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Transmission through Double Positive — Dispersive Double Negative Chiral Metamaterial Structure in Fractional Dimensional Space

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Abstract—This paper presents the frequency response of a stratified structure consisting of double-positive and dispersive double-negative chiral metamaterial layers. The structure is inserted between two half-spaces of fractional dimensions. Transfer matrix approach is used for the analysis. Dispersion within the double-negative chiral layers is realized by using Lorentz/Drude model. The effect of fractionality of the dimension is particularly investigated. Numerical results, for a five layer structure, are presented for various parametric values of the stratified structure and fractionality of the host media. It is shown that the fractionality of host media can be used as yet another parameter to control the frequency response of such a filtering structure. For integral values of dimensions, the results are shown to converge to the classical results thus validating the analysis.

1. INTRODUCTION

Fractional space has proved to be an extremely useful concept in many areas of physics, including electromagnetics, casting new problems and finding novel solutions to the existing ones [1–5]. Indeed, many problems already solved for integral dimensional space have recently been recast into fractional space paradigm [6–8]. The development made in the area of fractional calculus has been particularly helpful in carrying out such analyses [9–12]. It is worthwhile to mention that many natural objects, such as clouds, snowflakes, rough surfaces, cracks, turbulence in fluids, are aptly described by dimensions of fractional order [13]. Therefore, wave propagation in such media is best characterized by considering an effective space of non-integer dimensions. In many areas of application of electromagnetic theory such as remote sensing, communication and navigation, the study of wave propagation and scattering from fractal media becomes very important. Several investigations in this direction has been reported recently. For example, electromagnetic fields in fractional space are discussed in [14], the scattering of electromagnetic waves in fractal media is given in [15] and electromagnetic Green's function for fractional space is presented in [16]. Solutions for plane, cylindrical and spherical waves in fractal media are given in [17]. One way of realizing the fractional order dimensional space could be using the fractal media. In general, the fractal media cannot be considered as a continuous media, however, Tarasov [18] purposes a model and experimental testing for treating the fractal media as a continuous media thus paving the way for fractal media being considered as fractional space on all scales. Marwat and Mughal also used fractional dimension space for terahertz range of frequency in [19]. We, however, treated the problem theoretically and are unaware of any practical realization of the fractional space so far. We, in this article, report the effects of fractionality of the space on the frequency response of a stratified metamaterial structure.

Metamaterials are artificially engineered composites exhibiting peculiar electrical properties not otherwise found in the naturally occurring in constituent materials [20, 21]. Multilayered forms of the

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metamaterial are found to be suitable for design of many devices including polarizers and filters. Double-negative (DNG) metamaterials are structures having simultaneously negative values of permittivity and permeability, giving rise to the so called negative refraction phenomenon which in turns leads to negative direction of propagation in such materials [22, 23]. The concept of (DNG) media has achieved remarkable importance within the scientific and engineering communities due to their unusual properties observed for some microwave, millimeter and optical frequency bands. Chiral medium is another example of metamaterials constructed from numerous randomly oriented chiral elements; that is, the objects which can never be brought into congruence with their mirror images by any translation or rotation. A chiral medium provides a cross-coupling between the electric and magnetic fields the extent of which is determined by a chirality parameter [19, 24]. DNG chiral metamaterials, therefore, are defined as materials having both the negative permittivity and permeability and chirality in their characteristics [25, 26]. In double-positive (DPS) materials Permittivity and permeability are both positive and wave propagation is in the forward direction. The aim of present study is the theoretical analysis of electromagnetic wave propagation in a layered structure composed of alternate DPS-DNG chiral layers embedded in a host fractional medium. The DNG chiral materials are realized mathematically by employing the Lorentz/Drude models, which incidentally also allow the incorporation of frequency dispersive parameters [27, 28]. The transfer matrix method approach is used to determine the transmitted fields through the structure for various incidence angles and frequencies. The main attention is given to the effect of fractionality of the host medium on the overall frequency response of such a structure in relation with the permittivity and chirality of different media.

The problem geometry and formulation is presented in Section 2. Also by using transfer matrix method the expressions for reflected and transmitted power are derived in this section. In Section 3, numerical results for transmission characteristics of a five layered structure placed in various fractional dimensional spaces are presented. The cases for dispersive lossless and dispersive lossy layers are also incorporated. The results are also shown to agree with already published ones if the dimension of the substrate is taken to be of integral values. Finally, the paper is concluded in Section 4.

2. FORMULATION

Consider a planar layered structure sandwiched between two half-spaces as shown in Figure 1. It is assumed that the left half space and right half space have non-integral dimensions (specifically, in the z-direction of the Cartesian coordinates), whereas space occupied by each layer is assumed to be ordinary Euclidean space. Layers are of dispersive Double Negative chiral (DNG chiral) and/or Double Positive (DPS) metamaterial. Dispersion in DNG chiral layers is realized by employing Lorentz/Drude models with constitutive parameters given below [27]

$$\varepsilon(\omega) = \varepsilon_o\left(1 - \frac{\omega_{ep}^2}{\omega^2 + i\omega\delta_e}\right) \tag{1}$$

$$\mu(\omega) = \mu_o\left(1 - \frac{F_c\omega_{mp}^2}{\omega^2 - \omega_{mo}^2 + i\omega\delta_m}\right), \tag{2}$$

here ω_{ep} is the electric plasma frequency, δ_e the electric damping frequency, ω_{mp} the magnetic plasma frequency, ω_{mo} the magnetic resonance frequency, δ_m the magnetic damping frequency, and F_c the filling parameter. Quantities ε_o, μ_o are free-space constitutive parameters. It is also assumed that front face of the structure is located at $z = \mathrm{d}$. Thickness of a layer is denoted by d_m and the interface is termed as I_m, where $m = 1, 2, \ldots$ stands for the m-th layer or the m-th interface. Constitutive parameters for host mediums filling left and right half spaces are denoted as $\varepsilon_{h1}, \mu_{h1}$ and $\varepsilon_{h2}, \mu_{h2}$, respectively. The wavenumber and impedance of the left half space are $k_{h1} = \omega\sqrt{\mu_{h1}\varepsilon_{h1}}$, and $\eta_{h1} = \sqrt{\mu_{h1}/\varepsilon_{h1}}$, respectively. The wavenumber and impedance of the right half space are $k_{h2} = \omega\sqrt{\mu_{h2}\varepsilon_{h2}}$, and $\eta_{h2} = \sqrt{\mu_{h2}/\varepsilon_{h2}}$, respectively.

Interface located at $z = \mathrm{d}$ is excited by a linearly polarized time harmonic electromagnetic plane wave. The expressions for incident and reflected electromagnetic plane waves in left fractional half-space

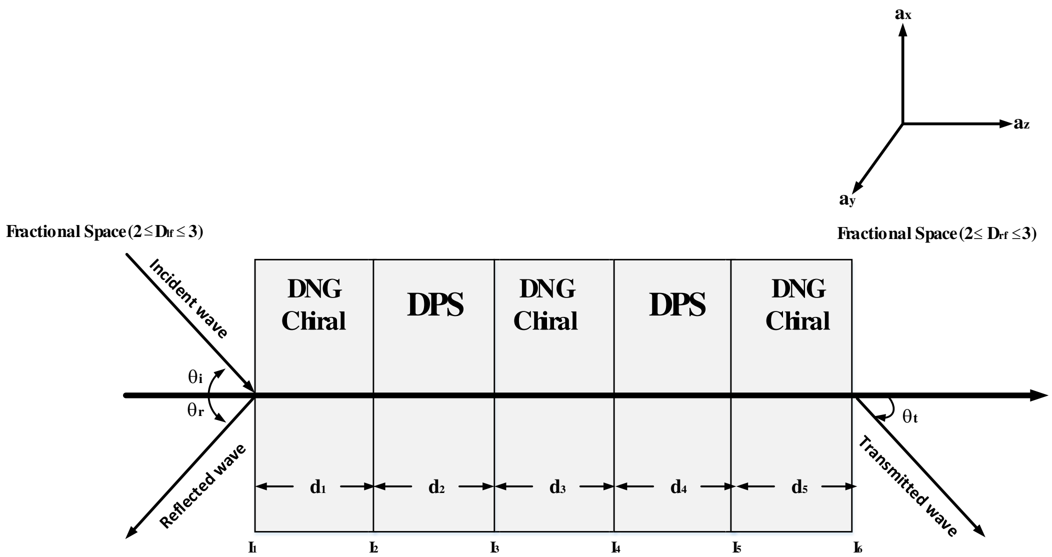

Figure 1. The layered structure of dispersive DNG chiral and DPS slabs sandwiched between two fractional half-spaces ($2 \leq D_{\text{lf,rf}} \leq 3$).

are [19]

$$\mathbf{E}_\text{i} = \left[E_{\shortparallel}^{+}(\hat{a_x}\cos\theta_\text{i} + \hat{a_z}\sin\theta_\text{i}) + E_{\perp}^{+}\hat{a_y}\right]\exp(-ik_{h1}(-\sin\theta_\text{i}x))H_1 \tag{3}$$

$$\mathbf{E}_\text{r} = \left[E_{\shortparallel}^{-}(\hat{a_x}\cos\theta_\text{r} - \hat{a_z}\sin\theta_\text{r}) + E_{\perp}^{-}\hat{a_y}\right]\exp(-ik_{h1}(-\sin\theta_\text{r}x))H_2 \tag{4}$$

$$\mathbf{H}_\text{i} = (1/\eta_{h1})\left[E_{\shortparallel}^{+}\hat{a_y} - E_{\perp}^{+}(\hat{a_x}\cos\theta_\text{i} + \hat{a_z}\sin\theta_\text{i})\right]\exp(-ik_{h1}(-\sin\theta_\text{i}x))H_3 \tag{5}$$

$$\mathbf{H}_\text{r} = (1/\eta_{h1})\left[-E_{\shortparallel}^{-}\hat{a_y} + E_{\perp}^{-}(\hat{a_x}\cos\theta_\text{r} - \hat{a_z}\sin\theta_\text{r})\right]\exp(-ik_{h1}(-\sin\theta_\text{r}x))H_4. \tag{6}$$

Here subscripts + and − represent fields traveling in forward and backward z-directions, respectively, and $\shortparallel, \perp$ represent parallel and perpendicularly polarized components of electric field vector. The incident angle with respect to normal to the planar interface is denoted as θ_i. The reflected angle θ_r can be calculated by using Snell's law of reflection. $E_{\shortparallel}^{-}$ and $E_{\perp}^{-}$ are unknown coefficients to be determined. According to the incident electric field given in Eq. (3), transmitted electric and magnetic fields in right fractional half space can also be written in terms of unknown coefficients as follows:

$$\mathbf{E}_\text{t} = \left[E_{\shortparallel}^{t+}(\hat{a_x}\cos\theta_\text{t} + \hat{a_z}\sin\theta_\text{t}) + E_{\perp}^{t+}\hat{a_y}\right]\exp(-ik_{h2}(-\sin\theta_\text{t}x))H_5 \tag{7}$$

$$\mathbf{H}_\text{t} = (1/\eta_{h2})\left[E_{\shortparallel}^{t+}\hat{a_y} - E_{\perp}^{t+}(\hat{a_x}\cos\theta_\text{t} + \hat{a_z}\sin\theta_\text{t})\right]\exp(-ik_{h2}(-\sin\theta_\text{t}x))H_6, \tag{8}$$

where θ_t is the transmitted angle, and $E_{\shortparallel}^{t+}$ and $E_{\perp}^{t+}$ are the unknown coefficients. The angle of transmission can be calculated by using the Snell's law of refraction. Quantities H_s $(s = 1, 2, 3, 4, 5, 6)$ used in above expressions are given below

$$\begin{cases} H_1 = (k_{h1}z\cos\theta_\text{i})^{n_\text{lf}}[\mathscr{H}_{n_\text{lf}}^2(k_{h1}z\cos\theta_\text{i})], \\ H_2 = (k_{h1}z\cos\theta_\text{r})^{n_\text{lf}}[\mathscr{H}_{n_\text{lf}}^1(k_{h1}z\cos\theta_\text{r})], \\ H_3 = (k_{h1}z\cos\theta_\text{i})^{nh_\text{lf}}[\mathscr{H}_{nh_\text{lf}}^2(k_{h1}z\cos\theta_\text{i})], \\ H_4 = (k_{h1}z\cos\theta_\text{r})^{nh_\text{lf}}[\mathscr{H}_{nh_\text{lf}}^1(k_{h1}z\cos\theta_\text{r})], \\ H_5 = (k_{h2}z\cos\theta_\text{t})^{n_\text{rf}}[\mathscr{H}_{n_\text{rf}}^2(k_{h2}z\cos\theta_\text{t})], \\ H_6 = (k_{h2}z\cos\theta_\text{t})^{nh_\text{rf}}[\mathscr{H}_{nh_\text{rf}}^2(k_{h2}z\cos\theta_\text{t})], \end{cases} \tag{9}$$

here $n_\text{lf,rf} = \frac{|3-D_\text{lf,rf}|}{2}$, $nh_\text{lf,rf} = \frac{|D_\text{lf,rf}-1|}{2}$ and $D_\text{lf,rf}$ are the dimensions of left and right fractional half spaces, respectively. Subscripts lf and rf are for left and right fractional half spaces, respectively. Here, dimensions of the two sides are allowed to be different, i.e., $D_\text{lf} \neq D_\text{rf}$ and taken as $2 \leq D_\text{lf,rf} \leq 3$. Moreover, $\mathscr{H}_{n_\text{lf}}^1, \mathscr{H}_{nh_\text{lf}}^1, \mathscr{H}_{n_\text{lf}}^2, \mathscr{H}_{nh_\text{lf}}^2, \mathscr{H}_{n_\text{rf}}^2$ and $\mathscr{H}_{nh_\text{rf}}^2$ are Hankel functions of first and second kind, respectively. Hankel function of second kind is used to represent positive traveling waves whereas Hankel function of first kind represents waves traveling in the negative direction [29].

A chiral medium can be described by the following constitutive relations [30]

$$\begin{cases} \mathbf{D} = \varepsilon\mathbf{E} + i\kappa\mathbf{H} \\ \mathbf{B} = \mu\mathbf{H} - i\kappa\mathbf{E} \end{cases} \tag{10}$$

where, ε, μ, κ are constitutive parameters of a chiral medium. The fields within each chiral layer are in linear combination of two waves propagating in opposite directions. Thus total electric and magnetic fields for circularly polarized wave in the m-th chiral layer can be written as

$$\begin{aligned} \mathbf{E}_m^+ &= E_{Lm}^+(\hat{a_x}\cos\theta_{Lm} + \hat{a_z}\sin\theta_{Lm} + i\hat{a_y})\exp(-ik_{Lm}(\cos\theta_{Lm}z - \sin\theta_{Lm}x)) \\ &\quad + E_{Rm}^+(\hat{a_x}\cos\theta_{Rm} + \hat{a_z}\sin\theta_{Rm} - i\hat{a_y})\exp(-ik_{Rm}(\cos\theta_{Rm}z - \sin\theta_{Rm}x)) \end{aligned} \tag{11}$$

$$\begin{aligned} \mathbf{E}_m^- &= E_{Lm}^-(-\hat{a_x}\cos\theta_{Lm} + \hat{a_z}\sin\theta_{Lm} + i\hat{a_y})\exp(-ik_{Lm}(-\cos\theta_{Lm}z - \sin\theta_{Lm}x)) \\ &\quad + E_{Rm}^-(-\hat{a_x}\cos\theta_{Rm} + \hat{a_z}\sin\theta_{Rm} - i\hat{a_y})\exp(-ik_{Rm}(-\cos\theta_{Rm}z - \sin\theta_{Rm}x)) \end{aligned} \tag{12}$$

$$\begin{aligned} \mathbf{H}_m^+ &= \left(\frac{-i}{\eta_m}\right) E_{Lm}^+(\hat{a_x}\cos\theta_{Lm} + \hat{a_z}\sin\theta_{Lm} + i\hat{a_y})\exp(-ik_{Lm}(\cos\theta_{Lm}z - \sin\theta_{Lm}x)) \\ &\quad + \left(\frac{i}{\eta_m}\right) E_{Rm}^+(\hat{a_x}\cos\theta_{Rm} + \hat{a_z}\sin\theta_{Rm} - i\hat{a_y})\exp(-ik_{Rm}(\cos\theta_{Rm}z - \sin\theta_{Rm}x)) \end{aligned} \tag{13}$$

$$\begin{aligned} \mathbf{H}_m^- &= \left(\frac{-i}{\eta_m}\right) E_{Lm}^-(-\hat{a_x}\cos\theta_{Lm} + \hat{a_z}\sin\theta_{Lm} + i\hat{a_y})\exp(-ik_{Lm}(-\cos\theta_{Lm}z - \sin\theta_{Lm}x)) \\ &\quad + \left(\frac{i}{\eta_m}\right) E_{Rm}^-(-\hat{a_x}\cos\theta_{Rm} + \hat{a_z}\sin\theta_{Rm} - i\hat{a_y})\exp(-ik_{Rm}(-\cos\theta_{Rm}z - \sin\theta_{Rm}x)) \end{aligned} \tag{14}$$

where $\eta_m = \sqrt{\mu_m/\varepsilon_m}$ is the impedance and $k_{Lm,Rm} = \omega\sqrt{\mu_m\varepsilon_m}(1 \mp \kappa_r)$ is propagation constant for left and right circular polarized waves in chiral media filling the m-th layer. Here, $\kappa_r = (\kappa/\sqrt{\mu_{rm}\varepsilon_{rm}})$ and θ_{Lm}, θ_{Rm} can be calculated by using Snell's law.

By imposing proper boundary conditions at interfaces, the relationship between incident, reflected and transmitted fields can be obtained by using transfer matrix summarized below.

$$\begin{bmatrix} E_{\shortparallel}^+ \\ E_{\perp}^+ \\ E_{\shortparallel}^- \\ E_{\perp}^- \end{bmatrix} = A \begin{bmatrix} E_{\shortparallel}^{t+} \\ E_{\perp}^{t+} \end{bmatrix} \tag{15}$$

where

$$A = [M_F][P][A_1]^n[M_E] \tag{16}$$

and

$$A_1 = [M_2][P][M_3][P] \tag{17}$$

Here M_F is matching matrix at the interface I_1 due to left fractional order dielectric medium and DNG chiral metamaterial and is located at $z = d$. The matrix M_E denotes matching matrix at interface I_6, i.e., interface due to DNG chiral metamaterial and right fractional order dielectric medium 1. Similarly, M_2 and M_3 are matching matrices at interfaces between DNG chiral and DPS layers and vice versa. Assuming n identical pairs of DNG chiral-DPS slabs, the expression (16) may be valid for any odd number of slabs $(2n+1)$. Therefore, from matrix A the reflection and transmission coefficients for any number of DNG chiral-DPS pairs can be obtained [31]. Moreover, all the above expression are derived for general non-integer dimension space. However, by inserting $D_{\text{lf,rf}} = 2$, one can recover the results for integer dimensional space. By setting $D_{\text{lf,rf}} = 2$, the order of Hankel function becomes $n_{\text{lf,rf}} = nh_{\text{lf,rf}} = \frac{1}{2}$. Now Hankel function of first kind can be expressed in exponential form as follows [32]

$$\mathscr{H}_{\frac{1}{2}}^1(\mathrm{z}) = \sqrt{\frac{2}{\pi \mathrm{z}}} e^{j(\mathrm{z})} \tag{18}$$

Similarly Hankel function of second kind can be expressed as

$$\mathscr{H}^2_{\frac{1}{2}}(\mathrm{z}) = \sqrt{\frac{2}{\pi \mathrm{z}}} e^{-j(\mathrm{z})} \tag{19}$$

By inserting Eqs. (18) and (19) in Eq. (9), we can achieve classical expressions for incident, reflected and transmitted fields in (Eqs. (3)–(8)). In expression (16), P is a propagation matrix which, for a layer of thickness d_m, can be written as

$$P = \begin{bmatrix} e^{-ik_{Lm}\mathrm{d}_m} & 0 & 0 & 0 \\ 0 & e^{-ik_{Rm}\mathrm{d}_m} & 0 & 0 \\ 0 & 0 & e^{ik_{Lm}\mathrm{d}_m} & 0 \\ 0 & 0 & 0 & e^{ik_{Rm}\mathrm{d}_m} \end{bmatrix} \tag{20}$$

For dielectric layers, relation between wave numbers becomes $k_{Lm} = k_{Rm} = k$. A consideration of the law of conservation of energy allows us to write the tangential components of the incident, reflected and transmitted field powers as follows [27]:

$$P_{\mathrm{i}z} = \left| \frac{E_{\mathrm{i}}^2}{\eta_{h1}} \right| \tag{21}$$

$$P_{\mathrm{r}z} = \left| \frac{(RE_{\mathrm{i}})^2}{\eta_{h1}} \right| \tag{22}$$

$$P_{\mathrm{t}z} = \left| \frac{(TE_{\mathrm{t}})^2}{\eta_{h2}} \right| \tag{23}$$

where R is the ratio of incident and reflected fields and T is the ratio of incident and transmitted fields. By denoting net power loss by P_{Loss}, the law of conservation of energy can be written in terms of R and T as

$$|R|^2 + \left| \frac{\eta_{h1}}{\eta_{h2}} \right| |T|^2 = 1 - P_{Loss} \tag{24}$$

where η_{h1} and η_{h2} are wave impedances of incident and transmitted media, taken same in our case.

3. NUMERICAL RESULTS AND DISCUSSION

The plots for transmittance, as a function of frequency of the incident electromagnetic wave for different values of the the fractionality of the host space are presented and discussed in this section. For all the plots, the structure is assumed to consist of five alternate layers of DPS and DNG chiral material placed in fractional space of varying order. The frequency range for the incident wave is selected such that both permittivity and permeability of chiral medium are negative in the specified range. In addition all results are provided for the normal incidence, i.e., ($\theta_{\mathrm{i}} = 0$). Furthermore, for all the results the incident electric field is assumed to be perpendicularly polarized. It may be noted that the conservation of power holds for all the results given in each figure. Two cases of dispersive lossless and dispersive lossy DNG chiral layers are treated separately. For each case, the results for different combinations of fractionality of the two half spaces are presented, i.e., when dimensions of both half spaces are non-fractional, when one of them is fractional and when both are fractional. The parameters of DPS layer are selected as $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$ and chirality of DNG chiral layer is also taken constant $\kappa = 0.5$ for all the cases. Moreover, the parameters used for the following figures are taken from [27, 28] and [33].

3.1. Transmission through Dispersive Lossless Layers

The permittivity and permeability for DNG chiral layers are taken from [1] and [2] that is, dispersion is taken into account. Here, damping frequency is taken zero, whereas $f_{mp} = 19\,\mathrm{GHz}$, $f_{ep} = 6\,\mathrm{GHz}$ and $f_{mo} = 4\,\mathrm{GHz}$ are assumed. Figure 2 presents transmittance versus the incident frequency when fractional dimension of the transmitted half space is changed from $D_{\mathrm{rf}} = 2$ to $D_{\mathrm{rf}} = 2.6$, while keeping the dimension of the incident host space fixed. The situation is reversed for Figure 3. It is clear from these figures that the structure overall behaves as bandpass filter and its passband narrows with increase in the fractionality of either of the half spaces. Moreover, it is noted that the passband transmittance decreases with the increase in the fractionality of either side of the half spaces, especially for the case of dimension mismatch of the two sides. Comparing Figures 2 and 3 also tells that the effect of increase in the dimension of incident half space is more pronounced on the passband transmittance compared with that of the transmitted half space. For the case of dimension matching, however, the passband is seen to narrow without any passband attenuation with increase in the dimension of both the sides. This suggests that the fractionality of host media can be an effective tool to control the passband width and behavior of a stratified metamaterial structure.

Figures 4 and 5 correspond to the results when magnetic plasma frequency of the DNG chiral layers is changed to $f_{mp} = 28$ GHz whereas all other parameters are kept the same. It is seen that the passband is much narrow, in this case,but the structure is not strictly bandpass and nonzero transmittance is

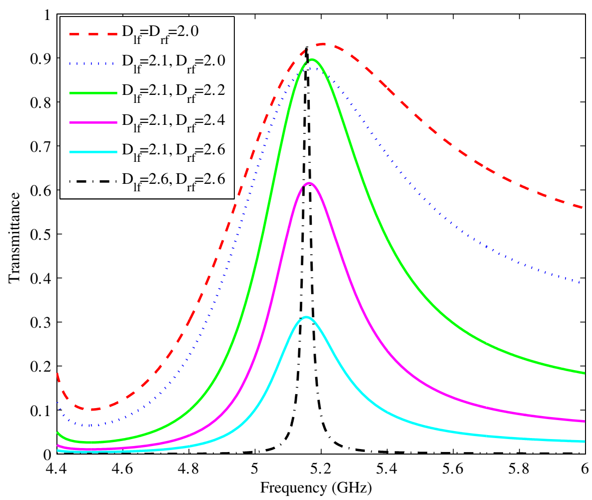

Figure 2. Transmittance for dispersive lossless structure with $\kappa = 0.5$, $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$, $f_{mp} = 19$ GHz, $f_{ep} = 6$ GHz, and $f_{mo} = 4$ GHz.

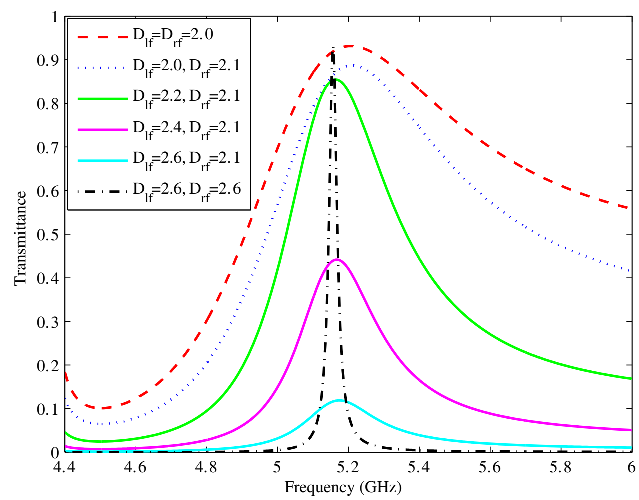

Figure 3. Transmittance for dispersive lossless structure with $\kappa = 0.5$, $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$, $f_{mp} = 19$ GHz, $f_{ep} = 6$ GHz, and $f_{mo} = 4$ GHz.

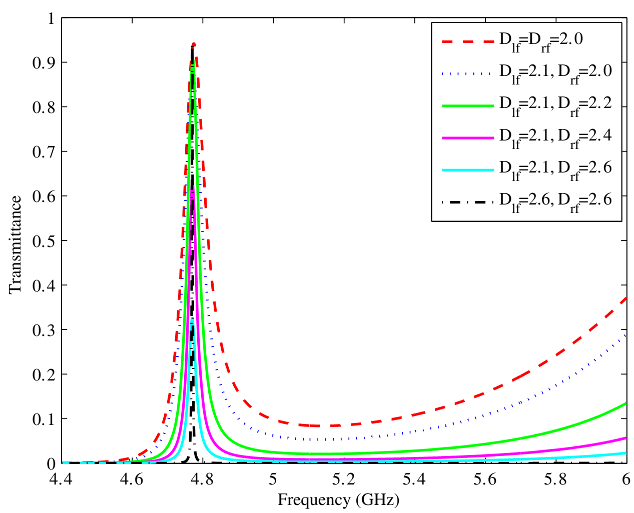

Figure 4. Transmittance for dispersive lossless structure with $\kappa = 0.5$, $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$, $f_{mp} = 28$ GHz, $f_{ep} = 6$ GHz, and $f_{mo} = 4$ GHz.

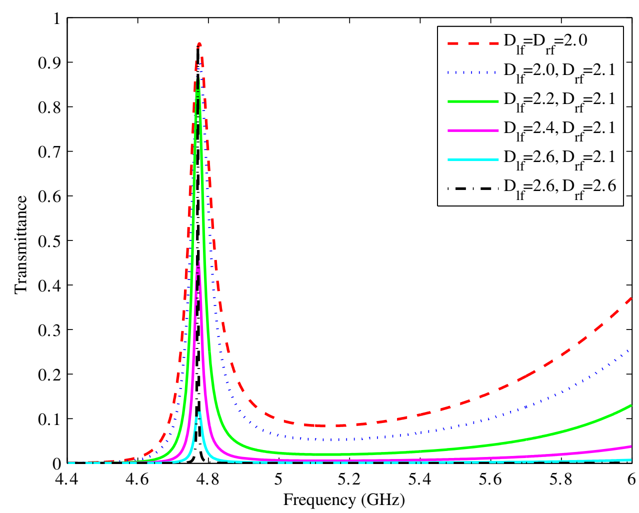

Figure 5. Transmittance for dispersive lossless structure with $\kappa = 0.5$, $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$, $f_{mp} = 28$ GHz, $f_{ep} = 6$ GHz, and $f_{mo} = 4$ GHz.

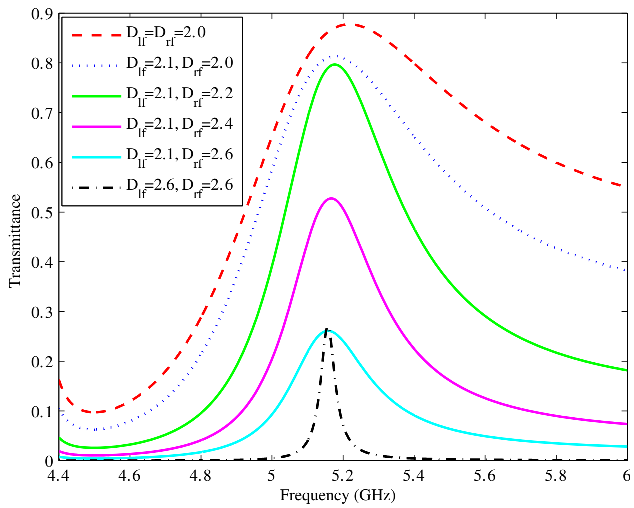

Figure 6. Transmittance for dispersive lossy structure with $\kappa = 0.5$, $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$, $f_{mp} = 19\,\text{GHz}$, $f_{ep} = 6\,\text{GHz}$, and $f_{mo} = 4\,\text{GHz}$.

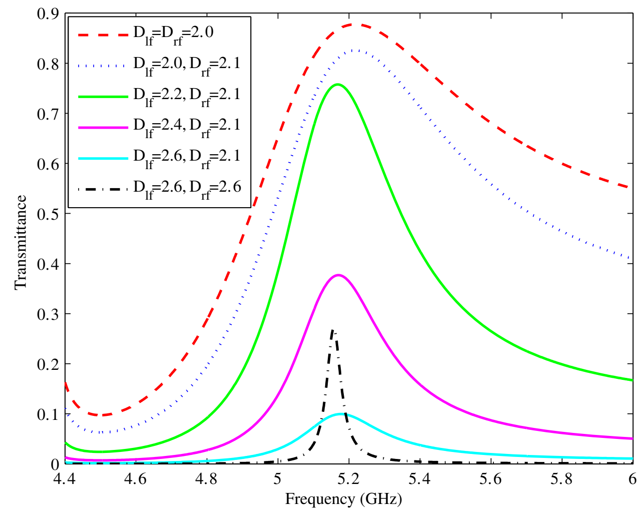

Figure 7. Transmittance for dispersive lossy structure with $\kappa = 0.5$, $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$, $f_{mp} = 19\,\text{GHz}$, $f_{ep} = 6\,\text{GHz}$, and $f_{mo} = 4\,\text{GHz}$.

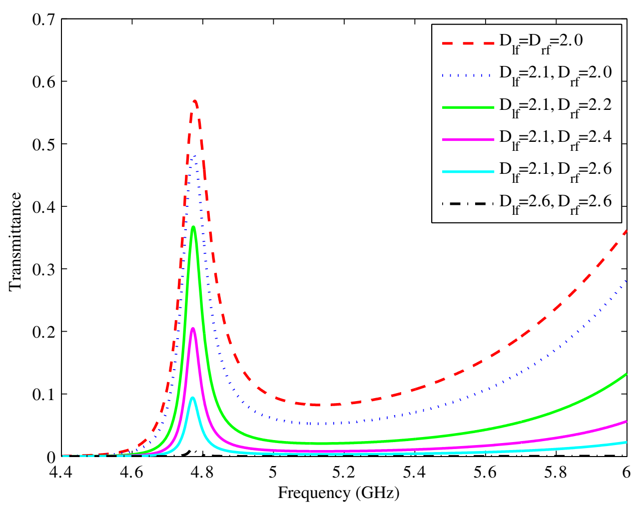

Figure 8. Transmittance for dispersive lossy structure with $\kappa = 0.5$, $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$, $f_{mp} = 28\,\text{GHz}$, $f_{ep} = 6\,\text{GHz}$, and $f_{mo} = 4\,\text{GHz}$.

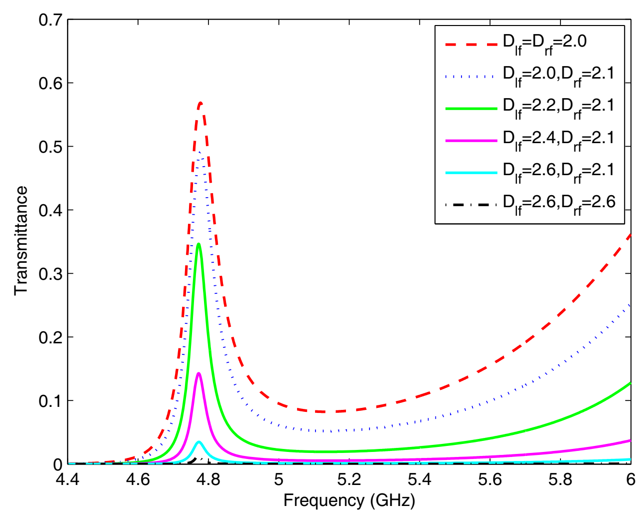

Figure 9. Transmittance for dispersive lossy structure with $\kappa = 0.5$, $\varepsilon_{rDPS} = 1.473$, $\mu_{rDPS} = 1$, $f_{mp} = 28\,\text{GHz}$, $f_{ep} = 6\,\text{GHz}$, and $f_{mo} = 4\,\text{GHz}$.

observed outside the passband. However, by increasing the dimension of either side of the structure, not only the passband is narrowed further but the sideband transmittance is also reduced. Especially when the dimensions of both sides of the structure are uniformly increased the sideband transmittance becomes almost zero.

3.2. Transmission through Dispersive Lossy Layers

For Figures 6–9, damping frequency in relations [1] and [2] is also taken into account with a value of $\delta_e = \delta_m = 10^8\,\text{Hz}$. The nonzero value of the damping frequency causes the DNG chiral layer to be lossy, this time. Again the transmittance as a function of incident frequency for two values of magnetic resonance frequencies and a set of fractional dimensions of either side of the structure is presented from Figures 6–9. A trend identical to that in the previous results is seen except that the passband transmittance is always seen to decrease with increase in the dimension of either of the half spaces whether matched or unmatched, in this case. Therefore, it can be argued that for the lossy stratified structure, the passband transmittance always reduces with increase in the fractional dimension of either side of the host media.

4. CONCLUSIONS

In this paper, frequency response of a multilayered structure composed of dispersive DNG chiral and DPS slabs is investigated with emphasis on fractional dimension of the left and right host spaces. Although the formulation is provided in generality, numerical results for a five layer structure are presented and discussed. The results show that the given structure acts as a bandpass filter whose characteristics, namely, passband width and passband transmittance, can efficiently be controlled by fractional dimensions of the left and right host spaces. Whereas, the center frequency of passband is shown elsewhere [28] to change with the thickness d_m of the layers in the stratified structure. Therefore, in addition to electrical parameters of a stratified structure, a topological parameter, namely, fractionality of the dimension of host space, also plays an important role and can be used to shape the frequency response of a given structure.

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