Optimization of Graded Materials for Broadband Radome Wall with DRR Control Using a Hybrid Method

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Abstract—A graded material structure is optimized for broadband radome application by using hybrid method in this paper. In the optimization, dynamic range ratio (DRR) of real permittivity and loss of material are taken into considerations. By using an analytical function, the optimization problem with the DRR constraint is converted to an unconstrained problem. The proposed hybrid method is a combination of trust region method (TRM) and genetic algorithm (GA). Firstly (TRM) is applied to optimize the dielectric constant distribution. Then the result of TRM is used as initial value of GA. GA is employed to improve the global property of the results provided by TRM. Because TRM has the advantage of fast searching speed and GA has the advantage of global property, the hybrid method has the feature of fast convergence speed. And the jitter property of GA permittivity distribution is moderated. The effectiveness of the hybrid is validated through the designs of two broadband radome walls. The minimum power transmission efficiency is 81.9% ranging from 1 GHz to 18 GHz for normal incidence.

1. INTRODUCTION

Radome has been widely applied to protect antenna against bad environments such as: terrible weather, high pressure, high temperature and so on [1, 2]. Both electromagnetic (EM) property and mechanical property are critical for a radome. Usually broadband radome wall is more challenging to design than that of narrow band because of the resonance characteristics of microwave. Though thin wall structure (i.e., short electrical size) can avoid the resonance characteristics, it's hard to meet the requirements of mechanical load. For past several years, attentions have been paid to the study of inhomogeneous media due to its potentiality for enlarging bandwidth [3–5]. Many methods have been proposed to design broadband graded media radome walls. Generally, they can be classified into three types of methods, such as analytical methods [6–9], hypotheses formulation methods [10, 11] and numerical optimization methods [11–13].

The analytical methods perform well in calculation efficiency and transmission efficiency. In [6, 7], an exponential function derived from Maxwell equation is utilized to design variations of dielectric constant across radome wall. The transmission efficiency is over 90% in X band at normal and 60° incident angles. In [8], closed-form design formulas are proposed to design inhomogeneous dielectric filter by using equivalent circuit methods. The drawback of the analytical methods is that it cannot control the minimum permittivity of the inhomogeneous media. For example, in [7], the permittivity of the outmost wall surface is 1.0 and near the outmost surface many points of the dielectric permittivity are less than 1.1. This structure has bad mechanical performance.

The hypotheses formula methods are relative simple. Firstly, the variation of the permittivity across the slab is predefined by a function. Then the best permittivity distribution will be found by sweeping

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the parameters of the function. In [10], a novel broadband graded porous Si_3N_4 wall structure with 4 layers is designed ranging from 1 GHz to 18 GHz. The power transmission efficiency is higher than 70% at normal incident angle. In [11], a broadband symmetrical porous structure is proposed. Its power transmission efficiency is higher than 80% with normal incident angle ranging from 1 GHz–100 GHz. The thermo-mechanical investigation shows that it meets the requirement for high temperature applications. But the dielectric loss has not been taken into consideration in this study. The transmission performance will degrade quickly once this is accounted.

The numerical optimization methods are more flexible than the above two types of methods. Nowadays, many commercial electromagnetic (EM) simulation software packages such as: ANSYS_HFSS, CST and so on, have their own optimization modules. They can simulate the transmission efficiency of single or multi-layer homogeneous materials. But it is difficult to construct a graded material model in these software packages because of the complexity of dielectric constant distribution. In [12], a multi-objective evolutionary algorithm is presented to design multi-layer dielectric filter. When the permittivity of each layer is given, its thickness is optimized at a certain incident angle. In [13], sequential quadratic programming method is employed to optimize wideband inhomogeneous planner layers. An integration equation is applied to constrain the minimum permittivity. The equation can ensure the average permittivity be over a certain value. However, the optimization results indicate that many points are equal to 1.0. The permittivity distribution is bad from the point view of mechanical strength for radome wall.

The convergence speed of stochastic method is low in [12] and the jitter of its optimum result is serious. But it has good global property. In [13], the optimization method is a deterministic method. It is a local optimization method. It has better convergence speed and the optimization result is smoother compared with stochastic method. While its objective function must be differentiable.

In order to speed up the optimization efficiency and control the minimum permittivity, a hybrid method which combines the trust region method (TRM) [14] and the genetic algorithm (GA) [15, 16] is proposed to design broad band graded materials radome wall in this paper. The distribution of permittivity solved by TRM is relatively smooth. And this distribution will be used as initial values for GA. Then the global property is improved by GA. Different from paper [13], power transmission factor is optimized rather than reflector factor. Because of dielectric loss of materials, the searching of minimum reflection factor is not equivalent to the searching of maximum transmission factor. In order to keep good mechanical performance, dynamic ratio range (DRR) of permittivity is defined by an analytical function. The constrained optimization problem is converted into an unconstrained optimization problem by using this analytical function. Moreover, the thickness of the outmost surface radome wall is fixed using a homogeneous dielectric.

The remainder of this paper is organized as follows: Section 2 deals with the EM performance analysis of graded materials. A hybrid optimization method of broadband graded materials is proposed in Section 3. Several optimization cases are given to validate the effectiveness of the hybrid method in Section 4. Conclusions are drawn in the final section.

2. EM PERFORMANCE ANALYSIS OF GRADED MATERIALS

A graded material slab can be approximated as a multi-layer dielectrics slab where the electrical size of each layer is very thin. EM analysis methods for multi-layer dielectric slab can be employed in the analysis of graded materials slab. Transfer matrix method [17] has been widely applied to analyze both transmission and reflection performance of multi-layer dielectrics slab. Compared with other methods [18, 19], it is simple and has acceptable accuracy.

The structure of a multi-layer dielectric slab is shown in Fig. 1. Consider a plane wave with an angle of incidence θ propagating through an N layers dielectric slab. Assume the relative permittivity and thickness of ith layer dielectric be $\varepsilon_i = \varepsilon_{ri}(1 - j \tan \delta_i)$ and d_i, respectively. The voltage-current matrix can be written as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^{N} \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}, \tag{1}$$
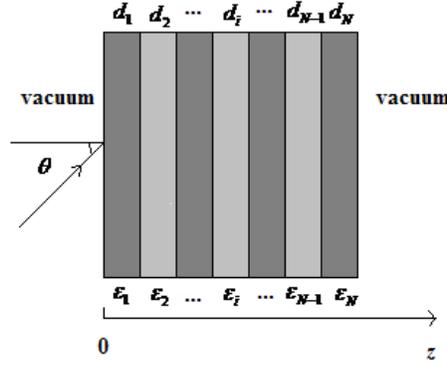
Figure 1. Structure of a multi-layer dielectric slab.

where

$$\begin{cases} A_i = D_i = \cos \beta_i d_i \\ B_i = jZ_i \sin \beta_i d_i \\ C_i = j \sin \beta_i d_i / Z_i \end{cases}, \tag{2}$$

$$Z_i = \begin{cases} \sqrt{\varepsilon_i - \sin^2 \theta}/(\varepsilon_i \cos \theta), & TE \\ \cos \theta / \sqrt{\varepsilon_i - \sin^2 \theta}, & TM \end{cases}, \tag{3}$$

where $\beta_i = k\sqrt{\varepsilon_i - \sin^2 \theta}$, k is the wave number in free space. The electric field transmission factor T and the reflection factor R are given by

$$\begin{cases} T = 2/(A + B + C + D) \\ R = (A + B - C - D)/(A + B + C + D) \end{cases}. \tag{4}$$

The power transmission efficiency is given by $|T|^2$. Eq. (3) indicates that when the incident angle is not normal (i.e., $\theta \neq 0$), the EM performance of TE polarization and TM polarization is different.

3. DESIGN OF BROAD BAND GRADED MATERIALS BY USING HYBRID METHOD

In this section, a hybrid optimization approach for broadband graded materials is proposed. First, dynamic range ratio (DRR) of permittivity is defined. Then the DRR constraint is removed by using an analytical function. The complex permittivity of the graded material is predicted. At the end of this section, two different objective functions are given.

Before the optimization process, the maximum real part of relative permittivity $\varepsilon_{r\,\max}$, the minimum real part of relative permittivity $\varepsilon_{r\,\min}$ and the maximum tangent loss $\tan \delta_{\max}$ should be given. $\varepsilon_{r\,\min}$ is decided by mechanical strength. The DRR of real part of relative permittivity is defined by

$$DRR = \frac{\varepsilon_{r\,\max}}{\varepsilon_{r\,\min}}, \tag{5}$$

where $\varepsilon_{r\,\max} = \max\{\varepsilon_{ri}\}$, $\varepsilon_{r\,\min} = \min\{\varepsilon_{ri}\}$ and $i = 1 \ldots N$.

In order to convert the optimization problem with DRR constraint into an unconstrained optimization problem, an analytical function is given by

$$\varepsilon_{ri} = \varepsilon_{r\,\max} - \varepsilon_{r\,\min}(DRR - 1)\sin^2 x_i, \tag{6}$$

where x_i is the optimization variable.

Graded material is usually achieved by porous material with different porosities or by mixture material with different complex permittivity. The accuracy of graded material effective complex

permittivity is critical for a successful design of wideband graded material. Many approaches [20–23] can be applied to predict effective complex permittivity of mixture or porous material, besides measurement methods. In this paper, the focus is on how to optimize wideband graded material, so a simple effective complex permittivity prediction method is applied. The relative permittivity of porous material can be approximated by [20]

$$\varepsilon_i = (1 - g_i) + g_i \varepsilon_{r\,\max}(1 - j \tan \delta_{\max}), \tag{7}$$

where g_i is the relative volume of space occupied by the $\varepsilon_{r\,\max}(1 - j \tan \delta_{\max})$, and it is an intermediate variable here. Using Eq. (7) and $\varepsilon_i = \varepsilon_{ri}(1 - j \tan \delta_i)$, g_i can be derived by

$$g_i = \frac{\varepsilon_{ri} - 1}{\varepsilon_{r\,\max} - 1}. \tag{8}$$

Then, the tangent loss can be derived by

$$\tan \delta_i = \frac{\varepsilon_{r\,\max}}{\varepsilon_{ri}} g_i \tan \delta_{\max}, \tag{9}$$

where $\tan \delta_{\max}$ is the maximum tangent loss corresponding to $\varepsilon_{r\,\max}$.

When x_i is given, $\tan \delta_i$ can be solved by combining Eqs. (6), (8) and (9). Therefore the relative permittivity ε_i can be expressed by x_i.

In the following part of this section, the optimization process of wideband graded material slab will be illustrated. The goal of design a broadband graded material slab is to find a permittivity distribution so that the minimum transmission factor of the slab is the maximum with a certain frequency range and incident angle. This problem can be solved by minimizing an objective function φ, which can be expressed as

$$\varphi(\mathbf{X}, \theta) = \max \left\{ \left(1 - \left|T_j^{pol}\right|^2\right) \right\}, \quad j = 1, \ldots, M; \; pol = TE, TM, \tag{10}$$

where $\mathbf{X} = [x_1, \ldots, x_i, \ldots x_N]$ is the optimization variables in (6), and T_j^{pol} is the transmission factor with incident angle θ, frequency f_j, and polarization pol. T_j^{pol} is calculated by using Eq. (4). The number of frequency sampling points is M.

φ is not a continuous function. It can be minimized by using random searching method. In this paper, φ is minimized by using GA. GA has good global property. Its objective function is flexible and does not need to be continuous. But it has a drawback of low searching efficiency. And according to our numerical results, the optimum permittivity distribution obtained by using GA usually has serious jitter.

In order to improve the optimization efficiency of GA and moderate the jitter of GA permittivity distribution, TRM is employed to initialize the starting points of GA. TRM does well in high searching speed because it is one of the gradient-based methods. The optimum permittivity distribution obtained by TRM result is relatively smooth. But TRM is a local optimization method, and the objective function should be a continuous differentiable formula. If we use the results of TRM as initial values for GA, it will speed up the GA and moderate the jitter of GA optimization result. The goal of TRM is to minimize the objective function which is given by

$$\phi(\mathbf{X}, \theta) = \sum_{j=1}^{M} \sum_{pol=TE}^{TM} (1 - |T_j^{pol}|^2)^2. \tag{11}$$

TRM needs the computation of the gradient and Hessian of the objective function. They can be calculated by finite-difference approximation because it is difficult to be analytically solved. Using the TRM, the local minimum of ϕ can be solved.

The flowchart of the hybrid method is illustrated in Fig. 2. In the flowchart, w_a and w_b are the convergence thresholds of TRM and GA respectively. The minimizing of ϕ is to search a complex permittivity distribution in order to minimize the distance between TE, TM power transmission factor curves and line 1. The minimizing of φ is to search a complex permittivity distribution so as to minimize the distance between TE or TM power transmission factor point and 1. Because the objective function (11) is weaker than (10), TRM is employed at the earlier phase of the optimization, and GA is used at the later phase.
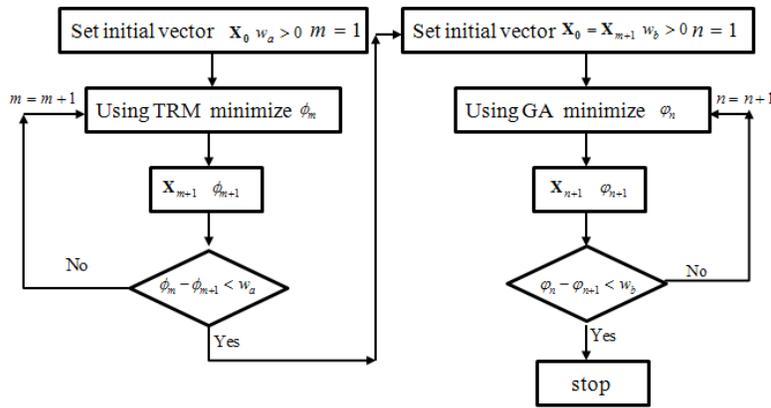
Figure 2. Flowchart of the proposed hybrid method.

4. NUMERICAL RESULTS AND DISCUSSIONS

In this section, several cases are studied to demonstrate the effectiveness of the hybrid method. In Section 4.1, the focus is on the optimization cases with normal incident angle, the results of different methods are compared. In Section 4.2, optimization cases with large incident angle will be studied.

The frequency ranges from 1 GHz to 18 GHz. The number of frequency sampling points is $M = 171$. In order to guarantee mechanical property of a graded material slab, silicon nitride (Si_3N_4) with a certain thickness is employed as its outmost layer with relative permittivity $\varepsilon_1 = 7(1 - j0.006)$. The maximum complex relative permittivity is the same as the outmost layer with $\varepsilon_{r\,\max} = 7$, $\tan\delta_{\max} = 0.006$. The ideal minimum real relative permittivity should be set to be $\varepsilon_{r\,\min} = 1$. But on the consideration of mechanical strength, the minimum real relative permittivity is set to be $\varepsilon_{r\,\min} = 1.2$. By using Eq. (5), the DRR of real relative permittivity is 5.83 in the optimization procedure. The total thickness of graded material wide band radome wall is $D = 20\,\text{mm}$.

4.1. Normal Incident Angle

For the case of normal incident angle, the power transmission efficiency performance of TM wave is the same as that of TE wave. Set the thickness of outmost layer be $d_1 = 1.2\,\text{mm}$.

Firstly, the objective function (11) is minimized by using TRM. Assume that the thickness of the remaining each layer d_i ($i = 2 \ldots N$) is equal. The starting point is $x_i = \pi/3$ ($i = 2 \ldots N$). The optimum complex permittivity distributions with different numbers of discrete layers $N = 60, 100$ are shown in Fig. 3. It shows that optimized results by using 60 layers agree well with the results by using 100 layers. Thus in order to save optimization time, we use 60 layers structure in the following numerical experiments.

The black square line in Fig. 4 gives the optimum power transmission efficiency by using TRM with $N = 60$. The minimum power transmission efficiency is 0.751 when the frequency is 18 GHz.

Secondly, in order to improve the optimum result provided by TRM, GA is applied to minimize the objective function (10) with the initial values provided by TRM. The thickness of each layer d_i ($i = 2 \ldots N$) is uniform and the discrete number is $N = 60$. The population size of GA is 200. The blue triangle line in Fig. 4 gives the optimum power transmission efficiency solved by the hybrid method. The minimum power transmission efficiency of hybrid method is 0.819 when the frequency is around 16 GHz.

In order to make a comparison, GA is directly applied to minimize the objective function (10) with random initializations. The thickness of each layer d_i ($i = 2 \ldots N$) is uniform and the discrete number is $N = 60$. The population size of GA is 200. The settings of this GA are the same as that of the GA in the hybrid method except the initial values. The red circled line in Fig. 4 gives the optimum power transmission efficiency solely solved by GA. The minimum power transmission efficiency of GA is 0.797 when the frequency is around 15 GHz.
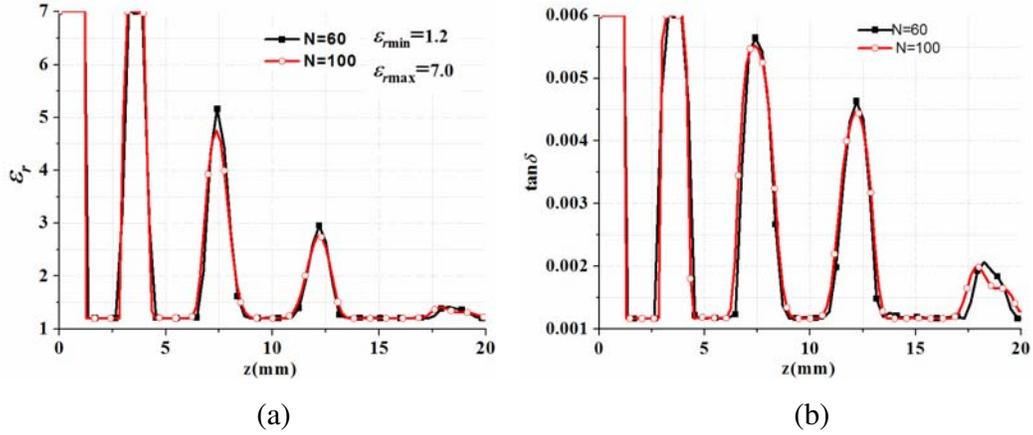
(a)   (b)

Figure 3. The relative permittivity distributions across the thickness optimized by TRM with dissected by different numbers of layers, DRR $= 5.83$, $\theta = 0°$, $d_1 = 1.2$ mm. (a) Real part of relative permittivity distributions. (b) Tangent loss distributions.
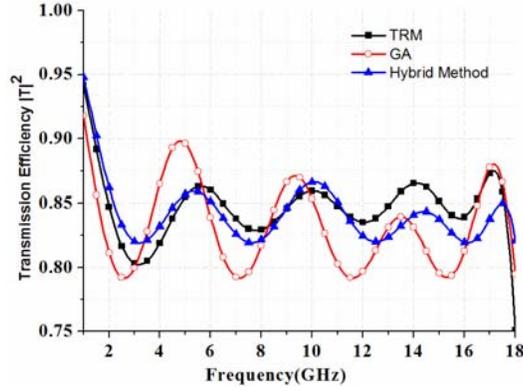

Figure 4. Comparison of optimum power transmission efficiency by using TRM, GA and the hybrid method with 60 layers structure, normal incidence, and $d_1 = 1.2$ mm.

Compared with the results shown in Fig. 4, the result provided by using the hybrid method has the best power transmission efficiency across the frequency range from the point view of the minimum power transmission efficiency.

The real relative permittivity distributions and the tangent loss across the thickness optimized by three different methods with DRR $= 5.83$ are shown in Figs. 5(a) and 5(b). It is observed that the relative permittivity distribution of TRM is the smoothest among the three results. The GA result has the largest oscillation. This is because TRM is a determination optimization algorithm and GA is a stochastic searching algorithm. If the jitter of relative permittivity distribution is too severe, it will be difficult to be realized by graded material. The result of hybrid method is very close to that of the TRM result.

Table 1 gives the comparisons of different methods on minimum power transmission factor and computational time. It indicates that the hybrid method has the best minimum power transmission factor. The hybrid method has better computational efficiency than GA.

4.2. Large Incident Angle

The hybrid method is applied to optimize the complex permittivity distributions when the incident angle is $\theta = 60°$. The thickness of the outmost layer is $d_1 = 0.8$ mm and $d_1 = 1.2$ mm, respectively. Each of the starting point of TRM is $x_i = \pi/3$ $(i = 2 \ldots N)$ and the population size of GA is 200. The

Table 1. Comparisons of different methods.

	TRM	GA	Hybrid method
Minimum $|T|^2$	0.751	0.797	0.819
Computational Time(s)	555	5136	1727
Convergence threshold	$w_a = 10^{-6}$	$w_b = 10^{-6}$	$w_a = 10^{-6}$, $w_b = 10^{-6}$
Other Parameters	$D = 20\,\text{mm}$, $d_1 = 1.2\,\text{mm}$, $\varepsilon_r \in [1.2, 7]$, $\theta = 0$, 1–18 GHz		

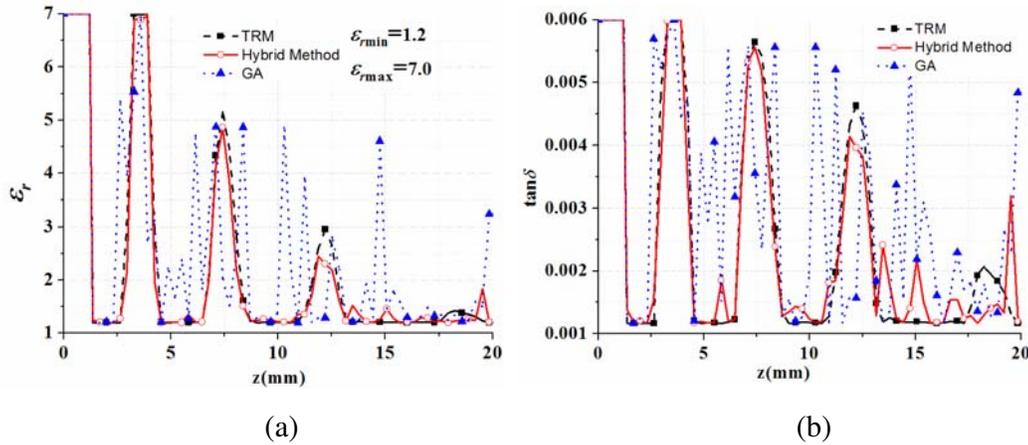

(a)   (b)

Figure 5. The relative permittivity distributions across the thickness by using TRM, GA and the hybrid method, with 60 layers structure, normal incidence, and $d_1 = 1.2\,\text{mm}$. (a) Real part of relative permittivity distributions. (b) Tangent loss distributions.
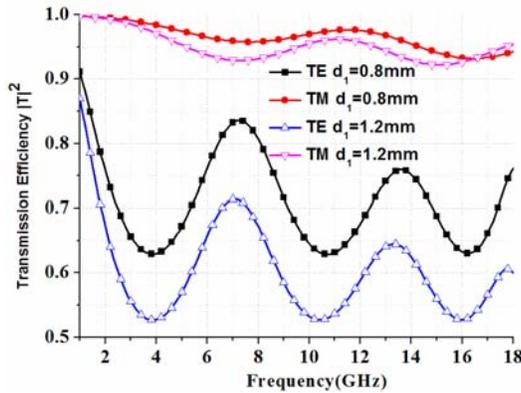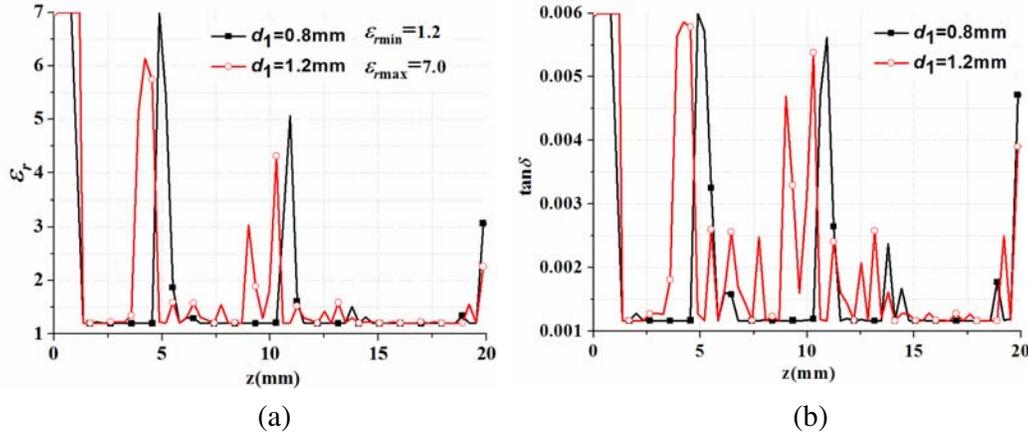

Figure 6. Power transmission efficiency by using the hybrid method with different thicknesses of the outmost layer, $N = 60$, $\theta = 60°$, $d_1 = 0.8\,\text{mm}$ and $d_1 = 1.2\,\text{mm}$.

convergence thresholds are $w_a = 10^{-6}$, $w_b = 10^{-6}$. The TE and TM optimum transmission efficiency curves with different outmost thicknesses are shown in Fig. 6. The minimum power transmissions of different polarizations and different outmost thicknesses are shown in Table 2. It shows that when the outmost layer becomes thicker, the optimum TE power transmission efficiency becomes lower. Because the tangent loss of the outmost layer is the maximum, the thickness of the outmost homogeneous layer should be as thin as possible when the mechanical strength requirement is satisfied.

The real part of relative permittivity distribution and the tangent loss distribution across the thickness are shown in Figs. 7(a) and 7(b). By comparing Fig. 6 and Fig. 7, it indicates that the peaks

Table 2. Comparisons on different outmost thicknesses.

Thickness of d_1	$\min\{|T_i^{TE}|^2\}$	$\min\{|T_i^{\mathrm{TM}}|^2\}$
$d_1 = 0.8\,\mathrm{mm}$	0.629	0.932
$d_1 = 1.2\,\mathrm{mm}$	0.527	0.922

(a)   (b)

Figure 7. The relative permittivity distribution across the thickness, $N = 60$, $\theta = 60°$, $d_1 = 0.8\,\mathrm{mm}$ and $d_1 = 1.2\,\mathrm{mm}$. (a) Real relative permittivity distribution. (b) Tangent loss distribution.

of TE power transmission efficiency correspond to the peaks of the permittivity distribution. Because the comparison between different methods is similar to that in Section 4.1, it isn't repeated here again.

5. CONCLUSIONS

A hybrid method with the combination of TRM and GA is proposed for broadband graded material optimization. This method employs the results of TRM as initial values of GA. Benefited from the fast convergence speed of TRM and global property of GA, this method has the advantages of fast convergence and less jittering in the optimum results. In the consideration of mechanical strength, DRR of permittivity constraint is added. The constrained problem is converted into an unconstrained problem by using an analytical function. The loss of the graded material is taken into account as well. The power transmission efficiency is directly optimized in an objective function. The effectiveness of the hybrid method performance is demonstrated by the designs of two 1 GHz–18 GHz wide band radome walls with different incident angles.

REFERENCES

1. Kozakoff, D. J., *Analysis of Radome Enclosed Antennas*, Artech House, Norwood, MA, 1997.
2. Meng, H. F. and W.-B. Dou, "A hybrid method for the analysis of radome-enclosed horn antenna," *Progress In Electromagnetics Research*, Vol. 90, 219–233, 2009.

3. Vegni, L. and A. Toscano, "Full-wave analysis of planar stratified with inhomogeneous layers," *IEEE Trans. Antennas Propag.*, Vol. 48, No. 4, 631–633, 2000.

4. Nair, R. U., M. Suprava, and R. M. Jha, "Graded dielectric inhomogeneous streamlined radome for airborne applications," *Electronics Lett.*, Vol. 51, No. 11, 862–863, 2015.

5. Nair, R. U. and R. M. Jha, "Electromagnetic design and performance analysis of airborne radomes: Trends and perspectives," *IEEE Antennas and Propag. Magazine*, Vol. 56, No. 4, 276–298, 2014.

6. Philippe, A. E., "Reflection and transmission of radio waves at a dielectric slab with variable permittivity," *IEEE Trans. Antennas Propag.*, Vol. 21, No. 2, 234–236, 1973.

7. Nair, R. U., S. Shashidhara, and R. M. Jha, "Novel inhomogeneous planar layer radome design for airborne applications," *IEEE Antennas Wirel. Propag. Lett.*, Vol. 11, 854–856, 2012.

8. Bilotti, F., A. Toscano, and L. Vegni, "Very fast design formulas for microwave nonhomogeneous media filters," *Microw. Opt. Tech. Lett.*, Vol. 22, No. 3, 218-221, 1999.

9. Toscano, A., L. Vegni, and F. Bilotti, "A new efficient method of analysis for inhomogeneous media shields and filters," *IEEE Trans. Electromagn. Compat.*, Vol. 43, No. 3, 394–399, 2001.

10. Chen, F., Q. Shen, and L. Zhang, "Electromagnetic optimal design and preparation of broadband ceramic radome material with graded porous structure," *Progress In Electromagnetics Research*, Vol. 105, 445–461, 2010.

11. Zhou, L., Y. Pei, R. Zhang, and D. Fang, "Optimal design for high-temperature broadband radome wall with symmetrical graded porous structure," *Progress In Electromagnetics Research*, Vol. 127, 1–14, 2012.

12. Venkatarayalu, N. V., T. Ray, and Y. B. Gan, "Multi-layer dielectric filter design using a multiobjective evolutionary algorithm," *IEEE Trans. Antennas Propag.*, Vol. 53, No. 11, 3625–3632, 2005.

13. Amirhosseini, M. K., "Wideband flat radomes using inhomogeneous planar layers," *International Journal of Antennas and Propagation*, Vol. 2008, 1–6, 2008.

14. Higham, D. J., "Trust region algorithms and time step selection," *SIAM Journal of Numerical Analysis*, Vol. 37, No. 1, 194–210, 1999.

15. Goldberg, D. E., *Genetic Algorithms in Search, Optimization & Machine Learning*, Addison-Wesley, Netherlands, 1989.

16. Deb, K., A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. on Evolutionary Computation*, Vol. 6, No. 2, 182–197, 2002.

17. Kong, J. A., *Electromagnetic Wave Theory*, John Wiley & Sons Inc., New York, 1986.

18. Khalaj-Amirhosseini, M., "Analysis of lossy inhomogeneous planar layers using finite difference method," *Progress In Electromagnetics Research*, Vol. 59, 187–198, 2006.

19. Zarifi, D., M. Soleimani, and A. Abdolali, "State transition matrix of inhomogeneous planar layers," *IET Microw., Antennas & Propag.*, Vol. 9, No. 4, 301–306, 2015.

20. Kuester, E. F. and C. L. Holloway, "A low-frequency model for wedge or pyramid absorber arrays-I: Theory," *IEEE Trans. Electromagn. Compat.*, Vol. 36, No. 4, 300–306, 1994.

21. Garnett, J. C. M., "Colors in metal glasses and metal films," *Trans. Roy. Soc.*, Vol. 53, 385–420, 1904.

22. Karkkainen, K. K. A. H., Sihvola, and K. I. Nikoskinen, "Effective permittivity of mixtures: Numerical validation by the FDTD method," *IEEE Trans. on Geoscience and Remote Sensing*, Vol. 38, 1303–1308, 2000.

23. Jebbor, N. and B. Seddik, "Effective permittivity of periodic campsite material: Numerical modeling by the finite element method," *Journal of Electrostatics*, Vol. 70, 393–399, 2012.