilon_0 - \varepsilon_1}{\varepsilon_0 + \varepsilon_1} \frac{1}{\sqrt{x^2 + (y + 2h_1)^2 + z^2}} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2} \frac{\varepsilon_0 - \varepsilon_1}{\varepsilon_0 + \varepsilon_1} \frac{1}{\sqrt{x^2 + (y + 2h_2 + 2h_1)^2 + z^2}} \\ & + \frac{\varepsilon_0 - \varepsilon_1}{\varepsilon_0 + \varepsilon_1} \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2} \frac{1}{\sqrt{x^2 + (y - 2h_1 - 2h_2)^2 + z^2}} \end{aligned} \quad (53)$$

Notice if there is no outer dielectric jacket material $\varepsilon_2 = \varepsilon_0$ we end up with only two terms (the source and one image in ε_1).

This same structure of images applies to the multipolar charges of the braid wires (the image placement is the same but the rate of decay increases with the order of the multipole). Hence we can modify the preceding potential distributions to include the image charges in order to construct the potential near the braid wires with the dielectric interfaces present. We can further approximate the sum of images generated by the multipole dielectric interfaces since the higher-order multipole moments have increasing rates of fall-off with distance. Such an image potential construction then allows us to match the equipotential boundary condition on the braid wires to determine the appropriate multipole moments in the presence of adjacent locally planar dielectric regions.

3. MAGNETIC COUPLING

The goal of this section is to formulate the transfer impedance (Z_T) or inductance per unit length L_T of the braid penetration as well as the inner coaxial impedance (Z_1) or inductance per unit length L_1.

In this section we begin with the simplification of a perfectly conducting braid.

3.1. Magnetic Flux Boundary Conditions & Braid Wire Currents

The currents on the braid wires within a strip carrier could be taken to be fixed along the wires if they are insulated from each other, or they could be allowed to vary due to contact between wires if the contact impedance were known. In the perfectly conducting case, contact between wires in a carrier strip would mean that the electric field or magnetic flux between wires vanishes and thus the net magnetic flux between wires in a carrier strip vanishes. (In the finitely conducting case the wire internal impedance also plays a role in selecting the current distribution of the wires in a carrier strip to prevent the combination of inductive and resistive voltage drops between wires in the carrier strip.) Incidental contact between carrier strips could allow the total current in a strip to vary. However, incidental contact between strips may involve a substantial impedance, and periodicity between strips in a cylindrically symmetric geometry might indicate that there is no preferred carrier. If the current is confined to the carriers, then the total wire current in a strip carrier is fixed, and for the cylindrically symmetric case would be taken to be the total braid current divided by the number of carriers; the individual wire currents could then be taken to conform with an arrangement that is dictated by no potential difference between wires in a carrier (at least over a braid period ℓ). Alternatively, if there is no contact between wires in a carrier, we could use the same current division between wires to assure no net magnetic flux over a period to prevent accumulation of voltage differences between individual wires along the cable, assuming they are connected together at the drive and load points. If we take the contour (54) to be along the surface of two of the braid wires over an axial period ℓ we can use periodicity to eliminate the ends of the contour and equate the integral of the vector potential along the braid wires. Using the potential representation $\underline{B} = \nabla \times \underline{A}$ the magnetic flux is defined by

$$\Phi = \int_S \underline{B} \cdot \underline{n} dS = \oint_C \underline{A} \cdot \underline{d\ell} \to \int_{C_{pj}} \underline{A} \cdot \underline{d\ell} = \text{constant for the braid wires} \tag{54}$$

where the contour C_{pj} extends over an axial period on the surface of the jth braid wire. Because the currents are assumed to be periodic over a short axial distance ℓ we construct the vector potential $\underline{A}$ to be also.

3.2. Magnetic Energy & Inductance Per Unit Length

The magnetic flux current relations in a two port can be written as

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} L_{11} & M_{12} \\ M_{21} & L_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \tag{55}$$

From Faraday's law in integral form

$$\oint_C \underline{E} \cdot \underline{d\ell} = -\int_S \frac{\partial}{\partial t} \underline{B} dS \to \frac{\partial V}{\partial z} = -\frac{\partial \Phi}{\partial t} \tag{56}$$

The voltage current relations in a two port inductive circuit can thus be written as

$$V_j(z+\ell) - V_j(z) = -\frac{\partial \Phi_j}{\partial t} = i\omega \Phi_j, \quad j = 1, 2 \tag{57}$$

In a reciprocal media the cross terms are equal $M_{12} = M_{21} = M$. The power removed from a periodic section is minus the derivative of the magnetic energy W_m

$$[V_1(z+\ell) - V_1(z)]\, I_1 + [V_2(z+\ell) - V_2(z)]\, I_2 = -\frac{d}{dt} W_m \tag{58}$$

Taking two sources we write the total magnetic field as $\underline{H} = \underline{H}_1 + \underline{H}_2$ and take the constitutive relation to be that of free space $\underline{B} = \mu_0 \underline{H}$. The 1 problem is defined to have a current I_1 on the center conductor and z directed with return on the braid and the chassis open circuited (no current); the 2 problem is defined to have a current I_2 on the chassis and z directed with return on the braid and the center conductor open circuited (no current). Equating the energies from the circuit and field gives

$$W_m = \frac{1}{2}L_{11}I_1^2 + MI_1I_2 + \frac{1}{2}L_{22}I_2^2 = \int_V \frac{1}{2}\mu_0 H^2 dV \tag{59}$$

The self inductances $L_{11} = \ell L_1$ and $L_{22} = \ell L_{sh}$ (where L_{sh} is the inductance per unit length in the outer transmission line) are the first and final terms

$$L_{11}I_1^2 = \ell L_1 I_1^2 = \int_V \frac{1}{2}\mu_0 H_1^2 dV \tag{60}$$

(L_{22} is similar). The mutual inductance M is the cross term

$$MI_1I_2 = \int_V \mu_0 \underline{H}_1 \cdot \underline{H}_2 dV \tag{61}$$

The sign of the mutual inductance in the preceding expressions depends on whether the magnetic flux generated in circuit 1 by the current I_2 has the same sign as the flux generated by current 1 in circuit 1; referring to Figure 4(b) we see that with $I_2 = -I_{sh}$ positive the field $\underline{H}_2 = \underline{H}_{sh}$ will reinforce $\underline{H}_1$ and the sign of M is positive. We take the current $I_2 = I_{\text{chassis}} = -I_{sh}$ to be negative (where I_{sh} is the positive z directed braid current) with $\Phi_2 = -\Phi_{sh}$ and define the transfer inductance by means of $M = L_T\ell$ (also in the magnetic problem we do not reverse the sign of the field even though we have reversed the sign of the current relative to the 2 problem)

$$L_T = -\frac{1}{\ell I_1 I_{sh}} \int_V \mu_0 \underline{H}_1 \cdot \underline{H}_{sh} dV \tag{62}$$

Now as shown in Figure 4(b) if we suppose the braid has a large aperture we see that the penetrant magnetic field $\underline{H}_{sh}$, for positive shield current I_{sh}, produces a magnetic flux around the center conductor of the inner line with the opposite sign as that produced by the center conductor current I_1 and thus the minus sign introduced in the mutual magnetic flux to make L_T positive for low optical coverage cables (the negative porpoising contribution will result in negative values of L_T for high optical coverage cables); the sign of the voltage source in the transmission line equation is positive for increasing current I_{sh} (the positive reference of this source voltage is on the positive z side of the elements since the inner transmission line equation is written as)

$$\frac{dV_1}{dz} + Z_1 I_1 = -i\omega L_T I_{sh} = Z_T I_{sh} \tag{63}$$

3.2.1. Self Inductance Per Unit Length & Vector Potential

Here we take the chassis to be open circuited and use the vector potential to replace the magnetic induction

$$\ell L_1 I_1^2 = \int_V \frac{1}{2}\underline{H}_1 \cdot \underline{B}_1 dV = \int_V \frac{1}{2}\underline{H}_1 \cdot (\nabla \times \underline{A}_1)\, dV \tag{64}$$

Then using Ampere's law $\nabla \times \underline{H} = \underline{J}$ and

$$\nabla \cdot (\underline{A}_2 \times \underline{H}_1) = \underline{H}_1 \cdot (\nabla \times \underline{A}_2) - \underline{A}_2 \cdot (\nabla \times \underline{H}_1) = \underline{H}_1 \cdot (\nabla \times \underline{A}_2) - \underline{A}_2 \cdot \underline{J}_1 \tag{65}$$

(in this case replacing 2 by 1) we find

$$\begin{aligned} \ell L_1 I_1^2 &= \oint_S (\underline{A}_1 \times \underline{H}_1) \cdot \underline{n} dS = \left(\int_{S_c} + \int_{S_w} + \int_{S_{chassis}} \right) \underline{A}_1 \cdot (\underline{n} \times \underline{H}_1)\, dS \\ &= \ell I_1 A_z(S_c) - \int_{S_w} \underline{A}_1 \cdot (\underline{n} \times \underline{H}_1)\, dS \end{aligned} \tag{66}$$

where we have dropped the chassis integral in the final line because $\int_{S_{\text{shassis}}} \underline{e}_z \cdot (\underline{n} \times \underline{H}_1) dS = 0$.

On the surface of the braid wires the surface current density is $\underline{K} = -\underline{n} \times \underline{H}$ (note here that the unit normal $\underline{n}$ points into the conductor) and then

$$L_1 I_1^2 = I_1 A_z (S_c) + \int_{S_w} \underline{A} \cdot \underline{K} dS/\ell \tag{67}$$

Now in the cylindrical case by symmetry we assume that the contribution from each of the N_s strip carriers is the same (only true on cylindrical conductor geometry, not on asymmetric geometries). This result in Eq. (67) defines the inductance per unit length L_1 in terms of the vector potential solution of the coax. The surface integral is over the wire conductors in the braid carrier strips for an axial braid period. In the perfectly conducting case the normal component of the magnetic field vanishes on the surface of the braid wires

$$\underline{n} \cdot \underline{B} = 0 \tag{68}$$

and hence the normal magnetic flux on any wire surface vanishes. This means that contour integrals on the conductor surface are independent of the integration path and only depend on the end points (because we can always add a vanishing closed contour integral to change to another path).

The electric field is $\underline{E} = -\nabla\phi + i\omega\underline{A}$ with axial component $E_z = -\partial\phi/\partial z + i\omega A_z = -i\Gamma\phi + i\omega A_z$ where the propagation constant of the interior cable transmission line $\Gamma = \omega\sqrt{L_1 C_1}$ is assumed to have a small value such that $\Gamma b \ll 1$. On the braid wire surfaces we take the scalar potential to vanish, and the tangential components of the electric field to vanish also

$$0 = \underline{n} \times \underline{E} = i\omega\underline{n} \times \underline{A} \tag{69}$$

Hence the tangential components of the vector potential vanish, and Eq. (67) becomes $L_1 I_1 = A_z(S_c)$. Thus the self inductance is determined from the magnetic flux passing between the center conductor and the braid wires. If the scalar potential is selected not to vanish on the braid then neither does the tangential vector potential and the inductance reverts back to Eq. (67).

3.2.2. Transfer Inductance Per Unit Length & Vector Potential

Using the vector potential gives

$$L_T = \frac{1}{\ell I_1 I_2} \int_V \mu_0 \underline{H}_1 \cdot \underline{H}_2 dV = \frac{1}{\ell I_1 I_2} \int_V \underline{H}_1 \cdot (\nabla \times \underline{A}_2)\, dV \tag{70}$$

Now using the identity in Eq. (65), setting $I_2 = -I_{sh}$ and $\underline{H}_2 = \underline{H}_{sh}$ with $\underline{A}_2 = \underline{A}_{sh}$, gives

$$L_T = \frac{1}{\ell I_1 I_{sh}} \oint_S \underline{A}_{sh} \cdot (\underline{n} \times \underline{H}_1)\, dS = -\frac{1}{\ell I_1 I_{sh}} \int_{S_c} \underline{A}_{sh} \cdot \underline{K}_1 dS - \frac{1}{\ell I_1 I_{sh}} \int_{S_w} \underline{A}_{sh} \cdot \underline{K}_1 dS \tag{71}$$

where periodicity again eliminates the end surfaces and the surface integral on the chassis is not present with the choice $\underline{A}_2$ because $\underline{A}_{sh} \sim A_{shz}\underline{e}_z$, with A_{shz} asymptotically constant on this perfect conductor (since there is no normal magnetic field), and there is no net current on the chassis in the 1 problem. On the surface of the center conductor S_c we can approximate the integral and find

$$L_T \approx -\frac{1}{I_{sh}} A_{shz} (S_c) - \frac{1}{I_1 I_{sh}} \int_{S_w} \underline{A}_{sh} \cdot \underline{K}_1 dS/\ell \tag{72}$$

The current $\underline{K}_1$ is negative z directed on the braid wires. Hence the final term is the appropriate subtraction (relative to the first term) of the potential $\underline{A}_{sh}$ on the braid wires weighted by the current density on the braid wires with a distribution appropriate to the interior coaxial mode.

Again on the braid wire surfaces if we take the scalar potential to vanish, and because the tangential components of the electric field vanish, the tangential components of the vector potential do also, hence Eq. (72) becomes $L_T I_{sh} \approx -A_{shz}(S_c)$. Thus the transfer inductance is determined from the magnetic flux passing between the center conductor and the braid wires due to the drive from the current on the outer transmission line; the negative sign results from the fact that this flux has the opposite orientation from the flux associated with the inner transmission line with positive current I_1.

3.2.3. Braid Wire Current Distribution

The above argument of zero net flux between braid wires, for determining the braid wire current distribution can be replaced by an energy argument. We determine the braid wire current distribution by minimizing the preceding magnetic energy

$$0=\frac{\partial}{\partial I_j}\left(L_1 I^2\right)=-\frac{\partial}{\partial I_j}\oint_S \underline{A}_1\cdot\left(\underline{n}\times\underline{H}_1\right)dS,\quad j=2,\ldots,N_sN_w \tag{73}$$

with the total braid wire current given

$$\sum_{j=1}^{N_wN_s} I_j=I \tag{74}$$

where N_w is the number of wires in a carrier strip and N_s is the number of carrier strips. This would have to be applied to determine the braid wire current distribution both for the problem where the inner coax is driven $I=I_c$ with $I_{sh}=0$ and for the problem where the outer coax is driven $I=I_{sh}$ with $I_c=0$. This approach results in a system of equations where the coefficients only involve integration on the conductor surfaces S. This is more useful in the mixed potential approach to follow. In the uniform cylindrical case the total strip current is constrained by symmetry to the total braid current divided by the number of carrier strips $\sum_{n=1}^{N_w} I_n=I/N_s$.

3.2.4. Evaluation of Energy Formulas Using Approximate Planar Form of Potentials

It is instructive to show how the preceding formulas can be approximated for nonuniform geometries. This is done by applying the preceding energy formulas stood off from the surface of the braid using an approximate planar evaluation of the potential near the braid to evaluate the self and transfer inductances.

Self Inductance Approximate Evaluation Let us decompose the volume into two regions, the auxiliary part V_0 and the part ΔV where $V=V_0+\Delta V$

$$L_{11}I_1^2=\ell L_1 I_1^2=\int_{V_0}\mu_0 H_1^2 dV+\int_{\Delta V}\mu_0 H_1^2 dV\approx\int_{V_0}\mu_0 H_0^2 dV+\int_{\Delta V}\mu_0 H_1^2 dV \tag{75}$$

where we have approximated the magnetic field in the volume V_0 (which has a surface spaced off from the braid wires) by the auxiliary field H_0 with $\underline{K}_0=-\underline{n}\times\underline{H}_0$ and $I_0=I_1$ on the center conductor. Using the vector potential representation and the identity in Eq. (65) (in the first integral with the 1 and 2 replaced by 0 and in the second integral with the 2 replaced by 1) and the divergence theorem (on the first S_0 integral, resulting from V_0, $\underline{n}$ points outward from the center conductor, but in the second S_0 integral, resulting from ΔV, $\underline{n}$ points inward toward the center conductor)

$$\begin{aligned}L_1 I_1^2\approx&\oint_{S_0}\left(\underline{A}_0\times\underline{H}_0\right)\cdot\underline{n}dS+\oint_{\Delta S}\left(\underline{A}_1\times\underline{H}_1\right)\cdot\underline{n}d\approx-\left(\int_{S_c}+\int_{S_0}\right)\underline{A}_0\cdot\left(\underline{n}\times\underline{H}_0\right)dS/\ell\\&-\int_{S_0}\underline{A}_1\cdot\left(\underline{n}\times\underline{H}_0\right)dS/\ell-\left(\int_{S_w}+\int_{S_{chassis}}\right)\underline{A}_1\cdot\left(\underline{n}\times\underline{H}_1\right)dS/\ell\end{aligned} \tag{76}$$

where we take the auxiliary potential to be z directed $\underline{A}_0=A_{0z}\underline{e}_z$ and we have approximated the magnetic field $\underline{H}_1$ by the auxiliary magnetic field $\underline{H}_0$ on the auxiliary surface S_0. We then drop the chassis integral because this conductor is here taken to be open circuited, the current density $\underline{K}_1=-(\underline{n}\times\underline{H}_1)$ is assumed to be nearly z directed on the chassis (and $\underline{A}_1\sim A_{1z}\underline{e}_z$ is constant on this conductor), and we take the auxiliary potential to vanish on S_0 and to have a constant value on the perfect center conductor ($\underline{n}$ points inward toward the center conductor on S_0)

$$L_1 I_1^2\approx A_{0z}\left(S_c\right)I_1-\int_{S_0}\underline{A}_1\cdot\left(\underline{n}\times\underline{H}_0\right)dS/\ell+\int_{S_w}\underline{A}_1\cdot\underline{K}_1 dS/\ell \tag{77}$$

The first term is the auxiliary problem magnetic flux per unit length $L_0 I_1$ times the current I_1. The third term vanishes if the vector potential is set consistently with the dynamic electric field of the problem; in general it may not vanish if a static representation is used (as discussed previously). In the second term the local approximations of the vector potential $\underline{A}_1 \sim A_{1z}\underline{e}_z$ at a distance from the braid wires can be used to evaluate the potential on the auxiliary shield surface S_0 (in this asymptotic expression $\underline{n}$ points in toward the center conductor)

$$A_{1z} \sim -\left[\left(\underline{\rho} - \underline{\rho}_m + \underline{n}A_b/B_0\right) \times \underline{B}_0\left(x_m, y_m\right)\right] \cdot \underline{e}_z \tag{78}$$

where the constant position A_b/B_0 is defined relative to the mean braid center line and we have approximated the field in the formula by the field of a solid shield conductor $\underline{B}_1 \sim (I_1/I_0)\underline{B}_0 = \underline{B}_0$ (in this case at the braid center line). Plugging this into the integral on S_0 will require us to set $\underline{\rho} \to \underline{\rho}_0$ on the auxiliary shield

$$\begin{aligned}(L_1 - L_0)\, I_1^2 &= \Delta L I_1^2 \approx -\int_{S_0} \underline{A}_1 \cdot (\underline{n} \times \underline{H}_0)\, dS/\ell \\ &\approx \mu_0 \int_{S_0} \left[\left\{\left(\underline{\rho}_0 - \underline{\rho}_m\right) \cdot \underline{n} + A_b/B_0\right\}\right] \left[\underline{e}_z \cdot (\underline{n} \times \underline{H}_0)\right]^2 dS/\ell \\ &= \mu_0 \int_{S_0} \left(d_0 + A_b/B_0\right) \underline{H}_0 \cdot \underline{H}_0 dS/\ell \end{aligned} \tag{79}$$

where d_0 is the distance from the auxiliary shield to the braid center line in Eq. (19).

Transfer Inductance Approximate Evaluation Decomposing the original volume which consists of both regions about the braid wires into the auxiliary volume V_0 from the center conductor outward toward the braid wires plus $\Delta V = V - V_0$ and approximating the field $\underline{H}_1$ in region V_0 by $\underline{H}_0$

$$M = L_T\ell \approx \frac{1}{I_0 I_2}\int_{V_0} \mu_0 \underline{H}_0 \cdot \underline{H}_2 dV + \frac{1}{I_1 I_2}\int_{\Delta V} \mu_0 \underline{H}_1 \cdot \underline{H}_2 dV \tag{80}$$

In the first V_0 integral we use Eq. (65) (with the 2 replaced by 0 and the 1 replaced by 2) to find

$$\begin{aligned}\int_{V_0} \mu_0 \underline{H}_0 \cdot \underline{H}_2 dV &= \oint_{S_0} \left(\underline{A}_0 \times \underline{H}_2\right) \cdot \underline{n} dS = -\left(\int_{S_0} + \int_{S_c}\right) \underline{A}_0 \cdot \left(\underline{n} \times \underline{H}_2\right) dS \\ &\sim -\int_{S_0} A_{0z}\underline{e}_z \cdot \left(\underline{n} \times \underline{H}_2\right) dS - \int_{S_c} A_{0z}\underline{e}_z \cdot \left(\underline{n} \times \underline{H}_2\right) dS \end{aligned} \tag{81}$$

where the final expression results because the auxiliary field is z directed. We can take the auxiliary potential A_{0z} to vanish on the auxiliary surface S_0 and the first term is zero. The auxiliary potential is constant on the center conductor, but because we again take the center conductor to be open circuited in the 2 problem the net current is zero and the second term vanishes.

In the second ΔV integral we use Eq. (65) to find

$$\begin{aligned}\int_{\Delta V} \mu_0 \underline{H}_1 \cdot \underline{H}_2 dV &= \oint_{\Delta S} \left(\underline{A}_2 \times \underline{H}_1\right) \cdot \underline{n} dS = -\left(\int_{S_0} + \int_{S_w} + \int_{S_{\text{chassis}}}\right) \underline{A}_2 \cdot \left(\underline{n} \times \underline{H}_1\right) dS \\ &\sim -\int_{S_0} \underline{A}_2 \cdot \left(\underline{n} \times \underline{H}_1\right) dS - \int_{S_w} \underline{A}_2 \cdot \left(\underline{n} \times \underline{H}_1\right) dS - \int_{S_{\text{chassis}}} A_{2z}\underline{e}_z \cdot \left(\underline{n} \times \underline{H}_1\right) dS \end{aligned} \tag{82}$$

where the final result follows because the potential approaches a scalar z directed quantity far from the local braid region on the chassis. Because the vector potential A_{2z} is a constant on the perfect chassis, and because we take the chassis to be open circuited in the 1 problem, the final integral vanishes

$$L_T\ell I_1 I_2 = -\left(\int_{S_0} + \int_{S_w}\right) \underline{A}_2 \cdot \left(\underline{n} \times \underline{H}_1\right) dS \tag{83}$$

Replacing the magnetic field $\underline{H}_1 \approx \underline{H}_0$ on S_0 (note here that $\underline{n}$ points inward on S_0 toward the center conductor since it arose from ΔV, however on S_w the unit vector $\underline{n}$ points into the wires) and setting $I_2 = -I_{sh}$ and $\underline{A}_2 = \underline{A}_{sh}$, gives

$$L_T \approx \frac{1}{I_0 I_{sh}}\int_{S_0} \underline{A}_{sh} \cdot \left(\underline{n} \times \underline{H}_0\right) dS/\ell + \frac{1}{I_1 I_{sh}}\int_{S_w} \underline{A}_{sh} \cdot \left(\underline{n} \times \underline{H}_1\right) dS/\ell \tag{84}$$

The current density on the auxiliary shield is $\underline{K}_0 = \underline{n} \times \underline{H}_0$, and the current density on these wires is $\underline{K}_1 = -\underline{n} \times \underline{H}_1$. The vector potential from the exterior drive $\underline{A}_{sh}$ can be selected so that on the perfect conductors of S_w, noting that $\underline{n} \times \underline{A}_{sh} = 0$ because of the vanishing of the tangential electric field on the braid perfect conductors, the final term vanishes ($\underline{n}$ points inward in this expression)

$$L_T \approx \frac{1}{I_0 I_{sh}} \int_{S_0} \underline{A}_{sh} \cdot (\underline{n} \times \underline{H}_0)\, dS/\ell \approx \frac{1}{I_0 I_{sh}} \int_{S_0} A_{shz} K_{0z} dS/\ell \tag{85}$$

where in the final expression we have used the fact that the auxiliary problem has z directed current density on the surface of the perfect conductors $\underline{K}_0 = K_{0z}\underline{e}_z$. In this case we then insert the form of the potential in the vicinity of the auxiliary shield (this asymptotic potential is on the interior side of the shield and has $\underline{n}$ pointing inward toward the interior center conductor region, but the field $\underline{B}_{sh0}$ is on the exterior side of the shield when the shield is replaced by a solid conductor at the auxiliary shield position)

$$A_{shz} \sim (A_c/B_0)\, \underline{e}_z \cdot [\underline{n} \times \underline{B}_{sh0}(x_m, y_m)] \tag{86}$$

to find

$$L_T \approx \frac{1}{I_0 I_{sh}} \int_{S_0} (A_c/B_0)\, \underline{e}_z \cdot [\underline{n} \times \underline{B}_{sh0}]\, \underline{e}_z \cdot (\underline{n} \times \underline{H}_0)\, dS/\ell \approx \frac{1}{I_0 I_{sh}} \int_{S_0} (\mu_0 A_c/B_0)\, \underline{H}_{sh0} \cdot \underline{H}_0 dS/\ell \tag{87}$$

3.3. Scalar Potential and Magnetic Vector Potential Construction

To set up the magnetic braid problem in a manner similar to the electric problem, we use a combination of the magnetic vector potential and magnetic scalar potential. The magnetic vector potential is used to represent the net current carried by each braid wire, while the magnetic scalar potential matches the boundary conditions on the wire surface. Because the magnetic scalar potential represents the difference problem where the braid wire carries no net current, the branch surfaces from the braid wires are avoided, while retaining the simplicity of the scalar description. We take the vector potential to be generated by current filaments at the centers of the wires and denote this potential as $\underline{A}_f$. This formulation includes both axial and transverse components of the current on the wire surfaces. The magnetic induction and magnetic field are then given by

$$\underline{B} = \mu_0 \underline{H} = -\mu_0 \nabla \phi_m + \nabla \times \underline{A}_f \tag{88}$$

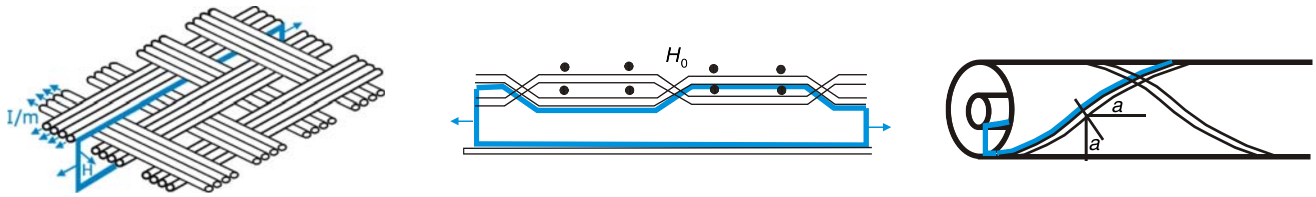
In the perfectly conducting case the magnetic scalar potential is used to restore the boundary condition of zero normal magnetic field on the metallic surfaces (68) on each of the braid wire segments. The magnetic flux Φ through a surface S from this representation is the sum

$$\Phi = \oint_C \underline{A}_f \cdot \underline{d\ell} - \mu_0 \int_S \nabla \phi_m \cdot \underline{n} dS \tag{89}$$

The φ directed magnetic flux of interest is that between a single wire of a single strip of the braid and the center conductor of the coax.

3.3.1. Flux Constants

From the preceding vector potential formulation, and in particular the local planar approximation, we see that our objective in that section was to evaluate the magnetic flux per unit length constants A_c/B_0 and A_b/B_0. These magnetic flux constants entered the preceding vector potential formulation to find the transfer impedance and self-impedance corrections. To find these using the present mixed potential approach we can drive a planar periodic representation of the braid with a uniform field that is tangential to the braid surface as illustrated in Figure 5(c). When driven from the exterior side of the braid we evaluate $-\ell A_c = \Phi$ where the surface S in the evaluation of the flux Φ (89) corresponds to that of the three drawings in Figure 7 and the contour surrounds S. In addition, when the planar braid is driven by the field from the interior side, then using the same contour $\ell(d_0 B_0 + A_b) = \Phi$ where d_0 is the normal distance from the center plane of the braid to the return leg of the contour (the edge of the surface S).



(a) (b) (c)

Figure 7. (a) Magnetic flux contour following a single braid wire in the planar braid approximation. (b) Cross section of flux contour following a single braid wire. (c) Magnetic flux contour in the coaxial geometry.

3.3.2. Self Inductance Per Unit Length Mixed Potentials

We now show that we can evaluate the self impedance from the energy argument directly using the mixed potentials on the surfaces of the wires. The inductance can be found from the stored magnetic energy by means of

$$L_1 \ell I_1^2 = \int_V \underline{H}_1 \cdot \underline{B}_1 dV = \int_V \underline{H}_1 \cdot \left(\nabla \times \underline{A}_{f1} - \mu_0 \nabla \phi_{m1}\right) dV \tag{90}$$

Using Gauss's law $\nabla \cdot \underline{B} = 0$ we can write

$$\nabla \cdot \left(\underline{A}_{f2} \times \underline{H}_1 - \phi_{m2} \underline{B}_1\right) = \underline{H}_1 \cdot \left(\nabla \times \underline{A}_{f2} - \mu_0 \nabla \phi_{m2}\right) - \underline{A}_{f2} \cdot \underline{J}_1 \tag{91}$$

(in this case with the 2 replaced by 1) and outside the conductors we find

$$L_1 \ell I_1^2 = \oint_S \left(\underline{A}_{f1} \times \underline{H}_1 - \phi_{m1} \underline{B}_1\right) \cdot \underline{n} dS = -\oint_S \underline{A}_{f1} \cdot \left(\underline{n} \times \underline{H}_1\right) dS = \oint_S \underline{A}_{f1} \cdot \underline{K}_1 dS \tag{92}$$

where the second term in the integrand vanishes on the PEC surfaces.

3.3.3. Transfer Inductance Per Unit Length Mixed Potentials

Using the mixed potential representation we can write

$$L_T = \frac{1}{\ell I_1 I_2} \int_V \mu_0 \underline{H}_1 \cdot \underline{H}_2 dV = \frac{1}{\ell I_1 I_2} \int_V \underline{H}_1 \cdot \left(\nabla \times \underline{A}_{f2} - \mu_0 \nabla \phi_{m2}\right) dV \tag{93}$$

Now using the identity (91) outside the perfect conductors gives

$$L_T = -\frac{1}{I_1 I_2} \oint_S \underline{A}_{f2} \cdot \left(\underline{n} \times \underline{H}_1\right) dS/\ell = \frac{1}{I_1 I_2} \oint_S \underline{A}_{f2} \cdot \underline{K}_1 dS/\ell = \frac{1}{I_1 I_{sh}} \oint_S \underline{A}_{fsh} \cdot \underline{K}_1 dS/\ell \tag{94}$$

where in the final equality we have taken $I_2 = -I_{sh}$, $\underline{A}_{f2} = -\underline{A}_{fsh}$.

If we take the filament in such a place that the potential on the chassis is asymptotic to $\underline{A}_{fsh} \sim A_{fshz} \underline{e}_z$ with A_{fshz} asymptotically constant on the chassis, then with no net current on the chassis in the 1 problem the surface integral will vanish and we find

$$L_T = \frac{1}{I_1 I_{sh}} \left(\int_{S_c} + \int_{S_w}\right) \underline{A}_{fsh} \cdot \underline{K}_1 dS/\ell \tag{95}$$

3.3.4. Braid Wire Current Distribution

In this case we can determine the braid wire current distribution by minimizing the preceding magnetic energy

$$0 = \frac{\partial}{\partial I_j}\left(L_1 I^2\right) = -\frac{1}{\mu_0} \frac{\partial}{\partial I_j} \oint_S \underline{A}_{f1} \cdot \underline{K}_1 dS, \quad j = 2, \ldots, N_s N_w \tag{96}$$

with the total braid wire current given by Eq. (74). This form is useful in this mixed potential approach to avoid having to integrate the magnetic flux off the conductor surfaces.

3.3.5. Magnetic Multipole Representation

With the preceding mixed potential representation we need the magnetic scalar potential representation for a sequence of magnetic line charge multipoles as illustrated in Figure 6 similar to the electric problem in addition to an electric current filament (and the field given by the magnetic vector potential). The solution for a varying magnetic line charge $q_m(s')$ is (44) with $q(s')/\varepsilon$ replaced by $q_m(s')/\mu_0$. If the charge is discretized as pulses of strength $q_{mn} = p_m^{(0)}$, length s_n, along an axis s aligned with the nth segment, we find (45) with q_n/ε replaced by q_{mn}/μ_0. The magnetic multipole moments can be defined in a similar way to the electric problem in Eq. (52) and lead to unknowns $p_{me}^{(m)}$ and $p_{mo}^{(m)}$ for each segment n. Note that we expect $q_{mn} = \underline{p}_m^{(0)}$ to vanish in the case of perfectly conducting braid wires, but we retain it since it may be required in the finitely conducting case, where magnetic flux could enter from a neighboring segment. There are $2M$ multipole coefficients for each of the N wire segments which must be determined from matching around the wire segments.

The electric current filament contribution is

$$\begin{aligned}\underline{A}_f &= \frac{\mu_0}{4\pi}\int \frac{I(s')}{|\underline{r}-\underline{r}'|}\underline{e}_{s'}ds' \\ &= \sum_{n=1}^{N}\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\frac{\mu_0 I_n}{4\pi}\underline{e}_{s_n}\ln\left[\frac{(s-s_n/2-ju_{sn}-kv_{sn})+|\underline{r}-\underline{r}_n^{+}-j\underline{u}-k\underline{v}|}{(s+s_n/2-ju_{sn}-kv_{sn})+|\underline{r}-\underline{r}_n^{-}-j\underline{u}-k\underline{v}|}\right]\end{aligned} \tag{97}$$

The boundary condition to be enforced on the perfectly conducting braid wires is Eq. (68). We sample this equation at $2M$ azimuthal points around each segment (we know that there is no net magnetic flux emanating from a segment in the perfect electric conductor case and therefore we expect the $m = 0$ coefficient to vanish) which generates $N \times (2M)$ equations for the $N \times (2M)$ unknowns ($2M$ multipole unknowns on each of the N segments) associated with the scalar potential ϕ_m. We also impose no net magnetic flux between wires in a carrier strip over a braid period. Furthermore, we want each carrier strip in the uniform coax to carrier net current I/N_s and for insulated wires the wire currents remain the same over the course of a period. These final conditions generate a further N conditions imposed on the coefficients I_n in the filament vector potential $\underline{A}_f$.

4. FINITE CONDUCTIVITY OF BRAID WIRES

We now consider modifications to the preceding magnetic problem when the braid wires have large but finite conductivity. The previous approach used to formulate the capacitance and inductance matrices can also be used in this case. Applying Faraday's law $\partial V/\partial z = i\omega\Phi$ to a section of line with the magnetic fluxes connected to the electric currents gives

$$i\omega\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = -\begin{pmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{pmatrix}\begin{pmatrix}I_1\\ I_2\end{pmatrix} = \begin{pmatrix}V_1(z+\ell)-V_1(z)\\ V_2(z+\ell)-V_2(z)\end{pmatrix} \tag{98}$$

In a reciprocal media the cross terms are equal $Z_{12} = Z_{21}$. The complex power removed from a periodic section is

$$\frac{1}{2}\left[V_1(z+\ell)-V_1(z)\right]I_1^* + \frac{1}{2}\left[V_2(z+\ell)-V_2(z)\right]I_2^* = -P \tag{99}$$

Faraday's law $\nabla\times\underline{E} = i\omega\underline{B}$ with the constitutive relation $\underline{B} = \mu\underline{H}$ (now μ can have a different value inside the conductors) now apply. Inside the conductor we ignore the displacement current term in Maxwell's equation $\nabla\times\underline{H} = \underline{J} = \sigma\underline{E}$ and outside we also ignore displacement currents $\nabla\times\underline{H} = 0$ since these were included in the electric problem. Then the time harmonic Poynting vector $\underline{S} = \underline{E}\times\underline{H}^*/2$ has divergence

$$\nabla\cdot\underline{S} = \frac{1}{2}\underline{H}^*\cdot\nabla\times\underline{E} - \frac{1}{2}\underline{E}\cdot\nabla\times\underline{H}^* = i\omega\frac{1}{2}\mu\underline{H}\cdot\underline{H}^* - \frac{1}{2}\underline{E}\cdot\underline{J}^* \tag{100}$$

where we have ignored the external electric energy term in this series impedance treatment. Integration over the transmission line volume and use of the divergence theorem then yields

$$\oint_S \underline{S}\cdot\underline{n}dS = i\omega\int_V \frac{1}{2}\mu\underline{H}\cdot\underline{H}^*dV - \int_{V_w}\frac{1}{2}\sigma\underline{E}\cdot\underline{E}^*dV = i\omega\int_{V_e}\frac{1}{2}\mu_0\underline{H}\cdot\underline{H}^*dV + \oint_{S_w}\underline{S}\cdot\underline{n}dS \tag{101}$$

where the unit normal points out of each region (including the braid wire region V_w in the final surface integral) and in this section we define the volume as including the internal braid wire region V_w, with $V = V_e + V_w$ and V_e is the external region. Note that we have left out the center conductor and chassis from the surface integrals since they are assumed to be perfect conductors with a vanishing normal Poynting vector component. The second equality uses the Poynting vector surface integral on the braid wires instead of replacing it with a volume integral inside the braid wires. This entire quantity corresponds to the power leaving a region of the line which is minus the average stored magnetic energy (times $-i\omega$) and minus the average losses associated with the braid conduction. Setting the complex power supplied P equal to minus the Pointing vector integral (101) gives

$$P = \frac{1}{2}Z_{11}I_1I_1^* + \frac{1}{2}Z_{12}I_2I_1^* + \frac{1}{2}Z_{21}I_1I_2^* + \frac{1}{2}Z_{22}I_2I_2^* = -\oint_S \underline{S} \cdot \underline{n} dS \tag{102}$$

Taking two sources we write the total fields as $\underline{H} = \underline{H}_1 + \underline{H}_2$ and $\underline{E} = \underline{E}_1 + \underline{E}_2$ to obtain

$$Z_{11}\left|I_1\right|^2 = -i\omega \int_V \mu \underline{H}_1 \cdot \underline{H}_1^* dV + \int_{V_w} \sigma \underline{E}_1 \cdot \underline{E}_1^* dV - i\omega \int_{V_e} \mu_0 \underline{H}_1 \cdot \underline{H}_1^* dV + \int_{S_w} (\underline{n} \times \underline{E}_1) \cdot \underline{H}_1^* dS \tag{103}$$

similarly for Z_{22}, and

$$Z_{12}I_1^*I_2 + Z_{21}I_1I_2^* = 2Z_T\ell \mathrm{Re}\left(I_2I_1^*\right) = -i\omega \int_V \mu\left(\underline{H}_1 \cdot \underline{H}_2^* + \underline{H}_2 \cdot \underline{H}_1^*\right) dV + \int_{V_w} \sigma\left(\underline{E}_1 \cdot \underline{E}_2^* + \underline{E}_2 \cdot \underline{E}_1^*\right) dV$$

$$= -i\omega \int_{V_e} \mu_0 \left(\underline{H}_1 \cdot \underline{H}_2^* + \underline{H}_2 \cdot \underline{H}_1^*\right) dV + \int_{S_w} \left[(\underline{n} \times \underline{E}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{E}_2) \cdot \underline{H}_1^*\right] dS \tag{104}$$

where the unit normal $\underline{n}$ in the braid wire surface integrals, which are denoted as unclosed, is taken to point inward and $Z_{12} = Z_{21}$ for reciprocal media.

4.0.6. Vector Potential Representation

If the external magnetic field is represented by the vector potential we can use the identity

$$\nabla \cdot (\underline{A}_2 \times \underline{H}_1^*) = \underline{H}_1^* \cdot (\nabla \times \underline{A}_2) - \underline{A}_2 \cdot (\nabla \times \underline{H}_1^*) = \underline{H}_1^* \cdot (\nabla \times \underline{A}_2) - \underline{A}_2 \cdot \underline{J}_1^* \tag{105}$$

(in this case with 2 replaced by 1) to find for the external part of the self impedance

$$Z_{11}^{ext} = Z_1^{ext}\ell = -i\omega \frac{1}{\left|I_1\right|^2} \int_{V_e} (\nabla \times \underline{A}_1) \cdot \underline{H}_1^* dV = i\omega \frac{1}{\left|I_1\right|^2} \oint_{S_e} \underline{A}_1 \cdot \left(\underline{n} \times \underline{H}_1^*\right) dS \tag{106}$$

where the surface integral is on the surface of the wire braid of the shield as well as on the center conductor and chassis (the outer chassis return for the self impedance will vanish because it is open circuited). Combining this with the internal term gives (the unit vector $\underline{n}$ points into the braid wires in the second term and out of S_e in the first term)

$$Z_{11} = Z_1\ell = -i\omega \frac{1}{\left|I_1\right|^2} \oint_{S_e} (\underline{n} \times \underline{A}_1) \cdot \underline{H}_1^* dS + \frac{1}{\left|I_1\right|^2} \int_{S_w} (\underline{n} \times \underline{E}_1) \cdot \underline{H}_1^* dS \tag{107}$$

Now if we construct the vector potential such that on the braid wire surfaces

$$i\omega\left(\underline{n} \times \underline{A}\right) = \underline{n} \times \underline{E} \tag{108}$$

then the surface integral terms on S_w cancel out. Thus we are left with (the unit vector $\underline{n}$ points into the center conductor)

$$Z_{11} = Z_1\ell = -i\omega \frac{1}{\left|I_1\right|^2} \int_{S_c} (\underline{n} \times \underline{A}_1) \cdot \underline{H}_1^* dS \tag{109}$$

Also using (105) external to the conductors we can write

$$\int_{V_e} \mu_0 \underline{H}_2 \cdot \underline{H}_1^* dV = \int_{V_e} (\nabla \times \underline{A}_2) \cdot \underline{H}_1^* dV = \oint_{S_e} (\underline{n} \times \underline{A}_2) \cdot \underline{H}_1^* dS \tag{110}$$

Then using this result (and with the indices interchanged) we find

$$\begin{aligned} Z_{12}I_2I_1^* + Z_{21}I_1I_2^* &= -i\omega \oint_{S_e} [(\underline{n} \times \underline{A}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{A}_2) \cdot \underline{H}_1^*]\, dS \\ &\quad + \int_{S_w} [(\underline{n} \times \underline{E}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{E}_2) \cdot \underline{H}_1^*]\, dS \end{aligned} \tag{111}$$

where the unit vector $\underline{n}$ points into the braid wires in the final integral. Again if on the braid wire surface S_w we have the relations from Eq. (108) then the braid wire surface integrals cancel and we find (where $\underline{n}$ points into the conductors)

$$Z_{12}I_2I_1^* + Z_{21}I_1I_2^* = 2Z_T\ell\mathrm{Re}\left(I_2I_1^*\right) = -i\omega \left(\int_{S_c} + \int_{S_{\mathrm{chassis}}}\right) [(\underline{n} \times \underline{A}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{A}_2) \cdot \underline{H}_1^*]\, dS \tag{112}$$

4.0.7. Mixed Potential Representation

Alternatively, if we have the external mixed potential representation in Eq. (88) we can write

$$Z_{11}I_1I_1^* = Z_1\ell\left|I_1\right|^2 = -i\omega \int_{V_e} \left(\nabla \times \underline{A}_{f1} - \mu_0 \nabla\phi_{m1}\right) \cdot \underline{H}_1^* dV + \int_{S_w} (\underline{n} \times \underline{E}_1) \cdot \underline{H}_1^* dS$$

Using

$$\nabla \cdot \left(\underline{A}_{f2} \times \underline{H}_1^* - \phi_{m2}\underline{B}_1^*\right) = \underline{H}_1^* \cdot \left(\nabla \times \underline{A}_{f2} - \mu_0\nabla\phi_{m2}\right) - \underline{A}_{f2} \cdot \underline{J}_1^* = \underline{H}_1^* \cdot \underline{B}_2 - \underline{A}_{f2} \cdot \underline{J}_1^* \tag{113}$$

(in this case with 2 replaced by 1) outside the conductors we find

$$Z_1\ell\left|I_1\right|^2 = -i\omega \oint_{S_e} \left[\left(\underline{n} \times \underline{A}_{f1}\right) \cdot \underline{H}_1^* - \phi_{m1}\underline{n} \cdot \underline{B}_1^*\right] dS + \int_{S_w} (\underline{n} \times \underline{E}_1) \cdot \underline{H}_1^* dS \tag{114}$$

These results require integration on the conductor surfaces but not out in the free space volume which is an advantage since we must solve the problem on these surfaces.

For the transfer impedance we begin with

$$Z_{12}I_2I_1^* + Z_{21}I_1I_2^* = -i\omega \int_{V_e} \left(\underline{H}_1^* \cdot \underline{B}_2 + \underline{H}_2^* \cdot \underline{B}_1\right) dV + \int_{S_w} [(\underline{n} \times \underline{E}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{E}_2) \cdot \underline{H}_1^*]\, dS \tag{115}$$

Using Eq. (113) outside the conductors gives

$$\begin{aligned} Z_{12}I_2I_1^* + Z_{21}I_1I_2^* &= 2Z_T\ell\mathrm{Re}\left(I_2I_1^*\right) = \int_{S_w} [(\underline{n} \times \underline{E}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{E}_2) \cdot \underline{H}_1^*]\, dS \\ &\quad -i\omega \oint_{S_e} \underline{n} \cdot \left[\left(\underline{A}_{f1} \times \underline{H}_2^* + \underline{A}_{f2} \times \underline{H}_1^*\right) - \left(\underline{B}_2^*\phi_{m1} + \phi_{m2}\underline{B}_1^*\right)\right] dS \end{aligned} \tag{116}$$

4.0.8. Perturbation Approximation with Vector Potential

We now want to approximate the integrals in the expressions for the impedance when the impedance length scale $Z_s/(\omega\mu_0)$ is small compared to the global transverse geometry of the cable. The surface impedance of the wires Z_s is $Z_s = (1-i)/(\sigma\delta)$ when the skin depth $\delta = \sqrt{2/(\omega\mu\sigma)}$ is small compared to the wire radius a. However, for large skin depths we take the surface impedance to be approximately $Z_s \approx 1/(\sigma\Delta)$ where the effective thickness of the braid is taken approximately as $\Delta = O(2a)$. When the impedance length scale is small the global current distribution is not significantly perturbed from the perfectly conducting case (in the circular cylindrical geometry it remains uniform even when this length scale become large). In this limit we can thus make use of the current distribution from the perfectly conducting solid shield cable of the same global geometry to determine the impedance per unit length of the braid using the approximate planar form of the vector potential near the braid.

We again take the external volume in the self impedance to be written as $V_e = V_0 + \Delta V$ and in the region V_0 we approximate the field as $\underline{H}_0$

$$Z_{11} \approx -i\omega \frac{1}{|I_0|^2} \int_{V_0} \mu_0 \underline{H}_0 \cdot \underline{H}_0^* dV - i\omega \frac{1}{|I_1|^2} \int_{\Delta V} \mu_0 \underline{H}_1 \cdot \underline{H}_1^* dV + \frac{1}{|I_1|^2} \int_{S_w} (\underline{n} \times \underline{E}_1) \cdot \underline{H}_1^* dS \tag{117}$$

Using Eq. (105) (with 2 replaced by 1) we can rewrite the second volume integral as

$$-i\omega \frac{1}{|I_1|^2} \int_{\Delta V} \underline{B}_1 \cdot \underline{H}_1^* dV = -i\omega \frac{1}{|I_1|^2} \left(\int_{S_0} + \int_{S_w} \right) (\underline{n} \times \underline{A}_1) \cdot \underline{H}_1^* dS \tag{118}$$

Assuming that the vector potential is set up to satisfy Eq. (108), we see that the S_w integrals cancel out and using the same identity in Eq. (105) (with 1 replaced by 0) on the first term yields

$$Z_{11} \approx -i\omega \frac{1}{|I_0|^2} \left(\int_{S_c} + \int_{S_0} \right) (\underline{n} \times \underline{A}_0) \cdot \underline{H}_0^* dS - i\omega \frac{1}{|I_1|^2} \int_{S_0} (\underline{n} \times \underline{A}_1) \cdot \underline{H}_1^* dS \tag{119}$$

The first term represents the perfectly conducting inductance contribution of the auxiliary problem $L_0\ell$. The second term represents the contribution from the braid wire geometry (versus the continuous shield) as well as the finite conductivity of the braid wires. We approximate the final term by taking the field as the field in the auxiliary problem $\underline{H}_1^* \to \underline{H}_0^*$ with $I_1^* \to I_0^*$

$$Z_{11} = Z_1 \ell \approx -i\omega L_0 \ell + i\omega \frac{1}{I_1 I_0^*} \int_{S_0} \underline{A}_1 \cdot (\underline{n} \times \underline{H}_0^*)\, dS \tag{120}$$

where we note that the unit normal points inward since it came from the ΔV integration. The entire contribution of the loss is contained in how the constant offset in the vector potential is changed to a complex value relative to the perfect electric conductor case. Now using the asymptotic form of the potential referenced to the mean braid radius in Eq. (78) we find

$$Z_1 = Z_0 + \Delta Z \approx -i\omega L_0 - i\omega \mu_0 \frac{1}{|I_0|^2} \int_{S_0} (d_0 + A_b/B_0)\, \underline{H}_0 \cdot \underline{H}_0^* dS/\ell \tag{121}$$

This is the same result as found previously except that the offset constant A_b/B_0 will now be complex to account for diffusion into the braid wires.

For the transfer impedance we take the external volume to be split into two parts $V_e = V_0 + \Delta V$, but in this case ΔV is contained between the two surfaces (and the braid wire surfaces) making up $\Delta S = S_{01} + S_{02} + S_w$ (plus the periodic surfaces separated by a periodic length ℓ), where S_{01} is near the braid on the center conductor side and S_{02} is near the braid on the chassis side. Then we can write

$$\begin{aligned} Z_{12} I_2 I_1^* + Z_{21} I_1 I_2^* \;=\; & -i\omega \left(\int_{V_0} + \int_{\Delta V} \right) (\underline{B}_1 \cdot \underline{H}_2^* + \underline{B}_2 \cdot \underline{H}_1^*)\, dV \\ & + \int_{S_w} [(\underline{n} \times \underline{E}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{E}_2) \cdot \underline{H}_1^*]\, dS \end{aligned} \tag{122}$$

We next drop the cross field interactions outside of the region about the braid ΔV ignoring the V_0 integration. Using Eq. (105) (as well as this identity with the 2 and 1 interchanged) we can then write

$$\begin{aligned} Z_{12} I_2 I_1^* + Z_{21} I_1 I_2^* \approx\; & -i\omega \oint_{\Delta S} [(\underline{n} \times \underline{A}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{A}_2) \cdot \underline{H}_1^*]\, dS \\ & + \int_{S_w} [(\underline{n} \times \underline{E}_1) \cdot \underline{H}_2^* + (\underline{n} \times \underline{E}_2) \cdot \underline{H}_1^*]\, dS \end{aligned} \tag{123}$$

Now assuming we have set up the potentials to satisfy Eq. (108) (for both the 1 and 2 fields) we see that the S_w surfaces cancel and we end up with (in the preceding equation $\underline{n}$ points out of the volume ΔV surrounding the braid wires)

$$Z_{12} I_2 I_1^* + Z_{21} I_1 I_2^* \approx i\omega \int_{S_{02}} \underline{A}_1 \cdot (\underline{n} \times \underline{H}_2^*)\, dS + i\omega \int_{S_{01}} \underline{A}_2 \cdot (\underline{n} \times \underline{H}_1^*)\, dS \tag{124}$$

Recalling the asymptotic form of the potential on S_{01} in Eq. (86) (in which $\underline{n}$ points consistently on S_{01} with the preceding equation) and the reciprocal form outside the braid (here $\underline{n}$ points consistently on S_{02} with the preceding equation and A_c/B_0 is the same constant as in Eq. (86) due to reciprocity, and $\underline{B}_0$ is the driving field from the center conductor interior)

$$A_{1z} \sim (A_c/B_0)\, \underline{e}_z \cdot [\underline{n} \times \underline{B}_0 (x_m, y_m)] \tag{125}$$

and approximating the fields by those of a solid shield $\underline{H}_2^* \approx \underline{H}_{sh0}^*$, $\underline{B}_{sh} \approx \underline{B}_{sh0}$, $\underline{H}_1^* \approx \underline{H}_0^*$, letting $I_1 = I_0$, $I_2 = -I_{sh}$, and $I_2^* = -I_{sh}^*$, gives

$$\begin{aligned} Z_{12} I_{sh} I_0^* + Z_{21} I_0 I_{sh}^* =&\ 2 Z_T \ell \mathrm{Re}\,(I_{sh} I_0^*) \\ \approx&\ -i\omega \int_{S_{02}} (A_c/B_0)\, [\underline{e}_z \cdot (\underline{n} \times \underline{B}_0)]\, [\underline{e}_z \cdot (\underline{n} \times \underline{H}_{sh0}^*)]\, dS \\ &-i\omega \int_{S_{01}} (A_c/B_0)\, [\underline{e}_z \cdot (\underline{n} \times \underline{B}_{sh0})]\, [\underline{e}_z \cdot (\underline{n} \times \underline{H}_0^*)]\, dS \end{aligned} \tag{126}$$

Bringing the two surfaces together near the mean braid location (which we will still denote as S_0), taking $Z_{12} = Z_{21} = Z_T \ell$, and since the $\underline{H}_0$ and $\underline{H}_{sh0}$ fields are those existing with a solid perfectly conducting shield (the first on the interior and the second on the exterior) they have the same phases as I_0 and I_{sh}, which can be taken as real

$$Z_T \approx -i\omega\mu_0 \,(A_c/B_0)\, \frac{1}{I_0 I_{sh}} \int_{S_0} \underline{H}_0 \cdot \underline{H}_{sh0} dS/\ell \tag{127}$$

This expression is the same as found previously for the transfer inductance except that the constant A_c/B_0 is now modified to a complex value accounting for the diffusion into the wires of the braid.

4.0.9. Examples of Impedance Parameters with Finitely Conducting Braid

In the case of lossy braid wires we expect the arguments for the perfectly conducting case to be modified in the sense that the planar braid constants A_b/B_0 and A_c/B_0 will become complex. Hence in the uniform coaxial case with auxiliary inductance per unit length $L_0(b_0) = \mu_0 \ln(b_0/a)/(2\pi)$

$$Z_T \sim -i\omega\mu_0 \,(A_c/B_0) \int_{S_0} \frac{1}{I_0 I_{sh}} \underline{H}_{sh0} \cdot \underline{H}_0 dS/\ell = -i\omega\mu_0 \left(\frac{A_c/B_0}{2\pi b} \right) \tag{128}$$

$$Z_1 \sim -i\omega L_0 \,(b_0) - i\omega\mu_0 \,(d_0 + A_b/B_0) \int_{S_0} \frac{1}{I_0^2} \underline{H}_0 \cdot \underline{H}_0 dS/\ell \sim -i\omega L_0 \,(b) - i\omega\mu_0 \left(\frac{A_b/B_0}{2\pi b} \right) \tag{129}$$

With an eccentric coax interior but the same cylindrical exterior the transfer impedance result is unchanged from Eq. (128), but with mean radius inductance per unit length $L_0(b) = \mu_0 \mathrm{Arccosh}[(a^2 + b^2 - d^2)/(2ab)]/(2\pi)$

$$Z_1 \sim -i\omega L_0 \,(b) - i\omega\mu_0 \left(\frac{A_b/B_0}{2\pi b} \right) (y_1'/y_c') \tag{130}$$

If an outer ground plane is also added to the eccentric coax the transfer impedance becomes

$$Z_T = -i\omega\mu_0 \left(\frac{A_c/B_0}{2\pi b} \right) \frac{y_c' - h_e}{y_1' - h} \tag{131}$$

If we rotate the azimuth of the eccentric coax with respect to the ground plane by $\pi/2$ then

$$Z_T = -i\omega\mu_0 \left(\frac{A_c/B_0}{2\pi b} \right) \frac{y_1' y_c' + h h_e}{y_1'^2 + h_e^2} \tag{132}$$

4.1. General Internal Field Representation

The internal field must also be represented in the finitely conducting case. The most convenient characterization may be to use Hertz potentials or the axial components of the two vector potentials within the braid wires. In this section z is a local coordinate along the axis of a braid wire segment. We assume that conduction currents dominate over displacement currents within the wires. The magnetic potential is taken as $\underline{\Pi} = \psi \underline{e}_z$ which is connected to the magnetic vector potential by $\underline{A} = \mu\sigma\underline{\Pi}$. The electric potential is taken as $\underline{\Pi}_e = \psi_e \underline{e}_z$ which is connected to the electric vector potential by $\underline{A}_e = \mu\sigma\underline{\Pi}_e$. The fields are given by

$$\underline{H} = \frac{1}{\mu\sigma}\left(\nabla\nabla\cdot\underline{A}_e + \gamma^2\underline{A}_e\right) + \frac{1}{\mu}\nabla\times\underline{A} = \nabla\nabla\cdot\underline{\Pi}_e + \gamma^2\underline{\Pi}_e + \sigma\nabla\times\underline{\Pi} \tag{133}$$

$$\underline{E} = \frac{1}{\mu\sigma}\left(\nabla\nabla\cdot\underline{A} + \gamma^2\underline{A}\right) + \frac{i\omega}{\sigma}\nabla\times\underline{A}_e = \nabla\nabla\cdot\underline{\Pi} + \gamma^2\underline{\Pi} + i\omega\mu\nabla\times\underline{\Pi}_e \tag{134}$$

where the scalar axial components satisfy the Helmholtz equation $(\nabla^2+\gamma^2)(\psi,\psi_e) = 0$ with propagation constant $\gamma^2 = i\omega\mu\sigma$.

The solutions of the Helmholtz equation can be used to find the fields in the nth cylindrical waveguide segment for a given cylindrical mode m as

$$(\psi,\psi_e) = e^{i\alpha z + im\varphi} J_m(\zeta\rho) \approx e^{i\alpha z + im\varphi} J_m(\gamma\rho) \tag{135}$$

where $\zeta^2 = \gamma^2 - \alpha^2$ and in the final expressions we have approximated the solution for the axial propagation constant α small compared to the internal propagation constant γ or $\alpha \ll \gamma$. This approximation assumes that the axial variation of the current density in the wire is slow compared to the skin depth. We will simplify the analysis at present by assuming this is valid, but it could conceivably be violated near the wire crossover point in the braid. Because the braid wires carry a net current in the same z direction (or $\underline{e}_{s_n}$ directions), this common mode current density should not concentrate near the region of crossover near-contact as oppositely directed currents would tend to do, and hence we might expect this approach to be approximately valid. The potential component ψ represents the axial component of the current density, whereas the potential component ψ_e represents the transverse components of the current. The potentials are then given by

$$\begin{pmatrix}\psi\\ \psi_e\end{pmatrix} \approx J_m(\gamma\rho)\left[\begin{pmatrix}F^e_{nm}(z)\\ G^o_{nm}(z)\end{pmatrix}\cos(m\varphi) + \begin{pmatrix}F^o_{nm}(z)\\ G^e_{nm}(z)\end{pmatrix}\sin(m\varphi)\right] \tag{136}$$

and the magnetic field is

$$H_z(\rho) \approx \gamma^2 J_m(\gamma\rho)\left[G^e_{nm}(z)\sin(m\varphi) + G^o_{nm}(z)\cos(m\varphi)\right] \tag{137}$$

$$\begin{aligned} H_\varphi(\rho) \approx\ & -\sigma\gamma J'_m(\gamma\rho)\left[F^e_{nm}(z)\cos(m\varphi) + F^o_{nm}(z)\sin(m\varphi)\right] \\ & + \frac{m}{\rho} J_m(\gamma\rho)\left[G^{e\prime}_{nm}(z)\cos(m\varphi) - G^{o\prime}_{nm}(z)\sin(m\varphi)\right] \end{aligned} \tag{138}$$

$$\begin{aligned} B_\rho(\rho) =\ & \mu H_\rho(\rho) \approx -\mu\sigma\frac{m}{\rho}J_m(\gamma\rho)\left[F^e_{nm}(z)\sin(m\varphi) - F^o_{nm}(z)\cos(m\varphi)\right] \\ & + \mu\gamma J'_m(\gamma\rho)\left[G^{e\prime}_{nm}(z)\sin(m\varphi) + G^{o\prime}_{nm}(z)\cos(m\varphi)\right] \end{aligned} \tag{139}$$

The radial electric field is

$$\begin{aligned} E_\rho(\rho) \approx\ & i\omega\mu\frac{m}{\rho}J_m(\gamma\rho)\left[G^e_{nm}(z)\cos(m\varphi) - G^o_{nm}(z)\sin(m\varphi)\right] \\ & + \gamma J'_m(\gamma\rho)\left[F^{e\prime}_{nm}(z)\cos(m\varphi) + F^{o\prime}_{nm}(z)\sin(m\varphi)\right] \end{aligned} \tag{140}$$

For $m = 0$ the total current on a wire segment from $2\pi a H_\varphi(\rho = a)$ is

$$I_n \approx -\sigma\gamma 2\pi a J'_0(\gamma a) F^e_{n0} = \sigma\gamma 2\pi a J_1(\gamma a) F^e_{n0} \tag{141}$$

Because we will take this to not vary on a segment we must have F^e_{n0} be a constant in z. In fact we will take the wires to be insulated from one another and this will be a constant along each wire.

Because of the high level of wire conductivity we need to set the normal component of the current at the surface equal to zero $J_\rho(\rho = a) = \sigma E_\rho(a) \approx 0$ and thus

$$i\omega\mu m J_m(\gamma a)\begin{pmatrix}-G^e_{nm}(z)\\ G^o_{nm}(z)\end{pmatrix} = \gamma a J'_m(\gamma a)\begin{pmatrix}F^{e\prime}_{nm}(z)\\ F^{o\prime}_{nm}(z)\end{pmatrix} \tag{142}$$

4.2. Matching of External Potentials with Internal Hertz Potentials

As discussed in the perfectly conducting case, it may be more straightforward externally to the braid wires to use the field representation involving a filament vector potential from each wire carrying the net current plus a scalar magnetic potential (88) to match the boundary conditions.

Using the preceding $J_\rho(\rho = a) = 0$ expressions (142) (to insert these into Eqs. (138) and (139) another derivative in z can be taken) we can write the magnetic field continuity conditions at $\rho = a$ as

$$\begin{aligned} H_z(a) &\approx \frac{\gamma a}{i\omega\mu m}\gamma^2 J_m'(\gamma a)\left[-F_{nm}^{e\prime}(z)\sin(m\varphi) + F_{nm}^{o\prime}(z)\cos(m\varphi)\right] \\ &= \left[-\frac{\partial \phi_m}{\partial z} + \frac{1}{\mu_0}\left(\nabla \times \underline{A}_f\right)\cdot \underline{e}_z\right]_m \end{aligned} \tag{143}$$

$$\begin{aligned} H_\varphi(a) &\approx -\sigma\gamma J_m'(\gamma a)\left(1 + \frac{1}{\gamma^2}\frac{\partial^2}{\partial z^2}\right)\left[F_{nm}^{e}(z)\cos(m\varphi) + F_{nm}^{o}(z)\sin(m\varphi)\right] \\ &= \left[-\frac{1}{\rho}\frac{\partial \phi_m}{\partial \varphi} + \frac{1}{\mu_0}\left(\nabla \times \underline{A}_f\right)\cdot \underline{e}_\varphi\right]_m \end{aligned} \tag{144}$$

$$\begin{aligned} B_\rho(a) &\approx -\mu\sigma\frac{m}{a}J_m(\gamma a)\left[1 + \left\{\frac{\gamma a J_m'(\gamma a)}{m J_m(\gamma a)}\right\}^2 \frac{1}{\gamma^2}\frac{\partial^2}{\partial z^2}\right]\left[F_{nm}^{e}(z)\sin(m\varphi) - F_{nm}^{o}(z)\cos(m\varphi)\right] \\ &= \left[-\mu_0\frac{\partial \phi_m}{\partial \rho} + \left(\nabla \times \underline{A}_f\right)\cdot \underline{e}_\rho\right]_m \end{aligned} \tag{145}$$

where the subscript m on the brackets means that only the $\sin(m\varphi)$ and $\cos(m\varphi)$ variations of the quantity is retained and the left hand sides represent Eqs. (137), (138), and (139) evaluated at $\rho = a$. Ordinarily we would expect that $[B_\rho]_0 = 0$ due to the absence of magnetic charge, however this net flux can arise as a result of net flux entering or leaving neighboring segments. The continuity of H_z leads to

$$\begin{pmatrix} -F_{nm}^{e\prime}(z) \\ F_{nm}^{o\prime}(z) \end{pmatrix} = \frac{m/a}{\sigma\gamma J_m'(\gamma a)}\frac{1}{2\pi}\int_0^{2\pi}\left[-\frac{\partial \phi_m}{\partial z} + \frac{1}{\mu_0}\left(\nabla \times \underline{A}_f\right)\cdot \underline{e}_z\right]\begin{pmatrix} 2\sin(m\varphi) \\ \varepsilon_m\cos(m\varphi) \end{pmatrix} d\varphi \tag{146}$$

and the expressions resulting from continuity of H_φ lead to

$$\left(1 + \frac{1}{\gamma^2}\frac{\partial^2}{\partial z^2}\right)\begin{pmatrix} F_{nm}^{e} \\ F_{nm}^{o} \end{pmatrix} = -\frac{1}{\sigma\gamma J_m'(\gamma a)}\frac{1}{2\pi}\int_0^{2\pi}\left[-\frac{1}{a}\frac{\partial \phi_m}{\partial \varphi} + \frac{1}{\mu_0}\left(\nabla \times \underline{A}_f\right)\cdot \underline{e}_\varphi\right]\begin{pmatrix} \varepsilon_m\cos(m\varphi) \\ 2\sin(m\varphi) \end{pmatrix} d\varphi \tag{147}$$

Using these the preceding B_ρ continuity condition can be grouped as

$$\begin{aligned} &-\frac{1}{2\pi}\int_0^{2\pi}\left[-\frac{1}{a}\frac{\partial \phi_m}{\partial \varphi} + \frac{1}{\mu_0}\left(\nabla \times \underline{A}_f\right)\cdot \underline{e}_\varphi\right]\begin{pmatrix} 2\sin(m\varphi) \\ \varepsilon_m\cos(m\varphi) \end{pmatrix} d\varphi \\ &+\frac{m}{\gamma a}\left\{\left(\frac{\gamma a J_m'(\gamma a)}{m J_m(\gamma a)}\right)^2 - 1\right\}\frac{1}{\gamma}\frac{\partial}{\partial z}\frac{1}{2\pi}\int_0^{2\pi}\left[-\frac{\partial \phi_m}{\partial z} + \frac{1}{\mu_0}\left(\nabla \times \underline{A}_f\right)\cdot \underline{e}_z\right]\begin{pmatrix} \varepsilon_m\cos(m\varphi) \\ -2\sin(m\varphi) \end{pmatrix} d\varphi \\ =\; &\frac{\gamma J_m'(\gamma a)}{\mu\left(\frac{m}{a}\right)J_m(\gamma a)}\frac{1}{2\pi}\int_0^{2\pi}\left[-\mu_0\frac{\partial \phi_m}{\partial \rho} + \left(\nabla \times \underline{A}_f\right)\cdot \underline{e}_\rho\right]\begin{pmatrix} \varepsilon_m\cos(m\varphi) \\ -2\sin(m\varphi) \end{pmatrix} d\varphi \end{aligned} \tag{148}$$

These final expressions (148) provide the connection between the tangential derivatives of the external potential and the radial derivative of the external potential. Note the terms in braces in (148) vanish in the limit $\gamma a \to 0$; alternatively the limit $\gamma a \to \infty$ produces $[B_n]_m \to 0$. The unknowns are the multipole pulse amplitudes $p_{me}^{(m)}$ and $p_{mo}^{(m)}$ in the representation for ϕ_{mn} for each segment n.

4.2.1. Symmetric Mode Currents

As in the vector potential treatment above we do not expect the net wire currents I_n to be determined from the preceding matching, but instead from the total injected current of the braid and a condition of no net voltage difference between wires over a braid period. In this finitely conducting case where we have an accurate treatment of the internal electric field inside the wires, we can thus apply the continuity voltage condition inside the wires

$$\int_{C_{pj}} \underline{E}^j \cdot \underline{d\ell} = \int_{C_{p1}} \underline{E}^1 \cdot \underline{d\ell}, \quad j = 2, \ldots, N_w N_s \tag{149}$$

where the contour C_{pj} extends over an axial period inside of the jth wire.

5. CONCLUSIONS

This paper discusses the first principles formulation of the electromagnetic cable braid penetration and propagation problem. Basic energy and power formulas are used to define the immittances in integral form and are applied to nonuniform geometries such as an exterior ground plane and an interior eccentric coax.

The detailed solution of the boundary value problems involved in the wire braid shield are set up using a basis of line multipoles along the braid wires. This approach leads to an efficient formulation of the periodic cell of the braid. In the electric problem images are used to treat adjacent dielectric boundaries. In the magnetic field problem we use Hertz potentials inside the wires and a combination of a magnetic scalar potential and filament vector potential (to carry the net wire current) outside of the wires. In this way the porpoising and hole penetration characteristics of the braid penetration arise in a self consistent way. We further simplify the braid geometry by mapping it to a planar surface so that the penetration, as well as the self immittance characteristics are reduced to certain intrinsic constants linked to the basic geometry of the braid wires.

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