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2D and 3D Far-Field Radiation Patterns Reconstruction Based on Compressive Sensing

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Abstract—The measurement of far-field radiation patterns is time consuming and expensive. Therefore, a novel technique that reduces the samples required to measure radiation patterns is proposed where random far-field samples are measured to reconstruct two-dimensional (2D) or three-dimensional (3D) far-field radiation patterns. The proposed technique uses a compressive sensing algorithm to reconstruct radiation patterns. The discrete Fourier transform (DFT) or the discrete cosine transform (DCT) are used as the sparsity transforms. The algorithm was evaluated by using 3 antennas modeled with the High-Frequency Structural Simulator (HFSS) — a half-wave dipole, a Vivaldi, and a pyramidal horn. The root mean square error (RMSE) and the number of measurements required to reconstruct the antenna pattern were used to evaluate the performance of the algorithm. An empirical test case was performed that validates the use of compressive sensing in 2D and 3D radiation pattern reconstruction. Numerical simulations and empirical tests verify that the compressive sensing algorithm can be used to reconstruct radiation patterns, reducing the time and number of measurements required for good antenna pattern measurements.

1. INTRODUCTION

Far-field radiation patterns are an essential part of antenna characterization. Usually, the process of measuring two-dimensional (2D) or three-dimensional (3D) radiation patterns is time consuming and expensive. For instance, a complete 3D radiation pattern with a spatial resolution in θ and ϕ of 2° requires 16200 measurements. The need for a new technique that will reduce the time required to measure a far-field radiation pattern is thus identified.

Several techniques have been proposed in literature where a small number of near-field samples are taken to reconstruct far-field radiation patterns [1–3]. For these kinds of measurement techniques, the software and hardware to perform near-field measurements are required. In case that only far-field measurements capabilities (software and hardware) are available, then the number of measurements depends on the resolution and accuracy desired.

Compressive sensing has been widely used to overcome sampling restrictions for a wide variety of applications [4–6]. The use of compressive sensing applied to radiation patterns measurements is limited. In [7], a method that uses compressive sensing to reconstruct the antenna radiation pattern is proposed. However, the measurements were performed by using randomly placed sensors which makes the method difficult for practical data measurements. A methodology that reduces the number of samples required in near-field to reconstruct far-field radiation pattern is proposed in [8]. An iterative algorithm is used to find the optimized samples required in near-field. None of these approaches have been applied to far-field measurements. In our previous work, compressive sensing was used to recover missing sections of far-field radiation patterns by sampling the available section of the radiation pattern and by using compressive sensing reconstructing the complete radiation pattern [9]. A different application of

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compressive sensing is presented in this paper, where by using few randomly-distributed samples in the far-field, far-field radiation patterns can be reconstructed.

Specifically, a new measurement technique based on compressive sensing is proposed to perform the measurement and the compression of far-field radiation patterns. The reconstruction algorithm uses the discrete Fourier transform (DFT) or discrete cosine transform (DCT) to recover the patterns. By using the proposed technique a complete reconstruction of either the 2D or 3D radiation pattern is obtained reducing the number of measurements and the time required to characterize an antenna. The number of random measurements required for an optimal reconstruction is calculated by using a priori information obtained from the simulation of the antenna radiation pattern and the compressive sensing algorithm.

2. COMPRESSIVE SENSING THEORY

Consider a signal $\mathbf{x}$ of size $N \times 1$ that can be represented in terms of a basis matrix $\boldsymbol{\psi}$ of size $N \times N$. Then $\mathbf{x} = \boldsymbol{\psi}^{-1}\mathbf{s}$, where $\mathbf{s}$ is a vector of size $N \times 1$, and $\boldsymbol{\psi}^{-1}$ is the inverse of $\boldsymbol{\psi}$. The signal $\mathbf{x}$ is said to be sparse if K ($K << N$) coefficients of the vector $\mathbf{s}$ are non-zero. In such cases, the signal $\mathbf{x}$ can be compressed by using an adequate transform as a basis function [10].

The number of measurements required to reconstruct the signal $\mathbf{x}$ can be reduced by taking $\mathbf{y} = \boldsymbol{\Omega}\mathbf{s}$ measurements, where $\mathbf{y}$ is a vector of size $M \times 1$, M is the number of measurements less than N, $\boldsymbol{\Omega} = \mathbf{A}\boldsymbol{\psi}$, and $\mathbf{A}$ is the measurement matrix used to keep the basis functions associated with the measurements. The approximation of the $\mathbf{x}$ signal in the sparse domain, $\hat{\mathbf{s}}$, is accomplished by solving the ℓ_1-minimization problem $\hat{\mathbf{s}} = \min \|\mathbf{s}\|_1$ such that $\mathbf{y} = \boldsymbol{\Omega}\hat{\mathbf{s}}$, with the ℓ_p-norm of a vector $\mathbf{f}$ defined as $\|\mathbf{f}\|_p = (\sum_{j=1}^{N} |f_j|^p)^{1/p}$, where $p \geq 1$ [10].

2.1. Compressive Sensing Applied to Radiation Patterns

A far-field radiation pattern is a representation of the intensity of the field with respect to θ and ϕ given by $f(\theta, \phi)$. Consider a 2D radiation pattern $f(\theta)$ that represents $\mathbf{x}$ in the compressive sensing algorithm. A sparse basis representation $\mathbf{s}$ of $f(\theta)$ is required to apply the reconstruction compressive sensing algorithm. Moreover, designing the measurement matrix is a key factor in the radiation pattern recovery using compressive sensing. The measurement matrix used should follow the restricted isometry property (RIP) to ensure recovery [10]. One measurement matrix that has been used in compressive sensing applications that follow the RIP is the random partial Fourier matrix [11–13]. The randomly distributed partial Fourier matrix is derived from the DFT given by

$$F(k) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi kn/N}, \tag{1}$$

where n is the index of the radiation pattern angle, k the index of the transform domain, and N the total number of samples.

In the case of a measured far-field radiation pattern, $f(\theta)$ is a vector of size $N \times 1$; therefore, the DFT can be represented as an operator in matrix form such that $\mathbf{s} = \boldsymbol{\psi}\mathbf{x}$, where $\mathbf{s}$ is in the transform domain that contains $K < N$ non zero values, and $\boldsymbol{\psi}$ is the matrix representation of the DFT given by

$$\boldsymbol{\psi}_{DFT} = \begin{bmatrix} e^{-j2\pi n_0 k_0/N} & \dots & e^{-j2\pi n_0 k_{N-1}/N} \\ \vdots & \ddots & \vdots \\ e^{-j2\pi n_{N-1} k_0/N} & \dots & e^{-j2\pi n_{N-1} k_{N-1}/N} \end{bmatrix}. \tag{2}$$

Another transform that can be used as a basis function is the discrete cosine transform (DCT). The DCT can be performed by using a matrix defined as

$$\boldsymbol{\psi}_{DCT} = \begin{bmatrix} 1 & \dots & 1 \\ \cos\left[\dfrac{\alpha_0 n_1 \pi}{2N}\right] & \ddots & \cos\left[\dfrac{(\alpha_{N-1} n_1)\pi}{2N}\right] \\ \cos\left[\dfrac{\alpha_0 n_{N-1} \pi}{2N}\right] & \dots & \cos\left[\dfrac{(\alpha_{N-1}) n_{N-1} \pi}{2N}\right] \end{bmatrix}, \tag{3}$$

where $\alpha_q = 2k_q + 1$.

The DFT or DCT matrix can be converted to a partial random matrix by randomly selecting the rows of the DFT or DCT matrices [13, 14]. The random rows are created by using a uniform random distribution. The measurement of the radiation pattern is performed by using M number of measurements where $M < N$ yielding a basis matrix $\mathbf{\Omega}$ with dimensions $M \times N$. As can be seen, sensing of the radiation pattern is already incorporated into the compressive sensing algorithm, where just M samples are randomly measured. Ultimately, the measured samples are used as the input to the compressive sensing algorithm to reconstruct the radiation pattern with a minimal reconstruction error.

In the case of 3D radiation patterns, $f(\theta, \phi)$, the goal is to reduce the number of samples in both the θ and ϕ directions. A traditional 2D compressive sensing algorithm converts the matrix into a long 1D vector to perform the reconstruction. Although this may work for other applications [10], vectorizing the matrix will reduce the number of measurements in only one direction and will add complexity to the reconstruction in the radiation pattern. A method where parallel compressive sensing is used in order to overcome vectorization limitations was proposed in [15], where the reconstruction is performed by using compressive sensing column by column. By using this method, however, the reduction of the number of measurements is performed only in one direction.

Assume that the matrix containing the 3D radiation pattern information is a matrix $N \times L$. A two-step process is proposed, where random measurements are taken in both directions resulting in a matrix $M \times P$, where $M < N$ and $P < L$, obtaining a reduction of the measurements needed in both θ and ϕ directions. The 3D radiation pattern reconstruction is performed first in the ϕ direction for each measured θ, where each 2D cut is reconstructed using parallel compressive sensing for each row [15]. Then, the 2D cuts are reconstructed by using the columns of the matrix resulting into a 3D radiation pattern with size $N \times L$. A representation of the measurement technique is shown in Fig. 1, where randomly-distributed samples are taken to reconstruct the far-field radiation pattern using compressive sensing.

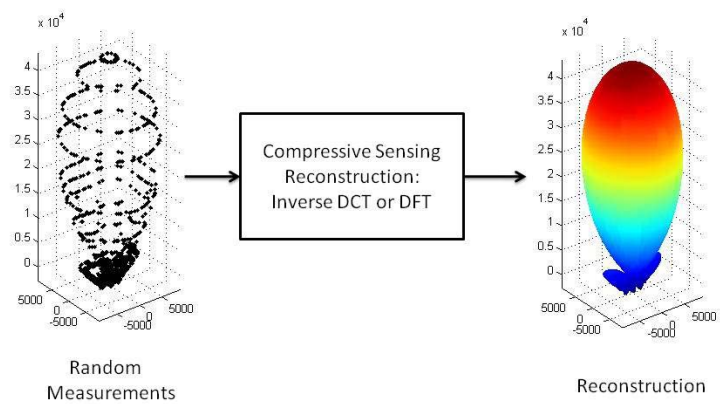

Figure 1. Compressive sensing algorithm for 3D radiation patterns.

The number of measurements required to successfully reconstruct an antenna radiation pattern is depended on the type of antenna used. Therefore, a priori information is required. This information is obtained by simulating the far-field radiation pattern and the number of samples required to successfully reconstruct the radiation pattern with low error as shown in the following sections. Once the number of measurements needed is identified, the randomly-distributed samples can be sorted to perform the anechoic chamber measurements to reduce the time required to take the measurements.

In order to evaluate the performance of the compressive sensing algorithm, the root mean square

error (RMSE) is used, defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\mathbf{f}_n - \hat{\mathbf{f}}_n)^2}, \tag{4}$$

where $\mathbf{f}_n$ is the simulated or measured radiation pattern point and $\hat{\mathbf{f}}_n$ the compressive sensing reconstruction of the radiation pattern point.

3. EVALUATION OF THE COMPRESSIVE SENSING RECONSTRUCTION ALGORITHM BY SIMULATION

The compressive sensing algorithm was evaluated by using 3 antennas: the half-wave dipole, the Vivaldi and the pyramidal horn. The total far-field electric field radiation patterns of the antennas were simulated in HFSS. Compressive sensing was applied to reconstruct the radiation pattern for each antenna with limited data points. The half-wave dipole and the horn antennas were modeled with a frequency of 1.35 GHz, and the Vivaldi antenna was modeled at 6 GHz. The electrical size of the half-wave dipole was 0.5, the horn antenna was 1.566, and the Vivaldi ranged from 0.166 to 1.278. The 2D radiation pattern for each antenna was simulated at a resolution of 2°; therefore, the total number of samples to be reconstructed is thus considered to be 180. Sketches of the HFSS models of the antennas are shown in Fig. 2. A test coaxial cable (CBL Mini-Circuits) was included in the model for each antenna. For a better visualization, the antennas are not scale with respect to each other.

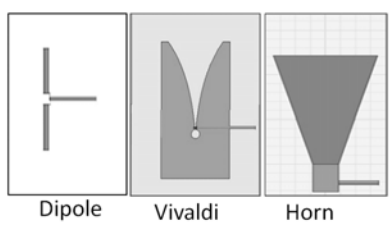

Figure 2. Antennas used to evaluate the compressive sensing algorithm.

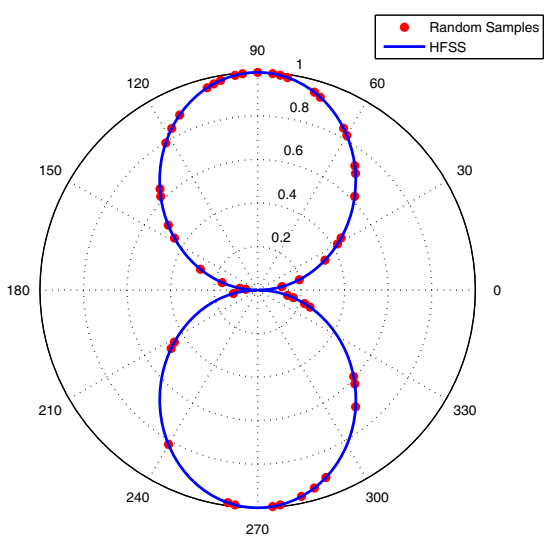

Figure 3. Randomly distributed measurements taken from the simulated radiation pattern of a horizonal half-wave dipole at $\phi = 0$.

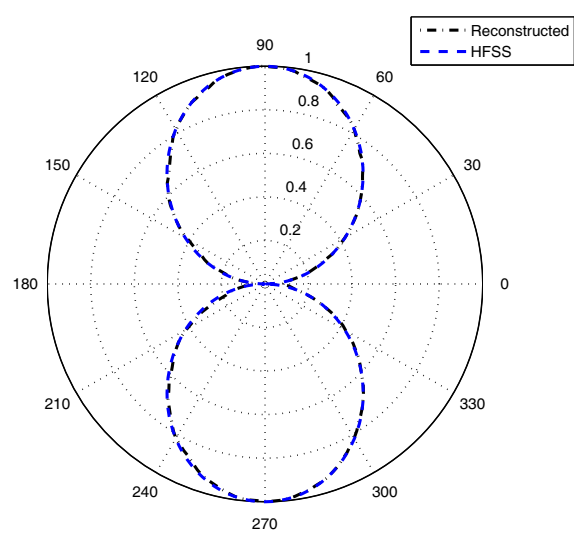

Figure 4. Simulated and reconstructed radiation patterns of a horizonal half-wave dipole at $\phi = 0$.

3.1. Half-Wave Dipole Antenna

Figure 3 shows the simulated 2D far-field radiation pattern for $\phi = 0$. Superimposed on the pattern are the samples used for reconstruction. The reconstructed radiation pattern using the DFT matrix for the compressive sensing reconstruction algorithm is shown in Fig. 4. In this case 50 samples where used to reconstruct the radiation pattern, that is 27% of the total number of samples with an RMSE of 2×10^{-3}.

There is a tradeoff between the number of measurements required to obtain a reconstruction of the radiation pattern and the RMSE of that reconstruction. Since a random distribution of measurements is used in the compressive sensing reconstruction, a Montecarlo simulation was performed where the RMSE of reconstruction was calculated as the number of measurements M was increased. The experiment was repeated 1000 times for each number of measurements and the RMSE averaged over the simulations. Fig. 5 shows the RMSE with respect to the normalized number of measurements using the DFT and the DCT. The reconstruction converges to the radiation pattern faster when using the DFT. This plot is used as a priori information to define the number of measurements required for a given RMSE.

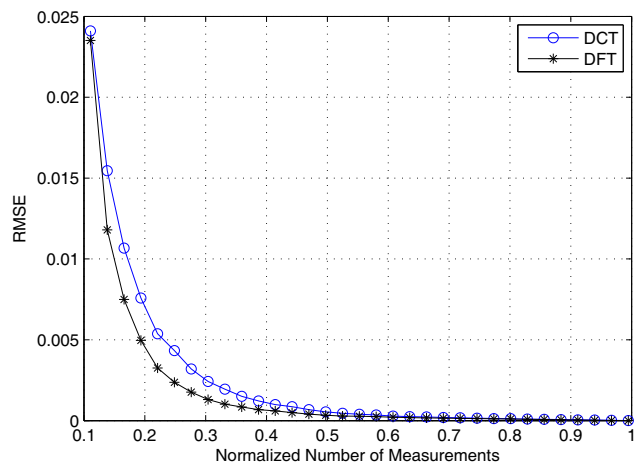

Figure 5. A Montecarlo simulation of the RMSE of the reconstructed radiation pattern for the horizontal half-wave dipole for $\phi = 0$.

3.2. Vivaldi Antenna

The half-wave dipole has a simple radiation pattern that can be very well reconstructed by using the proposed compressive sensing algorithm. A more complicated far-field radiation pattern was obtained by modeling a Vivaldi antenna. The H-plane principal cut of the 2D radiation pattern of the Vivaldi antenna was simulated with HFSS. The radiation pattern and the randomly distributed samples used for the compressive sensing reconstruction are shown in Fig. 6.

The reconstructed radiation pattern is shown in Fig. 7. In this case the DFT and 70 samples were used for reconstruction, 38% of the total samples that are required to obtain a reconstruction with a RMSE of 2.5×10^{-3}.

The RMSE was calculated by a Montecarlo simulation of 1000 realizations as the normalized number of measurements increased. Fig. 8 shows the RMSE comparison of the DFT and the DCT compressive sensing algorithms. It is seen that the RMSE of the reconstruction obtained when using the DFT converges to zero faster. However, due to the relative complexity of the pattern, more samples are required to obtain a good reconstruction compared to the half-wave dipole antenna pattern.

3.3. Pyramidal Horn Antenna

The simulated far-field radiation pattern and the randomly distributed measurements used for reconstruction are shown in Fig. 9. In this case, 80 measurements were used, 44% of the total data

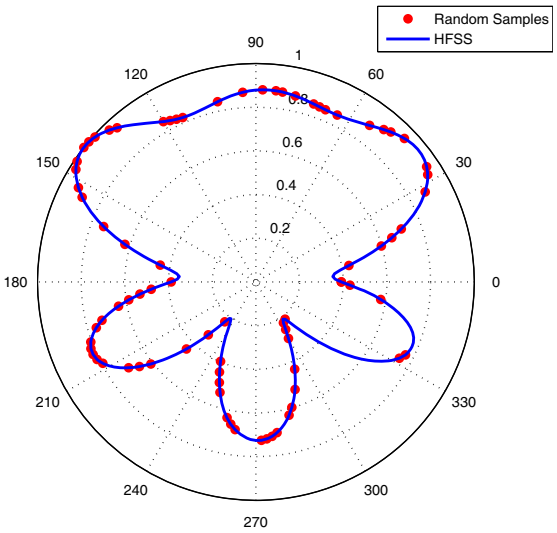

Figure 6. Simulated radiation pattern and randomly distributed samples used for the reconstruction of the Vivaldi antenna.

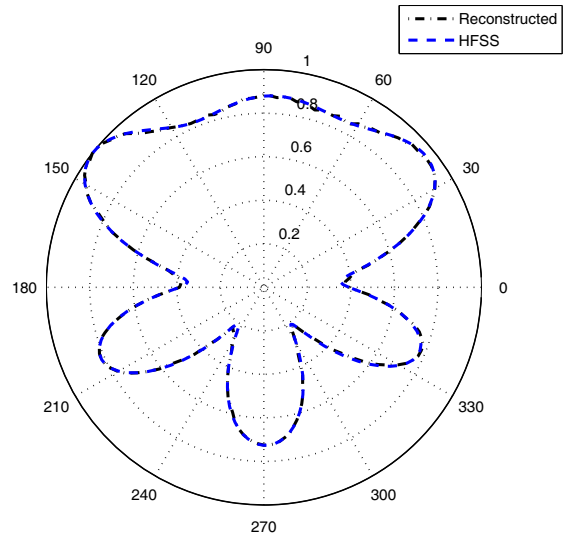

Figure 7. Simulated and reconstructed radiation patterns of the Vivaldi antenna.

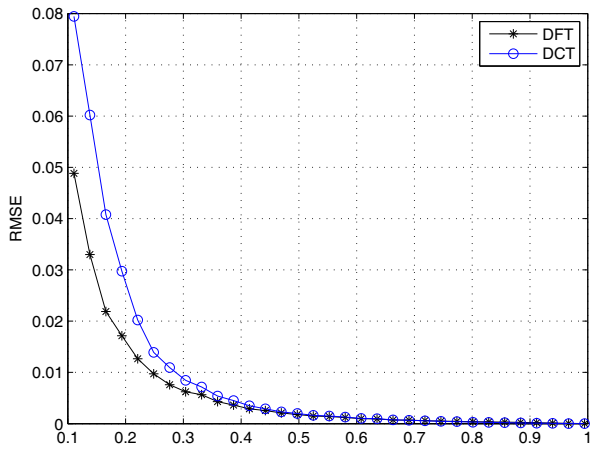

Figure 8. A Montecarlo simulation of the RMSE of the reconstructed radiation pattern of the Vivaldi antenna.

points. This yielded an RMSE of 1.3×10^{-3}.

Figure 10 shows the simulated radiation pattern compared to the reconstructed radiation pattern. A good approximation of the radiation pattern of the horn antenna is obtained.

The RMSE was calculated as the number of measurements was increased. A Montecarlo simulation of 1000 trials was used to obtain the RMSE of the radiation pattern reconstructed with the DFT and the DCT as a function of measurement points, M, used. The results are shown in Fig. 11. The two reconstruction basis matrices give similar results as they converge to zero with approximately the same number of measurements.

The 3D radiation pattern reconstruction of the horn antenna was evaluated by using the number of points required for a given acceptable RMSE as found in Fig. 11. A Montecarlo simulation was performed where the number of measurements along the θ was fixed to 30 as the number of measurements along ϕ was increased. The RMSE was calculated for each 2D radiation pattern cut $f(\phi)$ of the 3D pattern. The process was repeated 100 times to obtain the RMSE for each ϕ as the number of measurements was increased. Fig. 12 shows the results of the Montecarlo simulation. As it can be observed, the RMSE

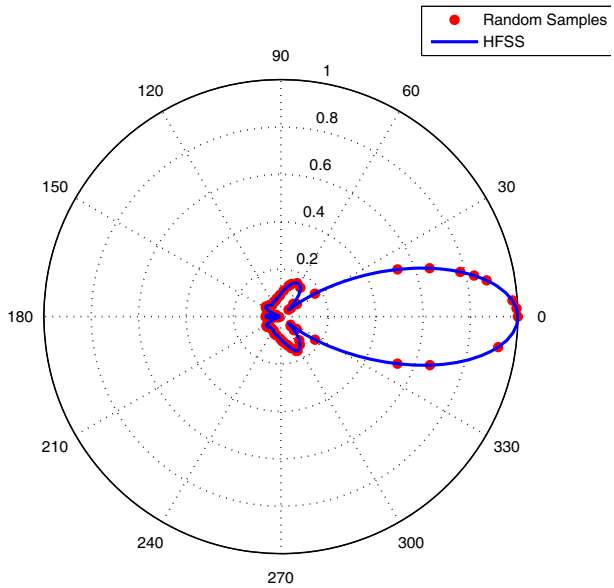

Figure 9. Simulated radiation pattern and randomly distributed measurements used for reconstruction of the horn antenna.

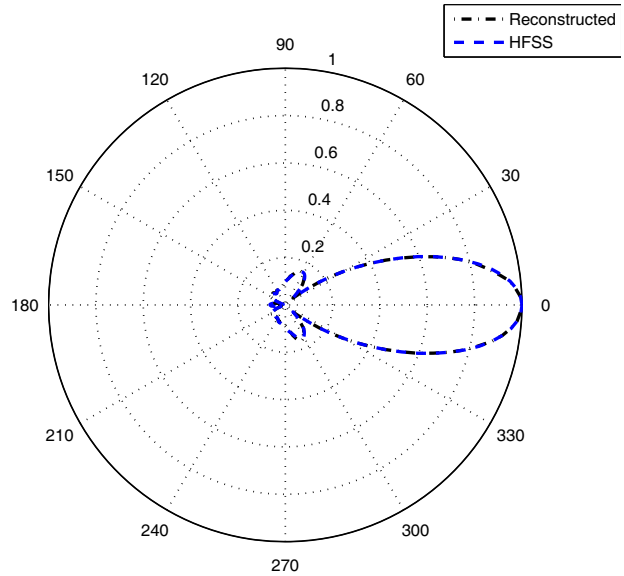

Figure 10. Simulated and reconstructed radiation patterns of the horn antenna.

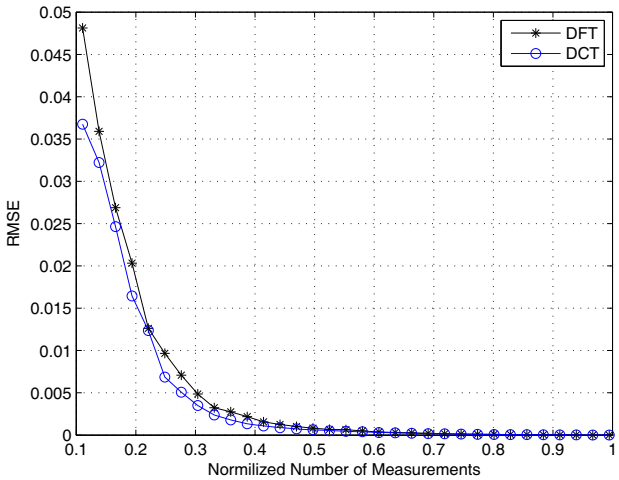

Figure 11. A Montecarlo simulation of RMSE of the reconstructed radiation pattern of the horn antenna.

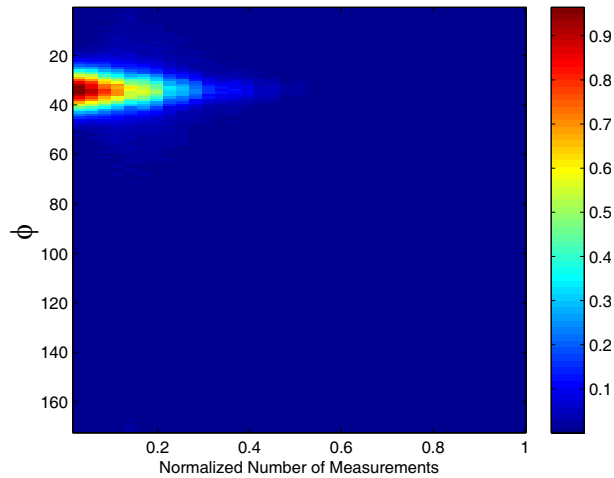

Figure 12. RMSE of the reconstructed 3D radiation pattern of the measured pyramidal horn antenna using the DFT.

approaches zero after 0.4 normalized or 40% of the number of measurements. The RMSE is higher at the region where ϕ is close to zero.

4. EMPIRICAL EVALUATION OF THE COMPRESSIVE SENSING RECONSTRUCTION ALGORITHM

A pyramidal horn antenna (having the same dimensions as the horn antenna simulated in HFSS) was measured to test the compressive sensing algorithm empirically. The transmitted frequency used was 1.35 GHz. In order to test the compressive sensing reconstruction algorithm, the radiation pattern was measured at a resolution of 2° in both the θ and ϕ directions. This measurement took approximately

5 hours in our anechoic chamber. The measured 2D radiation pattern and the randomly distributed samples used for reconstruction are shown in Fig. 13. Note the lack of back lobe measurement due to presence of the supporting tower in the anechoic chamber in addition thermal noise is being added to the experiment because of the hardware used.

The simulated a priori information was used to identify the number of samples required for a good reconstruction. Based on Fig. 11, 80 samples, that is 46% of the total samples, were used for the reconstruction (the same number of samples used in the simulated radiation pattern of the horn antenna). The 2D radiation pattern reconstruction and the measured radiation pattern are shown in Fig. 14.

Figure 15 shows the empirical radiation pattern and the randomly distributed samples used to reconstruct the radiation pattern. The compressive sensing algorithm was applied in θ and in ϕ. A

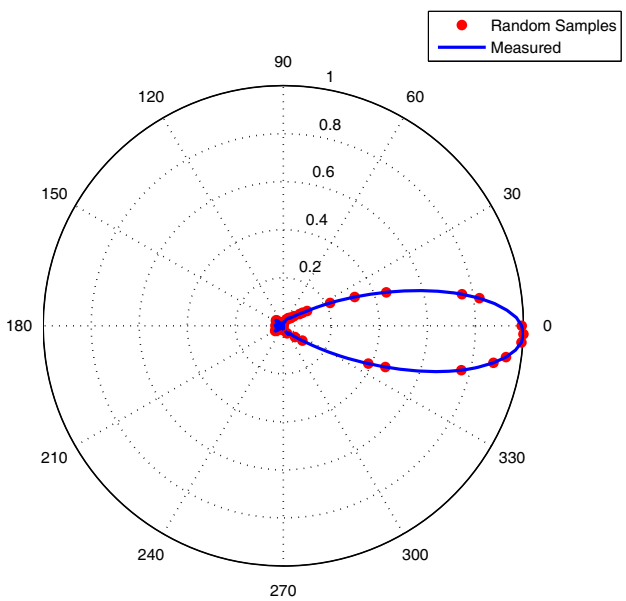

Figure 13. 2D radiation pattern of the measured pyramidal horn antenna and the random measurement samples used for reconstruction.

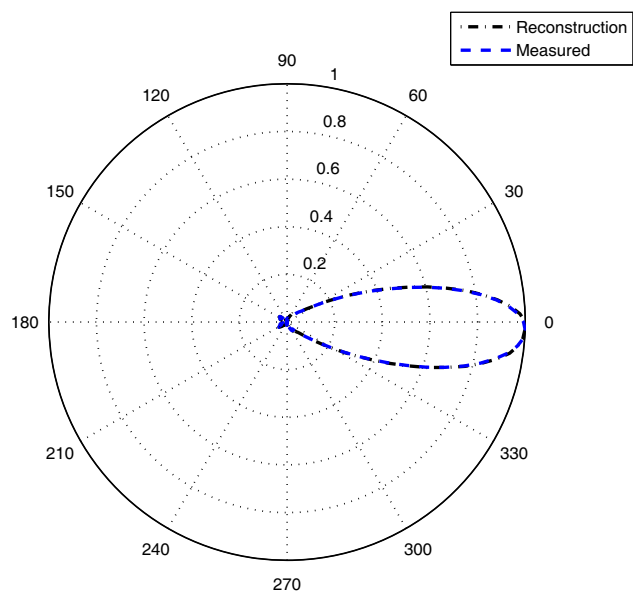

Figure 14. Measured and reconstructed radiation patterns of the pyramidal horn antenna.

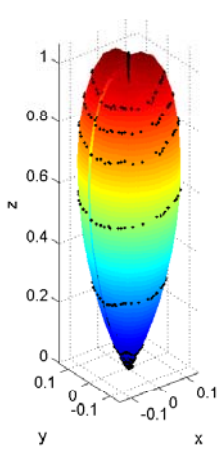

Figure 15. The empirical 3D radiation pattern and randomly distributed samples used for the reconstruction of the pyramidal horn antenna.

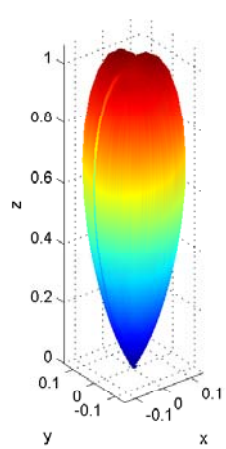

Figure 16. Reconstructed 3D empirical radiation pattern using the DFT of the pyramidal horn antenna.

reduced number of measurements was used in both direction: 30 measurements were used along θ and 70 measurements were used along ϕ. The number of measurements were chosen a priori. Since the DFT and the DCT provide similar results when reconstructing radiation pattern of the horn antenna, only the DFT was used for the 3D reconstruction. A total of 2100 samples taken randomly from a total of 16200 samples were used to reconstruct the 3D pattern. The reconstructed 3D radiation pattern is shown in Fig. 16. The compressive sensing reconstruction reproduces the original radiation pattern very well.

5. CONCLUSIONS

A compressive sensing algorithm that performs the reconstruction of 2D or 3D far-field radiation patterns was presented. The compression rate, and thus the number of random measurements required for a good reconstruction, depends on the complexity of the radiation pattern structure. It was observed that as the complexity of the antenna increased, the number of measurements required for a good reconstruction also increased. Two basis matrices, the DFT and the DCT, were evaluated for the reconstruction of radiation patterns. It can be concluded that the DFT matrix performs a better reconstruction of the radiation patterns. Information about the number of samples required to reconstruct the radiation pattern is obtain by simulating the radiation pattern and running the compressive sensing algorithm a priori. Numerical simulations indicates that 2D or 3D radiation patterns can be reconstructed using between 20% and 44% of the total number of measurements.

The algorithm was evaluated empirically on a pyramidal horn antenna, where 2D and 3D radiation patterns measured in an anechoic chamber were reconstructed well. The proposed reconstruction algorithm can thus be used to reconstruct 2D or 3D radiation patterns with a reduced number of required far-field measurements. This, in turn, reduces the time required to take the measurements.

The implementation of the algorithm requires the implementation of a new measurement paradigm that will perform random measurements in the anechoic chamber. By using the proposed methodology, the time required to measure radiation patterns can be reduced without significant loss of accuracy. The number of measurements required to obtain a good reconstruction using compressive sensing depends on the complexity of the radiation pattern. That is, it depends on the number of K non-zero coefficients in the sparsity domain representation of the radiation patterns.

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