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Analysis of Generic Near-Field Interactions Using the Antenna Current Green's Function

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Abstract—We investigate the problem of near field interactions with general antenna systems using the antenna current Green's function formalism recently proposed by the authors as a framework for the theoretical and computational analysis of the interaction problem. The paper focus is on conceptual and numerical issues related to the analysis of the electromagnetic response of generic devices to arbitrary illumination fields produced, for example, by nearby source or scattered by surrounding objects. We provide some method of moment numerical examples involving wire antenna systems substantiating the ACGF approach to the problem of near field excitations.

1. INTRODUCTION

There is a growing interest within the applied electromagnetic community in systems involving parts and subsystems interacting at close distance. Examples are compact antenna arrays, wireless devices working in dense and crowded electromagnetically changing environments such as MIMO and DoA applications, miniaturized circuits and radiators, near-field communication, and wireless energy transfer. A common denominator in all these applications is the existence of the problem of illumination by *near* fields. In theory, the near field is much more complex than the far fields or waveguide modes. Although it is not completely arbitrary but appears to enjoy a very specific mathematical structure of its own, e.g., see [1–4], understanding near-zone problems remains challenging compared with other limit cases (examples of the latter include far zone and waveguide-based excitations.).

Field illumination by a *generic* source at close distance involves not only propagating modes with wavelength roughly determined by the formula $\lambda = f/c$, but also *short*-wavelength components or *evanescent* modes that represent the rapid variation in the near field [5]. Note that for a specially prepared discretization mesh, it is possible to compute the response to any near field using full-wave analysis methods such as MoM [6]. However, for each new type of near field, e.g., different composition of short-wavelength components, it is necessary to change the mesh in order to obtain an accurate solution to the problem [7, 8]. Recently, some methods to compute the response of antennas to generic near fields were proposed but for special antennas [9, 10]. These methods, though useful, don't address the problem at the most general level, besides being already contained in the method presented here as a special case.

Investigation of the question whether there is a *general* method to compute the response of the antenna to such generic excitation led the authors to the development of the antenna current Green's function (ACGF) formalism. The works [11–13] contain the bulk of the theoretical foundations of the approach to the ACGF as an exact transfer function in space. Some applications to mutual coupling in arrays were reported in [14–16]. The numerical and experimental examples found in [13, 14], however,

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included only far field illumination. The present paper provides the first numerical study of the near-field interaction problem using the antenna current Green's function method.

The paper is organized as follows. Section 2 provides a motivation for the study of the problem of EM devices interactions with near field illumination by pointing out how this case arises in practice. Section 3 provides the theoretical background needed to understand the present paper in a self-sufficient manner. It reviews the concept of the ACGF, explains how the general theoretical presentation in [12] can be adapted for near field applications. Some of the main themes discussed there will be put into concrete demonstration in Section 4, which provides a detailed example comprised of wire antenna solved using the method of moment. We compute the ACGF and discuss its validity and physical significance within the specific context in which it appears. Finally, we end up with conclusion.

2. SOME PRELIMINARY MOTIVATIONS

Before starting the technical analysis itself, let us pause for a while to reflect on how the problem of near-field illumination of EM devices arises in practice. There are several possibilities. First, we may think of our device D as directly excited by another antenna S acting like a source producing near fields sufficiently strong to force D to respond by generating a signal at its output terminal. Figure 1 illustrates this case, which we call the S-Rx scenario.

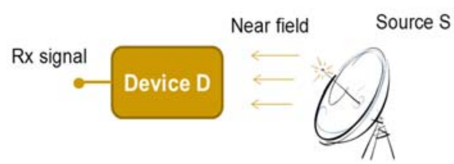

Figure 1. The S-Rx scenario. Interaction between a Source S close to Device D (typically an antenna system). The field impinging on Device D is a near field rich with short-wavelength components. The device responds to the near field by generating a received (Rx) signal.

In literature, such situation is dealt with in the following manner. Both Device D and Source S are treated as a *single* system in the full-wave solution of the problem in order to take into account the extent of mutual coupling between the two [6–8]. However, in practice one needs to study how D is working as an *independent* unit that can be embedded into various environments, where a source like S is nothing but one potential element of this environment among others. In other words, our main focus is the system D *itself*, while sources are relegated to the background. Therefore, the first natural step is to ignore the effect of the source S on the electromagnetic responsitivity of D and deal with the latter as a system interacting mainly with illumination fields. Here, the field is a near field and the device will behave according to this type of excitation. For example, one can assume that the device is electromagnetically shielded in a proper way or designed to minimize its interaction with nearby objects. After developing some basic understanding of the physics of how an independent D responds to arbitrary near field produced by S, we can move subsequently to the next stage where the effect of mutual coupling is included in the problem. This will most likely manifest itself as a *perturbation* on the characteristics studied in the previous step. Detailed investigations of mutual coupling will be tackled by the authors in a separate publication, but see [15, 16].

Let us move now to a more common problem encountered in real-life situations. Figure 2 depicts what we choose to call the S-O-Rx scenario. Here, the source antenna S is located in the far zone of the device D. The far field generated by S will first interact with an object O close to the device D. Next, the field scattered by O will impinge on D as a *near* field. Therefore, although the entire structure D + O is located in the far zone of the source S, then, provided we again focus attention on the device of interest D, the latter is eventually excited by near fields, not far fields. In most of the standard literature, the idealized scheme in which the receiving system D exists alone in free space (or above a ground plane) is assumed. In electromagnetically dense environments, such as highly populated urban areas, this is no longer satisfactory. Even more, in nanoscale problems there is always strong energy

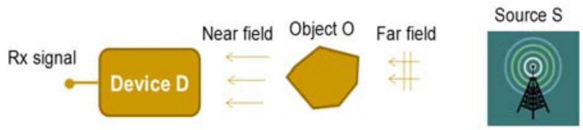

Figure 2. The S-O-Rx scenario. Interaction between a Source S far from a Device D (typically an antenna system) is mediated by a Object O placed in the vicinity of the device. The far field produced by the distant source will be converted upon interaction with Object O to a near field (scattered field) impinging in turn on Device D. The device responds to the near field by generating a received (Rx) signal.

exchange with the surrounding objects unless extreme care is taken to eliminate this interaction [17]. Therefore, we believe that there is a need to reexamine the electromagnetic problem as a whole from this perspective of a device interacting solely with some near field.

Interest in describing the antenna using the ACGF method derives from the well-known advantages of classical system theory in electrical engineering. Indeed, by finding a characteristic function (transfer or Green's function) of the device, we achieve

(i) Deeper understanding of the physics of the device. This is because the Green's function as a transfer function contains the theoretically and physically *complete* information needed to compute the output for *generic* input regardless to the environment and without the need to re-meshing.

(ii) Reduced computational efforts. This is because one may use the same device operating within very different systems and environments. Without re-performing a full-wave analysis of the device + new enviroment/system every time, we can directly compute the output by just specifying what is the illumination field at hand.

(iii) Although sometimes it is not easy to know the illumination field itself without further calculations, we can still use the Green's function productively in theoretical research, test, and devolvement, for example, by performing *statistical* analysis to predict the expected received signal in such-and-such scenarios or working conditions, e.g., DoA and MIMO applications often involve such kind of analysis.

(iv) Knowing the antenna current Green's function and employing it for near-field excitation calculations is a key step in finding new methods to compute and compensate for mutual coupling. This line of research will be treated elsewhere, e.g., see [16].

To our best knowledge, such study has not been systematically considered in the computational literature. The authors believe that it is convenient at the present time to spend considerable efforts on understanding the physical and computational aspects of near-field interactions in electromagnetic systems, both simple and complex. Although the topic of the near field is notoriously difficult, we hope that a group effort within the larger community will eventually produce significant progress. This paper is part of an initial contribution in this direction.

3. CONCEPTUAL AND COMPUTATIONAL ASPECTS OF THE ANTENNA CURRENT GREEN'S FUNCTION METHOD FOR NEAR-FIELD INTERACTIONS

In this section, we present the most important essentials of the ACGF formalism [12, 13] adapted for the needs of the current work, i.e., the study of near-field interactions.

3.1. Definition of the ACGF of a General Antenna System

Consider an arbitrarily shaped antenna system defined by a closed surface S with unit outward normal $\hat{n}$. For simplicity, we restrict the treatment to perfect electric conductor boundary condition. Therefore, only the electric field of the illumination is relevant. This illumination field will interact with the entire surface S and generate a current $\mathbf{J}(\mathbf{r})$. It was shown in [12, 13] that the received current at location $\mathbf{r}'$

can be expressed as

$$\mathbf{J}_{\mathrm{rx}}\left(\mathbf{r}'\right) = \int_S ds\, \bar{\mathbf{F}}^T\left(\mathbf{r}', \mathbf{r}\right) \cdot \mathbf{E}_t\left(\mathbf{r}\right), \tag{1}$$

where $\mathbf{E}_t(\mathbf{r}) = \hat{n} \times \mathbf{E}^{\mathrm{in}}(\mathbf{r})$ is the tangential component of the incident field on the surface S. In this expression, $\bar{\mathbf{F}}(\mathbf{r}', \mathbf{r})$ is the ACGF obtained in the transmitting mode, i.e., when a special Dirac-delta source (see [12] for the technical definition) is applied at location $\mathbf{r}$ while the produced current is observed at $\mathbf{r}'$. It was demonstrated in [13] that the receiving mode Green's function is obtained by merely taking the transpose operator T of the corresponding transmitting mode function. In other words, although relation (1) is used to predict the received signal for arbitrary illumination, it uses the transmitting mode Green's function computed only once for a special input, the surface Dirac-delta excitation. The inverse reciprocity theorem, outlined in [11] and formally proved in [13], states in effect that this is possible. However, note that this is not identical to typical reciprocity methods common in electromagnetic literature, which, to our knowledge, don't refer to any concept of antenna transfer of Green's function but work rather with fields and sources. For some comparison with literature, see [11].

3.2. The ACGF Approximation Techniques

The existence of the ACGF used in (1) was proved in [12] within the framework of distribution theory. The method of the proof was to actually construct the function using sequences of trial approximations and then showing that some subsequence does converge to the exact relation (1). Therefore, it is possible to exploit the existence proof itself for the study of the concrete implementation of the ACGF in numerical contexts. This topic will be briefly illuminated here.

The distributional ACGF $\bar{\mathbf{F}}(\mathbf{r}', \mathbf{r})$ is replaced by a sequence of regular dyadic functions $\bar{\mathbf{F}}_n(\mathbf{r}', \mathbf{r})$, $n = 1, 2, \ldots, \infty$. While these functions approximate the exact ACGF of the problem, they differ essentially from $\bar{\mathbf{F}}(\mathbf{r}', \mathbf{r})$ in being *ordinary* functions, rather than a distribution. In fact, each function $\bar{\mathbf{F}}_n(\mathbf{r}', \mathbf{r})$ can be represented as the current on the antenna generated in response to a special vector surface excitation function $f_S(\mathbf{r}', \mathbf{r})$. This special excitation belongs to a very wide range of localized fields all approximating in a certain sense a generalization of the concept of the Dirac delta function familiar in system theory. For example, any localized smooth pulse can work for this excitation.† Therefore, the ACGF of the antenna problem can be approximated by simply exciting the antenna by a sequence of surface Dirac functions and choosing a suitable approximation level to work with. The Method of Moment (MoM), Finite Element Method (FEM), Finite Difference Time Domain Method (FDTD), can all be used to perform this computation. For example, to confirm the method, FEM was used in [14] (with comparison with measurement) while the MoM was utilized in [13] in conjunction with the singularity expansion method.‡ It is possible, moreover, to rely on special measurement methods to obtain the ACGF. For instance, by exciting the antenna by an extremely concentrated field and measuring the induced current distribution, it might be possible to bypass the need to employ very dense mesh in the numerical solution.

3.3. The ACGF and Traditional Full-Wave Solvers

Since electromagnetic fields are either far or near fields, an arbitrarily complex field is most probably a near field. Commercial EM codes usually deal with two types of excitation, wave ports and far-field illumination. Both types of excitations necessitate generating a sufficiently fine mesh with a degree of resolution that can be quickly estimated from the operating frequency. The reason is that both far-field and wave port excitations involve *propagating* modes, which have a wavelength roughly around v/f, where v is the speed of propagation in the medium of interest and f the frequency. The situation, however, is very different with near field excitations. In this case, not only wavelengthes between ∞ and λ_0 are available, but also *short* wavelength components $0 < \lambda < \lambda_0$ corresponding to the *non*propagating modes. Those rapidly decaying field modes make the *a priori* prediction of the proper resolution of the full-wave numerical solution mesh very difficult for *generic* field excitation. In other words, only if the

† For the rigourous description of the set of allowed delta sequences, see [13].

‡ However, we pose the possibility that other special numerical methods need to be developed in order to perform this computation in a more accurate and systematic fashion.

nature of the field impinging on the antenna is known *in advance* can we specify the suitable mesh of the problem. We believe that in order to characterize a device in a manner that is *independent of the nature of the illumination field*, one must have something new in addition to the traditional numerical solver: The *transfer function* of the system, or, equivalently, the *antenna current Green's function*.

In our opinion, the reason why this new concept was not fully pursued during the last few decades relates to the nature of the research problems that have been deemed important by the practicing community. It has been widely believed that far fields interaction with antennas is the dominant type of interactions. This is certainly correct in many communication and radar applications operating in typical environments, where the idealization of properly isolated and shielded device can always be made. However, at the present time things look different compared with before. Indeed, urban environments are electromagnetically dense. Devices are always embedded in complex environments. Typical practical scenarios now involve multiple antennas and circuits constantly interacting with each others. Moreover, in nanoscale structures and metametrials, the smallness of the devices and unit cells forces interactions with the surroundings to become very critical (as an example, consider near-field nano-optics and subwavelength imaging [17].) In all such cases, two key terms come to mind: near field and mutual coupling. Interaction at short distances typically involve near fields.

4. NUMERICAL ANALYSIS OF NEAR-FIELD INTERACTIONS IN LINEAR WIRE ANTENNAS

As an a concrete example demonstrating the ACGF method for near field excitation, we consider a linear wire antenna system illuminated by an external near field $\mathbf{E}^{\rm ex}(\mathbf{r})$. In order to numerically solve the problem, a thin-wire electric field integral equation (EFIE) for the current distribution on the antenna is solved using the Method of Moment [6, 7]. The operator equation is written as

$$\hat{n} \times \mathbf{E}^{\rm ex}(\mathbf{r}) = \hat{n} \times \mathcal{L}\mathbf{I}(\mathbf{r}), \tag{2}$$

where $\mathbf{I}$ is the current on the wire with the latter's outward normal vector pointing along $\hat{n}$. The electromagnetic operator $\mathcal{L}$ is the one associated with the EFIE.

For simplicity, we work with a linear (triangular) basis function. The code is verified by comparison with the solver WIPL-D, which uses higher-order basis functions to model the current on wire segments [8].

The MoM expansion of the current is given by

$$I(z) = \sum_{l=1}^{N} I_l f_l(z). \tag{3}$$

Here we assume that the antenna is oriented along the z-axis and that the origin coincides with the middle point of the wire. The complex numbers I_n give the unknown current values at the mesh locations z_l, for $l = 1, 2, \ldots, N$, while $f_l(z)$ are the basis functions (for explicit expressions in the triangular case, see for example [7].) Using this numerical model, we intend to approximate the ACGF of the wire numerically as

$$F(z, z') = \sum_{l=1}^{N} I_l(z') f_l(z). \tag{4}$$

Note that the MoM current values I_l become functions of the location of excitation z'.

Let the tangential component of the external field be denoted by $E^{\rm ex}(z)$. From the fundamental formula of the ACGF (1), the current induced on the antenna can be given by

$$I(z') = \sum_{l=1}^{N} I_l(z') \int_{S_l} dz\, f_l(z) E^{\rm ex}(z), \tag{5}$$

where the integration is on the segment S_l on which the lth basis function $f_l(z)$ is defined. The integration is performed numerically using Gauss-Legendre technique with five points on each segment. Note that in order to use the Tx mode ACGF (4) to write (5), we have invoked the inverse reciprocity theorem [11, 13].

The expression (5) gives the main formula of the ACGF approach when the MoM is used as the computational medium through which we implement the exact theory in [12]. Its accuracy depends on how good the current values $I_n(z')$ are in modeling the response to a delta function at $z = z'$. In order to assess the method, we need also to know how good is the obtained ACGF in modeling the response of the antenna to rapid fluctuations or short-wavelength components present in the illuminating near field.

A convenient way allowing both goals to be achieved at once is to choose a delta sequence [19] (a sequence of ordinary functions approximating the idealized Dirac delta source) that consisting of decaying exponentials. One possible such choice is the sequence of functions

$$E^n(z) = (1/2)\, n \exp(-n|z|), \tag{6}$$

for $n = 1, 2, \ldots$. The main advantage in using the special sequence (6) is that it has exactly the same form of the *evanescent* modes radiated by a point source at the origin [2, 5]. As is well known in electromagnetic near field theory, the major difference between far fields and generic fields (for example, near fields, directed beams, scattered fields) is that the latter contain rich and complex mixture evanescent modes in addition to the typical propagating modes [2, 5]. Mathematically, a generic near field can be written as

$$\mathbf{E}\left(\mathbf{r}; \bar{\mathbf{R}}\right) = \mathbf{E}_{\mathrm{pr}}\left(\mathbf{r}; \bar{\mathbf{R}}\right) + \mathbf{E}_{\mathrm{ev}}\left(\mathbf{r}; \bar{\mathbf{R}}\right), \tag{7}$$

where $\mathbf{E}_{\mathrm{pr}}$ and $\mathbf{E}_{\mathrm{ev}}$ are the propagating and evanescent parts, respectively. Here, $\bar{\mathbf{R}}$ is a 3D rotation matrix specifying orientation of the local coordinate frame used in deciding the direction of the axis along which the splitting into propagating and evanescent modes is enacted [2].

Consequently, since it is this type of evanescent modes $\mathbf{E}_{\mathrm{ev}}(\mathbf{r}; \bar{\mathbf{R}})$ that represents the rapid part of generic near field variations, knowledge of the accuracy of the ACGF method in terms of exponentially-decaying excitations like (6) automatically provides information about how good the ACGF thus obtained in predicting the response of the antenna system to this level of short-wavelength details in the near field.

We may now use (5) to illustrate the applicability of this argument in actual examples. Let each ACGF approximation be written as

$$F^n\left(z, z'\right) := \mathcal{L}^{-1} E^n(z) = \sum_{l=1}^{N} I_l^n\left(z'\right) f_l(z). \tag{8}$$

for $n = 1, 2, \ldots$. It can be shown that all the requirements of the special delta sequence (6) stated in [13] are satisfied and therefore, according to the convergence theory of the ACGF, it is possible always to obtain production of the near field response in antennas using a sufficiently small exponential illumination field.§ Therefore, we conclude that the corresponding current values $I^n(z')$ obtained by (5) will converge to the exact solution (1) [12].

Consider a linear wire antenna with length 0.25λ and radius 0.001λ. We choose four members of the exponential delta sequence (6). Figure 3 provides the shape of the illumination field along the wire extension. The ACGF approximations corresponding to each exponential field are shown in Figure 4. In general, as n in the excitation fields of (6) becomes large, the sequence of pulses approaches the idealized delta function. Since the electric-field integral equations of this particular problem is *not* continuous (the EFIE operator is unbounded [18]), the convergence of the delta sequence functions (6) to the Dirac does not necessarily imply that the corresponding current distribution will converge. However, the analysis in [12] proved that convergence to the correct current *does* happen for all operators obeying the reciprocity theorem, which include in particular the EFIE operator of the antenna problem (2). Thus, from the purely numerical viewpoint, one can work with any approximation level of the exact ACGF by choosing the proper value n. In general, the higher the accuracy of the approximating ACGF, the larger is n.

To verify the prediction of the current distribution using the ACGF with a direct approach, we present in Figure 5 comparison with the scattering problem (2) solved directly using the MoM for two choices of n. In the MOM scattering code, we generate a nonuniform mesh appropriate to the shape of the exponential delta sequence member for each choice of n in the input field (6). For

§ The details of the proof are lengthy but straightforward and can be readily obtained by mimicking the argument of [12].

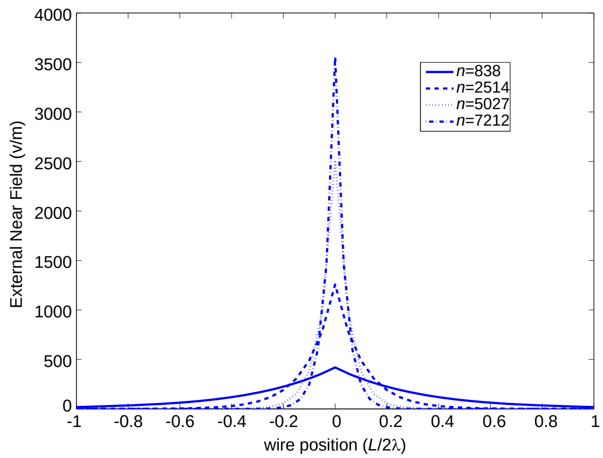

Figure 3. A sequence of exponentially localized near electric field excitations (as in (6)) approximating the Dirac delta functions applied around the center of a 0.25λ linear thin-wire antenna with radius 0.001λ.

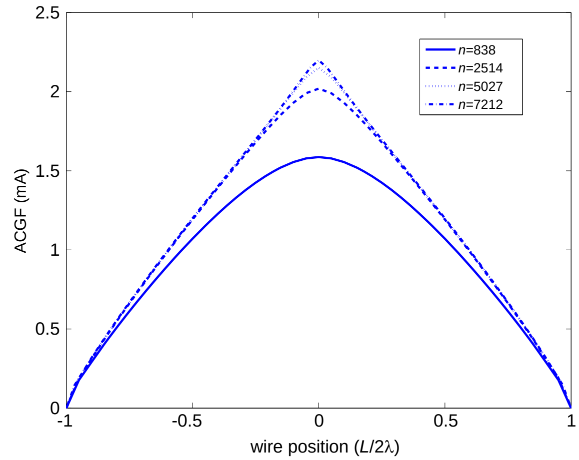

Figure 4. The approximations of the ACGFs of the antenna computed using the MoM in response to the excitations of in Figure 3.

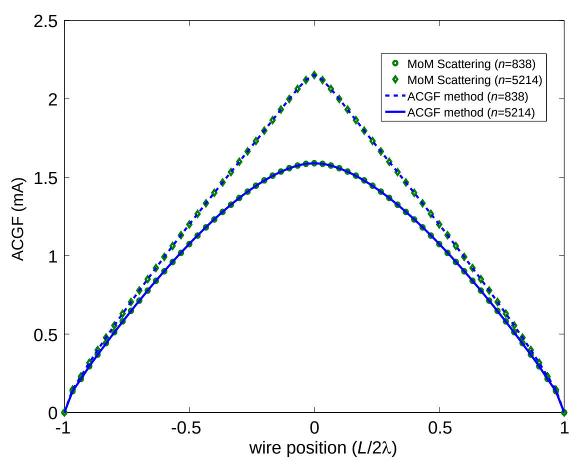

Figure 5. Comparison between the ACGF and direct MoM scattering code for the antenna described in the caption of Figure 3.

the computation of the ACGF in the transmitting mode, we use a single mesh sufficiently dense to provide accurate prediction for all the input excitation near field cases considered. Excellent agreement can be observed. Although this agreement in the convergence results with increasing n was proved mathematically in [12] and verified numerically for far-field illumination in [13], they are confirmed here numerically for non-plane wave, i.e., inhomogeneous or evanescent fields, for the first time. Note that since a generic electromagnetic field can always be written as a proper mixture of propagating and evanescent modes [2], the results here provide evidence that the ACGF can be used to compute the response to any excitation field. This is because the basic building blocks of this excitation, namely propagating and evanescent modes, are dealt with successfully using the ACGF method.

Although the method of moment was used in the present work to investigate the near-field ACGF approach, we mention that since the ACGF has been established on a rigourous basis going back to Maxwell's equations [12], it could be advantageous to develop special numerical methods or measurement procedures to obtain the Green's function in ways that go beyond conventional numerical methods.

In order to go beyond linear wire antennas, for example to consider more complex structures such as multilayered microstrip and dielectric resonator antennas, it is required to develop a general EM solver that can deal with the special excitations (the delta sequence) applied to the antenna in order to obtain its ACGF. Most commercial EM solvers provide only plane wave and wave port excitations, or sometimes special waves like Gaussian beams. In studying such problems, more factors come into the picture, for instance how the geometrical shape of the antenna interacts with spectral content of the non-standard NF applied to that part under consideration. Such detailed study, including questions of convergence, were dealt with theoretically in [12], and numerically for linear wire antennas here. The numerical study of 2D and 3D structures will be taken up in future publications.

5. CONCLUSION

We provided a new general view on the numerical computation of the antenna current Green's function introduced recently by the authors. The topic was discussed within the framework of near-field interaction where we studied how general antenna systems respond to an arbitrary near field. The ACGF is used as a transfer function in space to predict the response at a certain point (say port location) due to a generic near field, including short-wavelength components or evanescent modes. The Method of Moment was used to compute and verify the ACGF method and the problem of how to control the level of accuracy was discussed.

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