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Study of Optical Responses in Hybrid Symmetrical Quasi-Periodic Photonic Crystals

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Abstract—The light propagation through a one-dimensional symmetrical photonic structure, determined by the symmetric Silver mean Ag_4 distribution embedded between two Bragg structures Bg_{27} ($Bg_{27}/Ag_4/Bg_{27}$), is studied using the transfer matrix method (TMM). The focus lies on the investigation of the influence of symmetry of the structure as well as the dependence of transmission on the frequency, angle of incidence of the light striking the structure and symmetrical deformation of the structure. The deformation was introduced by applying a power law, so that the coordinates y of the deformed object were determined through the coordinates x of the non-deformed structure in accordance with the following rule: $y = x^{1+k}$. Here, k is the degree of the law. A comparison will be made with a symmetrical periodic structure having the same number of layers. All results will be discussed in relation with the k values. Indeed, in the case of low k values close to zero a monochromatic filter was obtained, and in the case of relatively high values, an omnidirectional mirror is obtained.

1. INTRODUCTION

The light propagation through quasi crystals has been conducted to understand the physical properties of these aperiodic materials. These materials are regarded to have a degree of order intermediate between crystals and disordered systems [1, 2]. The photonic properties of quasi crystals have a special interest because the disorder and complex symmetries in these materials make them suitable for the application in several optical devices such as polarization switching, microelectronic devices that are based on photons rather than on electrons, which potentially can be the electromagnetic analogue to semiconductors [3, 4].

The theoretical study of photonic quasi-periodic systems often applies the concept of quasiperiodic mathematical sequences. Especially, the photonic properties of one-dimensional photonic crystals have been extensively analyzed with this approach because they can be easily produced in reality, and we can apply transfer matrix methods to study the optical responses through these systems. Such one-dimensional systems can be relatively easily produced in reality. We can find a close resemblance of the theoretical and experimental results [5, 6].

Most of the investigations focus on the study of optical responses of quasiperiodic photonic crystals when the incident light wave is perpendicular to the layers of the stack and do not investigate the influence of oblique incidence on the transmission or reflection through the system. In addition, these systems are not symmetrical; we can think to study the effect of the geometrical symmetry of these materials, also the deformation that can be produced symmetrically on the geometrical thickness of layers.

However, in this paper we will study the optical properties of the structure at normal incidence, the oblique incidence and a small symmetrical deformation of geometrical thickness layers of 1D symmetrical quasiperiodic photonic crystal consisting of a sequence of layers made of two different refractive indices

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n_H and n_L, where the configuration of the layers is given according to the so-called Silver-mean sequence that is sandwiched between two Bragg sequences Before that, we remember that all the members of the family of metallic mean are obtained through an "inflationary scheme" that produces a binary chain originated by two primitive blocks H and L that are distributed according to the inflation scheme [7, 8]:

$$S_{j+1} = S_{j-1}^m S_j^n \tag{1}$$

where m and n are integers, and $j \geq 2$, S_i^m represents m adjacent repetitions of stack S_i.

The case $m = 1$ and $n = 2$ corresponds to the Silver-mean sequence that is given by the following relation [7, 9]

$$S_{j+1} = S_{j-1} S_j^2 \tag{2}$$

The outline of this paper is as follows: firstly we give an introduction to the transfer matrix method used for the calculation, then, we present our results and discuss them. This is followed by a brief conclusion.

2. METHOD OF CALCULATION

In this part, a brief summary will be given about the Transfer Matrix Method used in our calculations. This is an easy to implement method and allows the investigation of layered media. It allows calculating the reflection and transmission coefficients of these layered media [10]. The Transfer Matrix Method considers a field moving in the forward direction E_F and one in the backward direction E_B. By using appropriate boundary conditions, the relation between forward and backward moving fields at an interface is obtained. Transmission and reflection through a slab (Figure 1) are then obtained according to the two fields E_F and E_B.

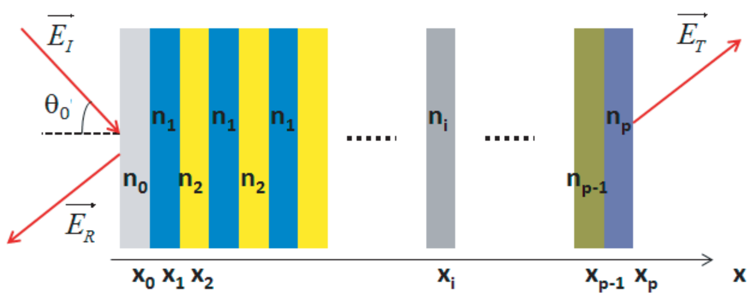

Figure 1. Propagation of light through a multilayer structure consisting of the two different materials with different indices of refraction.

The relation between the amplitudes of these electric fields with two different planes including a stratified medium is expressed by the following matrix product [11]:

$$\begin{bmatrix} E_F(x_0) \\ E_B(x_0) \end{bmatrix} = C_1 C_2 C_3 \ldots C_{p+1} \begin{bmatrix} E_F(x) \\ E_B(x) \end{bmatrix} \tag{3}$$

where C_j is the product of the propagation matrix C_{pr} and the interface matrix C_{int} given by:

$$C_{pr} = \begin{bmatrix} e^{i\varphi_{j-1}} & 0 \\ 0 & e^{-i\varphi_{j-1}} \end{bmatrix} \quad C_{int} = \frac{1}{t_i} \begin{bmatrix} 1 & r_j \\ r_j & 1 \end{bmatrix} \tag{4}$$

where t_j and r_j are, respectively, the Fresnel transmission and reflection coefficients between the $(j-1)$th and jth layers given by.

For TE polarization:

$$r_j = \frac{n_{j-1} \cos\theta_j - n_j \cos_{j-1}}{n_{j-1} \cos\theta_j + n_j \cos\theta_{j-1}} \tag{5}$$

$$t_j = \frac{2n_{j-1} \cos\theta_{j-1}}{n_{j-1} \cos\theta_j + n_j \cos\theta_{j-1}} \tag{6}$$

For TM polarization we get:

$$r_j = \frac{n_{j-1}\cos\theta_{j-1} - n_j \cos_j}{n_{j-1}\cos\theta_{j-1} + n_j\cos\theta_j} \tag{7}$$

$$t_j = \frac{1}{2}\frac{n_{j-1}\cos\theta_{j-1}}{n_{j-1}\cos\theta_{j-1} + n_j\cos\theta_j} \tag{8}$$

The value φ_{j-1} in Equation (4) indicates the change in the phase of the wave between the $(j-1)$th and jth interfaces and expressed by:

$$\varphi_0 = 0 \text{ and } \varphi_{j-1} = \frac{\lambda_0}{2\lambda}\pi\cos\theta_{j-1} \text{ for } j > 1. \tag{9}$$

By using the above equations, we can easily obtain the energy reflectance R as: $R = |r|^2$ for (S) and (P) polarizations.

3. RESULTS

In this paper, we will study the optical properties of a symmetrical one-dimensional hybrid quasiperiodic structure constructed of Silver mean (Ag) structure sandwiched between two Bragg structures (Bg) having the form $\mathrm{Bg}_n/\mathrm{Ag}_m/\mathrm{Bg}_n$ where n and m are the iteration number of each structure. Here the Bragg structure (Bg_n) is constructed as $H(LH)^n$. The configuration of layers of Silver-mean quasi periodic photonic crystal is generated by the recurrence relation

$$\begin{aligned} &S_{n+1} = S_{n-1}S_n^2 \\ &S_1 = H; \quad S_2 = LH; \quad S_3 = HLHLH; \quad S_4 = LHHLHLHHLHLH; \quad \ldots \end{aligned} \tag{10}$$

such that each term of the chain is formed by writing contiguously two replicas of the preceding term and adding its ancestry to the left of the replicas.

In an optical implementation of $\mathrm{Bg}_n/\mathrm{Ag}_m/\mathrm{Bg}_n$ sequence, the letters in the alphabet $\{L, H\}$ are realized as layers made of materials with refractive indices (n_L, n_H) and thicknesses (d_L, d_H), respectively. The material L has a low refractive index, while H has a high refractive index, which justifies the notation employed thus far. To properly compare the optical response we take advantage of the transfer-matrix technique. In what follows, we investigate the optical responses by using two dielectric materials TiO_2 and SiO_2 with refractive indices $n_L = n_{\mathrm{SiO}_2} = 1.45$ and $n_H = n_{\mathrm{TiO}_2} = 2.3$. In all our computations, the two refractive indices are varied independently from 0.3 µm to 1 µm which corresponds to the spectral range $\frac{\lambda_0}{\lambda} \in [0.5,\ 1.66]$. The reference wavelength is $\lambda_0 = 0.5\,\mu\mathrm{m}$ chosen to be in the spectral range $[0.3,\ 1]\,\mu\mathrm{m}$. The constitutive parameters of the unit cell are determined by the quarter-wave condition in order to reflect the wavelength λ_0 that lies within the photonic band gap

$$n_H d_H = n_L d_L = \frac{\lambda_0}{4} \tag{11}$$

This condition leads for an incidence angle $\theta_0 = 0$ to an identical optical wave path for the light in the two materials H and L of the stack [12, 13].

To obtain the symmetry of any structure, we need to use the following formula:

$$S_n = \left\{G_n\ ,\ \overline{G_n}\right\} \tag{12}$$

where S_n is the symmetrical structure, G_n the sequence that can be periodic or quasiperiodic, $\overline{G_n}$ the inverse of the sequence, and n the number of the iteration.

In Figure 2, we present an example of a symmetrical quasiperiodic structure having the form $\mathrm{Bg}_n/\mathrm{Ag}_m/\mathrm{Bg}_n$.

Now, to study the symmetrical system $\mathrm{Bg}_n/\mathrm{Ag}_m/\mathrm{Bg}_n$, we need to fix the iteration numbers n and m. These numbers must give a structure that has an optical response with a large photonic band gap (PBG) and can be compared, after this study, to other structures having approximately the same number of layers. For all these reasons we have fixed n to 27 and m to 4. So the studied structure in this work is $\mathrm{Bg}_{27}/\mathrm{Ag}_4/\mathrm{Bg}_{27}$ that gives a total number of layers equal to 192.

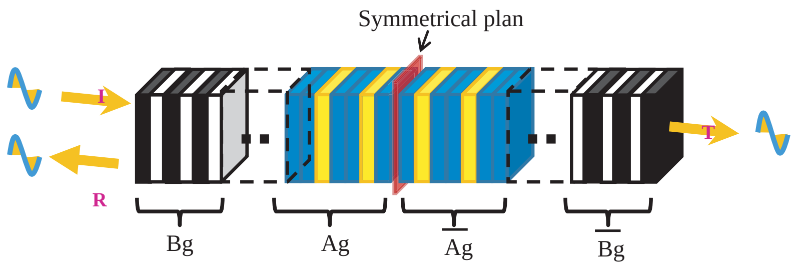

Figure 2. An example of a hybrid symmetrical quasi-periodic system having the form $Bg_n/Ag_m/Bg_n$.

3.1. Oblique Incidence

In this section, we will study the effect of oblique incidence on the reflection of the structure $Bg_{27}/Ag_4/Bg_{27}$. We represent in Figure 3 the variation of the reflection spectrum according to the normalized wavelength $\frac{\lambda_0}{\lambda}$ and the incidence angle θ_0 for both S-polarized (TE) mode and P-polarized (TM) mode.

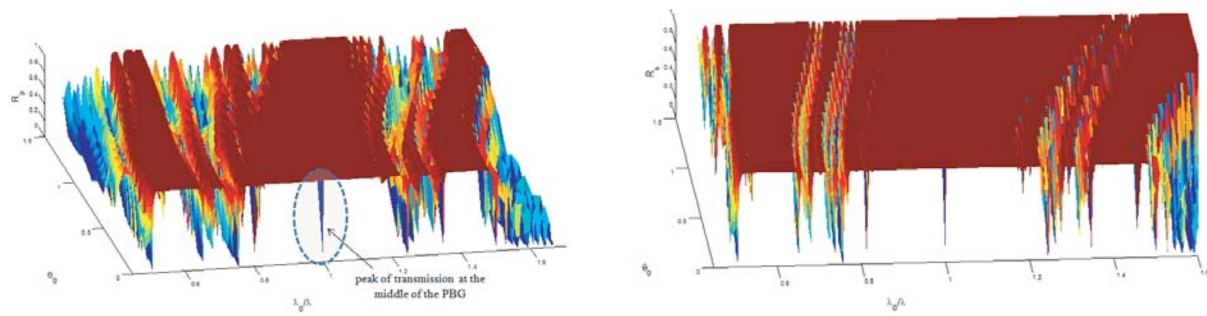

Figure 3. Variation of the reflection spectrum according to the normalized wavelength $\frac{\lambda_0}{\lambda}$ and the incidence angle θ_0 for both polarization TE and TM.

When we see these figures, the first thing that we note is the presence of an optical window in the center of the photonic band gap (PBG). The second note is the presence of a range of frequency where the reflection is insensitive to the incident angle. In the next part of this paper, we will study the origin of the optical window that appears at the center of the PBG, and then, the characteristic of the omnidirectional mirror that is constructed when the incident angle increases.

3.1.1. Origin of the Transmission Peak

To know where the origin of the optical window is, we present in Figure 4 the optical response in 2D of the system for different incident angles.

Here we note that the peak appears only when the incident angle is equal to 0. So, what is the cause that brings up this peak? To answer this question, we present in Figure 5 the optical response of the structure $Bg_{27}/Ag_4/Bg_{27}$ and each of its parts ($Bg_{27}/\overline{Bg_{27}}$ and $Ag_4/\overline{Ag_4}$) at normal incidence.

The most outstanding result of this study is the Bragg PBG containing the transmission peak that exists in the center of the spectral range of the symmetric $Bg_{27}/Ag_4/Bg_{27}$ system.

If we compare these results, we can observe that the origin of the mirror symmetry in the $Bg_{27}/Ag_4/Bg_{27}$ system is that the effect of the Bragg system is to make the frequency regions more localized and the middle half width of the peak equal to 10^{-6}.

If we compare this result to a symmetrical Bragg system, having approximately the same number of layers (190 layers), the width of the peak at the middle half is equal to 10^{-3}.

So we can say that symmetry plays an important role in obtaining a perfect monochromatic filter with the $Bg_{27}/Ag_4/Bg_{27}$ system. This phenomenon can be regarded as a generic feature of multilayer systems that own an internal symmetry.

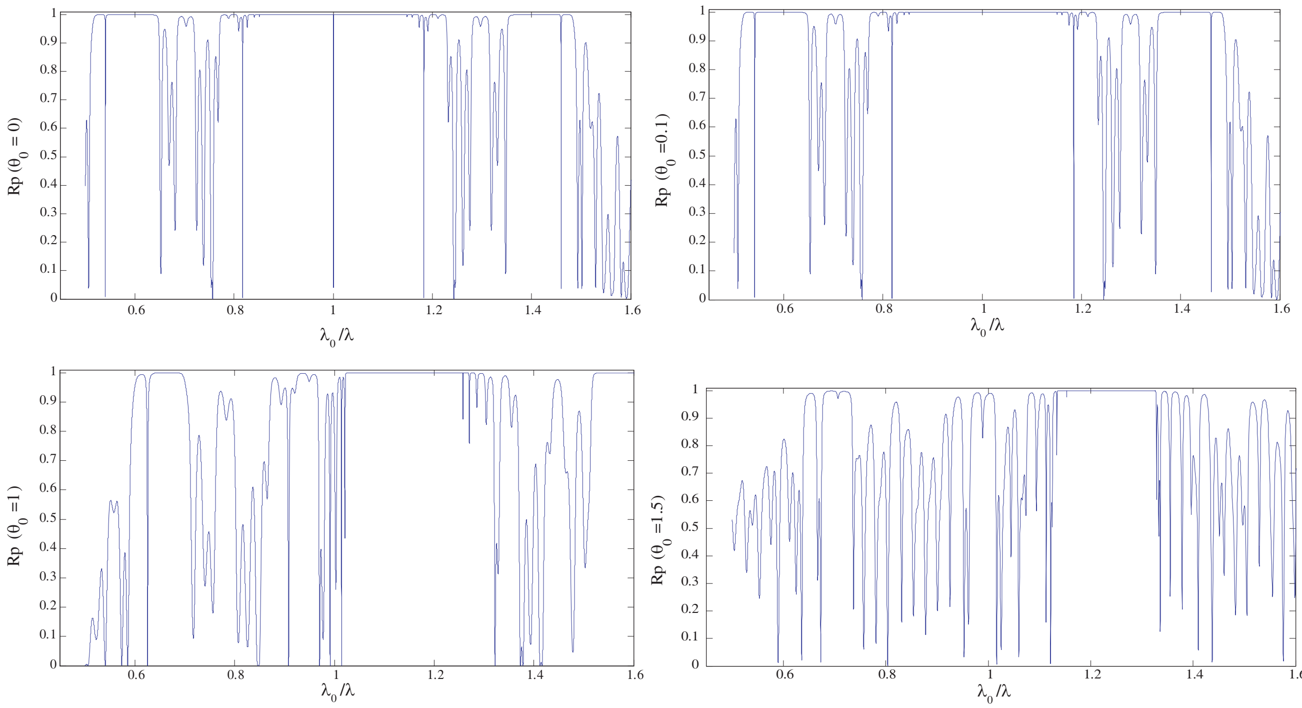

Figure 4. Spectra of reflectance Rp (*TM* mode) for different incident angles.

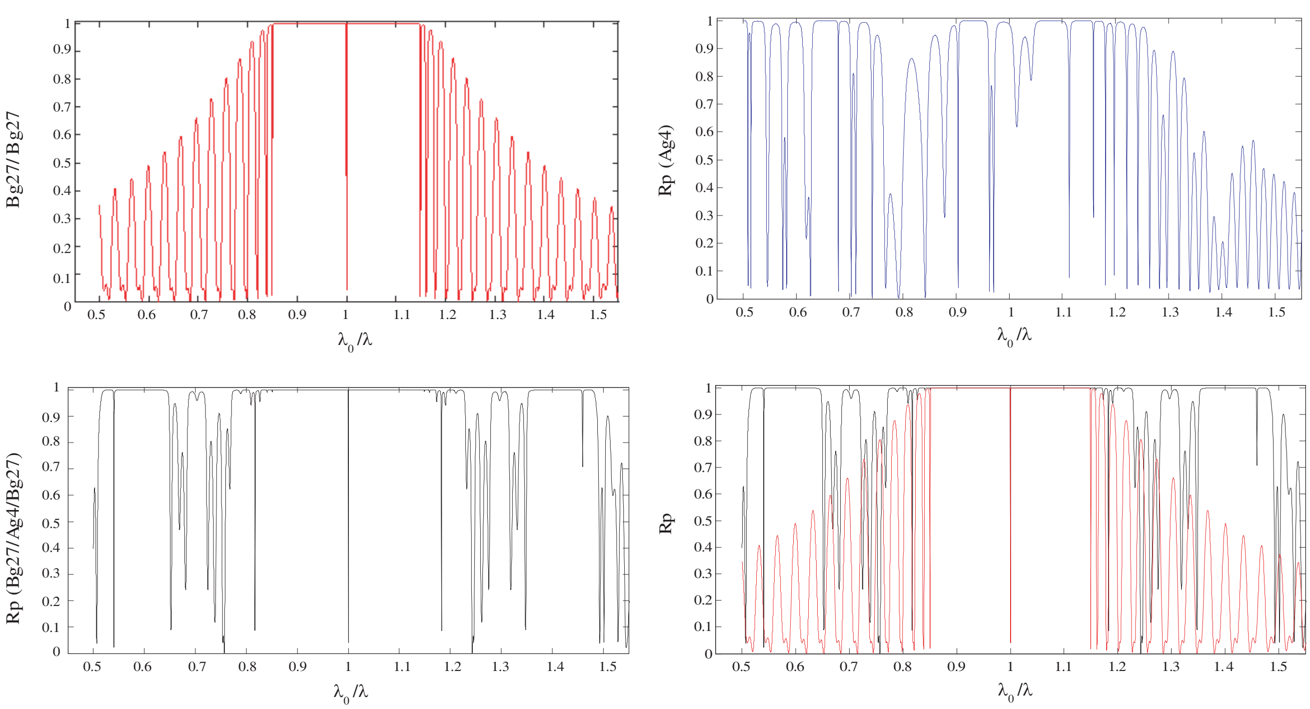

Figure 5. Reflection response of the structures: $Bg_{27}/\overline{Bg_{27}}$, $Ag_4/\overline{Ag_4}$ and $Bg_{27}/Ag_4/Bg_{27}$ at normal incidence.

3.2. The Omnidirectional Mirror

Now, we focus our interest on the bandwidth of the PBG. When we see Figure 3 again, we note that the *TM* polarization bandwidth is broader than at normal incidence, whereas the *TE* polarization bandwidth is narrower. From this figure, we can note that as the incident angle increases, the center wavelengths of both reflection bands shift to the shorter wavelength region. We also note that for a region of frequency, the omnidirectional PBG (OPBG) for the *TM* polarization is located within that for the *TE* polarization. Therefore, the OPBG for both *TE* and *TM* polarizations can be defined by the edges of the upper photonic band at the incident angle of $\pm\frac{\pi}{2}$ rad and the lower photonic band at the normal incidence. In Figure 6, we present the reflection response for *TE* and *TM* polarization in order to determine the range of OPBG (red region).

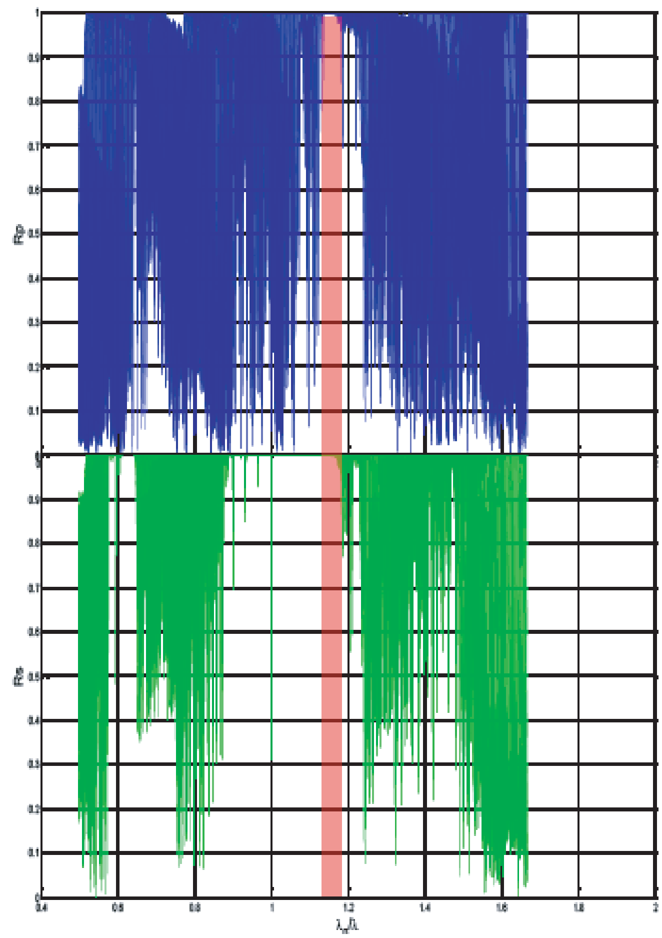

Figure 6. The reflection response for *TE* and *TM* polarizations according the normalized wavelength.

It can be seen from this figure that the width of the OPBG forbidden bands is equal to 0.04. When we compare this result with a symmetrical periodic system (Bragg system) having approximately the same number of layers (190 layers), we can find that, for the same conditions, we obtain 0.01 as the width at the middle half of the OPBG. Here we note that the quasiperiodic system $Bg_{27}/Ag_4/Bg_{27}$ ameliorates the bandwidth of the OPBG.

3.3. Deformation Effect

In this section, we investigate the optical properties of the deformed $Bg_{27}/Ag_4/Bg_{27}$ system and subsequently study how the OPBG frequency range varies with the symmetric deformation of the different layers of the system. The purpose of applying the symmetrical deformation is to keep the geometric properties of the global system; otherwise, the system will lose its symmetry. Here, the partial symmetric deformation was introduced by applying the power law, so that the coordinates y of the deformed object were determined through the coordinates x of the initial object in accordance with the following rule [10]:

$$y = x^{k+1} \tag{13}$$

Here k defines the degree of the law. In Figure 7(a), we present the initial symmetric system x ($Bg_{27}/Ag_4/Bg_{27}$) and in Figure 7(b) its symmetrical deformation y.

The initial optical phase thickness, when we apply the y function, is:

$$\varphi_i = \frac{\lambda_0}{\lambda}\pi \cos\theta_i \tag{14}$$

which takes the following form:

$$\varphi_i = \frac{\lambda_0}{\lambda}\pi \left[i^{k+1} - (i-1)^{k+1}\right] \cos\theta_i \tag{15}$$

after deformation.

Here i designates the ith layer. For the deformed system, the optical thickness of each layer becomes variable and depends on the ith layer and degree k. By applying the predicted deformation, the bandwidth of the PBG increases, and it becomes constant when $k \geq 0.05$. So we fix k to 0.05 and change the incident angle to see its effect with the polarization. The reflectance of TM- and TE-polarized light of the partially deformed quarter-wave stack is shown in Figure 8 as a function of normalized wavelength $\frac{\lambda_0}{\lambda}$ and incident angle θ_0 for two different values of the degree: $k = 0$ (Figure 8(a)) and $k = 0.05$ (Figure 8(b)).

We can see clearly that the bandwidth at the middle half of the OPBG is changed, and the center wavelengths of both reflection bands shift to the shorter wavelength region.

For $k = 0$, the bandwidth at middle half of the OPBG is equal to 0.04, and for $k = 0.05$, it is equal to 0.08. So the bandwidth is doubled for $k = 0.05$. If we take the symmetrical periodic photonic crystal having approximately the same number of layers (190 layers) and apply the deformation law $y = x^{k+1}$ with $k = 0.05$, we can find an OPBG having 0.006 as a bandwidth in the middle half and see the importance of the deformation effect.

Now, we focus our interest on the transmission peak at the middle of the PBG.

So we present in Figure 9 the reflection response of the deformed system at normal incidence.

It is clear, from these figures, that the optical window is absent when $k = 0.05$, and we can say that when we apply a variation of the thickness of each layer that converge to the center of the system, we can delete the localization of light that produces the transmission peak at the center of the PBG.

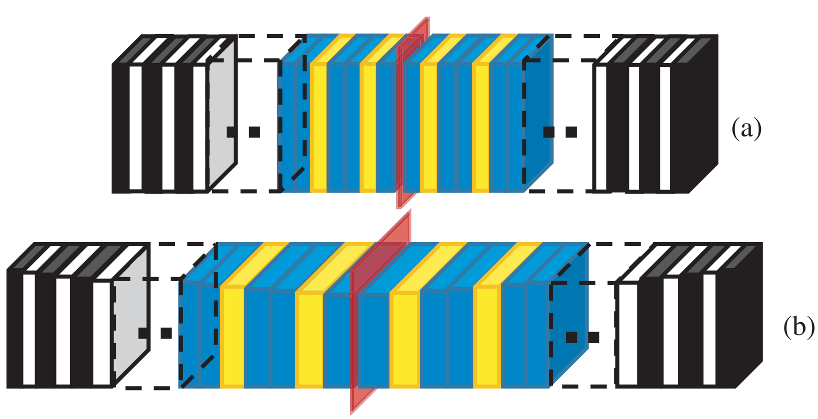

Figure 7. Example of symmetrical deformation of the structure $Bg_{27}/Ag_4/Bg_{27}$.

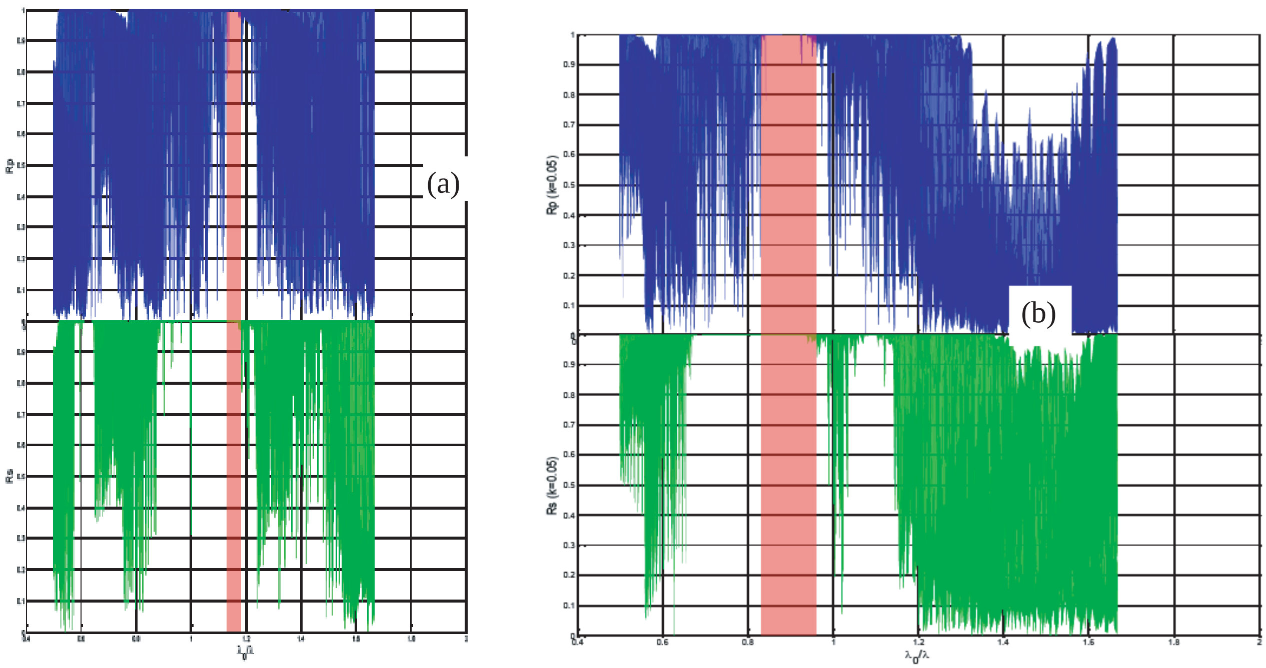

Figure 8. The reflection response for TE and TM polarizations according the normalized wavelength for two different values of the degree: (a) $k = 0$ and (b) $k = 0.05$.

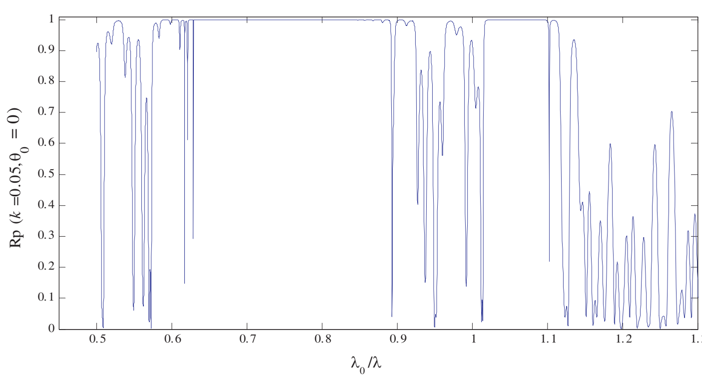

Figure 9. Reflection response of the deformed $Bg_{27}/Ag_4/Bg_{27}$ system at normal incidence and for $k = 0.05$.

4. CONCLUSION

In conclusion, we have studied the light wave propagation in the symmetric quasi-periodic photonic multilayer constructed with a symmetric Silver-mean sequence sandwiched between two Bragg sequences: $Bg_{27}/Ag_4/Bg_{27}$. We have shown that the symmetrical internal structure of the hybrid structure $Bg_{27}/Ag_4/Bg_{27}$ ameliorates the bandwidth of the PBG at oblique incidence compared to the periodic structure. We have demonstrated that in hybrid one-dimensional quasiperiodic systems having the form $Bg_{27}/Ag_4/Bg_{27}$, we can greatly enhance the transmission peak better than the periodic system and find a monochromatic filter with a high performance. Also, we have demonstrated that when we apply symmetrical deformation on the structure, we can obtain an OPBG larger than that obtained with a periodic system. In addition, this deformation has deleted the localization of the light that causes the presence of the transmission peak at the center of the photonic band gap.

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